Simplified robust adaptive control of a class of time-varying chaotic systems

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Abstract

Purpose – To develop a simplified robust control scheme for a class of nonlinear time-varying uncertain chaotic systems.

Design/methodology/approach – By means of input-to-state stability theory, a new robust adaptive control scheme is designed, which is simpler than the one proposed by Li *et al.* and applicable to a larger class of nonlinear systems. Only one parameter is adjusted in the controller and the scheme assures that all the signals remain bounded. The behavior of the proposed control scheme is also analyzed through simulations on the Rössler system.

Findings – By adjusting only one parameter in the controller and imposing only one mild assumption on the time-varying parameters, the proposed control algorithm assures that all the signal remain bounded and that the state of the original system will follow a desired trajectory defined either by the trajectory and its first time derivative, or given by a reference model.

Research limitations/implications – The results are limited to a particular class of nonlinear systems where the dimension of the input vector is equal to the order of the system (dimension of the state vector).

Practical implications – The main advantage of the proposed method is that the modification introduced leads to a substantially simpler adaptive robust controller whose practical implementation will be easier.

Originality/value – The contribution of the proposed method is in the simplification of the control algorithm applied to a class of nonlinear time-varying uncertain chaotic systems. This will be useful for control engineers to control complex industrial plants.

Keywords Control, Adaptive system theory

Paper type Research paper

1. Introduction

The study of chaos control starts with Ott *et al.* (1990). They present a method that stabilizes the chaotic system in one of the unstable periodic orbits of the attractor, by means of small disturbances. In the last decade, several ideas and methods have arisen to control chaos, looking for the stabilization in a desired state or the following of a desired trajectory. Within these strategies, we can cite the linearization by state feedback proposed by Shi-Hua and Yu-Ping (2003) applied to Lorenz system, the stabilization around unstable points by using a PI regulator presented by Jiang *et al.* (2002), the adaptive backstepping strategy described by Ge and Wang (1999), the passivity equivalence technique suggested by Yu (1999) and the adaptive control strategy using invariant manifold developed by Tian and Yu (2000).

In the case of nonlinear time-varying and chaotic systems, the work by Li *et al.* (2003) presents a robust adaptive tracking control so that the state of the original system asymptotically tracks a desired trajectory by means of the adjustment of only one parameter. A small variant of this result can be found in Estrada and Duarte-Mermoud (2004).

In reference Li *et al.* (2003), propose a robust adaptive controller based on Lyapunov stability theory for a particular class of nonlinear systems which include chaotic and time-varying plants. They introduce two assumptions under which the controller guarantees global asymptotic tracking of a desired trajectory whose first derivative has to be known. Assumption 2 is quite restrictive since reduces the class of plants that can be controlled by this controller. Moreover, not always is possible to know (measure) the derivative of the trajectory to be followed.

In this paper, based on the results of Li *et al.* (2003), we propose a substantially simpler robust adaptive control method for a larger class of nonlinear systems. The solution is given for two cases; first when the derivative of the desired trajectory is known (measured) and second when the desired trajectory is given by a model reference. In the following section, by means of input-to-state stability theory, a new robust adaptive control scheme is designed involving the adjustment of only one parameter, resulting in boundedness of all the signals in the adaptive system. Besides, the state of the nonlinear time-varying system asymptotically tracks the desired trajectory in both cases abovementioned. In Section 3, simulations on the Rössler system are presented to verify the stability of the resulting adaptive system. Finally, in Section 4, some conclusions are drawn.

2. Robust adaptive control design

Let us consider the nonlinear plant described by the following relationship:

$$\dot{x}(t) = f(x) + F(x)\boldsymbol{\theta}(t) + u(t) \tag{1}$$

where $x \in \Re^n$ corresponds to the state vector of the system, which is assumed to be accessible, $f(\cdot): \Re^n \to \Re^n$ and $F(\cdot): \Re^n \to \Re^{n\times p}$ are known continuously differentiable functions with $F(0) = 0$ and $f(0) = 0$. $u(t) \in \Re^n$ is the input to the plant and $\boldsymbol{\theta}(t) \in \Re^p$ is the unknown time-varying parameter vector. The unknown parameter vector $\boldsymbol{\theta}(t) \in \Re^p$ is known to vary between upper and lower bounds so that $\underline{\boldsymbol{\theta}}_i \leq \boldsymbol{\theta}_i(t) \leq \bar{\boldsymbol{\theta}}_i$ for $i = 1, 2, \ldots, p$,, i.e. we assume they belong to a bounded and closed set, as stated in the following assumption.

