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Mathematical models for optimizing production chain planning in salmon farming

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Abstract

The salmon farming production chain is structured in four consecutive phases: freshwater, seawater, plant processing, and distribution and marketing. The phases interact in a pull manner, freshwater stocks fish to meet seawater's demand, seawater produces to meet plant processing biomass demand, and the processing plant produces to satisfy consumers' demand. Freshwater planning decisions are in regard to which freshwater center the fish should be located depending on the state of development of the fish. The goal is to satisfy seawater's demand while minimizing costs. In the seawater phase, the fish are first placed in seawater centers, and then sent to the processing plant as they approach suitable harvest conditions. The goal of seawater is to maximize harvested biomass while satisfying processing plant's demand. This paper presents two mixed-integer linear programming models—one for the freshwater phase and another for the seawater phase. These models are designed in such a way that the production planning is well integrated and more efficient and incorporates the requirements of the farm operator's freshwater and seawater units (biological, economic, and health-related constraints) ensuring that production in both phases is better coordinated. The development of the two models was based on the farming operations of one of the main producer farms in Chile. Preliminary evaluations of the models indicate that they not only succeed in enforcing constraints that are difficult to be met by manual planning but also led to more effective results in terms of the objectives set out.

Keywords: production chain; salmon farming; planning; integer programming

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1. Introduction

Industrial scale production of salmon has been one of Chile's most important economic activities, the main source of employment in the southern regions of the country, and its primary food product to export. In just over 10 years, the industry's foreign shipments grew from 135,000 tonnes valued at US$ 538 million in 1996 to 397,000 tonnes worth a total of US$ 2.241 billion in 2007. Until the crisis of 2008, salmon farming ranked third in national significance after the mining and forestry sectors. At the international level, Chile had become the second largest producer of salmon and the largest producer of trout, accounting for 40% of total world salmon production. The two main export destinations were Japan and the United States (SalmonChile, 2009).

Since mid-2008, however, Chilean fish farmers have struggled to cope with a major outbreak of salmon disease that had a serious blow to the industry's economy. The spread of the infectious salmon anemia (ISA) virus, first detected in July 2007, incurred huge losses to operators in southern Chile, and with the world economic crisis compounding its difficulties the recovery of this sector is still ongoing. The production in 2010 just reached 255,000 tonnes due to early harvesting in the previous years and high mortality rates. For the employment impact, some 20,000 direct jobs were lost in the country's 10th and 11th regions by 2010. Currently, the recovery of the industry seems to be in its last stage. During the past year, the industry has shown huge improvements by achieving the production levels of 2008.

The salmon farming production chain consists of four consecutive phases: freshwater, seawater, plant processing, and distribution and marketing. In this work, we focus on the first two phases that account for the total cultivation time. We assume that production requirements from the last two phases are exogenous to our model. The freshwater phase covers the first stages of salmon growth in rivers and lakes, whereas the seawater phase consists of the fish-fattening period before harvest. The freshwater and seawater phases are coordinated through the quantity of fish transferred from the former to the latter. The production planning and the coordination of the cultivation phases include decisions regarding when and how much to stock fish, update fish stocks according to mortality and growth, and how to manage the transfers so that the exogenous processing plant demand is met while the various operating and fish weight constraints at each stage of the planning horizon are also satisfied. Although the phases are coordinated, the freshwater goal is to satisfy the seawater production requirements at a minimum cost and the seawater goal is to maximize the amount of biomass harvested.

To produce on an industrial scale, operators must be capable of attaining high levels of coordination and planning that will allow them to exploit resources sustainably while complying with prevailing production specifications and the standards of international markets where most of the industry's output is traded. Such planning is highly complex given the long production horizon of approximately three to four years and the enormous number of interrelated decisions. For instance, decisions about cultivation must take into account that fish of different species and state of development need to be cultivated separately to guarantee optimal growth. There are also requirements in the fish weight to allow transfers and harvest, as well as a maximum cultivation time for each phase. Anticipated harvests of fish were responsible for the loss of 360 tonnes of biomass in 2007, or equivalently a loss of $1.4 million dollars in revenue. To complicate matters further, the salmon disease crisis has forced the adoption of strong measures for controlling and improving the situation over time. These include fallowing fish farms, coordinating management of stocking and harvesting, strict biosecurity procedures for fish transport, new products, vaccination, and medications. Faced with these new considerations, a mathematical model that could facilitate the coordination and planning of salmon farming activities would afford the industry a much needed support for efficient decision making. For an illustration of some quantifiable benefits of the models, see Section 4.4, item 4 (Total biomass harvest and satisfaction of demand).

