Influence of roughness on the magnetostatic modes of ferromagnetic nano-wires

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Abstract

Magnetic nano-entities in their own or as parts of artificial structures have recently been the subject of a large research effort. In order to understand their dynamic behavior, in particular their microwave response as well their spin-wave modes are of great interest. The latter subject is fairly understood in the bulk and for ferromagnetic thin films, but is under development for nano-structures. One aspect of interest is the influence of roughness on the spin-wave modes of these nano-entities. In this work this is studied in the magnetostatic approximation for nano-wires of elliptical cross section that have rough surfaces. The method is based on a perturbation treatment of extinction equations. The conclusion is that to first order surface roughness may well account for experimental frequency shifts of spin modes of nano-wires.

Keywords: Nanomagnetism; Ferromagnetic; Roughness; Spin waves; Magnetostatic

1. Introduction

Recently, a large research effort has been underway on the magnetic properties of structured magnetic materials, whose components have some dimensions on the nano-scale. An issue associated with nano-structures, in which we have been involved, is the description of the magnetic response to microwave fields of several types of nano-components like nano-wires [1] and nano-spheres [2], and collections of them [3,4]. The linear response of such entities is controlled by their spin-wave collective spectrum, which was analyzed in detail in those References. The description of the microwave response of bulk ferromagnets, as well as thin and ultra-thin ferromagnetic films is well developed, while the analogous description in nano-particles is under development: a few other representative references are [5–10].

The present study focuses on one aspect of interest of these nano-wires: the influence of their surface roughness on their spin-wave collective spectrum. As remarked in Ref. [11], the influence of roughness on the micro-magnetic properties of ferromagnetic particles has not been well studied. To some varying degree, roughness will always be present in these nano-structures, and one can envision an important influence of roughness in some aspects of their magnetic properties.

In this work, we determine analytical corrections to the spin-wave collective spectrum of nano-wires with surface roughness, within the magnetostatic approximation. The technique used corresponds to a perturbation analysis of a method based on the extinction equations theorem [12,13]. The latter is an effective method for determining the eigenfrequencies and corresponding eigenvectors of spin waves of nano-wires of arbitrary cross-sections (the nano-wires are magnetized along their long axis with an applied DC magnetic field H_0).

The starting unperturbed cross section of the ferromagnetic cylinder was chosen as elliptic since the magnetostatic surface modes of these ferromagnetic cylinders do have well separated frequencies in the long wavelength limit along the long axis ($k = 0$): they lie in the range $\Omega \in$

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$\gamma\{\sqrt{H_0(H_0+4\pi M_s)}, (H_0+2\pi M_s)\}$ (γ is the absolute value of the gyromagnetic factor, M_s the saturation magnetization). The nondegeneracy of the frequencies, simplifies the analytic perturbation treatment, as is familiar from time independent perturbation theory in Quantum Mechanics.

Explicit results, in terms of eigen-frequencies and eigenvectors, are obtained for surfaces whose roughness is described by a single Fourier component: to first order only selected modes will change their frequencies and shape. Note that the effect of a more general surface roughness can be described as a simple superposition of the results obtained for a single Fourier mode.

2. The formalism

In this paper we confine our attention to nano-wires of perturbed elliptical cross sections. Within the magnetostatic approximation, we look for spin-wave modes of frequency Ω with infinite wavelength along the z direction ($k=0$), i.e. $\mathbf{M}(\mathbf{x},t)=M_s\hat{z}+Re[(m_x(\mathbf{r})\hat{x}+m_y(\mathbf{r})\hat{y})e^{-i\Omega t}]$, with $\mathbf{r}\equiv(x,y)$. Then the time-dependent dipolar field generated by the spin motions is given by $\mathbf{h}=-\nabla\varphi_M$, with $\varphi_M(\mathbf{r})$ the magnetostatic potential. The components of transverse magnetization are $4\pi\mathbf{m}=(\mu_1(\Omega)-1)\mathbf{h}-i\mu_2(\Omega)\hat{z}\times\mathbf{h}$, which result from the linear limit of the Landau–Lifshitz equations ($\mu_1(\Omega)\equiv 1+\Omega_H\Omega_M/(\Omega_H^2-\Omega^2)$, $\mu_2(\Omega)\equiv-\Omega\Omega_M/(\Omega_H^2-\Omega^2)$, $\Omega_H\equiv\gamma H_0$, and $\Omega_M\equiv 4\pi\gamma M_s$). We derived a set of integral equations in the form of contour integrals around the periphery of the wire [13]. A homogeneous version of this set of equations allows to determine in this periphery the magnetic potential $\varphi_M(\mathbf{r})$, the only component (z) of the vector potential $A(\mathbf{r})$, as well as the frequencies Ω of the modes (the magnetic induction follows from the vector potential, i.e. $\mathbf{B}=\nabla\times(A\hat{z})$). The homogeneous equations are

