Extended hydrodynamics from Enskog's equation: The bidimensional case

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Abstract

A heat conduction problem is studied using extended hydrodynamic equations obtained from Enskog's equation for a simple case of two planar systems in contact through a porous wall. One of the systems is in equilibrium and the other one in a steady conductive state. The example is used to put to test the predictions which have been made with a new thermodynamic formalism.

Keywords: Kinetic theory; Enskog dense gases; Heat conduction; Nonequilibrium thermodynamics

1. Introduction

Today Enskog's original kinetic theory is known as the standard Enskog theory (SET) [1–3] because after the pioneering work of van Beijeren and Ernst [4] there are several new versions of Enskog's theory collectively called revised Enskog's theory (RET) [5]. Among the latter there are versions that have been extended to describe condensed matter [6]. To Navier–Stokes level both SET and RET lead to the same results [4,7], whether or not an external force is present.

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In the present article we make use of *extended hydrodynamic equations* for the bidimensional case [9]. They are more complete than a linear approximation but still they are the result of an approximation scheme that we explain elsewhere. Using a strategy as in Ref. [8] and approximations defined in Ref. [9] we obtain in Section 5 the same hydrodynamic equations for SET and RET.

In this article we apply our extended hydrodynamics to a one-dimensional steady heat conductive state. There is much work on this as, for e.g., the experimental results in Ref. [10] or the theoretical ones in Refs. [11–13]. Recently Kim and Hayakawa [14] studied this problem for hard core and Maxwellian particles using Boltzmann's equation combined with Chapman–Enskog's method. They tried a test and criticized the analysis of the nonequilibrium steady-state thermodynamics (SST) proposed by Sasa and Tasaki [15]. In the last reference the authors state that if there is gas in a one-dimensional heat conductive configuration in contact, through a porous wall, with an equilibrium gas state, then a pressure difference must appear in the direction of the heat flow. We analyze this double system making use of the extended hydrodynamic equations derived from Enskog's equation using Grad's moment expansion method [16]. Our conclusions differ from those in Ref. [15].

The organization of the present article is as follows: in Section 2 the configuration of these systems is drawn schematically, in Section 3 the condition for the two systems to be in contact via the central porous plate is introduced: the upper and lower plates are normal plates; the central plate has many small pores through which the gas can pass. In Section 4 we give the basic equations used in this paper. Comments are in Section 5. Finally, our discussion and conclusions are written in Sections 6 and 7, respectively.

2. Definition of the system

Sasa and Tasaki [15] proposed an interesting system consisting of a nonequilibrium steady-state subsystem in contact with a subsystem in equilibrium as explained below. This system has three plates as shown in Fig. 1 and there is gas between them. The upper and lower plates (plates 1 and 3) are normal plates. The central plate (plate 2) has pores through which gas can pass.

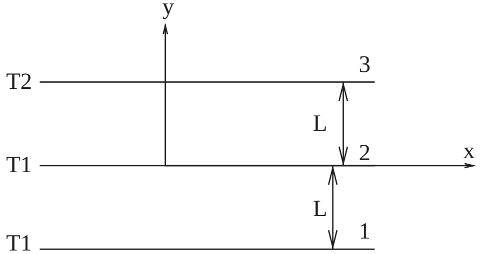

Fig. 1. The plates of the system as described in the text.

Following Sasa and Tasaki, we consider the system consisting of three infinite parallel plates 1, 2 and 3 separated by a distance L. The Y-axis is defined perpendicular to them while an X-axis is placed on plate 2. The pores in plate 2 are distributed homogeneously. Plates 1 and 2 have fixed temperature T_1 while plate 3 has a different (fixed) temperature T_2.

After a sufficiently long time, by effusion, some of the gas passes through plate 2 and the gas between plates 1 and 2 reaches an equilibrium state. The system between the plates 2 and 3 reaches a nonequilibrium steady state with translation symmetry along the X-axis.

