Fuzzy Arithmetic for the DC Load Flow

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***Abstract*—The consideration of uncertainties in the future system operation is a key aspect in current planning methodologies. In this context, load flow studies based on probabilistic theory, fuzzy numbers, and Monte Carlo simulations have been proposed in the literature. This work analyzes in a novel way the application of fuzzy number arithmetic for the DC load flow problem. In this sense, fuzzy sets theory is reviewed and is showed that there are two alternative and valid procedures to perform a subtraction. A key aspect for the right selection of one of these procedures is the independency or dependency among the involved variables. Assuming input data as independent variables, this work is focused on analyzing the fuzzy subtraction between dependent state variables of the system, such as voltage phase angle. Accordingly, two new valid alternative methodologies are proposed and applied to case studies. Results are compared to previous related works showing the consistency of the proposed methodologies. Future research will be focused on the consideration of ohmic losses and AC network modeling in the field of expansion planning studies.**

***Index Terms*—Fuzzy sets, load flow analysis, power system planning, uncertainty.**

I. Introduction

LOAD flow calculation is one of the fundamental tools for power system operation analysis and planning, by allowing the simulation of the system steady state operation for a specific set of generation and load values. The most common approach to solve the load flow problem is the use of deterministic values for the input variables. For this situation, the obtained results are also deterministic and unique. This model is commonly known as the deterministic load flow (DLF). When the system representation involves the existence of uncertainty in its variables, its solution is usually faced by means of multiple DLF calculations. Examples of this situation occur when network data are located in a certain range or the exact value for loads/generations is unknown. The load inaccuracy can be originated in midterm projections due to inaccurate forecasts of prices, regional developments, and industry location. Uncertain generations can be produced by machines with variable energy source such as the wind and hydro power plants. Although the strategy of performing multiple cases is valid and commonly used in the industry, this task implies the use of a huge computational effort and its related man-hours, which are desirable to reduce.

The need for tools that incorporate uncertainty in some system variables has been widely recognized by researchers focused on system planning. In this way, nowadays, there are different alternatives to facing this problem, such as the probabilistic load flow (PLF), Monte Carlo simulations (MCS), and the fuzzy load flow (FLF) [1]–[10].

The first notion of a probabilistic load flow appears in the 1970s. Borkowska *et al.* [1], [2] proposed a simplified model whose main assumptions were: 1) the electric system can be represented by a linear approximation (DC) and 2) the active power demands are treated as random independent variables. The generation dispatch is solved defining a distribution function based on expert criteria. Under this approach, it is possible to adjust the variations of the total demand by considering the previous defined power injections in the system. Because of the consideration of loads as independent variables, the density functions of the load flow through the transmission lines can be determined by convolution. Afterwards, this method was extended to a network model on alternate current (AC) [3], [4].

The assumption of independence among the loads at different nodes, in general, is not realistic. Allan *et al.* [5] developed a theory allowing the incorporation of linear dependency among the loads. Da Silva [6] proposed a method that makes use of a linear dependency among the loads and the linearized load flow. It solves the problem combining Monte Carlo simulations and convolution. They also incorporated the operational policies of the company. Dopazo [7] used correlations in order to consider dependency among the loads. In this work, a Gaussian distribution for the lines flows and busbar voltages is assumed. Thus, the only requirement is the calculation of the associated variances and averages. Later, Allan [8] demonstrated that the assumption related to the assignment of a normal distribution for the output variables, even in the case where all the input variables have a normal distribution, is unrealistic. This is due to the nonlinear nature of the load flow equations. They also proposed a new algorithm for the PLF called boundary load flow (BLF). This approach allows obtaining estimations of the flows for multiple linearization points, overcoming the errors introduced by the models that use one point of linearization. The BLF allows finding ranges of values for the state variables and for the output variables, using the available ranges of values of the input variables. The method used for determining the variables intervals is heuristic. Dimistrovski [9] analyzed the BLF and proposed an algorithm with a more precise mathematical formulation, making possible to obtain the boundary values of the random variables intervals. The accuracy of results is improved, but the computational effort increases, too. As an alternative to

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the BLF, Leite da Silva [10] proposed the combined use of both Monte Carlo simulations and load flow equations with multiple linearizations. The algorithm applies as a criterion for the use of the load distribution functions to define the different linearization points. In this context, the Monte Carlo simulations, with intensive use of computational time, have been used as a validation tool for the different developed proposals [8], [10].

Another family of algorithms for load flow calculation under uncertainty is based on the fuzzy set theory. In this case, uncertainty is supposed to be originated by a vague or inaccurate concept, which is not the case of the probabilistic models highly related to the statistical behavior of a phenomenon. A clear advantage of this approach is the easy incorporation of expert knowledge to explain uncertainties. In [11], Miranda proposed modeling the loads and the active power injected by generators as fuzzy variables, with the purpose of representing a qualitative or linguistic originated uncertainty, which is not well represented by probabilistic models. Typical input fuzzy variables are: active and reactive power at the loads and the active power injected by generators. The output fuzzy variables correspond to: magnitude and phase angle of busbar voltages, reactive and active line flows, and the ohmic losses. Two models were proposed to solve the problem: DC load flow and AC load flow. The methodology applied in [11] is similar to the one used on the probabilistic models. As steady initial point, a DLF is solved considering the injected power in the busbars as the central value of the fuzzy numbers. The output fuzzy values are obtained applying linearization around the selected steady initial point. The *sum* operation with fuzzy numbers, not explicitly mentioned in [11], is shown in [12] and it would consist of max-min convolution. It is important to note that the subtraction of fuzzy numbers is not described in [11], which makes it difficult to reproduce the results obtained in the presented examples. Nevertheless, from [13], it would be deduced that the treatment used for the subtraction of fuzzy numbers is to sum the opposite (additive inverse) of the second number with the first one. The major drawback observed in the previously described methods is that final results are obtained by means of linearization around a selected point instead of applying on an explicit way fuzzy operations over the variables with uncertainty.

