Modeling public transport corridors with aggregate and disaggregate demand

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ABSTRACT

Microeconomic public transport models aimed at maximizing social benefits usually consider demand in an aggregate manner. In this paper we examine the effect of this approach on the optimal values of frequency and vehicle size by comparison with models where demand is described in detail as a matrix of flows between every station in a single line service. The theoretical analysis and the numerical examples suggest that the spatially aggregated model underestimates optimal frequency and overestimates vehicle size.

1. Introduction

Microeconomic models of public transport (usually a single line) are based on aggregated demand description, that is, total ridership along a line and average journey length (Mohring, 1972; Jansson, 1980; Jara-Díaz and Gschwender, 2003). This way, aggregation is spatial rather than temporal, as most public transport systems are designed for representative periods of large demand. The goal is to maximize social benefit or minimize total costs (considering both users and operators), finding optimal levels for the decision variables such as frequency, vehicle size, number of bus stops and spacing between lines.

Presently, it is feasible to capture more precise demand patterns in cases where either the payment device or the specialized infrastructure in buses and stations can provide not only passenger counts, but also exact identification of the origin and destination of the passenger journey (Radio Frequency Identification cards along with wide-range devices on bus doors, cameras, etc.). In this paper we examine the effects of having different levels of aggregation regarding demand information in order to explore improvements on the recommendations and conclusions obtained from classical microeconomic models. Thus, we establish the optimal conditions for the relevant decision variables (frequency and vehicle size) on a public transport corridor with inelastic demand, in cases where the demand data is only available at an aggregated level (at the level of an entire line, or ridership per direction of movement) as well as cases in which it is feasible to obtain more detailed information on the demand structure, like origin-destination matrices at the level of bus stops or number of passenger who board and alight each bus at each stop.

In the following section analytical expressions for the optimal frequency and vehicle size are developed for each case. A theoretical comparison is presented in Section 3 regarding optimal frequencies, and numerical experiments are conducted in Section 4. We close with some relevant comments, conclusions and further research in the final section.

2. Single line models with aggregate and disaggregate demand

2.1. Demand description

In what follows, a linear corridor is used as a representation of a generic public transport system, which can correspond to either a single isolated bus line or a line inserted within an existing network of fixed topology (number and position of bus routes). The corridor is one-dimensional, with two terminals (1 and N) and two directions of circulation: direction 1 from 1 to N and direction 2 from N to 1, as shown in Fig. 1. We assume that passengers arrive randomly to the stations, at a fixed rate, a reasonable assumption for high frequency corridors (Seddon and Day, 1974; Danas, 1980). The demand is treated parametrically in the proposed formulations. Our purpose is to find the optimal value for the design variables (in these developments, optimal frequency f and vehicle size K) in order to maximize the social welfare of the system, which in the case of inelastic demand is equivalent to minimizing the total cost, taking into consideration both users and operators. Users' costs will include both waiting and in-vehicle travel time, both of which depend on frequency, the latter because of the effect of boarding and alighting at stations. As the location of bus stops along the corridor is given, access time is not considered.

As mentioned, the major objective of this paper is to compare the analytical expressions obtained for the optimal design variables under different demand aggregation levels. First, a model

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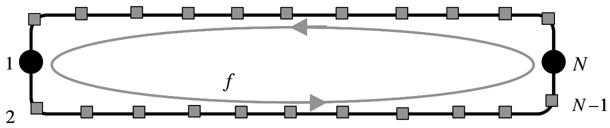

Fig. 1. Generic public transport corridor.

based upon *aggregated demand* is formulated in two versions, one whereby only the total cycle demand y is know (*Model 1* or *M1*) and a second model relying on aggregated demand information per direction of circulation, say y_1 and y_2 on direction 1 and 2, respectively (*Model 2* or *M2*). In both cases the average journey length is assumed to be known. Then, a model relying on *disaggregated demand* is presented (*Model 3* or *M3*) assuming that a stop-to-stop OD matrix is available considering N stations in one direction ($N-1$ segments), as shown in Fig. 1. Going from station 1 to N is called direction 1; direction 2 goes from station N to 1.

In the models we describe next, the following parameters are assumed known and fixed:

L: Length of the corridor [km].

R_k: Bus movement travel time under normal service between stations k and $k+1$, including acceleration and deceleration times at bus stops [min].

β: Marginal passenger boarding time [s/pax].