We assume that the typical distances between pores is very small and that the diameter of the pores is also very small, so that the ratio between such lengths and the mean free path is much smaller than unity. Having no external force and no hydrodynamic velocity there is no heat flux parallel to the plates. The system is in a static configuration.

3. The contact condition

In general, there is a difference between the temperatures of the plates and the temperatures of the gas in contact with them. This well-known effect is called thermal slip. However, for simplicity sake, we assume that the temperature of plate 2 and the gas in contact on both sides of it are equal, namely, we are neglecting the Knudsen layer.

The velocity and the peculiar velocity of the gas will be denoted by $\mathbf{c}$ and $\mathbf{C}$, respectively. The condition that there is no mass flux through plate 2 is

$$\int_{-\infty}^{\infty} \mathrm{d}c_x \int_{0}^{\infty} \mathrm{d}c_y\, c_y f_{\text{equil}} + \int_{-\infty}^{\infty} \mathrm{d}c_x \int_{-\infty}^{0} \mathrm{d}c_y\, c_y f_{y=0} = 0\,, \tag{1}$$

where

$$f_{\text{equil}} = n_{\text{eq}} \left[\frac{1}{2\pi T_1}\right] \exp\left[-\frac{C^2}{2T_1}\right]$$

is the equilibrium distribution function associated to the gas between plates 1 and 2 and n_{eq} is the uniform number density in this same region while $f_{y=0}$ is the nonequilibrium distribution function between plates 2 and 3 evaluated at $y = 0$. Next, it is necessary to see how the two distributions satisfy condition (1).

Imposing condition (1) yields

$$n_{\text{eq}} = \frac{1}{2}[n(0) + P_{yy}] - \delta \frac{\chi}{2\mathrm{Kn}}[n(0) + P_{yy}]n(0) + \delta^2 \frac{\chi^2}{2\mathrm{Kn}^2}[n(0) + P_{yy}]n(0)^2\,. \tag{2}$$

In addition, the total mass conservation law for the system is

$$n_{\text{eq}} + \int_0^1 n(y)\,\mathrm{d}y = 2\,. \tag{3}$$

Above we are using dimensionless fields and dimensionless variables in general. The fields n (number density), P_{ij} (pressure tensor), $\vec{Q}$ (net heat flux vector), and T (temperature) generally depend on the coordinate y, where p_{ij} and $\mathbf{q}^k$ are the symmetric and traceless part of the pressure tensor and the kinetic part of the heat flux vector, respectively. These hydrodynamic fields are defined according to the following sum rules:

$$\int f \,\mathrm{d}\mathbf{c} = n(y) , \tag{4}$$

$$\int \mathbf{C}(y) f \,\mathrm{d}\mathbf{c} = 0 , \tag{5}$$

$$\int \tfrac{1}{2} C(y)^2 f \,\mathrm{d}\mathbf{c} = n(y)T(y) , \tag{6}$$

$$\int C_i(y)C_j(y) f \,\mathrm{d}\mathbf{c} = n(y)T(y)\delta_{ij} + p_{ij}(y) , \tag{7}$$

$$\int \tfrac{1}{2} C(y)C^2(y) f \,\mathrm{d}\mathbf{c} = q_y^{\mathrm{k}}(y) . \tag{8}$$

We also use the following dimensionless numbers

$$\text{Knudsen number, } \mathrm{Kn} = \frac{8\sqrt{2}}{\pi}\frac{\ell}{L}, \quad \delta = \frac{\sigma}{L} = \mathrm{Kn}\,\rho_0 ,$$

where σ is the particle's diameter, ℓ the mean free path at equilibrium and ρ_0 is the mean area density.