In this paper, we propose two planning models, one for the freshwater and another for the seawater phase. The two optimization formulations, both designed before the onset of the salmon disease crisis, are coordinated with each other through the quantity of fish transferred from the freshwater to seawater phase. To the best of our knowledge, this is the first attempt to connect the planning of the two cultivation phases while taking into account the goal and cultivation requirements of each individual phase. The development of the two models was based on the farming operations of Multiexport Foods S.A., the third largest producer in Chile as of 2007 with production facilities located mainly in the country's 9th, 10th, and 11th regions. Regarding the state of advancement of the modeling process, only partial implementation of the seawater model and partial validation of the freshwater model were completed before the collapse of the industry. We present numerical results for both the models that illustrate their power in terms of accomplishing their individual objectives and reducing significantly the time and resources required for the planning process compared to the current manual approach.

2. The state of the art: production planning models in aquaculture

Aquaculture production planning has been studied by various authors, who have adopted a range of approaches to the subject employing different modeling and solution tools. The production problem displays many of the complexities typical of renewable resources in which renewal periods are fixed, and uncertainty in growth and mortality must be taken into account in the planning process (see Bjørndal et al., 2012).

Two complementary visions of modeling fish culture coexist in the literature. The biological approach focuses on issues such as population average growth and mortality by Brett (1979) and Mangel and Stamps (2001), cultivation conditions by Bjørnsson (1994), and feeding and diseases by Cho and Bureau (1998). The operational approach, on the other hand, attempts to capture considerations relevant to operating conditions, including work force and capital requirements, facility location, and stocking, harvest, and optimal cultivation time decisions, incorporating in its specifications the economic variables that affect profitability. The merging of the two visions has produced planning models that facilitate evaluation of alternative strategies and future scenarios to support integrated decision making. One of the first works in this area is Bjørndal (1988) that develops an analytical continuous-time bioeconomic model to calculate the optimal harvesting time for farmed Atlantic salmon under different cost scenarios.

To capture the uncertainty in the fish growth, a Markov chain approach is used in which future uncertain growth depends solely on the latest growth measurement. Through this approach, growth rates are computed and considered as deterministic inputs for the planning models. As for mortality, this has not received as much attention as the growth modeling and has been modeled as a constant rate depending on the state of development of the fish. Motivated by this gap in the literature, we developed a stochastic version of the freshwater model in a second paper to be submitted. Sparre (1976) was the first to model the structured growth of fish using the Markov approach, formulating and comparing linear and dynamic programming models for optimizing the harvest. More recently, Leung and Shang (1989) and Leung et al. (1990) investigated the application of this approach with dynamic programming, capturing stochasticity into the growth process of a shrimp farming operation to generate decisions regarding cultivation times, inventories, and harvesting times.

Other studies, such as those conducted by Jensson and Gunn (2001), combine the Markov approach to model growth with linear programming to create a long-term production planning model in which integer variables are then introduced to obtain a detailed short-term model that determines the harvesting schedule for the seawater phase. Forsberg (1996) utilizes integer linear programming in a multiperiod model for long-term production planning that features discrete weight-structured growth classes and capacity and operating constraints. Yu and Leung (2005) develop a network flow formulation for planning a shrimp operation with multiple production centers and stocking–harvesting cycles in the planning horizon subject to biological and economic constraints.

In studies of harvesting techniques, Forsberg (1999) compares the economic benefits of two harvest strategies for an Atlantic salmon population, showing that weight grading of fish and harvesting only those that have reached market weight yield higher profits than harvesting on the basis of average weight. Yu et al. (2006) also analyze the economic benefits of partial harvests.

As for studies involving actual fish farm applications, Gasca-Leyva and Veliov (2008) examine the influence of population heterogeneity on cultivation time, validating their conclusions at a tilapia farming operation in the Yucatan, Mexico, where they found that introducing heterogeneity leads to longer cultivation times. Yu et al. (2006) also present a real-world application of the above-described techniques. Using the data from a commercial shrimp operation in Hawaii with multiple production centers and evaluation periods, they demonstrate that partial harvesting produces greater profits than a batch harvest strategy. Cisternas et al. (2012) introduced a mathematical model to plan the maintenance and change of cage nets in seawater centers with the goal of minimizing resource utilization. This model was actually implemented in the same company where we carried out our work. Begen and Puterman (2003) developed an optimization tool based on the operations of a Vancouver-based company. The model supports decisions related to fish supply allocation to various end products in order to maximize profit.

What all the above models have in common is that they deal with a single link in the production chain in which individuals enter a production unit, remain there until attaining the required growth characteristics, and are then transferred to the following unit, regardless of the harvesting method. In the present study, we approach the planning problem as a multipart process in which the fish move through the various phases or stages as they attain the required growth characteristics to be transferred from one phase or stage to the next, until finally they meet the requirements for delivery to the processing plant. We also assume multiple centers (units) at each phase or stage and the possibility of multiple stocking–harvesting cycles over the planning horizon.