λ_{kl}: Trip rate between stations k and l [pax/h] (used for M3). This demand is assumed fixed over the studied period, defining a trip matrix of the form:

$$\begin{bmatrix} 0 & \lambda_{12} & \cdots & \lambda_{1N} \\ \lambda_{21} & \ddots & & \vdots \\ \vdots & & & \lambda_{N-1N} \\ \lambda_{N1} & \cdots & \lambda_{NN-1} & 0 \end{bmatrix},$$

Additionally, for M3 also, the following functions need to be defined for the formulation of the cost functions:

Passenger boarding rate at station k, whose destination is among stations l_1 and l_2 inclusive [pax/h]: $\lambda_k^+(l_1, l_2) = \sum_{l=l_1}^{l_2} \lambda_{kl}$
Passenger alighting rate at station k, whose destination is among stations l_1 and l_2 inclusive [pax/h]: $\lambda_k^-(l_1, l_2) = \sum_{l=l_1}^{l_2} \lambda_{lk}$

Thus, from these functions we can define the following quantities:

Passenger boarding rate at station k, direction 1: $\lambda_k^{1+} \equiv \lambda_k^+(k+1, N) = \sum_{l=k+1}^{N} \lambda_{kl}$
Passenger alighting rate at station k, direction 1: $\lambda_k^{1-} \equiv \lambda_k^-(1, k-1) = \sum_{l=1}^{k-1} \lambda_{lk}$
Passenger boarding rate at station k, direction 2: $\lambda_k^{2+} \equiv \lambda_k^+(1, k-1) = \sum_{l=1}^{k-1} \lambda_{kl}$
Passenger alighting rate at station k, direction 2: $\lambda_k^{2-} = \lambda_k^-(k+1, N) = \sum_{l=k+1}^{N} \lambda_{lk}$

We assume that the boarding process dominates over the alighting process, and therefore, in the model only the first phenomenon (quantified through the parameter β) is considered.

From these definitions, we can relate the demand defined in the context of the three models as follows:

$$y = y_1 + y_2 = \sum_{k=1}^{N} (\lambda_k^{1+} + \lambda_k^{2+}). \tag{1}$$

Besides, the vehicle arrival distribution to the stations is crucial for a correct computation of the passenger waiting time, which decreases as the headways become more regular. Following Delle Site and Filippi (1998), two possible bus arrival patterns to stations are considered, namely scheduled service (regular headways) and random bus arrivals (Poisson process). The former is associated with systems with low variability in both running and passenger transfer times at bus stops, for instance special bus segregated corridors with efficient transfer operations at stops. In this case the average waiting time turns out to be half of the headway. Random arrivals are characteristic of networks with high variability in running times (poorly controlled bus systems); in this case the expected waiting time is equal to the average headway.

Recalling that the pursued objective is to minimize the total system cost expression with respect to the relevant design variables (frequency and vehicle size), next we define analytically the different cost components considered for the three models, from both the users and operator standpoints. The former comprises the waiting and in-vehicle time costs while the latter comprises two operational cost expressions associated with distance and time, respectively.

2.2. Users' costs

The *waiting time cost* (C_w) is the product between the total expected waiting time experienced by all customers and the subjective value of waiting time (P_w). If x is an auxiliary binary variable (equals to 1 if buses arrive Poisson, 0 if buses arrive at constant headway, as discussed earlier) we can compute the waiting time cost component as follows:

$$C_w = P_w \frac{1+x}{2} \frac{y}{f}, \tag{2}$$

where f is the operational frequency, computed as the inverse of the headway. Expression (2) applies to all models, regardless of the demand aggregation level applying expression (1).

The *in-vehicle time cost* (C_v) is the product between the total expected in-vehicle time and the subjective value of the in-vehicle time (P_v).Unlike the waiting time component C_w, C_v adopts a different form depending on the demand aggregation level. For the M1 model, in-vehicle travel time is expressed as a fraction of the total cycle time, i.e. the ratio between the average journey length l and the total route length 2 L (Mohring, 1972; Jansson, 1980; Jara-Díaz and Gschwender, 2003). On the other hand, for the M2 model, it is possible to split this component per direction, as the average journey length on direction i (namely l_i) of journey over the route length (L). Moreover, the cycle time t_c is computed as the sum of the running time by direction (denoted as R_i) and the total dwelling time at stations (computed as the total expected passenger boarding time). The latter component is computed as the product between the average number of passengers boarding a vehicle, y_i/f, and the marginal boarding time. Analytically, for M1:

$$C_v = P_v \frac{l}{2L} \left(R_1 + R_2 + \beta \frac{y}{f} \right) y. \tag{3}$$