4. Balance equations

The basic concrete equations solved here are the following:

- In the case of the linearized Boltzmann–Grad method (LBG): $P_{yy}(y) \equiv P_{yy} =$ constant, $P_{xy}(y) \equiv P_{xy} =$ constant, $Q_y(y) \equiv Q_y =$ constant,

$$n(y)T(y) = P_{xx} = P_{yy}, \quad \frac{\mathrm{d}T(y)}{\mathrm{d}y} + \frac{2Q_y}{\mathrm{Kn}\sqrt{\pi T(y)}} = 0 . \tag{9}$$

- In the case of the Enskog–Grad method (EG): $P_{yy}(y) \equiv P_{yy} =$ constant, $Q_y(y) \equiv Q_y =$ constant, $P_{xy}(y) = p_{xy}(y) = 0$,

$$P_{yy} = -\left[1 + \frac{\delta}{\mathrm{Kn}}\chi n(y)\right] p_{xx}(y) + \left[1 + 2\frac{\delta}{\mathrm{Kn}}\chi n(y)\right] n(y)T(y) , \tag{10}$$

$$Q_y = \left[1 + \frac{3}{2}\chi n(y)\frac{\delta}{\mathrm{Kn}}\right]q_y^k(y) - \delta^2\frac{2}{\sqrt{\pi}\,\mathrm{Kn}}\chi n(y)^2\sqrt{T(y)}\frac{\mathrm{d}T(y)}{\mathrm{d}y}\ , \tag{11}$$

$$\begin{aligned} -\frac{1}{2}\frac{\mathrm{d}q_y^k(y)}{\mathrm{d}y} = & -\frac{8}{\sqrt{\pi}\,\mathrm{Kn}}\chi\sqrt{T(y)}\left[n(y)p_{xx}(y) - \frac{q_y^k(y)^2}{128T(y)^2}\right] \\ & + \frac{\delta}{4\mathrm{Kn}}\chi\left[5q_y^k(y)\frac{\mathrm{d}n(y)}{\mathrm{d}y} + 3n(y)\frac{\mathrm{d}q_y^k(y)}{\mathrm{d}y}\right] \\ & - \frac{\delta^2}{\sqrt{\pi}\,\mathrm{Kn}}\chi n(y)\sqrt{T(y)}\left[2\frac{\mathrm{d}n(y)}{\mathrm{d}y}\frac{\mathrm{d}T(y)}{\mathrm{d}y} + \frac{1}{2}\frac{n(y)}{T(y)}\left(\frac{\mathrm{d}n(y)}{\mathrm{d}y}\right)^2\right. \\ & \left. + n(y)\frac{\mathrm{d}^2T(y)}{\mathrm{d}y}\right]\ , \end{aligned} \tag{12}$$

and

$$\begin{aligned} & - T(y)\frac{\mathrm{d}p_{xx}(y)}{\mathrm{d}y} - p_{xx}(y)\frac{\mathrm{d}T(y)}{\mathrm{d}y} - n(y)T(y)\frac{\mathrm{d}T(y)}{\mathrm{d}y} + \frac{T(y)}{n(y)}p_{xx}(y)\frac{\mathrm{d}n(y)}{\mathrm{d}y} \\ & - \frac{p_{xx}(y)}{n(y)}\frac{\mathrm{d}p_{xx}(y)}{\mathrm{d}y} \\ & = -\frac{4}{\sqrt{\pi T(y)}\mathrm{Kn}}[n(y)T(y) - p_{xx}(y)]q_y^k(y) \\ & \quad + \frac{\delta}{\mathrm{Kn}}\chi\left[\frac{7}{2}n(y)T(y)\frac{\mathrm{d}p_{xx}(y)}{\mathrm{d}y} + \frac{7}{2}T(y)p_{xx}(y)\frac{\mathrm{d}n(y)}{\mathrm{d}y} + 2p_{xx}(y)\frac{\mathrm{d}T(y)}{\mathrm{d}y}\right. \\ & \quad \left. - 8n(y)T(y)^2\frac{\mathrm{d}n(y)}{\mathrm{d}y} - 7n(y)^2T(y)\frac{\mathrm{d}T(y)}{\mathrm{d}y}\right]\ . \end{aligned} \tag{13}$$

The substitution of $\delta = 0$ and $\chi = 1$ in the EG equations yields the equations corresponding to the nonlinearized Boltzmann–Grad method (NLBG).