3. Salmon production planning

3.1. Brief description of the production chain

As already noted, the salmon farming production chain is structured around four phases. The first two phases—freshwater and seawater—are set in production centers located at river, lake, or sea sites where the fish are cultivated in ponds (hatcheries) or cages. Each of them is characterized by a particular state in the development of the fish and responds to a corresponding set of needs so that healthy growth of the population is ensured. The two final phases consist of postharvest procedures including slaughter, final product processing, and distribution to the final consumer (Cluster del Salmon, 2008). The complete production period is relatively lengthy, extending over three to four years from selection of reproducers to final product, with little flexibility to adapt to unforeseen events.

Next, we give a brief description of the four phases of the salmon industry production chain in the order of their occurrence.

1. *Freshwater phase (8–13 months)*: This phase accounts for 11% of the total production costs. Freshwater begins with the selection of reproducers at the seawater centers to obtain salmon eggs. These are then sent to the hatchery where they are fertilized and incubated. After the fish hatched out, they begin the alevin state and are cultivated for two to four months until they weigh enough (6–15 g) to be relocated to a river or lake for smoltification. At this stage, they begin the morphological process of adapting to a salt water environment, and on reaching the required weight (40–125 g) over a period of five to six months they are transferred to an estuary center. There, the smoltification process is completed over one to three months, producing salmon smolts of 100–200 g that are ready for transfer to the seawater phase. See Section 5 for a detailed description of this phase.
2. *Seawater phase (13–22 months)*: Once introduced into the seawater centers, the smolts are cultivated for 13–22 months until they reach market size and are ready for harvest. At that point, the live fish are delivered to holding facilities where they are stored for a few days before being slaughtered, and perhaps also gutted at harvest centers for further processing. This phase accounts for approximately 62% of the total production cost. See Section 4 for a detailed description of this phase.
3. *Plant processing phase*: The fish arrive gutted or ungutted at the processing plant, where they are cleaned and calibrated. They are then filleted and either cut into portions or further processed into other value-added products, such as cold or hot smoked salmon. The final products are bagged and packed in boxes, weighed and labeled, and stored in refrigeration facilities to await distribution. The cost of this phase is 11% of the total production cost.
4. *Distribution and marketing phase*: This phase entails the logistic process of shipping out the final products for marketing to wholesalers and retailers, both at home and abroad. The cost of this phase accounts for 16% of the total production cost. Shipments are transported by surface or air depending on the destination. Since most of Chile's salmon exports consist of frozen products that are sent to more than 30 countries around the world, maintaining the cold chain through to the buyer's location is crucial for guaranteeing product quality.

3.2. Current company planning

The salmon farming production chain is a pull system. At Multiexport Foods, the freshwater and seawater phases are planned independently, the only linkage being the latter's fish requirement that must be supplied by the former, and the two planning processes are carried out manually by the company's planning teams.

The processing plant's fish biomass requirements are planned in accordance with demand for the final product and communicated to the seawater phase. The stocking and harvesting operations for a 48-month horizon are planned there as a function of the sea-site capacities, using mortality and growth models to project biomass over the cultivation period. Due to the nature of the salmon market, it is assumed that all harvested biomass could be sold in the form of a final salmon product. Therefore, the main objective of seawater phase is to maximize harvested biomass subject to satisfy processing plant's biomass requirements for each month.

The seawater planning results are then sent to the freshwater phase, which plans its stocking operations and transfers between stages on the basis of available capacity and time estimates for each stage. The requirements thus derived are then projected backwards, adjusting for historical mortality, to arrive at approximate dates for hatchery stocking. These dates will determine the types of fish to be stocked given that this decision must take into account the time of year.

A notable aspect of the production chain is that the planning objectives of the two phases do not coincide. Although the seawater phase attempts to maximize harvested biomass and supply the biomass requirement of the processing plant, the freshwater phase aims at minimizing costs while satisfying the seawater requirements. It is very difficult to coordinate the different stages in both phases, so that the right amount of salmon is obtained at the different stages when needed. A model approach will provide significantly better solutions than the current manual method, as will be shown in the results obtained. All these aspects prompted the authors to develop the planning problem strategy adopted here for designing two separate optimization models, one for each phase of the production process, which communicate through the requirements of fish from the seawater to freshwater phase.

4. Seawater planning model

In this section, we present the various elements that are the basis of the development of our optimization model for planning the seawater phase. Since the production chain is a pull system, we begin our analysis with this phase—the more downstream of the two phases at the cultivation end of the chain whose operations are "pulled" by the processing plant's biomass requirement.

4.1. Description of the seawater phase

Seawater production planning at Multiexport includes scheduling the fish stocking and harvesting dates for each of the producer's seawater centers over a five-year planning horizon. The objective is to maximize harvested biomass over the horizon and meet the processing plant's biomass demand while satisfying a range of biological, economic, and environmental constraints.