For M2:

$$C_v = P_v \left[\frac{l_1}{L} \left(R_1 + \beta \frac{y_1}{f} \right) y_1 + \frac{l_2}{L} \left(R_2 + \beta \frac{y_2}{f} \right) y_2 \right]. \tag{4}$$

For M3 no approximation is needed; travel time t_{kl} for each OD pair (k,l) is given by

$$t_{kl} = \begin{cases} \sum_{i=k}^{l-1} \left(R_i + \beta \frac{\lambda_i^{1+}}{f} \right) & \text{if } k < l \\ \sum_{i=l+1}^{k} \left(R_{i-1} + \beta \frac{\lambda_i^{2+}}{f} \right) & \text{if } l < k \end{cases} \tag{5}$$

Then, multiplying (5) times λ_{kl}, adding over all OD pairs and multiplying by P_v. the total in-vehicle travel time cost is obtained in monetary units. Analytically,

$$C_v = P_v\left\{\sum_{k=1}^{N}\sum_{l=k+1}^{N}\left[\sum_{i=k}^{l-1}\left(R_i+\beta\frac{\lambda_i^{1+}}{f}\right)\right]\lambda_{kl} + \sum_{k=1}^{N}\sum_{l=1}^{k-1}\left[\sum_{i=l+1}^{k}\left(R_{i-1}+\beta\frac{\lambda_i^{2+}}{f}\right)\right]\lambda_{kl}\right\}. \quad (6)$$

2.3. Operator cost

In this computation we take into account two components for the operator cost, as some items are better represented on a temporal basis (e.g. labor) and others over a spatial basis (running cost, maintenance, etc.). Following Jansson (1980) and Oldfield and Bly (1988), a linear dependency on the vehicle capacity K is assumed for the operator cost functions. Let us denote $c(K)$ as the cost per vehicle-hour ($/veh-h) and $c,(K)$ as the cost per vehicle-kilometer ($/veh-km). Analytically,

$$c(K) = c_0 + c_1K \qquad c'(K) = c'_0 + c'_1K. \quad (7)$$

Therefore, the operator cost can be expressed as

$$C_o = c(K)F + c'(K)vF, \quad (8)$$

where v is the commercial speed and F is the fleet size given by frequency f times cycle time t_c discussed in Section 2.2. Thus, (8) can be rewritten as a function of t_c and f as follows:

$$C_o = c(K)ft_c + 2c'(K)fL. \quad (9)$$

Finally,

$$C_o = f\left[c(K)\left(R_1+R_2+\beta\frac{y}{f}\right)+2c'(K)L\right], \quad (10)$$

which applies for the three models.

2.4. Optimal value of the frequency and the vehicle capacity

The total cost minimization problem comprises the joint minimization of both users' and operators' costs, encompassing Eqs. (2), (3), (4), (6) and (10). In order to find the optimal value of the variables f and K, first order conditions (FOC) are applied. The vehicle capacity is adjusted in order to accommodate the demand of the most loaded segment along the corridor, q_{max}, which can be easily obtained from the OD matrix in M3. For models M1 and M2, this value must be assumed to be known (or accurately estimated). Then, by defining a safety factor $\eta \in (0,1]$ in order to have reserve capacity to absorb the randomness in the demand, and computing $K = q_{max}/\eta f$, the FOC yield the following values for the optimal frequency for M1, M2 and M3, respectively:

$$f^* = \sqrt{\frac{P_e\frac{1+x}{2}y + P_v\beta\frac{l}{2L}y^2 + c_1\frac{q_{max}}{\eta}\beta y}{c_0(R_1+R_2)+2c'_0L}}, \quad (11)$$

$$f^* = \sqrt{\frac{P_e\frac{1+x}{2}y + P_v\beta\left(\frac{l_1}{L}y_1^2+\frac{l_2}{L}y_2^2\right) + c_1\frac{q_{max}}{\eta}\beta y}{c_0(R_1+R_2)+2c'_0L}}, \quad (12)$$

$$f^* = \sqrt{\frac{P_e\frac{1+x}{2}y + P_v\beta\left(\sum_{k=1}^{N}\sum_{l=k+1}^{N}\lambda_{kl}\sum_{i=k}^{l-1}\lambda_i^{1+} + \sum_{k=1}^{N}\sum_{l=1}^{k-1}\lambda_{kl}\sum_{i=l+1}^{k}\lambda_i^{2+}\right) + c_1\frac{q_{max}}{\eta}\beta y}{2\left(c_0\sum_{k=1}^{N-1}R_k + c'_0L\right)}}. \quad (13)$$