5. The pressure difference between the equilibrium and nonequilibrium sides

All the results we describe in what follows were obtained using perturbation methods choosing $T_2 > T_1$ and using $\varepsilon = (T_2 - T_1)/T_1$ as the perturbation parameter. We solve the system of equations and their boundary (contact) conditions up to ε^6. We choose $\delta = 0.001$. In such case Kn is in inverse proportion to ρ_0. We choose Henderson's expression [17] as the concrete expression for χ:

$$\chi = \frac{1 - (7/16)\rho_0}{(1 - \rho_0)^2}\ . \tag{14}$$

We calculate the pressure in both sides of plate 2. Using P_{yy} for the nonequilibrium steady-state side and the pressure P_{eq} which is estimated by the state equation for the equilibrium side

$$P_{\mathrm{eq}} \equiv \left[1+\delta \frac{2}{\mathrm{Kn}} \chi n_{\mathrm{eq}}\right] n_{\mathrm{eq}} \,, \tag{15}$$

the pressure difference ΔP is defined by

$$\Delta P = P_{yy} - P_{\mathrm{eq}} \,. \tag{16}$$

Note that $n(0) = n_{\mathrm{eq}}$ and $\Delta P = 0$ to first order in ε. Hence, we rewrite our results in the following way:

$$n(0) = n_{\mathrm{eq}} \left[1 + \lambda_n \frac{Q_y^2}{n_{\mathrm{eq}}^2}\right] , \tag{17}$$

$$\Delta P = \lambda_{\Delta P} \frac{Q_y^2}{n_{\mathrm{eq}}} \,, \tag{18}$$

where λ_n and $\lambda_{\Delta P}$ are constants. Furthermore, it is possible to rewrite P_{yy}:

$$P_{yy} = n(0)\left[1+\delta \frac{2}{\mathrm{Kn}} \chi n(0)\right]\left[1+\lambda_p^{yy} \frac{Q_y^2}{n(0)^2}\right] . \tag{19}$$

Tables 1and 2 give the values of these constants for $\varepsilon = 0.05$ and 0.1, respectively. The value and sign of ΔP depend on ε and Kn. Table 3 gives the value of $\lambda_{\Delta P}$ obtained by first-order EG, i.e., up to δ for $\varepsilon = 0.05$ and 0.1, respectively. In this case, the pressure difference also exists and its value and sign depend on ε and Kn, too.

In the case of LBG, since $p_{xx} = 0$, $\lambda_n = \lambda_{\Delta P} = \lambda_p^{yy} = 0$. There is no pressure difference in this case.

On the other hand, for the case of NLBG, the substitution of $\delta = 0$ in Eqs. (2), (10)–(19) leads to $\lambda_n = \frac{1}{256}$, $\lambda_{\Delta P} = -\frac{1}{256} < 0$ and $\lambda_p^{yy} = -\frac{1}{128}$ where for λ_n and $\lambda_{\Delta P}$ it is correct to consider only up to second order in Q_y. Hence the osmotic pressure difference does exist and its value is constant and negative. Furthermore, for the case of EG, $\lambda_{\Delta P}/\lambda_n = -2 - \delta(4/\mathrm{Kn})n_{\mathrm{eq}} \neq -2$.

Table 1
The values of λ_n, $\lambda_{\Delta P}$ and λ_p^{yy}/λ_n for $\varepsilon = 0.05$ in the case of EG

Kn	$\lambda_n \times 10^{-2}$	$\lambda_{\Delta P} \times 10^{-2}$	λ_p^{yy}/λ_n
0.005	−0.360727	1.43059	−3.45019
0.01	0.6175405	−0.37938	−1.17319
0.02	0.5891347	−0.51577	−1.66426
0.05	0.4851223	−0.46437	−1.87508
0.1	0.4309258	−0.44057	−1.93856
0.2	0.4158376	−0.41159	−1.96948

Table 2
The values of λ_n, $\lambda_{\Delta P}$ and λ_p^{yy}/λ_n for $\varepsilon = 0.1$ in the case of EG

