Tracks Emerging by Forcing Langton's Ant with Binary Sequences

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The well-known "ant" defined by C. Langton on a grid with black and white squares is forced by periodical binary sequences $\{r_m\}$, as follows: i) The ant turns 90° to the left (right) if it enters a white (black) square and if $\{r_m\} = 0$ (Langton's case); and ii) the directions are reversed if $\{r_m\} = 1$; in both cases the color of the square is inverted as the ant proceeds. Changing the sequences $\{r_m\}$, we obtain a plethora of different, periodical tracks. Thousands of runs, some of them differing only by one bit, never rendered the same pattern. Also, an ant moving from a white to a black domain may experience reflection, refraction or sliding on the black-white-border.

Key Words: cellular automata; Langton's ant; emergence

1. INTRODUCTION

A cellular automaton model that has caught considerable attention is Langton's ant (see [1–7]). Its behavior is defined on a two-dimensional square grid. The ant heads in one of four possible directions. If it enters a white square, it turns 90° to the left and paints this square black. If it enters a black square, it turns 90° to the right and paints this square white. Disordered patterns appear until 9977 iterations. Then, unexpectedly, the ant moves periodically in one of the four possible diagonal directions. It has been rigorously shown [7] that the temporal period τ on this periodical pattern is $\tau = 104$ steps.

It is interesting that Langton's ant can be considered as a very simple description of a physical particle in a scattering environment. In fact, in so-called Lorentz lattice gases appearing in statistical mechanics, scatterers can be assumed to be distributed on a lattice; a scatterer is changed after a particle interacts with it ([8,10]; see also [6]). Langton's ant would correspond—in a highly simplified model—to a particle that is scattered 90° to the right or 90° to the left, these directions being reversed (due to modification of the scatterer) after each scattering event. Related descriptions can be used for charged particles within inhomogeneous magnetic fields, e.g., in a turbulent magnetized plasma [8].

Previous studies [2,4,5] have been devoted to a particular generalization of Langton's algorithm. In this generalization, n states $k = 0, 1, 2,…, n − 1$ are considered, instead of

just two states. After the ant leaves a cell in state k, this state changes to $k + 1 \pmod{n}$. A rule-string (length n) of 0's and 1's is given. If r_k is the kth bit of that string, then the ant turns to the right if $r_k = 1$ and to the left if $r_k = 0$. It could be shown that the ant's track is always unbounded, provided the rule-string contains at least one 1 and one 0. An interesting result of the generalization to n states with $n \geq 3$ is the appearance of a large variety of complex patterns with bilateral symmetry.

In this contribution we present a different type of generalization. It consists in maintaining the number of states $n = 2$, but introducing infinitely long periodical sequences $\{r_m\}$ of 0's and 1's, which are used for forcing the ant as follows; the mth element of that sequence is considered at the mth iteration ($m = 0, 1, 2,…$); if $r_m = 0$, then the ant behaves as in Langton's case; if $r_m = 1$, then the ant will turn to the left if it enters a black square and to the right if it enters a white square. As in Langton's algorithm, the square will always change its color after the ant turns by 90°.

The sequence $\{r_m\}$ is obtained here with the iteration

$$x_{m+1} = (x_m + p)\bmod 1, \, m \in \mathbb{N}. \tag{1}$$

The x_m and the parameter p are rational numbers in the interval [0, 1], $p = 1$ having the same effect as $p = 0$. We set $r_m = 0$ if $0 \leq x_m \leq 0.5$ and $r_m = 1$ otherwise. x_0 is set to 0.2. Langton's ant corresponds to $p = 0$. Note that $p \in [0.5, 1]$ produces the same result as $p' = 1 - p$, $p' \in [0, 0.5]$ with $x'_0 = 1 - x_0$. In fact, the sequences generated with p are mirror images of those generated with p' because the 0's and 1's in the forcing sequences are interchanged. We considered $p \in [0, 0.5]$.