An expression similar to (11) was previously found by Jansson (1980). Instead of assuming known the maximum load q_{max}, the author uses the average load ($\frac{l}{2L}y$) amplified by a factor greater than 1, in order to have enough capacity to accommodate demands above the average. Notice that in real systems, the operators normally have a good estimation of the value of q_{max} regardless of the aggregation level of the demand, which validates the three expressions.

3. Analytical comparison

By looking at expressions (11), (12) y (13), we can observe that the only difference in the optimal frequency values appears in the term associated with the in-vehicle travel time within the square root because in-vehicle time cost is quadratic with the demand. Therefore, the relevant comparison involves

$$\frac{l}{2L}y^2, \quad (11a)$$

$$\frac{l_1}{L}y_1^2 + \frac{l_2}{L}y_2^2, \quad (12a)$$

$$\sum_{k=1}^{N}\sum_{l=k+1}^{N}\lambda_{kl}\sum_{i=k}^{l-1}\lambda_i^{1+} + \sum_{k=1}^{N}\sum_{l=1}^{k-1}\lambda_{kl}\sum_{i=l+1}^{k}\lambda_i^{2+}.. \quad (13a)$$

Note first that in systems where the stop time at stations is fixed ($\beta = 0$), the three formulae (Eqs. (11)–(13)) provide the same result. On the other hand, by writing the average journey time length as a function of the disaggregated quantities we obtain

$$l_1 = \frac{L}{y_1(N-1)}\sum_{k=1}^{N}\sum_{l=k+1}^{N}\lambda_{kl}(l-k),$$

$$l_2 = \frac{L}{y_2(N-1)}\sum_{k=1}^{N}\sum_{l=1}^{k-1}\lambda_{kl}(k-l),$$

$$l = \frac{L}{(y_1+y_2)(N-1)}\left[\sum_{k=1}^{N}\sum_{l=k+1}^{N}\lambda_{kl}(l-k) + \sum_{k=1}^{N}\sum_{l=1}^{k-1}\lambda_{kl}(k-l)\right].$$

First, we focus our analysis in models M1 and M2. Let us define D_i as the total distance traveled along direction i, that is, $D_i = l_iy_i$ that yields

$$D_1 = \frac{L}{(N-1)}\sum_{k=1}^{N}\sum_{l=k+1}^{N}\lambda_{kl}(l-k),$$

$$D_2 = \frac{L}{(N-1)}\sum_{k=1}^{N}\sum_{l=1}^{k-1}\lambda_{kl}(k-l).$$

Then (11a) and (12a) can be rewritten as follows:

$$\frac{l}{2L}y^2 = \frac{D_1+D_2}{2L}(y_1+y_2),$$

$$\frac{l_1}{L}y_1^2 + \frac{l_2}{L}y_2^2 = \frac{D_1}{L}y_1 + \frac{D_2}{L}y_2.$$

After some algebraic work, comparing these two expressions is equivalent to analyzing the sign of the expression $(y_1-y_2)(D_2-D_1)$.Then, expression (11) will be larger than (12) if either $(y_1 > y_2)\land(D_2 > D_1)$ or $(y_1 < y_2)\land(D_2 < D_1)$, which is equivalent to say that the traveled distance is the largest on the smallest demand direction of movement. On the other hand, if the largest demand direction matches the largest traveled distance (which is a very reasonable intuitive assumption), the most aggregated model (expression (11)) underestimates the optimal frequency compared with that obtained from the model that differentiates both directions behind expression (12).