At the time of this study, Multiexport operated 46 seawater centers equipped with an aggregated total of 994 sea cages. The locations were scattered along an extended geographic area in southern Chile and are therefore subject to a range of different water temperature and sunlight conditions. Consequently, growth and mortality rates varied considerably from site to site. The centers can be classified by size as "single," for those with just one module (14 cages), or "double," if they contain two modules (28 cages). Planning decisions at the company are made on a monthly basis in two stages. First, dates are proposed for the harvesting of existing stocks at the various centers and for future stocking operations and their respective harvests. Next, the centers where future stocking will take place are defined. For each month in the planning horizon, any given center may be in one of three possible states: empty (available for stocking), fallow (required rest period between consecutive stocking operations during which no fish cultivation may take place), or stocked (at least one cage contains fish). The planners then draw up a feasible Gantt chart for the entire planning horizon that reflects the states of all seawater centers. Further details on the operations of Multiexport's seawater division as of 2008 are given below.

Seawater stocks: For each cage in centers with existing salmon stocks at the beginning of the planning horizon, the number of fish and the average weight are known. Growth and mortality support models were available for projecting future fish numbers and growth (see Section 4.2). The processing plant's biomass demand is also known for each month in the planning horizon. These data are used to project the specific harvest dates for each cage, subject to constraints regarding minimum and maximum harvest weights, health considerations (fallow periods), use of centers, and satisfaction of demand in each period.

Other operating considerations include the restriction that each cage should be harvested only once so that no cage has only a small number of salmon, as this would complicate the management of the process. In addition, given the high running costs of a center in operation even when it contains few fish, the various cages at a given site must be harvested at contiguous intervals. Ideally, harvest periods will be completed within three to five months. Also, since the fish weight that generates the best return on the raw resource is 4600 g, the harvest of individuals outside a valid weight range, and around 4600 g is to be avoided. Another consideration is that salmon growth and feed conversion rates decline over time, eventually reaching a point where maintaining fish at a center becomes relatively costly. This is compounded by the fact that as the individuals reach sexual maturation, the quality of the flesh deteriorates and the risk of death increases. In light of this, an ideal grow-out period range has been established.

Finally, for economic and plant production feasibility reasons, best practices indicate that the total biomass harvest should not vary significantly from one month to the next.

Future stocking operations: Decisions regarding future stocking operations include when and how much salmon should be stocked into seawater centers to meet the production plan over the planning horizon. They are closely related to the seawater harvest decisions since the latter determines fish availability at the centers and whether or not the availability will satisfy demand.

Once the stocking requirements are defined, they are communicated to the freshwater planners, who are responsible for ensuring that the requested quantities are available for transfer to the seawater centers on the indicated dates. The decisions are made at an aggregate level by module or center, given that it is impossible to know with certainty the initial weight and quantity of the fish in each of the cages stocked in each period. Once the fish are unloaded at a sea site, the site manager distributes them among the cages according to their stage of development, ensuring that individuals of different sizes are not placed together, thus promoting growth and reducing mortality.

These decisions must also take into account the permitted ranges for the stocking–harvesting cycle, fish weight, and grow-out periods already defined for the existing stocks. For the first months of any planning horizon, some stocking operations are required to be previously determined (fixed stockings). Both the dates and quantities of the stocking operations will be practically impossible to modify. The source of this inflexibility is either existing commitments with the freshwater phase, biological considerations, or external supply agreements with other operators for eggs, alevins, or smolts. The planning horizon is therefore divided into four consecutive groups of months: the first group, consisting of the initial months, in which some stockings are fixed and cannot be altered; the second group in which stocking dates can be delayed no more than a single period; the third group, in which the dates can be advanced or delayed by a maximum of one period; and the fourth group for which there are no fixed stocking dates.

Use of seawater centers: Once the future stocking dates have been projected, the planners must decide which centers will be stocked. Since growth and mortality depend on temperature and sunlight, the decision must consider the specific characteristics offered by each center's geographical location. "Candidate sites" are those that are available—that is, that are active, have unoccupied modules, and are not in a fallow period between stocking–harvesting cycles. During the periods that can last up to six months, no stocking should be planned to permit recovery of seafloor conditions.

Finally, if the stocking necessary to satisfy demand is greater than the capacity of the active and available centers, the activation of new centers must be considered. The model allows us to evaluate the tradeoff of obtaining more biomass with the high cost (around 2 million dollars) needed in investment of new centers.

4.2. Planning support models

Good planning of the production chain requires support models that can estimate how much biomass should be extracted in each planning horizon period. The ones used by Multiexport were proposed by the company's planning personnel based on expert knowledge. Monthly sampling is conducted to check these models and keep them adjusted to the actual performance of the stocked cages.

Growth model: This model is used to project the future weight of the fish in a seawater center. This projection is based on the initial weight of the fish, monthly average water temperature, and amount of sunlight (which is assumed to depend on the day of the year). Various models for estimating salmon growth can be found in the literature. Multiexport planners chose the formulation they considered to be best suited to the company's operations.