We have chosen the iteration given above because it easily renders complicated long periodicities. In addition, as we will show below, p allows to determine the smallest possible period T by writing $p = N/T$, where $N, T \in \mathbb{N}$ have no common divisors. To show this, we consider first that the iteration (1) can be written as follows:

$$x_{m+1} = (x_0 + mp)\bmod 1. \tag{2}$$

$T \in \mathbb{N}$ is (by definition) the smallest length of a period iff T is the minimum integer satisfying

$$x_{(n+1)+T} = x_{(n+1)}, \, n \in \mathbb{N}. \tag{3}$$

Using Eq. (2), we now rewrite the right side of Eq. (3) by setting $m = n$, and the left side by setting $m = n + T$. We obtain

$$(x_0 + p(n + T))\bmod 1 = (x_0 + np)\bmod 1. \tag{4}$$

Eq. (4) is valid iff pT is a natural number, which we call N. Thus, $p = N/T$ and, owing to the minimality condition for T, N and T lack a common divisor.

Note that iteration (1) can lead to additional complexity periods due to aliasing. As an example, consider $p = 0.26 \approx 0.25$. If one performs just a few iterations, one obtains repetitive sequences of the period of $p = 0.25$, namely of blocks 0011. These sequences, however, will occasionally be affected by aliasing, which will cause a 1 or a 0 to be skipped. Nevertheless, the aliasing itself is periodical (it modulates periodically, so to say, the sequences of 0011-blocks), its period being 50, as can be seen by writing $p = 0.26 = 13/50$, i.e., $p = N/T$ with $T = 50$. The frieze rendered by this aliasing is thus more complex than for that of the simple case $p = 0.25$.

We will always let the ant start in a completely white array of cells, except in cases (exemplified in this report in Figure 3) in which we examine the effect on the ant's track as it moves from a white into a black array.

2. ANT-MADE "FRIEZES"

Figure 1 illustrates part of the overwhelming variety of frieze-like patterns drawn by the ant after an initial disordered transient. The patterns are rotated differently, so that they are all displayed horizontally. Note that the width of the ant's tracks differ so much that each of them had to be displayed on our figures using drastically different scales. For example the width (in numbers of cells after rounding down to the next integer) of the patterns displayed on Figure 1 are 3080 (a), 1935 (b), 2232 (c), 1378 (d), 1403 (e), 97 (f), 129 (g), 925 (h), and 2122 (i). Altogether, we observed widths ranging between 2 ($p = 1/4$) and 3964 (p = 0.349262 cells. A systematic relationship between the widths and the parameter p (other than a higher probability for complex and wide friezes for larger denominators T of p) could not be found.