In [14], Saraiva *et al.* compared the results obtained in [11] with Monte Carlo simulations. Recently, Bijwe *et al.* [15] incorporated to the FLF models with consideration of the reactive power limits, voltage dependent loads, and uncertainty on the network parameters. They also integrate the concept of BLF [9] in the problem solution.

The existing correlation among the loads on a DC load flow is treated by Saraiva [16]. Afterwards, this proposal is improved by the inclusion of AC load flow, where the correlation among active loads, reactive loads, and active with reactive loads is allowed [17]. Recently, Matos and Gouveia [18] proposed to use a DC FLF to achieve transmission system adequacy. They also define new risk indexes denominated repression and severity indexes. These can be used in security studies or network expansion planning.

Additionally, specialized versions of FLF have been developed for distribution systems. In [19], a non-iterative load flow is solved for radial distribution networks using fuzzy sets and interval arithmetic. Bijwe [20] solves a fuzzy distribution power flow for weakly meshed systems. He combines a previous developed algorithm for deterministic load flow calculation in distribution networks with the BLF presented in [9].

A different alternative to solve the FLF is reported by the authors in [21], by using fuzzy number arithmetic in an explicit way, instead of linearization around a specific point. The use of α*-cuts* for the sum and subtraction operations is proposed, allowing more realistic results for the FLF. However, difficulties were faced in problems with meshed networks.

This research is focused on solving the FLF problem for any network topology using fuzzy number arithmetic on a direct way. Consequently, the calculation of a central value DLF and linearization around the selected steady initial point is not needed. An analysis of subtraction between fuzzy numbers is also presented, and two methodologies to solve the DC FLF considering dependency or independency among variables are proposed. For the development of these proposed models, a complete independency in the input data is assumed, so the major interest of this work is focused on analyzing the operation management with the dependent variables generated during the calculation process. Readers are assumed to have a basic knowledge of fuzzy sets.

This work is organized in five sections. Section II presents the fuzzy sets theory and the sum and subtraction operations. Section III shows the proposed methodology for solving the fuzzy load flow. In Section IV, case examples and its validation are presented. Section V presents the main conclusions and future work in this field.

II. Fuzzy Sets and Fuzzy Numbers

A. Basic Concepts

A fuzzy set A_d subset of a *universal set U* can be represented by an ordered pair composed by a generic element x and its membership value, that is,

$$A_\alpha = \{(x, \mu_{A_d}(x))\,|x \in U\}. \tag{1}$$

Additionally, an α*-cut* of a fuzzy set A_d is a classical set A_α that contains all the elements in U with a membership value in A_d greater or equal than α, that is,

$$A_\alpha = \{x \in U\,|\mu_{A_d}(x) \geq \alpha, \alpha \in [0,1]\}. \tag{2}$$

B. Fuzzy Number Arithmetic

A fuzzy number $\tilde{A}$ is a fuzzy subset in U that fulfills the following conditions [22]: 1) $\tilde{A}$ is normal, 2) $\tilde{A}$ is convex, 3) $\tilde{A}$ has a bounded support, and 4) every α*-cut* of $\tilde{A}$ is a closed interval in U.

1) Fuzzy Numbers Representation Through α*-Cuts:* A special type of fuzzy number, frequently used in practice, is the one with triangular shape. In (3), the membership function for a fuzzy number $\tilde{A}$ is presented:

$$\mu_A(x) = \mu_A(x; a, b, c) = \begin{cases} \frac{(x-a)}{(b-a)}, & \text{if } a \leq x \leq b \\ \frac{(c-x)}{(c-b)}, & \text{if } b \leq x \leq c \\ 0, & \text{if } x > c \vee \text{x} < \text{a}. \end{cases} \tag{3}$$

The α-*cuts* representation in a triangular shaped fuzzy number is achieved building α dependent functions for both left and right edges of the triangle. In this way, the α-*cuts* for the fuzzy number $\tilde{A}$ are defined by

$$A_\alpha = \left[a_\alpha^-, a_\alpha^+\right] \tag{4}$$

where

$$a_\alpha^- = (b-a)\alpha + a \tag{5}$$
$$a_\alpha^+ = (b-c)\alpha + c. \tag{6}$$

2) Sum and Subtraction of Fuzzy Numbers: Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers and $A_\alpha = [a_\alpha^-, a_\alpha^+]$, $B_\alpha = [b_\alpha^-, b_\alpha^+]$ its respective α-*cuts*. Then, the sum of $\tilde{A}$ with $\tilde{B}$, $\tilde{A}+\tilde{B}$, is a fuzzy number with α-cuts defined by

$$C_\alpha = (A+B)_\alpha = \left[a_\alpha^- + b_\alpha^-, a_\alpha^+ + b_\alpha^+\right] = \left[C_\alpha^-, C_\alpha^+\right]. \tag{7}$$

Since a fuzzy number is defined by its α-*cuts*, the fuzzy number $\tilde{C} = \tilde{A} + \tilde{B}$ is well defined in accordance with (7). If two fuzzy numbers with membership functions $\mu_A(x) = \mu_A(x; a_1, b_1, c_1)$ and $\mu_B(x) = \mu_B(x; a_2, b_2, c_2)$ are considered, in general, the membership function for the *sum* is defined by

$$\mu_{A+B}(x) = \mu_{A+B}(x; a_1+a_2, b_1+b_2, c_1+c_2). \tag{8}$$

In this case, the resulting support is $c_1 - a_1 + c_2 - a_2$, which corresponds to the *sum* of each number support.