The most important parameter of the model is the "specific growth rate," $SGR(P, \tau, D)$, which is a function depending on the weight P of the fish, the temperature of the water τ, and day of the year D. The function $SGR(P, \tau, D)$ is not easy to compute, but an estimation of its values was given by the company planners as an input to the model. The model also has some adjustment parameters that allow for experimentation with different scenarios: $AJP_j \in [0, 1]$, the cage adjustment parameter and $AJP_c \in [0, 1]$, the center adjustment parameter.

Assuming that cage j of center c is not harvested in month t, the average weight of the cage, $PJAU^{jc}_{t+1}$, on the first day of month $t+1$ can be computed using the following recursive formula:

$$PJAU^{jc}_{t+1} = PJAU^{jc}_{t}\left[1 + AJP_j \cdot AJP_c \cdot \frac{SGR\left(PJAU^{jc}_{t}, \tau_t, D_t\right)}{100}\right]^{|t|},$$

where $|t|$ is the number of days in month t, $t+1$ is the month following month t, D_t is the first day of month t, and τ_t is the average monthly temperature.

On the other hand, if cage j of center c is harvested on day d of month t, a slightly different formula is used. The average harvest weight in $t+1$, denoted $PCOS_{t+1}$, can be computed as

$$PCOS^{jc}_{t+1} = FC \cdot PJAU^{jc}_{t}\left[1 + AJP_c \frac{SGR\left(PJAU^{jc}_{t}, \tau_t, D_t\right)}{100}\right]^{d},$$

where FC is the harvest weight factor, that is, the fraction of the salmon weight that is effectively processed in the plant after deduction of preprocessing losses. Planners at Multiexport have set $FC = 0.93$.

Mortality model: The mortality model estimates the number of salmon in a cage at a future date given its initial number of stocked individuals. Regarding the parameters, the expected mortality percentage M^{jc}_t is known for each cage j at each center c in each month t. In a similar fashion to the growth model, the mortality model has two adjustment parameters: the cage adjustment factor $AJN_j \in [-1, 1]$ and the center adjustment factor $AJN_c \in [-1, 1]$.

Hence, if we denote as N^{jc}_t the number of salmon in month t, the number of salmon in month N^{jc}_{t+1} is given by

$$N^{jc}_{t+1} = N^{jc}_t\left[1 - M^{jc}_t \cdot (1 + AJN_j) \cdot (1 + AJN_c)\right].$$

4.3. Optimization model

We now describe the planning model of the seawater problem, which was formulated using mixed-integer linear programming. The assumptions and considerations included in the model are examined in this subsection.

Planning horizon: In line with Multiexport's planning methodology, the model assumes monthly decision periods and a planning horizon of five years.

Stocking and harvesting of a center: The model assumes that all cages at a single center are stocked in the same month. The duration of the stocking period is therefore one month. The harvest period is assumed to last at most three months, during which all cages at the center must be harvested.

Valid harvest ranges: The permitted average weight range is defined as $P_{\min} = 3200$ g to $P_{\max} = 5200$ g and the grow-out period range is specified by the values m, M ($m < M$). Thus, if a center is stocked in t and the harvest period begins in s, then $s \in [t+m, t+M+2]$. In practice, the values used are $m = 13$, $M = 20$.

Data for projecting weight and mortality: The number of salmon and their average weight are known for the cages at centers with existing stocks, but the states of initial cage for future stocking operations states are not known, and an approximation must therefore be used. Thus, it is assumed

Table 1
Index sets for the variables and parameters of the model

Index sets	
$\mathcal{T} = \{1, 2, 3, \ldots, T\}$	Periods T (months) in planning horizon
$SFR \subseteq \mathcal{T}$	Unmodifiable fixed future stocking periods
$SFS \subseteq \mathcal{T}$	Fixed future stocking periods that can be delayed one month
$SFY \subseteq \mathcal{T}$	Fixed future stocking periods that can be advanced or delayed one month
$\mathcal{C}$	Seawater grow-out centers
J_c	For each $c \in \mathcal{C}$, its set of cages
$\mathcal{C}_f$	Seawater grow-out centers with salmon stocks at the beginning of the planning horizon
$\mathcal{C}_e$	Seawater grow-out centers with no salmon stocks at the beginning of the planning horizon
$J_c^f \subseteq J_c$	For $c \in \mathcal{C}_f$, the set of cages in c with salmon stocks at the beginning of the planning horizon
J_c^{inf}	Cages in c with initial stocking weight P_{inf}
J_c^{med}	Cages in c with initial stocking weight P_{med}
J_c^{sup}	Cages in c with initial stocking weight P_{sup}
$\mathcal{F}_C$	Pairs of valid grow-out period range for centers, that is, $(t, s \in \mathcal{F}_C)$ if and only if $t, s \in \mathcal{T}$ and $t + m \leq s \leq t + M$
$\mathcal{F}_J$	Triplets of valid grow-out period range for cages, that is, $(t, s, t') \in \mathcal{F}_J$ if and only if $t, s, t' \in \mathcal{T}$, $(t, t') \in \mathcal{F}_C$ and $t + m \leq s \leq t' + 2$. The interpretation of this set is that if a center is stocked in month t and its harvesting cycle starts in t', then all its cages must be harvested in $\{t', t' + 1, t' + 2\}$