We investigated the effect of very small perturbations of the forcing. As smallest possible perturbation we changed the first (or the last) binary digit in each period of $\{r_m\}$. Figure 2 shows the result of such changes on the ant's tracks of Figure 1 [first digit changed in Figure 2(a–c); last digit in Figure 2(d–i)]. Some perturbed tracks (Figure 2) are reminiscent of the unperturbed ones (Figure 1), whereas others are not at all. Altogether, we performed 2000 runs with different forcings and we never obtained the same pattern. We considered 500 values of p equidistant in the interval [0, 0.5]. In addition, we generated 500 random values of p in that interval. Finally, we perturbed the 1000 values of p by inverting one binary digit. For comparisons of the patterns, we relied on our visual judgement, refraining from automated inspection. For the visual comparisons, we defined patterns as "different" if they had the same elements, but distributed in different ways [as Figures 1(e) and 2(e)], of if they contained different elements (as all patterns in Figure
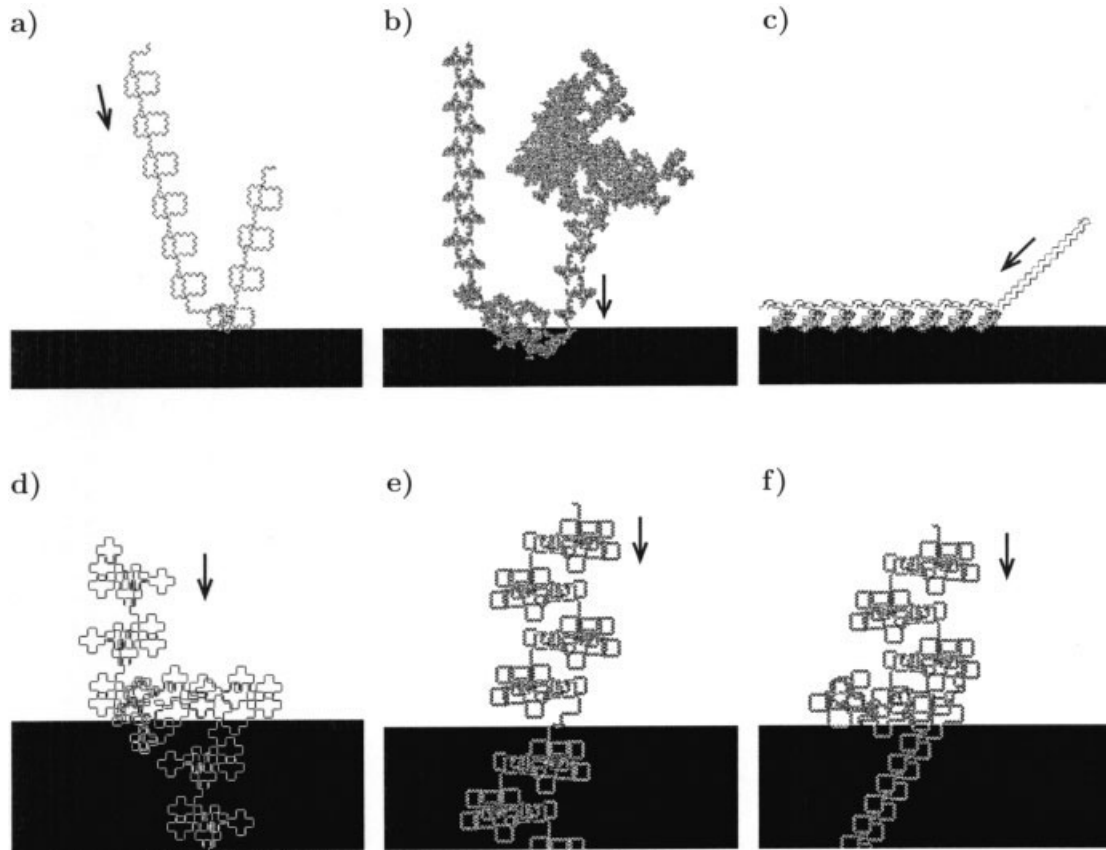
FIGURE 1
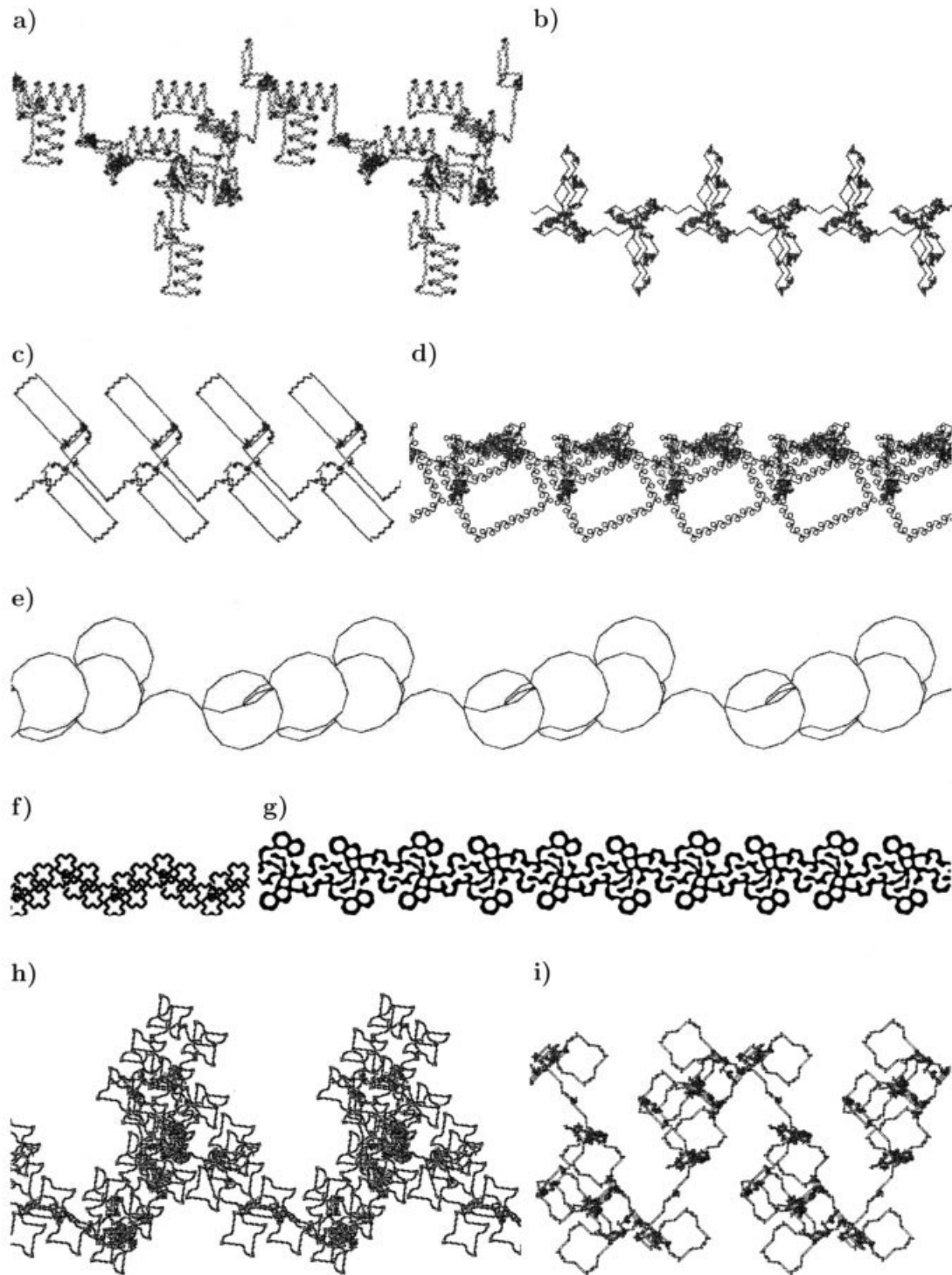
Frieze-like tracks generated by periodical forcing of Langton's ant. $p = 0.15799$ (a), $p = 0.34293$ (b), $p = 0.09013$ (c), $p = 0.34911$ (d), $p = 0.12932$ (e), $p = 0.276$ (f), $p = 0.207$ (g), $p = 0.3488$ (h), and $p = 0.05101$ (i).