Now, let us compare formulae (12a) and (13a), i.e. M2 and M3. By writing (12a) in a disaggregated way we have:

$$\frac{l_1}{L}y_1^2 + \frac{l_2}{L}y_2^2 = \frac{D_1}{L}y_1 + \frac{D_2}{L}y_2$$
$$= \left(\sum_{k=1}^{N}\lambda_k^{1+}\right)\left(\sum_{k=1}^{N}\sum_{l=k+1}^{N}\lambda_{kl}\frac{l-k}{N-1}\right)$$
$$+ \left(\sum_{k=1}^{N}\lambda_k^{2+}\right)\left(\sum_{k=1}^{N}\sum_{l=1}^{k-1}\lambda_{kl}\frac{k-l}{N-1}\right)$$

The previous expression must be compared against (13a). A priori, it seems not possible to perform any comparison between both expressions under a generic situation. In order to approach to the solution, let us to examine some interesting particular cases. First, let us examine the case of equal trip rates in each direction, that is

$$\lambda_{kl} = \begin{cases} \lambda_1 & \text{if } k < l \\ \lambda_2 & \text{if } k > l \end{cases}$$

In such a case, (12a) and (13a) yield the same result given by

$$\frac{N^2(N^2-1)}{12}(\lambda_1^2 + \lambda_2^2). \tag{14}$$

Moreover, expression (11a) becomes

$$\frac{N^2(N^2-1)}{24}(\lambda_1 + \lambda_2)^2. \tag{15}$$

Note that (15) is always lower or equal than (14). Besides, in this case the average length of trip is the same in both directions.

Let us now see the case in which the number of stations equals three ($N = 3$). In this case, (12a) and (13a) become, respectively (by simplicity, we analyze only direction 1):

$$(\lambda_{12} + \lambda_{13} + \lambda_{23})\left(\frac{\lambda_{12}}{2} + \lambda_{13} + \frac{\lambda_{23}}{2}\right), \tag{12b}$$

$$\lambda_{12}(\lambda_{12} + \lambda_{13}) + \lambda_{13}(\lambda_{12} + \lambda_{13} + \lambda_{23}) + \lambda_{23}^2. \tag{13b}$$

The comparison then is reduced to

$$2\lambda_{12}\lambda_{23} + \lambda_{13}\lambda_{23}, \tag{12c}$$

$$\lambda_{12}^2 + \lambda_{23}^2 + \lambda_{12}\lambda_{13}. \tag{13c}$$

Then, the relative value of the trip rates λ will determine the value of the optimal frequencies. Two particular cases:

(a) If $\lambda_{12} = \lambda_{23}$, (12c) is equivalent to (13c) and both optimal frequencies turn out to be the same.
(b) If $\lambda_{13} = \lambda_{23} \equiv \lambda_1$ and $\lambda_{23} \equiv \lambda_2$, then if $\lambda_1 > \lambda_2$, (12c) is larger than (13c) and consequently (12) is larger than (13); the other case is analogous.

Therefore, we cannot establish *a priori* a ranking among optimal frequencies obtained from the different models. That ranking mostly depends upon the value of the matrix cells. It seems that matrix heterogeneity increases the probability of getting different values for the optimal frequencies from the various proposed models, which suggests that the concentration of trips is a relevant issue when comparing the analytical recommendations obtained from the different aggregation level models. In the next section, we complement these analytical insights with the conclusions from some numerical examples.

4. Numerical comparison

In this section we conduct some numerical computations of the optimal design variables (frequency, vehicle and fleet size) obtained by applying the different demand aggregation models (M1, M2 and M3). We concentrate our analysis on two numerical cases. One is the study of a public transport corridor in Santiago, Chile, called Los Pajaritos, from where we have origin-destination demand matrix for the most demanded morning peak hour on a typical day of operation (MTT, 1998). Los Pajaritos is a corridor of 7 km., with 9 segments and 10 stations along each direction. The second example is the experiment proposed by Delle Site and Filippi (1998), where also a detailed station to station origin destination demand matrix is available. The matrices used in both examples were properly generated from real data of affluence at the level of stations. In both examples, the assumed parameters are those shown in Table 1 next.

Example 1. Los Pajaritos Corridor. The morning peak hour OD matrix and the associated load profile are shown in Table 2 and Fig. 2, respectively. Load imbalance between both directions of

Table 1
Summary of the assumed model parameters

Parameter	Value
N	10
Running time between station (min)	1
Distance between stations (km)	0.5
Subjective value of waiting time ($/h)	2700
Subjective value of travel time ($/h)	900
c_0 (CLP/h)	1800
c_1 (CLP/h-seat)	30
c_0' (CLP/km)	400
c_1' (CLP/km-seat)	1
Boarding time β (s/pax)	5
Safety factor η	0.9

The currency CLP is Chilean Pesos (US $ 1 ≈ CLP 500).