that all the cages are stocked with a quantity N_0 of individuals and the average initial weight per cage for any center follows a distribution validated by company planners, which is as follows: 25% of cages have an average initial weight of $P_{inf} = 50$ g; 50% have an average initial weight of $P_{med} = 100$ g; and the remaining 25% have an average initial weight of $P_{sup} = 150$ g. This distribution is based on the fact that salmon weight is normally distributed.

Next, we describe the decisions, the constraints, and the objective function of the seawater optimization model. See Tables 1 and 2 for a description of the index sets and parameters of the model. Definitions of the decision variables can be found in Table 3.

The constraints of the model are as follows.

1. *Requirements for centers with salmon stocks at the beginning of the planning horizon.*
At any center c not empty at the beginning of the planning horizon, the harvest period must begin in a month within the maximum seawater grow-out period:

$$\sum_{t \leq M} X_{ct} = 1 \qquad \forall\, c \in \mathcal{C}_f.$$

Any cage i with salmon at the start of the planning horizon must be completely harvested in a month within the maximum seawater grow-out period:

$$\sum_{t \leq M+2} x_{cit} = 1 \qquad \forall\, c \in \mathcal{C}_f \; \forall\, i \in J_c^f.$$

Table 2
List of all the parameters of the model

Parameters	
SFR_t	Number of modules already planned for stocking in month t, where $t \in SFR$
SFS_t	Number of modules already planned for stocking in month t, where $t \in SFS$
SFY_t	Number of modules already planned for stocking in month t, where $t \in SFY$
d_c	Fallow period in months of center c. If the harvest at c is completed in $t \in \mathcal{T}$, the following stocking–harvesting cycle cannot start until $t + d_c$ where $c \in \mathcal{C}$
Tam_c	Number of modules at center c. We assume that $Tam_c \in \{1, 2\}$ where $c \in \mathcal{C}$, meaning that there are only two classes of centers. This is in accord with current practice at the company
P_t	Projected demand for month t in kilograms, where $t \in \mathcal{T}$
Cap_t	Maximum processing plant capacity in kilograms, where $t \in \mathcal{T}$
η	Share of harvest for a given month that can be retained for delivery to the plant the following month
δ	Minimum proportion of demand that must be satisfied each month
ε_{sup}	Maximum permitted percentage increase in biomass delivered to the production plant in a given month relative to the preceding month
ε_{inf}	Maximum permitted percentage decrease in biomass delivered to the production plant in a given month relative to the preceding month
β_{cit}	Amount of biomass from cage i that will be processed at the plant in month t, where the cage was harvested in the same month, with $c \in \mathcal{C}_f$, $i \in J_c^f$ and $t \in \mathcal{T}$
β_{cits}	Amount of biomass from cage i that will be processed at the plant in month t, where the cage was stocked in t and harvested in s, with $c \in \mathcal{C}$, $i \in J_c$ and $t, s \in \mathcal{T}$
α_p	Penalty coefficient in objective function for falling short of demand

Table 3
List of all the variables of the model

Binary variables for centers in $\mathcal{C}_f$	
X_{ct}	1 if harvest of center $c \in \mathcal{C}_f$ starts in month t, 0 otherwise
x_{cit}	1 if cage $i \in J_c^f$ is harvested in month t, 0 otherwise
Binary variables for future stockings	
W_{cts}	1 if at center c all cages are stocked in month t and the harvest period starts in month s, 0 otherwise
$w_{citst'}$	1 if cage $i \in J_c$ is stocked in month t and harvested in month s and harvest of center c begins in month t', 0 otherwise
Binary variables for fixed stockings	
S_{cts}	1 if center $c \in \mathcal{C}$ is stocked in month $s \in \{t, t+1\}$ to satisfy a fixed stocking requirement for month $t \in SFS$, 0 otherwise
A_{cts}	1 if c is stocked in $s \in \{t-1, t, t+1\}$ to satisfy a fixed stocking requirement for month $t \in SFY$, 0 otherwise
Continuous variables for biomass estimation	
y_t	Quantity of biomass delivered in month t
h_t	Quantity of biomass retained in month t for delivery to the processing plant in month $t + 1$
HG_t	Margin representing the difference between quantity of biomass demanded and quantity actually delivered in month t