Assumption 1. The unknown parameter vector $\boldsymbol{\theta}^{\mathrm{T}}(t) = [\boldsymbol{\theta}_1(t), \boldsymbol{\theta}_2(t), \ldots, \boldsymbol{\theta}_p(t)] \in \Re^p$ belongs to a bounded and closed set Ω where $\Omega = [\underline{\boldsymbol{\theta}}_1, \bar{\boldsymbol{\theta}}_1] \times [\underline{\boldsymbol{\theta}}_2, \bar{\boldsymbol{\theta}}_2] \times \cdots \times [\underline{\boldsymbol{\theta}}_p, \bar{\boldsymbol{\theta}}_p]$, with $\underline{\boldsymbol{\theta}}_i, \bar{\boldsymbol{\theta}}_i$ for $i = 1, 2, \ldots, p$ unknown constants representing the lower and upper bounds on the components of vector $\boldsymbol{\theta}(t) \in \Re^p$.

From Assumption 1, we can immediately write:

$$\|\boldsymbol{\theta}(t)\| = \left(\sum_{i=1}^{p} \boldsymbol{\theta}_i(t)^2\right)^{1/2} \leq \left(\sum_{i=1}^{p} \max[|\underline{\boldsymbol{\theta}}_i|^2, |\bar{\boldsymbol{\theta}}_i|^2]\right)^{1/2} \equiv \beta \tag{2}$$

where $\beta \in \Re$ is an unknown but constant parameter.

The objective is to determine a bounded input $u(t)$ such that $x(t)$ behaves in some desired fashion. The goal in this study is that $x(t)$ follows a desired trajectory $x_d(t)$. In the solution, we distinguish two cases; the first where $x_d(t)$ and $\dot{x}_d(t)$ are known, and the second where $x_d(t)$ is given by a reference model.

2.1 Case when the derivative of the desired trajectory is known

We first address the case when the first time derivative $\dot{x}_d(t)$ of the desired trajectory $x_d(t)$ is known (measurable). It is assumed that $\dot{x}_d(t)$ and $x_d(t)$ are continuous and bounded functions. Defining the tracking error:

$$e(t) = x(t) - x_d(t) \tag{3}$$

and subtracting $\dot{x}_d(t)$ from both sides of equation (1) we can write:

$$\dot{x}(t) - \dot{x}_d(t) = \dot{e}(t) = f(x) - \dot{x}_d(t) + F(x)\boldsymbol{\theta}(t) + u(t) \tag{4}$$

Next, we choose the control law:

$$u(t) = -f(x) + \dot{x}_d(t) - e(t) + \alpha(e, x, \hat{\beta}) \tag{5}$$

where:

$$\alpha(e, x, \hat{\beta}) = \frac{-F(x)\mu(e, x)\hat{\beta}^2}{\|\mu(e, x)\|\hat{\beta} + \varepsilon\|e\|^2} \tag{6}$$

and:

$$\mu^{\mathrm{T}}(e, x) = e^{\mathrm{T}}F(x) \tag{7}$$

with $\varepsilon > 0$. The adaptive law for $\hat{\beta}$ is chosen as:

$$\dot{\hat{\beta}}(t) = \gamma\|\mu(e, x)\| \tag{8}$$

with $\hat{\beta}(t_0) > 0$ and $\gamma > 0$. Then we can state the following theorem.

Theorem 2.1. If Assumption 1 is satisfied by system (1), then the control law given by (5)-(7) with $0 < \varepsilon < 1$ and the adaptive law given by equation (8) will guarantee that all the signals of the adaptive system will remain bounded and $\lim_{t\to\infty} e(t) = \lim_{t\to\infty} (x(t) - x_d(t)) = 0$.

Proof. Replacing equations (5) and (6) in equation (4), we can write:

$$\dot{e}(t) = -e(t) + F(x)\boldsymbol{\theta}(t) - \frac{F(x)\mu(e, x)\hat{\beta}^2}{\|\mu(e, x)\|\hat{\beta} + \varepsilon\|e\|^2} \tag{9}$$

For systems (8) and (9), $\boldsymbol{\theta}(t)$ can be seen as input to the system. Then for the unforced system ($\boldsymbol{\theta}(t) = 0$) the equilibrium state is $e(t) = 0$ and $\hat{\beta}(t) = 0$. In order to prove that systems (8) and (9) is input-to-state stable (Khalil, 2002, p. 175), we choose the following continuously differentiable function:

$$V(e, \tilde{\beta}) = \frac{1}{2}e^{\mathrm{T}}e + \frac{1}{2\gamma}\tilde{\beta}^2 \tag{10}$$

where $\tilde{\beta}(t) = \hat{\beta}(t) - \beta \in \Re$ and $\gamma > 0$ is the adaptive gain given in equation (8).