On the other hand, for the subtraction of $\tilde{A}$ with $\tilde{B}$, $\tilde{A} - \tilde{B}$ there are two procedures, later described in this section. The respective α-cuts for the subtraction are defined in (9) and (10) [22], [23]:

$$(A-B)_\alpha = \left[\min\left(a_\alpha^- - b_\alpha^-, a_\alpha^+ - b_\alpha^+\right), \max\left(a_\alpha^- - b_\alpha^-, a_\alpha^+ - b_\alpha^+\right)\right] \tag{9}$$
$$(A-B)_\alpha = \left[a_\alpha^- - b_\alpha^+, a_\alpha^+ - b_\alpha^-\right]. \tag{10}$$

Taking (9) and executing necessary algebraic operations, a simpler formula is obtained for the membership function of the subtraction in (11) at the bottom of the page.

For this case, the resulting support is $(c_1 - a_1) - (c_2 - a_2)$, which corresponds to subtract the support of each fuzzy number.

Likewise, for (10), it is also possible to obtain a simpler general expression:

$$\mu_{A-B}(x) = \mu_{A-B}(x; a_1 - c_2, b_1 - b_2, c_1 - a_2). \tag{12}$$

However, in this case, the resulting support is $(c_1 - a_1) + (c_2 - a_2)$, which corresponds to *sum* the support of each fuzzy number.

Similarly, an interesting problem consists of determining the opposite (additive inverse) of a fuzzy number. In order to obtain this result, it is supposed that the fuzzy number $\tilde{A}$ has the following membership function: $\mu_A(x) = \mu_A(x; 0; 0; 0)$. Using (9), the fuzzy number $\tilde{A} - \tilde{B}$ will be given by

$$(A-B)_\alpha = \left[\min\left(-b_\alpha^-, -b_\alpha^+\right), \max\left(-b_\alpha^-, -b_\alpha^+\right)\right] \tag{13}$$
$$-B_\alpha = (-1)\cdot B_\alpha = \left[-b_\alpha^+, -b_\alpha^-\right]. \tag{14}$$

Hence, the membership function for the opposite of a fuzzy number B is established by

$$\mu_{(-1)B}(x) = \mu_{(-1)B}(x; -c_2, -b_2, -a_2). \tag{15}$$

Otherwise, if the fuzzy number $\tilde{A}$ is added to the opposite of $\tilde{B}$ defined in (15), it is also possible to find a generic formula for calculating the subtraction between two fuzzy numbers. In this case, the membership function takes the following structure:

$$\mu_{A+(-1)B}(x) = \mu_{A+(-1)B}(x; a_1 - c_2, b_1 - b_2, c_1 - a_2). \tag{16}$$

This result is similar to the one obtained in (12). Usually, when systems of equations are solved, it is common to carry out additions of numbers with opposed of other numbers. In these situations, it is important to remember that the result will be executed according to (12) or (16).

The operations defined in (8), (11), and (12) are applied to triangular, trapezoidal fuzzy numbers, symmetrical or not, without difficulty. However, while subtractions are performed with (11), it may happen that central values of the fuzzy number are located out of the support; in these cases, it is necessary to correct these values and assign them the maximum or minimum value of the interval.

Next, two examples showing the application of (11) and (12) are described. The first one consists of subtracting a fuzzy number with its opposite. Initially, the operation is made using (12); later on, the subtraction is carried out using (11). The obtained results are shown in Fig. 1. The used values correspond to those defined in [21]. In Fig. 1, it is also observed that the opposite of a fuzzy number is its mirror image with respect to the ordinate axis. Moreover, the sum of a fuzzy number with its opposite is a fuzzy number with a nonzero support. This result has a central value in zero, but its support is equal to double the original number. This would indicate that the opposite according to (16) is not symmetrical respect to the "+" operation for two fuzzy numbers. This means that fuzzy number zero (0,0,0) cannot be obtained by (16). On the contrary, if (11) is applied, the result is $\mu_{B-B}(x) = \mu_{B-B}(x; 0; 0; 0)$. This means that when the explicit definition for the subtraction is used, the

$$\mu_{A-B}(x) = \begin{cases} \mu_{A-B}(x; a_1 - a_2, b_1 - b_2, c_1 - c_2), & \text{if } a_1 - a_2 \leq c_1 - c_2 \\ \mu_{A-B}(x; c_1 - c_2, b_1 - b_2, a_1 - a_2), & \text{if } a_1 - a_2 \geq c_1 - c_2 \end{cases} \tag{11}$$

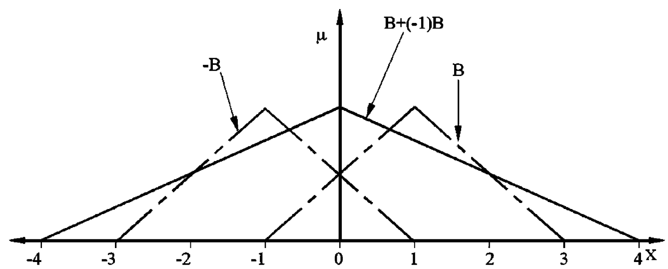

Fig. 1. Opposite of a fuzzy number and the sum between the fuzzy number with its opposite.

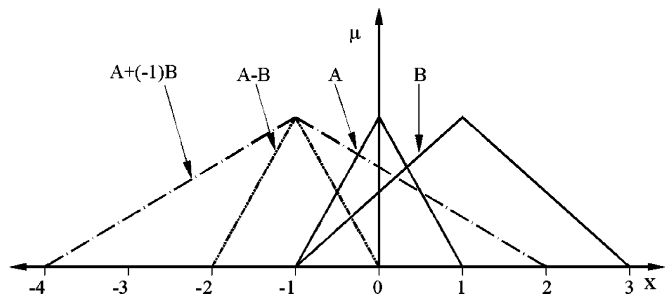

Fig. 2. Operations with fuzzy numbers.

zero fuzzy number can be obtained, which is the reason why the operation according to (11) is symmetrical for the sum.