$$0=\oint_C dl[H_t^0(\mathbf{r},\mathbf{r}')A(\mathbf{r})-B_n^0(\mathbf{r},\mathbf{r}')\varphi_M(\mathbf{r})], \tag{1}$$

$$0=\oint_C dl[H_t^I(\mathbf{r},\mathbf{r}')A(\mathbf{r})-B_n^I(\mathbf{r},\mathbf{r}')\varphi_M(\mathbf{r})], \tag{2}$$

where C is the boundary curve, $\mathbf{r}'$ represents an arbitrary point inside the sample for the first equation, and one outside the sample for the second, and $H_t^{0,I}(\mathbf{r},\mathbf{r}')$ and $B_n^{0,I}(\mathbf{r},\mathbf{r}')$ are Green's functions terms [13]. A sensible method of solution of the latter system of equations requires a good choice of the $\mathbf{r}'$. Once this homogeneous set of equations is solved on C, the inhomogeneous equations allow to calculate the magnetostatic potential of the modes everywhere [13] (these equations take a particularly simple form in terms of complex variables ($z=x+iy$, $\bar{z}=x-iy$)).

2.1. Rough elliptical cross section

Since the goal is to solve for the frequencies and shape of long wavelength modes of wires with cross sections close to elliptical, we performed a conformal transformation to elliptical coordinates [13], $z=x+iy=(c/2)\cosh(\xi+i\theta)$. Contours of constant $\xi=\xi_0$ are ellipses centered at the origin, with semi major axis $a=(c/2)\cosh\xi_0$ (along the x axis) and semi minor axis $b=(c/2)\sinh\xi_0$ (along y). Starting from Eqs. (1) and (2) written in complex variables, and after some steps that use appropriately the freedom of choice of the $\mathbf{r}$ [13], one obtains the following set of extinction equations in elliptical coordinates:

$$0=\int_0^{2\pi} d\theta\, \exp[-m(\xi(\theta)\pm i\theta)][B(\theta)\mp H(\theta)], \tag{3}$$

$$0=\int_0^{2\pi} d\theta \cosh[m(\xi(\theta)\pm i\theta)]\{(h\pm\omega)B(\theta) \mp(h+1\pm\omega)H(\theta)\} \tag{4}$$

with m a positive integer, $B(\theta)=i dA(\theta)/d\theta$ and $H(\theta)=d\varphi_M(\theta)/d\theta$, $\xi(\theta)$ describes the curve C, and dimensionless measures of frequency and applied magnetic field $\omega=\gamma\Omega/4\pi M_s$, and $h=H_0/4\pi M_s$ were introduced.

Since Eqs. (3), (4) involve angular integrations over unknowns and functions that depend on the angle variable θ, we introduce angular Fourier representations for them. We define $f_{|m|}^{\pm}(\theta)\equiv\exp[\pm|m|\xi(\theta)]$ and expand it, as well as $B(\theta)$ and $H(\theta)$, in terms of Fourier series, i.e. $B(\theta)=\sum_l B_l\exp(il\theta)$, etc. Then, the extinction equations (3), (4) become equations on the unknown Fourier coefficients, i.e. the $B_l's$ and $H_l's$.

Furthermore, defining $X_{2|l|-1}\equiv H_{-|l|}$, $X_{2|l|}\equiv H_{|l|}$, $Y_{2|l|-1}\equiv B_{-|l|}$ and $Y_{2|l|}\equiv B_{|l|}$, with $l>0$, the pair of extinction equations (3) and the pair (4) are written as matrix equations, respectively:

$$0=PY-QX, \tag{5}$$

$$0=(hV-\omega D)Y+((h+1)D-\omega V)X, \tag{6}$$

with P, Q, V and D, matrices that depend on the shape of the boundary (on the $f_{|m|}^{-}(j)'s$), and where the dependence on frequency ω and applied field h has been shown explicitly. Inverting Eq. (5) and replacing it into Eq. (6), one obtains the following eigenvalue problem for the eigenfrequencies and eigenvectors of the modes:

$$0=(M-\omega S)Y. \tag{7}$$

3. The perturbation scheme

The idea is to solve perturbatively Eq. (7) for the frequencies and eigenvectors of the modes.