FIGURE 2
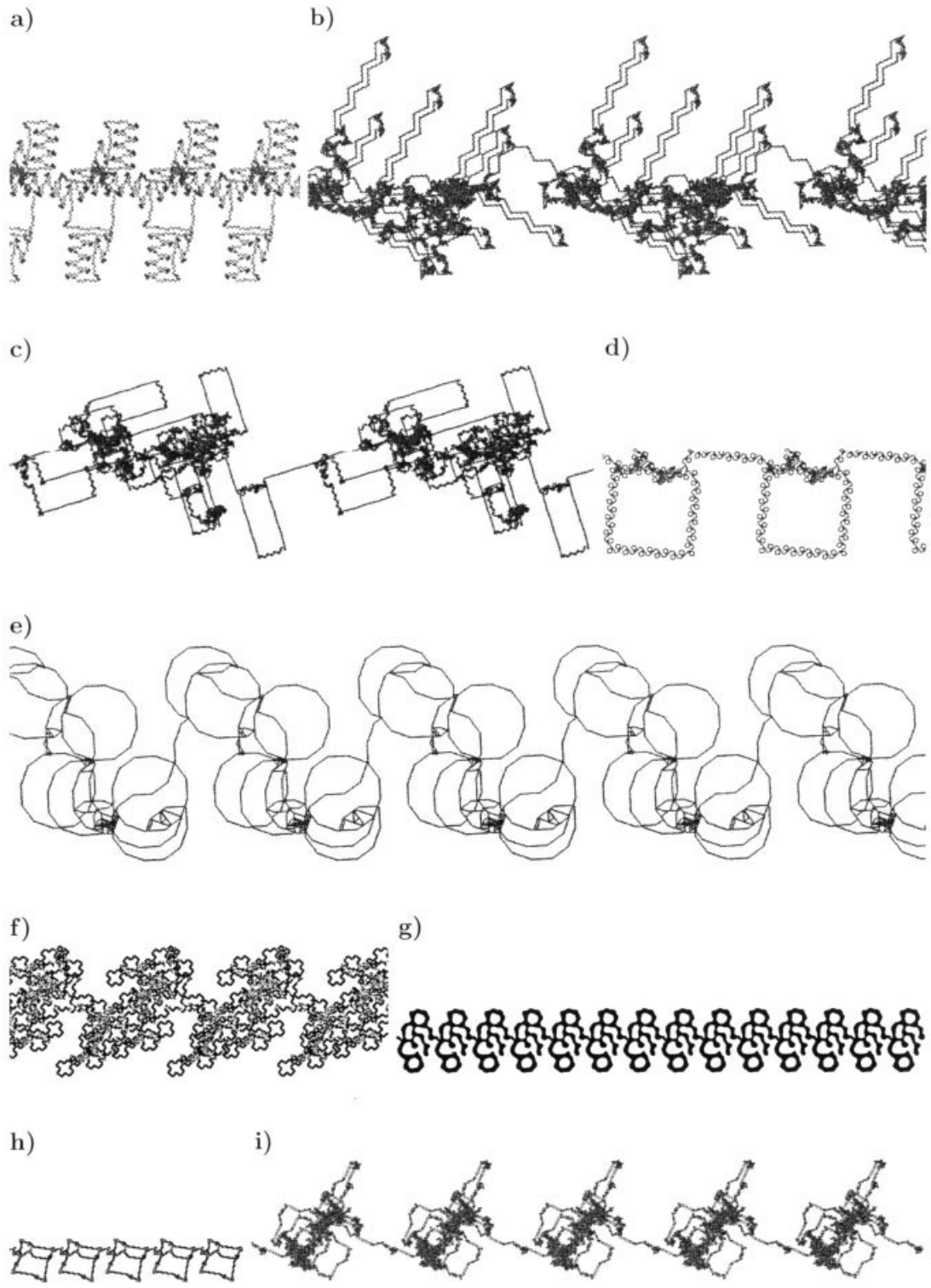
Frieze-like tracks generated by periodical forcing of Langton's ant. The forcing periods differ for each case a) through i) in Figure 1 only by one bit.