The second example consists of subtracting any two fuzzy numbers. The used values are those presented in [21]. Fig. 2 shows the graphs for the subtraction operations determined with (11) and the subtraction obtained by (16). It is clearly appreciated that the results are different. This divergence on the results produces suspicion regarding which formulation, (11) or (16), is the more adequate for performing a fuzzy subtraction.

In general, relation (12) is of common use on the calculation of a subtraction between fuzzy numbers [13], [24]. From another point of view, the use of (11) or (12) for the subtraction operation depends on the origin of the uncertainties involved in the operation. Equation (11) seems to be consistent with the subtraction of two fuzzy numbers highly dependent. Instead (12) is more appropriate for subtraction of fully independent fuzzy numbers.

C. Independent Variables

The behavior of independent variables is well known in the probability theory. For instance, let X_1 and X_2 be two independent variables whose parameters are $\left(\mu_1, \sigma_1^2\right)$ and $\left(\mu_2, \sigma_2^2\right)$, respectively. The new random variable obtained from the sum or subtraction of these random variables, $X_1 \pm X_2$ distributes with parameters $\left(\mu_1 \pm \mu_2, \sigma_1^2 + \sigma_2^2\right)$ [25]. From the previous concept, an analogous treatment can be applied to fuzzy numbers. In this sense, when two fuzzy independent numbers are added or subtracted, the resulting support should be the sum of the supports of each fuzzy number. This result is obtained when (12) is applied, which is consistent with the probability theory. However, for the case of the subtraction, if (11) is applied, the resulting support is lower than the one obtained from (12). Therefore, it is not recommended to use (11) for the subtraction of two fuzzy independent numbers.

D. Dependent Variables

Usually, the numbers obtained from operations performed over independent variables are used in further calculations.

Let $\Im_N$ be the set of the fuzzy numbers. Considering an element $A \in \Im_N$, and the numbers $\alpha, \beta \in \Re^+$, a sum operation can be defined as

$$\tilde{C} = \alpha \cdot \tilde{A} + \beta \cdot \tilde{A}. \tag{17}$$

Applying (9), it is easy to demonstrate what is defined by

$$\tilde{C} = (\alpha + \beta) \cdot \tilde{A}. \tag{18}$$

In this way, it is concluded that addition operation behaves in accordance with the theoretical framework, even when fuzzy numbers are dependent variables.

Consider now the subtraction between dependent fuzzy numbers described in the following:

$$\tilde{D} = \alpha \cdot \tilde{A} - \beta \cdot \tilde{A}. \tag{19}$$

Applying (9), it can be demonstrated that $\tilde{D}$ is given by

$$\tilde{D} = \begin{cases} (\alpha - \beta) \cdot \tilde{A}, & \text{if } \alpha \geq \beta \\ |\alpha - \beta| \cdot (-1)(\tilde{A}), & \text{if } \alpha < \beta. \end{cases} \tag{20}$$

From (20), it is deduced that $(-1) \cdot \tilde{A}$ operates in the same way as (15), that is, as the opposite of $\tilde{A}$. Additionally, if $\alpha \geq \beta$, the resulting support given by (20) is smaller than the one obtained when the variables are independent. This result is consistent with the probabilities theory, which states that the variance of the sum or subtraction of two dependent random variables is $\sigma_1^2 + 2\text{cov}(X_1, X_2) + \sigma_2^2$ [25]. If the correlation among the variables is negative, the covariance $\text{cov}(X_1, X_2)$ is negative, which implies a smaller variance in the result. Therefore, it can be concluded that subtraction performed with (11) delivers adequate results for dependent fuzzy numbers.

For the case of subtraction operation by means of (12), the obtained result is

$$\mu_D = \mu_D(x; \alpha \cdot a - \beta \cdot c, [\alpha - \beta] \cdot b, \alpha \cdot c - \beta \cdot a). \tag{21}$$

Equation (21) shows that (12) does not operate adequately with dependent variables. Given the previous analysis, it is not recommended to use (12) for subtractions of fuzzy dependent numbers.

Considering the first example presented in section B, it is natural to think that the opposite of $\tilde{B}$ is completely dependent of $\tilde{B}$. Hence, the subtraction between these numbers should be the fuzzy number $\tilde{0}$. In this sense, the more adequate equation to perform this calculation must be (11). For the case of the second example, using (11) implies supposing that fuzzy numbers $\tilde{A}$ and $\tilde{B}$ are dependant, while the application of (12) requires to know that both numbers are independent.

Operations defined in section B are applied without difficulty to triangular, trapezoidal fuzzy numbers, symmetrical or not. However, when subtractions are performed applying (11), it is possible that central values of fuzzy numbers are located out of its respective support; in these cases, this is adjusted by an assignation of the interval limit value according to each case.

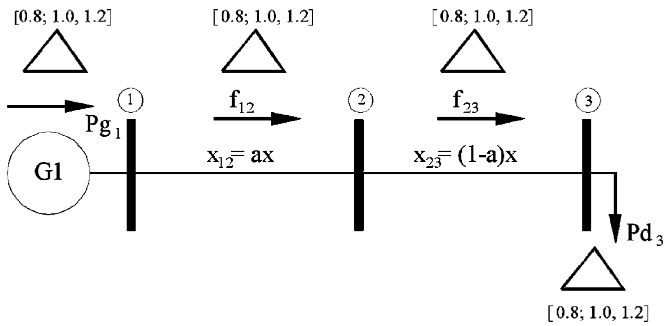

Fig. 3. Three-node system.