Order zero: The zeroth order problem corresponds to an unperturbed elliptic cross section, for which the eigenvalue problem of Eq. (7) becomes

$$0=(M^{(0)}-\omega^{(0)}S^{(0)})Y^{(0)}. \tag{8}$$

The matrix $M^{(0)}-\omega^{(0)}S^{(0)}$ is block diagonal, with blocks of 2×2. The nth block easily renders the frequencies of the perfect elliptical cross section

$$\omega_{\pm}^{(0,n)} = \pm\sqrt{(h+1/2)^2 - \mathrm{e}^{-4|n|\xi_0}/4}. \tag{9}$$

Likewise, the eigenvectors $Y_{\pm}^{(0,n)}$ corresponding to the frequencies $\omega_{\pm}^{(0,n)}$ have only two nonzero components: $(Y^{+}_{2|n|-1}, Y^{+}_{2|n|}) = (p_n, q_n)$, and $(Y^{-}_{2|n|-1}, Y^{-}_{2|n|}) = (q_n, p_n)$, with $p_n \equiv \mathrm{e}^{-|n|\xi_0}$ and $q_n \equiv 2(h + 1/2 + \omega_{+}^{(0,n)})\mathrm{e}^{|n|\xi_0}$.

Order one: The mode to be perturbed to first order is the nth mode $Y_{\pm}^{(0,n)}$ with positive frequency $\omega_{+}^{(0,n)}$, its correction to first order is called $Y^{(1)}$. Thus, to first order Eq. (7) implies:

$$\begin{aligned} 0 &= (M^{(0)} - \omega_{+}^{(0,n)} S^{(0)}) Y^{(1)} \\ &\quad + (M^{(1)} - \omega_{+}^{(0,n)} S^{(1)}) Y_{+}^{(0,n)} - \omega^{(1)} S^{(0)} Y_{+}^{(0,n)}. \end{aligned} \tag{10}$$

$Y^{(1)}$ is expanded in terms of the set of the zeroth order eigenmodes $Y_{\pm}^{(0,j)}$, i.e. $Y^{(1)} = \sum_{\pm,j=1}^{L} a_j^{\pm} Y_{\pm}^{(0,j)}$. The members of this basis set are mutually orthogonal through the matrix $S^{(0)}$, i.e. $(Y_{\pm}^{(0,j)})^T \cdot S^{(0)} \cdot Y_{\pm}^{(0,l)} \equiv U_{\pm\pm}^{jl} = 0$ if j, l are different, and the $\pm$ signs differ. Multiplying Eq. (10) by $(Y_{+}^{(0,n)})^{\mathrm{T}}$ on the left, one obtains the first order correction to the frequency of order n

$$\omega_n^{(1)} = \frac{V_{++}^{nn}}{U_{++}^{nn}}, \tag{11}$$

with V_{++}^{nn} an appropriate matrix element. Similarly, multiplying Eq. (10) by $(Y_{\pm}^{(0,l)})^{\mathrm{T}}$ on the left, with $l \neq n$, expressions for the coefficients a_l^{+} are obtained, i.e. $Y^{(1)}$ is determined.

Application to a single mode roughness: The results of the previous perturbation scheme, are applied now to the simplest type of roughness, i.e. one described in terms of a single Fourier spatial mode. Thus, we assume that the perturbed elliptical cross section is given by $\xi(\theta) = \xi_0 + \xi_j \mathrm{e}^{\mathrm{i}j\theta} + \xi_{-j}\mathrm{e}^{-\mathrm{i}j\theta}$, with j a given integer. Using Eq. (11), corrections to the frequency to first order are

$$\omega_n^{(1)} = \frac{n p_n^2}{2\omega_{+}^{(0,n)}} [\Delta\xi_0 p_n^2 \delta_{j0} - Re(\xi_j)\delta_{j(2n)}], \tag{12}$$

with $\Delta\xi_0$ the change of ξ_0 ($j = 0$). Except for the special case $j = 0$ (which amounts to an elliptical cross section of different semi-axis), it is clear that the frequency of the nth mode is perturbed by a single Fourier mode of order j such that $j = 2n$.

One can also obtain explicit expressions for the expansion coefficients $a_l^{\pm}$ of the first order correction of the eigenvectors ($Y^{(1)}$): to first order, the nth mode only acquires a correction of an lth Fourier mode type, if $l = j \pm n$, or $l = n - j$, with j the Fourier index of the perturbation of the surface.

One can then run some numerical estimates of these first order changes of frequencies for a reasonable surface roughness: for parameters corresponding approximately to the experiments of Ref. [14] one gets frequency shifts of the order of 0.1 GHz, over frequencies which are of the order of 10 GHz, thus showing the order of magnitude to be expected from the effect of roughness on the frequency spectrum.

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