Table 2
OD matrix, Los Pajaritos corridor

From/To	1	2	3	4	5	6	7	8	9	10
1		600	189	165	64	44	342	605	726	395
2	3620		11	10	4	3	20	35	42	23
3	790	38		5	2	1	10	18	22	12
4	1585	75	82		0	0	2	4	5	3
5	281	13	14	14		2	13	24	29	16
6	186	9	10	9	8		13	22	27	15
7	264	13	14	13	12	9		12	14	8
8	2631	125	135	130	117	86	107		36	19
9	337	16	17	17	15	11	14	18		67
10	4425	211	228	218	197	144	180	232	200	

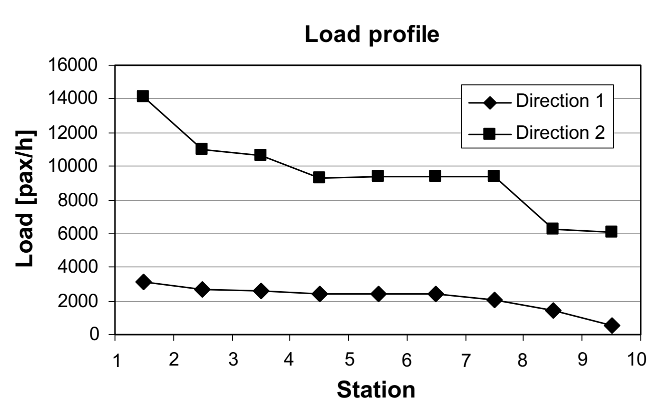

Fig. 2. Load profile, Los Pajaritos corridor.

Table 3
Optimal design variables, Los Pajaritos corridor

Model	Operation	C_w (CLP/min)	C_v (CLP/min)	C_o (CLP/min)	C_{tot} (CLP/min)	Frequency (veh/h)	Fleet size (veh)	Cap veh (pax/veh)
M1	Scheduled	2152	42.594	23.784	68.530	215	94	74
M2		1876	40.500	25.834	68.210	247	103	64
M3		1876	40.500	25.833	68.209	247	103	64
M1	Poisson	4022	41.525	24.755	70.302	230	98	69
M2		3561	39.775	26.701	70.037	260	107	61
M3		3561	39.775	26.700	70.036	260	107	61

movement is evident. Results are summarized in Table 3. Models M2 and M3 show similar results while M1 clearly underestimates not only the optimal frequency but also the optimal fleet size.

Example 2. Delle Site and Filippi (1998). The morning peak hour OD matrix and its load profile are shown in Table 4 and Fig. 3. In this case, demand imbalance between directions is concentrated in only a group of stations. Results for M1, M2 and M3 are summarized in Table 5, which shows that optimal frequency increases with the level of demand of information available, suggesting that lesser information will result in an underestimation of the optimal frequency. The fleet size remains unchanged. As the difference among models only happens in travel time, Table 6 shows the results of a sensitivity analysis increasing both travel time value Pv to 1800 \$/h and travel time between stations to 3 min (which could represent a scenary of traffic congestion), in order to amplify the weight of the in-vehicle time in the total cost function. The difference in optimal frequency becomes larger and the optimal fleet size is now different for the three models.

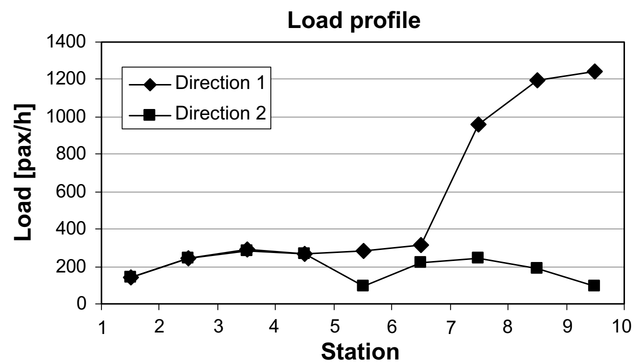

Fig. 3. Load profile, Delle Site and Filippi (1998) example.