III. Fuzzy Load Flow

A. Theoretical Analysis of the Fuzzy Load Flow

In Section II, two different formulas for the calculation of fuzzy numbers subtraction were presented. Likewise, it was demonstrated that (11) should be used when the numbers to subtract are dependent. Moreover, (12) should be used when the numbers are independent.

A way of working with dependent variables using (11) is operating with the state variables of the system. In this sense, it is known that voltage angles of busbars depend on the net power injected in the nodes. In this way, some calculation regarding voltage angles implies operating with dependent variables. Traditionally, a way of facing this problem consists of keeping equations as a function of independent variables. However, the result may be quite complicated in some cases, as it is a case of determining system losses. In this section, a small system is analyzed, and the feasibility of applying (11) in the calculation of line flows using the busbar voltage angles is shown. Then, a three-node radial system is considered, input data in p.u., MVA base of 100, which is shown in Fig. 3. The membership function for load at busbar 3 is

$$\mu(Pd_3) = \mu(Pd_3; p_a, p_b, p_c). \quad (22)$$

Transmission line series reactances are stated as a function of parameter "a", with $a \in [0,1]$. Considering a DC model, the theoretical solution for the fuzzy load flow should be consistent with

$$\tilde{P}g_1 = \tilde{f}_{12} = \tilde{f}_{23} = \tilde{P}d_3. \quad (23)$$

This would imply that the membership function of line flows must have the same values and shape that the load has.

Considering the solution using the conventional fuzzy operations, with busbar 1 as reference, the set of algebraic equations that solve the problem is

$$\tilde{\theta}_2 = x_{12}\left(-\tilde{P}d_3\right) = ax\left(-\tilde{P}d_3\right) \quad (24)$$

$$\tilde{\theta}_3 = (x_{12}+x_{23})\left(-\tilde{P}d_3\right) = x\left(-\tilde{P}d_3\right) \quad (25)$$

$$\tilde{f}_{12} = -\frac{\tilde{\theta}_2}{x_{12}} = \tilde{P}d_3 \quad (26)$$

$$\tilde{f}_{23} = \frac{\tilde{\theta}_2 - \tilde{\theta}_3}{x_{23}} = \frac{ax\left(-\tilde{P}d_3\right)}{(1-a)x} - \frac{x\left(-\tilde{P}d_3\right)}{(1-a)x}. \quad (27)$$

In (24)–(26), the subtraction operation is not required. Therefore, those equations are not subjected to possible calculation errors. However, in (27), the subtraction operation is required. Using (12) and applying it in (27), the membership function for the flow f_{23} is obtained:

$$\mu(f_{23}) = \mu\left(f_{23}; \frac{p_a - ap_c}{1-a}, p_b, \frac{p_c - ap_a}{1-a}\right). \quad (28)$$

It is observed that (28) does not match with the theoretical result given in (23). Basically, the problem is originated when (12) is applied for the calculation of f_{23}. This is because (12) is used for the subtraction $\theta_2 - \theta_3$, where θ_2 and θ_3 are *dependent variables* ($\theta_2 = a \cdot \theta_3$). On the other hand, when applying (11) to determine (28), the following is obtained:

$$\mu(f_{23}) = \mu(f_{23}; p_a, p_b, p_c). \quad (29)$$

In this case, the result matches with the expected values in theoretical and physical terms. In this way, for this case, (11) delivers adequate results dealing with *dependent state variables*.

B. Fuzzy DC Load Flow Using Arithmetical Operations

Considering the previous results, two methodologies to solve the FLF are analyzed. In the first one, line flows are calculated applying (11). This means, considering dependent state variables. In the second one, line flows are calculated from the purchased and injected power, as a function of independent variables. Once final results for each methodology are obtained, a comparison between both methodologies is performed looking to determine the applicability of (11) in FLF studies. In both cases, it is supposed that the fuzzy input data are independent.

1) Methodology A. DC Load Flow With Dependent Variables: This methodology corresponds to the classical formulation of linear load flow or DC load flow and consists of the following steps.

a) Determine the fuzzy active power injection $\tilde{P}$ at each node:

$$\tilde{P} = \tilde{P}_G - \tilde{P}_D \quad (30)$$

where $\tilde{P}_G$ corresponds to the vector of fuzzy active power generated and $\tilde{P}_D$ is the fuzzy load vector. A fully independency is assumed between the source of uncertainty for the considered power injections and loads. This calculation is performed by means of (12).

b) Calculate the fuzzy voltage phase angle at each node using the DC load flow approximation:

$$\tilde{\theta} = B^{-1} \cdot \tilde{P} = X \cdot \tilde{P}. \quad (31)$$

c) Calculate the fuzzy active power flows in the system branches (lines and transformers):

$$\tilde{f}_{pq} = \frac{(\tilde{\theta}_p - \tilde{\theta}_q)}{x_{pq}} \quad (32)$$

where x_{pq} is the series reactance of the branch $p - q$.

In (32), subtraction is calculated with (11), because angles are dependent variables.

2) Methodology B. DC Load Flow With Independent Variables: The procedure steps are as follows.

a) Calculate fuzzy voltage angles at the busbars:

$$\tilde{\theta} = B^{-1} \cdot \tilde{P} = X \cdot \tilde{P}. \tag{33}$$

In (33), (12) is used because each $\tilde{P}_i$ is supposed to be independent of $\tilde{P}_j \; \forall \, i \neq j$.

b) Fuzzy active power flows in system branches are calculated as

$$\tilde{f} = Y \cdot A^t \cdot \tilde{\theta} = Y \cdot A^t \cdot X \cdot \tilde{P} = T \cdot \tilde{P} \tag{34}$$

where Y is the primitive nodal admittance matrix and A the branch-bus incidence matrix. Similarly to (33), (12) is used to calculate (34).