The chosen cage harvest dates must be related to the center harvest cycle dates:

$$X_{ct} \leq x_{cit} + x_{cit+1} + x_{cit+2} \qquad \forall\, c \in \mathcal{C}_f,\ \forall\, t \in \mathcal{T} : t \leq M,\ \forall\, i \in J_c^f,$$

$$x_{cit} \leq X_{ct-2} + X_{ct-1} + X_{ct} \qquad \forall\, c \in \mathcal{C}_f,\ \forall\, t \in \mathcal{T} : t \leq M + 2,\ \forall\, i \in J_c^f.$$

2. *Requirements for stocking–harvesting cycles.*
   A maximum of three stocking–harvesting cycles are permitted for each center *c* over the planning horizon:

$$\sum_{(t,s)\in\mathcal{F}} W_{cts} \leq 3 \qquad \forall\, c \in \mathcal{C}.$$

The cages at each center must also satisfy the preceding condition:

$$\sum_{(t,s,s')\in\mathcal{F}_J} w_{citss'} \leq \sum_{(t,s)\in\mathcal{F}_C} W_{cts} \qquad \forall\, c \in \mathcal{C}\ \forall i \in J_c.$$

A center $c \in \mathcal{C}_f$ cannot start a new stocking–harvesting cycle until the harvest period and any required fallow periods have been completed:

$$X_{ct} + \sum_{(t',s)\in\mathcal{F}_C} W_{ct's} \leq 1 \qquad \forall\, c \in \mathcal{C}_f,\ \forall\, t, t' \in \mathcal{T},\ \text{such that } t' \leq t + 2 + d_c.$$

Stocking–harvesting cycles at any center must not overlap and established fallow periods must be observed:

$$\sum_{\substack{(t',s') \in \mathcal{F}_C \\ t \in I(t', s'+2+d_c)}} W_{ct's'} \leq 1 \quad \forall\, c \in \mathcal{C},\ \forall\, t \in \mathcal{T} : 1 + (m + 2 + d_c) \leq t \leq T - (m + 2 + d_c).$$

The cages in any center may be harvested only in the months constituting a harvest period established for that center:

$$W_{cts} = w_{citss} + w_{cits+1s} + w_{cits+2s} \qquad \forall\, c \in \mathcal{C},\ \forall\, i \in J_c,\ \forall\, (t, s) \in \mathcal{F}_C.$$

3. *Maximum quantity retained from one month to the next.*
   Part of the biomass harvested in any month can be retained for delivery to the processing plant the following month. However, this quantity h_t must not exceed a certain percentage η of the

harvest:

$$
\begin{aligned}
&h_t \le \eta \left(\sum_{c\in\mathcal{C}_f} \sum_{i\in J_c^f} \beta_{cit} x_{cit} \right) && \forall t \in \mathcal{T} : t < m,\\
&h_t \le \eta \left(\sum_{c\in\mathcal{C}_f} \sum_{i\in J_c^f} \beta_{cit} x_{cit} + \sum_{c\in\mathcal{C}} \sum_{i\in J_c} \sum_{(s,t,t')\in\mathcal{F}_J} \beta_{cist} w_{cistt'} \right) && \forall\, t \in \mathcal{T} : m \le t \le M+2,\\
&h_t \le \eta \left(\sum_{c\in\mathcal{C}} \sum_{i\in J_c} \sum_{(s,t,t')\in\mathcal{F}_J} \beta_{cist} w_{cistt'} \right) && \forall\, t \in \mathcal{T} : t > M+2.
\end{aligned}
$$

4. *Calculation of the quantity of biomass delivered to the processing plant.*
   The quantity of biomass y_t delivered to the processing plant is calculated as a function of each month's cage harvest:

$$
\begin{aligned}
&y_1 = \sum_{c\in\mathcal{C}_f} \sum_{i\in J_c^f} \beta_{ci1} x_{ci1} - h_1, &&\\
&y_t = \sum_{c\in\mathcal{C}_f} \sum_{i\in J_c^f} \beta_{cit} x_{cit} + h_{t-1} - h_t && \forall\, t \in \mathcal{T} : 1 < t < m,\\
&y_t = \sum_{c\in\mathcal{C}_f} \sum_{i\in J_c^f} \beta_{cit} x_{cit} + \sum_{c\in\mathcal{C}} \sum_{i\in J_c} \sum_{(s,t,t')\in\mathcal{F}_J} \beta_{cist} w_{cistt'} + h_{t-1} - h_t && \forall\, t \in \mathcal{T} : m \le t \le M+2,\\
&y_t = \sum_{c\in\mathcal{C}} \sum_{i\in J_c} \sum_{(s,t,t')\in\mathcal{F}_J} \beta_{cist} w_{cistt'} + h_{t-1} - h_t && \forall\, t \in \mathcal{T}\ t > M+2.
\end{aligned}
$$