Kn	$\lambda_n \times 10^{-2}$	$\lambda_{\Delta P} \times 10^{-2}$	λ_p^{yy}/λ_n
0.005	−0.405391	1.49646	−3.19368
0.01	0.614064	−0.37163	−1.16022
0.02	0.590175	−0.51564	−1.66031
0.05	0.486046	−0.40465	−1.87365
0.1	0.431343	−0.43093	−1.93785
0.2	0.416123	−0.41182	−1.96124

Table 3
The values of $\lambda_{\Delta P}$ for $\varepsilon = 0.05$ and 0.1 in the case of the first order EG

Kn	$\lambda_{\Delta P} \times 10^{-2}$ $\varepsilon = 0.05$	$\lambda_{\Delta P} \times 10^{-2}$ $\varepsilon = 0.1$
0.005	1.208777	1.269384
0.01	−0.417305	−0.410453
0.1	−0.431207	−0.431631

We analyze the pressure difference ΔP from another point of view. It is sufficient to calculate ΔP up to ε^2. It is given by

$$\begin{aligned}\Delta P &= \varepsilon^2 \frac{\pi\,\mathrm{Kn}^2}{4096}\left[215\rho_0^2 - \frac{52}{\chi}\rho_0 - \frac{1}{\chi^2}\right] \\ &= \varepsilon^2 \frac{\pi\,\mathrm{Kn}^2}{4096(7\rho_0-16)^2}[15335\rho_0^4 - 69024\rho_0^3 + 81344\rho_0^2 - 9216\rho_0 - 1024].\end{aligned} \tag{20}$$

It is seen that the sign of ΔP changes from negative to positive approximately at $\rho_0 = 0.2$, whereas it is always negative in the NLBG and to first order in the EG's case.

Besides the system far from equilibrium, we are also interested in a region extremely close to the equilibrium condition. Therefore, we analyze the case without the strong nonlinear term, namely, we eliminate the terms involving $q_y^k(y)^2$ and $p_{xx}(y)q_y^k(y)$ in Eqs. (12) and (13). In this case, up to δ,

$$P_{yy} = n(0) - \delta\frac{\sqrt{\pi}Q_y}{16n(0)}\left(\frac{\mathrm{d}n(y)}{\mathrm{d}y}\right)_{y=0},$$

$$\Delta P = -\delta\frac{\sqrt{\pi}Q_y}{32n(0)}\left(\frac{\mathrm{d}n(y)}{\mathrm{d}y}\right)_{y=0}. \tag{21}$$

Table 4
The values of λ_n, $\lambda_{\Delta P}$ and λ_p^{yy}/λ_n for $\varepsilon = 0.05$ in the case of EG without a strong nonlinear term

Kn	$\lambda_n \times 10^{-2}$	$\lambda_{\Delta P} \times 10^{-2}$	λ_p^{yy}/λ_n
0.005	−0.552682	1.59241	−2.72540
0.01	0.3430882	−0.16459	−1.00243
0.02	0.2577696	−0.22015	−1.63486
0.05	0.1182556	−0.11280	−1.87030
0.1	0.0612808	−0.05996	−1.93731
0.2	0.0311473	−0.03082	−1.96915

Table 5
The values of λ_n, $\lambda_{\Delta P}$ and λ_p^{yy}/λ_n for $\varepsilon = 0.1$ in the case of EG without a strong nonlinear term

Kn	$\lambda_n \times 10^{-2}$	$\lambda_{\Delta P} \times 10^{-2}$	λ_p^{yy}/λ_n
0.005	−0.598881	1.66316	−2.60733
0.01	0.340322	−0.15760	−0.98090
0.02	0.259368	−0.22086	−1.62996
0.05	0.119439	−0.11385	−1.86873
0.1	0.061951	−0.06598	−1.93656
0.2	0.031501	−0.03117	−1.96879