C. Dependence Between Input Data

When dependencies among input data are detected, the problem may be faced defining a dependency matrix D, which relates the dependency that could exist among input data. Then

$$\tilde{P} = \tilde{P}_G - \tilde{P}_D = D \cdot \tilde{P}_b. \tag{35}$$

Dimension of D is $(n-1) \times m$, where m is the number of independent input data and P_b is the vector of independent power (bases). For instance, a system is supposed with four buses where the generated power in bus 2 depends on 20% of the demand at bus 2, and the consumed power at bus 3 depends on 50% of the demand at bus 4. Thus, the dependency matrix is

$$D = \begin{bmatrix} 0.2-1 & 0 \\ 0 & -0.5 \\ 0 & -1 \end{bmatrix}; \quad \tilde{P}_b = \begin{bmatrix} P_{d2} \\ P_{d4} \end{bmatrix}. \tag{36}$$

Later, it is required to modify (33) and (34) to get the problem in terms of the independent variables; this means

$$\tilde{\theta} = X \cdot D \cdot \tilde{P}_b = \hat{X} \cdot \tilde{P}_b \tag{37}$$

$$\tilde{f} = T \cdot D \cdot \tilde{P}_b = \hat{T} \cdot \tilde{P}_b. \tag{38}$$

This mechanism may be applied, without distinction, to both methodologies A and B proposed in the previous section. However, (38) is not necessary for methodology A.

Alternatively to this procedure, there is a second way to solve the problem. In this alternative, it is not required to use the dependency matrix D. Equation (11) can be directly applied to operate with dependent variables. However, it is required to keep identified the variables that are dependent of the independent ones. The procedure is explained through an example. Picking up the example stated on the previous paragraph and supposing the following membership functions for power injections/purchases: $\mu(P_{g2}; 1, 1.25, 1.5)$, $\mu(P_{d2}; 5, 6.25, 7.5)$, $\mu(P_{d4}; -7, -6, -5)$, $\mu(P_{d3}; -7/2, -6/2, -5/2)$ the operations required to calculate the phase angle in a certain busbar are given by the following expression:

$$\begin{aligned} \theta &= 0.08 \cdot (P_{g2} - P_{d2}) + 0.06 \cdot P_{d3} + 0.04 \cdot P_{d4} \\ &= (P_{d2}; -0.48, -0.40, -0.32) \\ &\quad + \mu(P_{d4}; -0.56, -0.48, -0.40) \\ &= \mu(f; -1.04, -0.88, -0.72). \end{aligned} \tag{39}$$

In this case, the subtraction between $\tilde{P}_{g2}$ and $\tilde{P}_{d2}$ is performed with (11) given that they are dependent. Finally, the obtained results in these operations are added. The interesting thing about this last method (method A) is that it is not required to know the dependency level to perform the operation. It is just enough to know if there exists dependency among variables. For the load flow calculation, it would be required to know the variables that are supposed to be dependent in order to operate them in a separate way in the calculation process.

D. Considerations About the Slack Busbar

The consideration of the slack busbar in the FLF involves the analysis of two main aspects. The first aspect requires to state how it is going to treat the phase angle of the slack busbar. The second aspect deals with the treatment of the resulting power injections in the slack busbar. For the phase angle, the authors have chosen to define that there is not uncertainty associated to this state variable; while for the case of power, the choice of declaring the uncertainty in all the system buses, including the slack busbar, has been taken. In this sense, the slack busbar must absorb (balance) the uncertainty produced by all the independent power injections (purchases) in the system. For the case of real systems with capacity limits, the input data should be adjusted in such a way that, if the slack generator does not have enough capacity, the lacking uncertain power is distributed among generators that have a capacity surplus. The mechanism is similar to the one proposed by Dimitrovski and Tomsovic in [26].

IV. Numerical Validation and Example Cases

In this section, results obtained by computational simulations are presented. The simulations were performed applying both models for networks with radial and meshed topologies.

A. Radial System of Five Nodes

This system consists in a radial network of five busbars. Fuzzy loads are modeled at busbars 2, 3, 4, and 5 with a unique membership function given by $\mu(P; 0.4, 0.5, 0.6)$ p.u. and a series reactance of 0.1 p.u. for each line with an MVA base of 100. Fuzzy generations are modeled in busbar 4 $\mu(P_{g4}; 0.55, 0.65, 0.75)$ p.u. and busbar 5 with $\mu(P_{g5}; 0.35, 0.40, 0.45)$ p.u. Once the proposed model was applied, Fig. 4 shows the network with the obtained results.

Table I shows the obtained results for both methods A and B; in this case, results are the same for both methods. The obtained results for the proposed methods correspond to the theoretical result, which can be inferred by visual inspection in Fig. 4. These results allow concluding that solutions found by FLF are adjusted to what is expected for this type of problem. In this way, it is concluded that for radial networks, both proposed methodologies deliver results consistent with the physical properties of the problem.

B. Meshed System of Five Nodes

The system shown in Fig. 5 consists in a meshed network of five nodes [28]. A fuzzy load is supposed for busbars 2, 4, and 5 with membership functions: $\mu(P_{d2}; 1.15, 1.175, 1.2)$, $\mu(P_{d4}; 0.8, 0.825, 0.85)$, and $\mu(P_{d5}; 1.1, 1.025, 1.05)$ and a

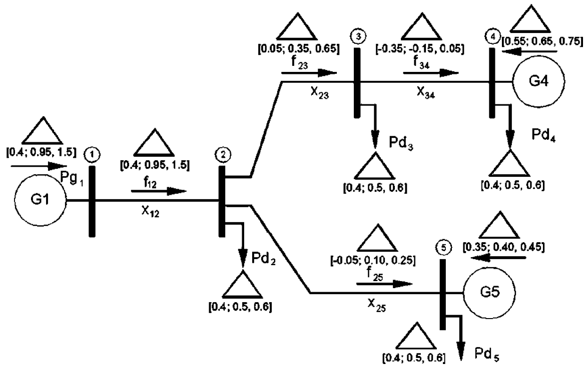

Fig. 4. Five-node radial network with results from the proposed methodology.