The first time derivative of equation (10) along the system trajectory equations (8) and (9) is given by:

$$\frac{\mathrm{d}V(e,\tilde{\beta})}{\mathrm{d}t} = -e^{\mathrm{T}}e + e^{\mathrm{T}}F(x)\boldsymbol{\theta}(t) - \frac{e^{\mathrm{T}}F(x)\mu(e,x)\hat{\beta}^{2}}{\|\mu(e,x)\|\hat{\beta} + \varepsilon\|e\|^{2}} + \frac{1}{\gamma}\tilde{\beta}\dot{\hat{\beta}} \tag{11}$$

Using equation (7) we have:

$$\frac{\mathrm{d}V(e,\tilde{\beta})}{\mathrm{d}t} = -\|e\|^{2} + \mu^{\mathrm{T}}(e,x)\boldsymbol{\theta}(t) - \frac{\|\mu(e,x)\|^{2}\hat{\beta}^{2}}{\|\mu(e,x)\|\hat{\beta} + \varepsilon\|e\|^{2}} + \frac{1}{\gamma}\tilde{\beta}\dot{\hat{\beta}} \tag{12}$$

From Assumption 1 and equation (2), we can write the following inequality:

$$\mu^{\mathrm{T}}(e,x)\boldsymbol{\theta}(t) \leq \|\mu(e,x)\|\beta \tag{13}$$

Moreover, the following inequality can be established for the third term of the right hand side of equation (12):

$$\begin{aligned} -\frac{\|\mu(e,x)\|^{2}\hat{\beta}^{2}}{\|\mu(e,x)\|\hat{\beta} + \varepsilon\|e\|^{2}} &= \|\mu(e,x)\|\hat{\beta}\left(-1 + \frac{\varepsilon\|e\|^{2}}{\|\mu(e,x)\|\hat{\beta} + \varepsilon\|e\|^{2}}\right) \\ &\leq \|\mu(e,x)\|\hat{\beta}\left(-1 + \frac{\varepsilon\|e\|^{2}}{\|\mu(e,x)\|\hat{\beta}}\right) \end{aligned} \tag{14}$$

Replacing equations (13) and (14) in equation (12), we get:

$$\frac{\mathrm{d}V(e,\tilde{\beta})}{\mathrm{d}t} \leq -(1-\varepsilon)\|e^{2}\| - \|\mu(e,x)\|\tilde{\beta} + \frac{1}{\gamma}\tilde{\beta}\dot{\hat{\beta}} \tag{15}$$

Finally, replacing the adaptive law given by equation (8) in equation (15) we obtain:

$$\frac{\mathrm{d}V(e,\tilde{\beta})}{\mathrm{d}t} \leq -(1-\varepsilon)\|e^{2}\| \tag{16}$$

If we chose ε such that $(1-\varepsilon) > 0$ then $\dot{V} \leq 0$. Since, conditions of Theorem 4.19 (Page 176 of (Khalil, 2002)) are fulfilled, system (8) and (9) is input-to-state stable and therefore if $\boldsymbol{\theta}(t)$ is bounded, then $e(t)$, $\tilde{\beta}(t)$ and $x(t)$, $\hat{\beta}(t)$ are bounded. From this fact, we can conclude that all the signals of the adaptive system remain bounded. Integrating both sides of equation (16), we can conclude that $e(t)$ is a square integrable signal, i.e. $e(t) \in L^{2}$. From equation (4) it follows that $\dot{e}(t)$ is bounded, since it depends on bounded functions. Using Barbalat Lemma (Narendra and Annawamy, 1989), we can conclude that $e(t) \to 0$ when $t \to \infty$. Therefore, the controller given by equations (5)-(7) and the adaptive law given by equation (8), guarantees that system (1) asymptotically tracks the desired signal $x_d(t)$. □

Remark 2.1. Unlike the controller proposed in Li *et al.* (2003), the one proposed here makes only assumptions on the time-varying parameter vector $\boldsymbol{\theta}(t) \in \Re^{n}$. No further assumptions on the form of $f(x)$ are made as it is done in Li *et al.* (2003). Therefore, the class of systems being controlled by the controller proposed here is larger than the one proposed in Li *et al.* (2003).

Remark 2.2. A closer look at Rössler system reveals that Assumption 2 stated in Li *et al.* (2003) is not satisfied and therefore that controller cannot be readily applied. In order to reach the control goal, an additional term is used in Li *et al.* (2003) in the control law but no explanation on its role is given. On the contrary, the controller proposed here can be readily applied to Rössler system, as will be shown in Section 3.