Assuming that the derivative $\mathrm{d}n(y)/\mathrm{d}y$ of the density at plate 2 has the same sign as Q_y (this is normally correct), ΔP is always negative. Evaluating up to δ^2 and ε^2 yields

$$\Delta P = \varepsilon^2 \delta \frac{\pi\,\mathrm{Kn}}{128\chi}[9\chi\rho_0 - 2] = \varepsilon^2 \delta \frac{\pi\,\mathrm{Kn}}{128\chi}\left[\frac{9\rho_0(16 - 7\rho_0)}{16(1-\rho_0)^2} - 2\right],$$

$$\lambda_n = \frac{\chi}{16}\left[1 - \frac{9}{2}\chi\rho_0\right], \quad \frac{\lambda_p^{yy}}{\lambda_n} = -2 + 2\chi\rho_0 + 9(\chi\rho_0)^2 . \tag{22}$$

As $0 < \rho_0 < 1$, ΔP is positive but $\lambda_p^{yy}/\lambda_n \neq -2$.

Furthermore, we calculate λ_n, $\lambda_{\Delta P}$ and λ_p^{yy}/λ_n up to δ^2 and ε^6. Tables 4 and 5 give the values of these constants for $\varepsilon = 0.05$ and 0.1, respectively. The value and sign of ΔP depends on ε and Kn, too.

6. Discussion

In Ref. [15] the authors argue that there is a pressure difference at plate 2, namely the pressure on one side of the plate is different from that on the other side. They call this new pressure, which acts on the central plate, the “flux induced osmosis” (FIO).

We consider the existence of FIO proposed in Ref. [15] identifying ΔP as the pressure difference defined in Section 5.

In Ref. [15] the following criteria are stated:

(1) $\Delta P > 0$ regardless of the sign of Q_y.

(2) P_{yy} is a function of the nonequilibrium quantities: T_1, the nonequilibrium steady heat flow Q_y, and it is related to the equilibrium quantity P_{eq} as long as the nonequilibrium and equilibrium temperature at both sides of plate 2 coincide

$$\frac{n(0)}{n_{\mathrm{eq}}} = \left(\frac{\partial P_{yy}}{\partial P_{\mathrm{eq}}}\right)_{T_1, Q_y}, \tag{23}$$

where T_1 is the thermodynamic temperature of plate 2. In Ref. [14], for a system of hard spheres and of maxwellian particles—which obey Boltzmann's equation and which obey the BGK equation [18]—using the Chapman–Enskog method, it is shown that criterion 1 in Ref. [15] is valid but criterion 2 is not valid.

On the other hand, for the hard disk's system, from our present scheme based on Enskog's equation we obtain that

(1) Criterion 1 in Ref. [15] is not obeyed in the case of NLBG: ΔP is always *negative* independent of the sign of Q_y.

(2) In the case of LBG, (23) is valid. However, substitution of (17) and (19) into (23) leads to $\lambda_p^{yy}/\lambda_n = -2$. This is correct only in the case of NLBG. Hence, criterion 2 in Ref. [15] is not satisfied.

In the formulation of SST it is assumed that the number of particles and the size of the system is infinite but that the number density of the system is finite [15]. This condition implies that the terms $O(\delta^2)$ in the collision terms can be neglected. Table 3 still indicates that under such condition the osmotic pressure difference is not always positive. Especially when the system is extremely close to equilibrium, Eq. (21) implies that ΔP is always negative.

However when the system is extremely close to equilibrium, Eq. (22) implies that ΔP is always positive. This result coincides with the hard sphere case in Ref. [14].

Furthermore, the condition under which there is no heat flux at the porous wall is

$$\int_{-\infty}^{\infty} \mathrm{d}c_x \int_0^{\infty} \mathrm{d}c_y \frac{c_y}{2} C^2 f_{\mathrm{equil}} + \int_{-\infty}^{\infty} \mathrm{d}c_x \int_{-\infty}^{0} \mathrm{d}c_y \frac{c_y}{2} C^2 f_{y=0} = 0\,. \tag{24}$$