TABLE I
POWER FLOWS FOR METHOD A AND B IN THE RADIAL SYSTEM

Line	Node 1	Node 2	F(a)	F(b)	F(c)	Support
f_{12}	1	2	0,400	0,950	1,500	1,100
f_{23}	2	3	0,050	0,350	0,650	0,600
f_{34}	3	4	-0,350	-0,150	0,050	0,400
f_{25}	2	5	-0,050	0,100	0,250	0,300

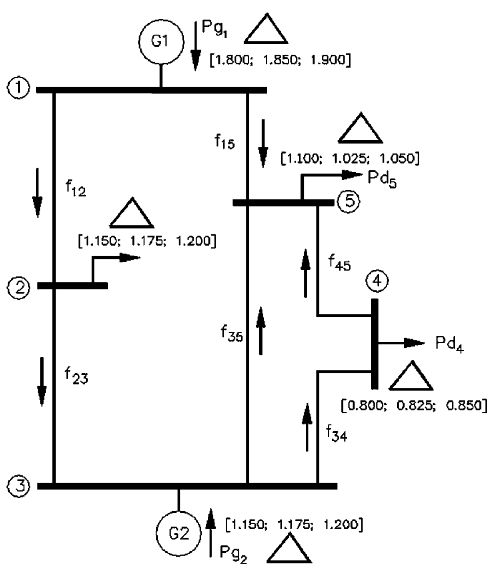

Fig. 5. Five-node meshed network.

TABLE II
LINE FLOWS FOR METHOD A IN THE MESHED SYSTEM

Line	Node 1	Node 2	F(a)	F(b)	F(c)	Support	ARMS [%]
f_{12}	1	2	0,855	0,900	0,945	0,091	0,491
f_{15}	1	5	0,895	0,950	1,005	0,109	0,625
f_{32}	3	2	0,255	0,275	0,295	0,041	0,062
f_{34}	3	4	0,568	0,575	0,582	0,014	0,072
f_{35}	3	5	0,314	0,325	0,336	0,023	0,005
f_{54}	5	4	0,232	0,250	0,268	0,036	0,068

TABLE III
LINE FLOWS FOR METHOD B IN THE MESHED SYSTEM

Line	Node 1	Node 2	F(a)	F(b)	F(c)	Support	ARMS [%]
f_{12}	1	2	0,855	0,900	0,945	0,091	0,491
f_{15}	1	5	0,895	0,950	1,005	0,109	0,625
f_{32}	3	2	0,241	0,275	0,309	0,068	0,392
f_{34}	3	4	0,555	0,575	0,595	0,041	0,161
f_{35}	3	5	0,305	0,325	0,345	0,041	0,204
f_{54}	5	4	0,227	0,250	0,273	0,045	0,182

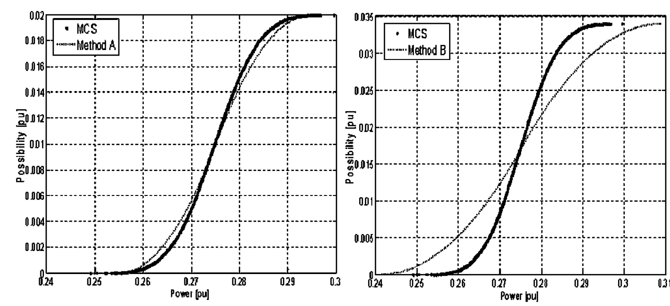

Fig. 6. Comparison of cumulative possibility functions for line flow 2–3. (Left) Method A with MCS. (Right) Method B with MCS.

TABLE IV
LINE FLOWS WITH MONTE CARLO SIMULATION IN THE MESHED SYSTEM

Line	Node 1	Node 2	Mean	Std Dev
f_{12}	1	2	0,900	0,010
f_{15}	1	5	0,950	0,012
f_{32}	3	2	0,275	0,007
f_{34}	3	4	0,575	0,005
f_{35}	3	5	0,325	0,005
f_{54}	5	4	0,250	0,006

fuzzy generation at busbars 1 and 3 with a membership function $\mu(P_{g3}; 1.150, 1.175, 1.200)$. The series reactance for each line has a value of 0.1 p.u. with an MVA base of 100.

On Tables II and III, results of the line flows for methodologies A and B are detailed, verifying that even if central values match up, the extremes of fuzzy numbers are different. It is also observed that supports of the fuzzy numbers associated to the line flows are often lower in the case of method A. In other words, method A provides a more conservative value of uncertainty.

Additionally, MCS were carried out for this case in order to compare the obtained results with methods A and B, as it is usually performed in other studies such as in [8] and [10]. In this case from the obtained results shown in Table IV, it is observed that mean values obtained with MCS are close to the ones of methods A and B. In order to verify the consistency of the proposed methods with MCS, the average root mean square (ARMS) error [29] for each line was calculated. In this context, comparing method A with MCS, the value of the ARMS average for all the lines is 0.22% while for method B is 0.34%. In Fig. 6, cumulative possibility function of line flow 2–3 obtained by MCS are compared with the ones of the possibility triangular function. The similarity of both possibility functions (MCS with methods A and B) is observed, providing an additional consistency proof for the proposed methods.