2.2 Case when the derivative of the desired trajectory is not measurable

Let us now consider the case when the first derivative of the desired trajectory is not measurable. In this case, we consider a reference model defined as:

$$\dot{x}_d(t) = f_d(x_d) + r(t) \tag{17}$$

where $f_d(x_d) \in \Re^n$ is a linear or nonlinear vector function, such that the origin of equation (17) is asymptotically stable ($r(t) \equiv 0$) and $r(t) \in \Re^n$ is a piecewise, continuous and uniformly bounded reference input.

As before, we define the tracking error $e(t) = x(t) - x_d(t)$ as in equation (3), and we choose this time the control law as follows:

$$u(t) = -f(x) - e(t) + f_d(x_d) + r(t) + \alpha(e, x, \hat{\beta}) \tag{18}$$

where $\alpha(e, x, \hat{\beta})$, $\mu(e, x)$ and $\dot{\hat{\beta}}$ are given by equations (6)-(8), respectively. Notice that $\dot{x}_d(t)$ it is not explicitly used in the control law (18). Now, we can state the following corollary from Theorem 2.1.

Corollary 2.1. If Assumption 1 is satisfied by system (1), then the control law given by equations (18), (6) and (7) with $0 < \varepsilon < 1$, together with the adaptive law given by equation (8), with$\hat{\beta}(t_0) > 0$ and $\gamma > 0$, will guarantee that all the signals of the adaptive system will remain bounded and $\lim_{t\to\infty} e(t) = \lim_{t\to\infty} (x(t) - x_d(t)) = 0$, where $x_d(t)$ is the desired trajectory given by equation (17).

Proof. Subtracting equation (17) from equation (1), we obtain:

$$\dot{x}(t) - \dot{x}_d(t) = \dot{e}(t) = f(x) + F(x)\boldsymbol{\theta}(t) - f_d(x_d) - r(t) + u(t) \tag{19}$$

Replacing equation (18) in equation (19), we get:

$$\dot{e}(t) = -e(t) + F(x)\boldsymbol{\theta}(t) - \frac{F(x)\mu(e,x)\hat{\beta}^2}{\|\mu(e,x)\|\hat{\beta} + \varepsilon\|e\|^2} \tag{20}$$

which has exactly the same form as equation (9). Thus, the rest of the proof follows along exactly the same lines as in Theorem 2.1. □

Remark 2.3. With minor modifications, similar results for Theorem 2.1 and Corollary 2.1 can be obtained if in control laws (5) and (18) the term $A_m e(t)$ is used instead of the term $-e(t)$, where $A_m \in \Re^{nxn}$ is any asymptotically stable matrix.

Remark 2.4. In the previous developments, constant adaptive gain γ was used in the adaptive law (8). It can be shown that the same results hold if a positive definite time-varying adaptive gain is introduced (Narendra and Annawamy, 1989).

3. Simulation results

In order to verify the behavior of the proposed method, we will control the Rössler system (Strogatz, 2000). This system can be expressed as:

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -x_2 - x_3 \\ x_1 \\ x_1 x_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ x_2 & 0 & 0 \\ 0 & 1 & -x_3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} + u \tag{21}$$

where $u = [\mathrm{u}_1 \quad \mathrm{u}_2 \quad \mathrm{u}_3]^{\mathrm{T}} \in \Re^3$ is the control vector. If we define:

$$f(x) = \begin{bmatrix} -x_2 - x_3 \\ x_1 \\ x_1 x_3 \end{bmatrix}, \quad F(x) = \begin{bmatrix} 0 & 0 & 0 \\ x_2 & 0 & 0 \\ 0 & 1 & -x_3 \end{bmatrix}, \quad \boldsymbol{\theta}(t) = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \tag{22}$$

the system (2.1) can be expressed in the form (1). The Rössler system exhibits chaotic behavior when $a = b = 0.2$, $c = 5.7$ and $u = 0$ (Strogatz, 2000).

For simulations, we will choose the initial state vector as:

$$x(0) = [\,2 \quad -4 \quad -0.3\,]^{\mathrm{T}}$$

and the time-varying parameter vector as:

$$\boldsymbol{\theta}(t) = [\,0.2(1 - \sin(t)) \quad 0.2\cos(t) \quad 5.7\,]^{\mathrm{T}}.$$

Clearly, the time-varying parameter vector $\boldsymbol{\theta}(t)$ satisfies Assumption 1 and it can be verified that $\beta = \sqrt{32.69}$. We choose the initial condition for the estimate $\hat{\beta}(0) = 0.2$.