Table 4
OD matrix Delle Site and Filippi (1998), example

From/to	1	2	3	4	5	6	7	8	9	10
1		29	14	64	4	3	3	1	1	25
2	14		15	70	4	4	3	1	1	27
3	5	5		49	3	3	2	1	0	19
4	8	7	4		18	15	12	4	3	111
5	74	63	35	0		5	4	1	1	37
6	4	4	2	0	0		5	2	1	50
7	1	1	0	0	0	3		20	16	636
8	8	6	3	0	0	26	5		7	262
9	16	14	7	0	0	58	11	0		77
10	13	11	6	0	0	47	9	0	10	

In both numerical examples, the optimal frequencies and fleet sizes obtained from M1 were systematically smaller than those obtained from M2 and M3; and these latter yield different results for the fleet size in the second example only. We developed several other numerical experiments reproducing the identified analytical conditions in Section 3, whereby (11) is higher than (12), or (12) is higher than (13), in which the results for the optimal frequencies did not differ significantly. Therefore, as a general rule, it seems that the better represented is transit demand, the larger the optimal frequency and the smaller the vehicle size. This makes even more dramatic Jansson's (1984) observation regarding the underestimation of optimal frequency and overestimation of vehicle size when users' costs are not taken into account, considering that he was using an M1 type model. Both the analytical developments and

Table 5
Optimal design variables Delle Site and Filippi (1998), example

Model	Operation	C_w (CLP/min)	C_v (CLP/min)	C_o (CLP/min)	C_{tot} (CLP/min)	Frequency (veh/h)	Fleet size (veh)	Cap veh (pax/veh)
M1	Scheduded	1538	2563	2910	7011	31	13	45
M2		1491	2536	2975	7002	32	13	44
M3		1425	2499	3074	6998	34	13	42
M1	Poisson	2344	2354	3560	8258	41	16	35
M2		2302	2341	3610	8253	42	16	34
M3		2240	2324	3688	8252	43	16	33

Table 6
Modified optimal design variables Delle Site and Filippi (1998), example

Model	Operation	C_w (CLP/min)	C_v (CLP/min)	C_o (CLP/min)	C_{tot} (CLP/min)	Frequency (veh/h)	Fleet size (veh)	Cap veh (pax/veh)
M1	Scheduled	1545	11.857	3869	17.271	31	31	45
M2		1471	11.773	4000	17.244	33	33	43
M3		1375	11.662	4194	17.231	35	35	40
M1	Poisson	2460	11.496	4541	18.497	39	38	36
M2		2384	11.453	4646	18.483	40	39	35
M3		2278	11.392	4806	18.476	42	41	34

the numerical examples suggest that the larger the cost associated with in-vehicle travel time, the more likely is that the various aggregated models predict different (smaller) values for the optimal frequencies.

5. Conclusions

In this paper we have examined the advantages of having detailed demand information when using classical public transport microeconomic models. We have established the optimal conditions for frequency on a public transport corridor with inelastic demand, in cases where the demand data is only available at an aggregated level (at the level of an entire line, or ridership per direction of movement) as well as cases in which it is feasible to obtain more detailed information on the demand structure, like origin-destination matrices at the level of bus stops or number of passenger who board and alight each bus at each stop.

We have developed two analyses, one which is purely analytical and another based on the result of applying the different aggregation level models to two examples in which real public transport data for peak periods are available. From the analytical part we clearly identified those terms in the optimal frequency expression that generates the differences among the models. However, conclusions with respect to the ranking among frequencies obtained in each model could only be established from the analysis of the empirical results, which show that the better represented is transit demand, the larger the optimal frequency and the smaller vehicle size. For a more intuitive explanation of the phenomenon, note that in the aggregate model one assumes an average load on the bus that applies throughout the route, along with an average amount of delay per stop to let people get on and off. But in fact, most people ride on the section of the route along which most people board, and therefore suffer more than would be expected from delay coming from their fellow passengers getting on and off. That amplification of dwell time delay – captured by M3 – pushes the optimal solution toward higher frequency and smaller buses, so that riders suffer less from the on and off delays caused by their fellow riders. This reinforces Jansson's (1984) and Jara-Díaz and Gschwender (2003) observation regarding the underestimation of optimal frequency and overestimation of vehicle size when users' costs are not taken into account properly. The larger the cost associated with in-vehicle travel time, the more likely is that the various aggregated models predict different (smaller) values for the optimal frequencies.

The analysis presented here motivates a deeper analysis of the analytical expressions, and also and more importantly, more numerical examples to validate our conclusions by discarding some possible cases that could be imposed numerically in the analytical developments, but not very likely to occur in reality. Overall, however, our results suggest that the underestimation of optimal frequency and overestimation of vehicle size when not accounting for users' costs fully is even more important than predicted by Jansson (1984).

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