C. IEEE Reliability Test System of 24 Nodes

With the aim of comparing the obtained results by the proposed algorithms and the results obtained with another fuzzy method proposed in the international literature, a test system of real size, IEEE RTS 24, is analyzed. This system is composed of 24 buses and 38 branches. The used data were obtained from

TABLE V
LINE FLOWS FOR METHOD A IN THE IEEE RTS SYSTEM

Line	Node 1	Node 2	F(a)	F(b)	F(c)	F(4)	Support
11	7	8	428,8	463,4	486,6	521,3	92,5
23	14	16	-441,6	-426,4	-416,3	-401,1	40,5
34	19	20	-621,9	-456,6	-346,4	-181,1	440,8

TABLE VI
LINE FLOWS FOR METHOD B IN THE IEEE RTS SYSTEM

Line	Node 1	Node 2	F(a)	F(b)	F(c)	F(d)	Support
11	7	8	428,8	463,4	486,6	521,3	92,5
23	14	16	-511,6	-443,9	-398,8	-331,1	180,5
34	19	20	-623,3	-456,9	-346,1	-179,7	443,5

TABLE VII
LINE FLOWS FOR METHOD [18] IN THE IEEE RTS SYSTEM

Line	Node 1	Node 2	F(a)	F(b)	F(c)	F(d)	Support
11	7	8	428,8	463,4	486,6	521,3	92,5
23	14	16	-511,3	-443,8	-398,8	-331,3	180,0
34	19	20	-504,7	-427,2	-375,4	-297,8	206,9

[18], and bus 23 was elected as a slack bus. Tables V–VII show the obtained results.

In this system, the possibility functions used are trapezoidal and the terms $F(a_1)$, $F(a_2)$, $F(a_3)$, and $F(a_4)$ correspond to the cuts $\alpha = \{0, 1, 1, 0\}$, respectively. The results shown in Tables V and VI have the same behavior observed for the meshed case of section B. This means that method A presents a more conservative and bounded uncertainty. Table VII shows the results obtained in [18]; these are similar to the ones presented in Table VI (method B), except for line 19–20. The difference between these methods is attributed to the fact that the model presented by Matos [18] considers the constraint $\sum P_i = 0$, which restricts uncertainty. If this constraint is not considered, the results are similar to the ones obtained with method B.

D. Chilean Central Interconnected System

An application of the proposed methodologies in larger real power systems is also provided. The analysis is performed on the Central Interconnected System (CIS) of Chile. The CIS covers an area of 2000 km in length, and supplies a peak load of 6850 MW in 2008. The major load is in Santiago, Chile's capital, located in the central part of the system. The annual peak load growth rate, during the last ten years, has been approximately 5.0% on average. For modeling purposes, the system comprises 101 busbars, 110 lines, and 27 transformers. The uncertainty assigned to each load or power generation located at the busbars is 2.5%, and trapezoidal functions were considered for this case. It is supposed that there is independency among load input data, and also among load and power injections. All network parameters are available in [30].Tables VIII and IX show the results obtained by methods A and B for three lines of the system.

The results present a similar tendency to what was observed in the systems of sections B and C; this means, the uncertainty obtained for method A is lower than the one obtained in method B. In these tables, line 18 represents the power line Carrera Pinto-Diego de Almagro located at the northern part of the system, which is characterized by power flows in the southern direction because of the small amount of demand. On the other hand, lines 49 and 96 represent the power lines Quillota-San Luis and Cerro Navia-Los Almendros, respectively, located near by the major load center of the system (Santiago). It can observed that for line 18, the uncertainty is lower than the other ones due to its almost unique dependency on power generations, while lines 49 and 96 show high influence of the variation on the major load center.

TABLE VIII
LINE FLOWS FOR METHOD A IN THE CIS

Line	Node 1	Node 2	F(a)	F(b)	F(c)	F(d)	Support
18	3	2	-3,231	-3,147	-2,979	-2,895	0,337
49	8	44	-8,361	-8,210	-7,906	-7,755	0,607
96	11	57	-4,209	-4,064	-3,775	-3,630	0,579

TABLE IX
LINE FLOWS FOR METHOD B IN THE CIS

Line	Node 1	Node 2	F(a)	F(b)	F(c)	F(d)	Support
18	3	2	-3,231	-3,147	-2,979	-2,895	0,337
49	8	44	-8,376	-8,217	-7,899	-7,740	0,636
96	11	57	-4,289	-4,104	-3,734	-3,549	0,740

Finally, for this real case, comparing the difference of the supports for all the lines between both methods, an average difference of 20.8% of method A with respect to method B is obtained. In other words, uncertainties achieved by method B are systematically greater than method A.

V. CONCLUSIONS

In this work, a novel theoretical analysis of arithmetical fuzzy operations for the solution of the fuzzy DC load flow problem is presented. It has been demonstrated that the main conceptual difficulty when fuzzy operations are performed is the subtraction operation.

Assuming input data as independent variables, this work is focused on analyzing the fuzzy subtraction between dependent state variables of the system, such as voltage phase angle. This property is strongly related with the dependency of the fuzzy numbers involved during the DC load flow algorithm operations. Accordingly, two valid alternative methodologies are proposed to solve the fuzzy DC load flow and applied to case studies based on radial and meshed networks.

Obtained results are compared to previous related works [18] showing the consistency of both proposed methodologies. The case studies allow concluding for radial systems both methods deliver the same results. On the other hand, for meshed networks, uncertainties achieved by method B are systematically greater than method A. Additionally, method A shows a more similar behavior compared to the general MCS approach, and it is not required to know the dependency level among variables to perform the operation.

Finally, a novel discussion of concepts about two valid alternative ways that deal with the arithmetic operation subtraction between dependent and independent fuzzy numbers is presented.

Future research will be focused on the application of the proposed methodologies in the case of the DC load flow with ohmic losses and the AC load flow, in the context of expansion planning studies. Besides, more concluding arguments will be studied in order to recommend one of the proposed methodologies (A or B).

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