In this study, we will consider the following reference model:

$$\dot{x}_d(t) = -x_d(t) + r(t) \tag{23}$$

with:

$$r(t) = [\,0 \quad \cos(t) \quad \sin(t)\,]^{\mathrm{T}}$$

and:

$$x_d(0) = [\,0 \quad 1 \quad 0\,]^{\mathrm{T}}.$$

Figure 1 shows the Rössler system state, model reference state and the estimate $\hat{\beta}(t)$ when the controller based on the Corollary 2.1 is designed and applied. In all the simulations, we have chosen $\varepsilon = 0.8$ and the adaptive gain $\gamma = 2$. It can be observed that all the signals remain bounded and the state error converges to zero, that is, the system state tracks the model reference state. In Figure 2, the evolution of control signals are shown.

4. Conclusions

In this paper, we have introduced some modifications on the adaptive robust controller proposed by Li *et al.* (2003), obtaining a substantially simpler controller. This controller can be applied to a larger class of nonlinear plants, including chaotic systems with time-varying unknown parameters. Using input-to-state stability theory, it is demonstrated that the proposed controller, adjusting only one parameter, assures that all the signals in the adaptive system will remain bounded and the state of the original system will follow a desired trajectory defined either by the trajectory and its first time derivative, or given by a reference model.

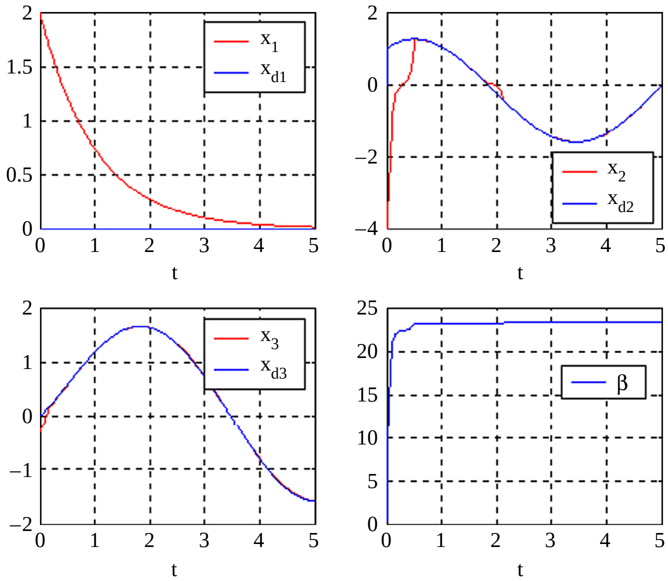

Figure 1.
Evolution of the Rössler system sate $x(t)$, model reference sate $x_d(t)$ and estimate parameter $\hat{\beta}(t)$ with the controller based on Corollary 2.1 with $\varepsilon = 0.8$ and adaptive gain $\gamma = 2$

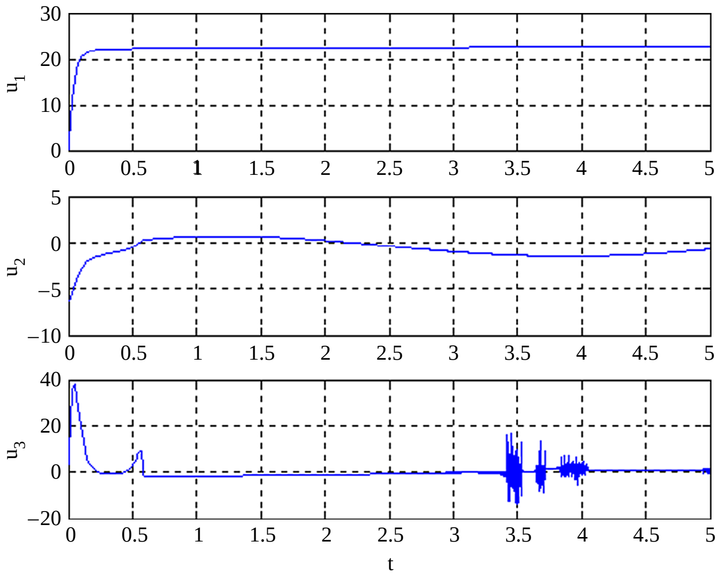

Figure 2.
Evolution of the Rössler system control $u(t)$, with the controller based on Corollary 2.1 with $\varepsilon = 0.8$ and adaptive gain $\gamma = 2$

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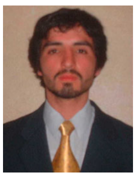

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