FIGURE 3

"Optical" properties of the ant as it collides with a black domain. Reflection for $p = 0.229$ (a) and $p = 0.427$ (b). Border sliding for p = 0.036 (c). Refraction for $p = 0.243$ (d), $p = 0.281$ (e,f). (e) and (f) differ in the distance of the starting point to the black-white border.

1). In many cases the pattern changed completely upon minimum perturbations [compare e.g., Figures 1(h) and 2(h)]. In addition, we observed that elements of the patterns obtained without aliasing are restructured, often in a very complex way, forming the patterns obtained with aliasing due to small perturbations of p.

Another aspect we investigated was the behavior at $p \rightarrow 0$. At very small p, the forcing consists of long sequences of 0's, alternating with long sequences of 1's. In each of these sequences, the ant behaves as the well-known, unperturbed ant described by Langton, thus moving in one of four diagonal directions. Each time the 0's switch to 1's or vice versa, the ant reenters a transient phase to consequently turn again in one of the four diagonal directions. The length of the transient before turning, as well as the angle of turning depends on the phase of Langton's pattern at which the binary switch occurs. Therefore, the path finally looks like that of a car driver who has gotten lost turning around erratically the blocks of an unknown city.

Our method of producing periodical forcing sequences, as described by Eq. (1), may seem arbitrary; therefore, we tested other periodical sequences $\{r_m\}$ as well, such as j 0's ($j \in \mathbb{N}$) followed by a single 1, then by j 0's and so on. Using such sequences, the patterns again are different from each other and have an aesthetical impact similar to those shown in Figure 1.