Using Eq. (1) yields

$$q_y^k(0) = \sqrt{\frac{2}{\pi}}[n_{\mathrm{eq}} - n(0)]\,. \tag{25}$$

The above condition is not satisfied without introducing a difference between the temperature of the gas and the porous wall except in the LBG case. Therefore, except in the LBG case, it must be difficult to maintain the equilibrium state between plates 1 and 2 even if the heat conductivity of plate 2 is extremely high. Here, as a rough simplification, let us introduce the temperature of the gas in contact with plate 2, $T_g \neq T_1$. Substituting $\delta = 0$ in Eqs. (2), (10)–(19), using the no-mass-flux condition given by an equation similar to (1), yields

$$\Delta P = n(0)\left[1 - \frac{\sqrt{T_g}}{2} - \frac{1}{2\sqrt{T_g}}\right] + \left[\frac{1}{\sqrt{T_g}} - 2\right]\frac{Q_y^2}{256n(0)}\ . \tag{26}$$

As $T_1 < T_g < T_2$ it follows that $1 < T_g < 1 + \varepsilon$, and putting $q_y^k(y)^2 = Q_y^2 = 0$ in the right side of Eq. (12), not only for NLBG but also for BGL, $\Delta P < 0$. The behavior of the pressure difference changes qualitatively even if we restrict the analysis to Boltzmann's regime. This implies that the estimation of ΔP is a very delicate problem. Even if one can prepare the walls which satisfy Eq. (23) and estimate the pressure difference in Enskog's regime, it is difficult to know what physical meaning lies behind such case.

7. Conclusions

We have analyzed a simple nonequilibrium steady-state system inspired by Ref. [15]. Our study refers only to a hard disk system and analyzes in great detail its behavior using our extended hydrodynamic equations [9] using various approximations. Since we obtain that the osmotic pressure difference is negative in many cases for which Eq. (23) is not satisfied we cannot agree with Ref. [15].

We have assumed that the pores in plate 2 are small enough and we have not considered the problem about reflections on the wall at all. As we point out in the last part of Section 6, the boundary (contact) condition is very delicate. We recognize that a more sophisticated analysis is necessary.

However, in cases when strong nonlinearities can be neglected and the system is quite close to equilibrium—so that higher-order terms in ε do not contribute—the osmotic pressure is positive. This implies the possibility of the existence of FIO.

In addition, in the full-paper [15], the authors point out that Eq. (23) is directly related to the condition at the wall and this condition is essential to construct the formalism of SST in a complete form which gets a new nonequilibrium extensive quantity which determines the degree of nonequilibrium. Therefore, to clarify the problem, the measure of the pressure difference is done only for the case of a wall obeying Eq. (23). Within this context, they still recognize the results of Ref. [14] as implying the existence FIO. Hence we also think that it is worth estimating the pressure difference starting from Eq. (23). If one were to analyze the problem in such a way, then the boundary (contact) condition would have to be reconsidered to solve the kinetic equation. In other words, one would have to evaluate ΔP under the rather complex conditions required by kinetic theory that would lead to satisfy Eq. (23). This has not been done.

The SST formalism is quite interesting and the present study has only put to test the possible existence of FIO.

Finally, we briefly comment about extended irreversible thermodynamics [19–21] (EIT). For an ideal gas, Refs. [20,21] studied a problem quite similar to the one in the present article. In Ref. [20], the authors estimated the pressure difference without considering a special wall. They assumed that the direction of the heat flow is parallel to the interface and their results are very interesting. Furthermore, in Ref. [21] the authors estimated the difference between the pressures which are parallel and perpendicular to the heat flow.

For a hard disk, applying LBG to the systems of Refs. [20,21], it is easily possible to get equations similar to (9) and to see that the pressure difference predicted by Ref. [20] is positive and that the difference predicted by Ref. [21] is zero. In both cases, of course, it is totally unnecessary to use conditions (1) and (3).

Acknowledgements

The author wishes to express his sincere gratitude to Prof. P. Cordero who has guided him into the investigation using kinetic theory, given him the basic theme related to the present investigation and conducted many important discussions. The author also wishes to thank Prof. Dino Risso and Prof. Rodrigo Soto for many helpfull hints. The scholarship from *Mecesup UCh 0008 project* is gratefully acknowledged.

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