Further runs were performed by letting the ant move from a grid containing only white cells into a grid consisting only of black cells. A transformation of the shape of the frieze under such conditions is illustrated in Figure 3. Depending on the forcing sequence and on the distance d between the starting point and the black-white border, we found reflection [Figure 3(a,b)], border sliding [Figure 3(c)], or refraction [Figure 3(d–f)]. The latter may be such that a parallel displacement between the ingoing and the outgoing path [Figure 3(d,e)] occurs.

The angle and the parallel shift of a pattern, as well as the number of disordered steps before reflection or refraction occurs, are not only sensitive to p but also to the distance d

(compare e.g., Figure 3e with Figure 3f). We randomly picked 100 p-intervals of length 10^{-4} within [0, 0.5]. In each of these 100 intervals we looked at the patterns for 10 equidistant values of p. Doing so, we never detected an interval of p within which the "optical behavior" was qualitatively independent of p. We also investigated the dependence of the "optical behavior" on the distance d. For this we chose randomly 50 values of p within [0, 0.5] (with the constraint that the pattern periods were below 10^3 steps) and set—for each p—all possible values of d for one period of the pattern. The high sensitivity we had observed for changes of p was not observed for changes in d. In fact, there exist intervals of d in which the same behavior was observed. The longest d-interval of this sort had a length of 16 cells. The least frequent optical effect is border-sliding [as shown in Figure 3(c)], but even this was found in d-intervals, the longest having a length of 3 cells. Within larger d-intervals (the largest had a length of 29 cells), the difference in behavior consisted solely in parallel shifts of the pattern after refraction. Nevertheless, at the edges of all d-intervals with equal or parallel-shifted behavior, we observed drastic quantitative changes, such as that shown in Figure 3(e,f), or qualitative changes, such as changes from reflection to border sliding, or border-sliding to refraction, etc.

3. DISCUSSION

Instead of the variety of symmetrical patterns obtained in a previous generalization of Langton's ant [3–5], we obtain here an amazing plethora of frieze-like patterns.

It is noteworthy that the smallest possible change in the forcing (one bit in the binary sequence) leads to a completely different pattern. This result reminds us of dynamical systems with "riddled basins" [11,12], which render different attractors upon small changes of a parameter. Regarding the changes of patterns upon changes of the distance d between the starting point and a white-black border, we observed a behavior reminding us of "fractal basins," i.e., basins consisting of intervals having finite lengths. However, the number of possible d (smaller than the period length of the pattern) is too small to determine a fractal dimension.

There is another interesting outcome of this work: the periodical forcing, in some cases, considerably reduces the number N_T of transient steps occurring before a periodical track emerges. One example is $N_T = 35$ [Figure 3(f)] or $N_T = 705$ [Figure 3(c)], compared to the 9977 disordered transient steps of Langton's ant. In some cases we found the astonishing result $N_T = 0$ [e.g., Figure 1(f)], meaning the ant does not hesitate one single step before marching on its period-

ical path. In other cases, N_T is highly increased, compared with Langton's ant; one example is $N_T = 7528047$ [Figure 1(c)]. The one-digit perturbed ant required, in general, even larger transients, e.g., $N_T > 10^7$ [Figure 2(b)]. Another quantity that assumes drastically different values, depending on the forcing, is the width of the path, which we found to vary between 2 and 3964 cells. (Note: we also found ants deleting their own track for $t \rightarrow \infty$, e.g., for $p = 13/16$, thus implying a path width equal to zero).

For future work we propose to interpret a periodic sequence of 0's and 1's to mean: leave the color unchanged when 0 but change the color when 1.

We close with the remark that we are dealing here with a novel prototype of complex pattern formation resulting from extremely simple rules. Most striking is the extreme sensitivity of the results upon perturbations of the input. For example, one out of $T = 10^5$ bits was changed in Figure 1(a–d), rendering Figure 2(a–d). Furthermore, the variety of patterns clearly exceeds that resulting from other automata with simple algorithms, such as Wolfram's automata [13] or the game of life.

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