5. *Deliveries to processing plant must not exceed its capacity.*
   Monthly deliveries to the processing plant must not exceed the plant's production capacity Cap_t:

$$y_t \le \mathrm{Cap}_t \qquad \forall\, t \in \mathcal{T}.$$

6. *Satisfaction of biomass demand.*
   Biomass demand P_t, as determined by the monthly plan, must be satisfied:

$$y_t + HG_t \ge P_t \qquad \forall\, t \in \mathcal{T}.$$

The margin variable HG_t allows nonsatisfaction of demand, which will be penalized in the objective function. However, a minimum proportion δ, $0 \le \delta \le 1$, of monthly demand must be satisfied:

$$y_t \ge \delta P_t \qquad \forall\, t \in \mathcal{T}.$$

7. *Smoothing of biomass deliveries to processing plant.*
   The quantity of biomass y_t delivered over the entire planning horizon must not vary significantly between consecutive months:

$$y_t \geq (1 - \varepsilon_{inf}) y_{t-1} \qquad \forall\, t \in \mathcal{T},$$

$$y_t \leq (1 + \varepsilon_{sup}) y_{t-1} \qquad \forall\, t \in \mathcal{T}.$$

8. *Definition of variables used to impose fixed stockings.*
   Based on the previously requested demand requirements submitted to the freshwater phase, the deliveries that must be made from this phase are fixed for the first months of the planning horizon:

$$\sum_{\substack{s \in \mathcal{T} \\ (t,s) \in \mathcal{F}_C}} W_{cts} = S_{ctt} \qquad \forall\, t \in \mathcal{T} : t \in SFS,\ t-1 \notin SFS,\ \forall\, c \in \mathcal{C},$$

$$\sum_{\substack{s \in \mathcal{T} \\ (t,s) \in \mathcal{F}_C}} W_{cts} = S_{ctt} + S_{ct-1t} \qquad \forall\, t \in \mathcal{T} : t-1, t \in SFS,\ t+1 \notin SFY,\ \forall\, c \in \mathcal{C},$$

$$\sum_{\substack{s \in \mathcal{T} \\ (t,s) \in \mathcal{F}_C}} W_{cts} = S_{ctt} + S_{ct-1t} + A_{ct+1t} \qquad \forall\, t \in \mathcal{T} : t-1, t \in SFS,\ t+1 \in SFY,\ \forall\, c \in \mathcal{C},$$

$$\sum_{\substack{s \in \mathcal{T} \\ (t,s) \in \mathcal{F}_C}} W_{cts} = A_{ct+1t} + A_{ctt} + S_{ct-1t} \qquad \forall\, t \in \mathcal{T} : t \in SFY,\ t-1 \in SFS,\ \forall\, c \in \mathcal{C},$$

$$\sum_{\substack{s \in \mathcal{T} \\ (t,s) \in \mathcal{F}_C}} W_{cts} = A_{ct+1t} + A_{ctt} + A_{ct-1t} \qquad \forall\, t \in \mathcal{T} : t-1, t, t+1 \in SFY,\ \forall\, c \in \mathcal{C},$$

$$\sum_{\substack{s \in \mathcal{T} \\ (t,s) \in \mathcal{F}_C}} W_{cts} = A_{ctt} + A_{ct-1t} \qquad \forall\, t \in \mathcal{T} : t-1, t \in SFY,\ t+1 \notin SFY,\ \forall\, c \in \mathcal{C},$$

$$\sum_{\substack{s \in \mathcal{T} \\ (t,s) \in \mathcal{F}_C}} W_{cts} = A_{ct-1t} \qquad \forall\, t \in \mathcal{T},\ t-1 \in SFY,\ t, t+1 \notin SFY,\ \forall\, c \in \mathcal{C}.$$

9. *Satisfaction of fixed stocking restrictions.*
   For each of the first months in the planning horizon, the amounts of fish fixed by the seawater plan must be stocked:

$$\sum_{c \in \mathcal{C}} \sum_{\substack{s \in \mathcal{T} \\ (t,s) \in \mathcal{F}_C}} Tam_c W_{cts} = SFR_t \qquad \forall\, t \in SFR.$$

For the months in which stocking can be delayed by one month, the amounts fixed by the seawater plan must be stocked:

$$\sum_{c\in\mathcal{C}} \sum_{\substack{s \in \mathcal{T} \\ (t,s) \in \mathcal{F}_C}} Tam_c W_{cts} = SFS_t \qquad \forall\, t \in SFS.$$

For the months in which stocking can be delayed or advanced by one month, the amounts fixed by the seawater plan must be stocked:

$$\sum_{c\in\mathcal{C}} \sum_{\substack{s \in \mathcal{T} \\ (t,s) \in \mathcal{F}_C}} Tam_c W_{cts} = SFY_t \qquad \forall\, t \in SFY.$$

The objective of the optimization model is to define stocking and harvesting dates for each center, which maximize the harvested biomass in the period while satisfying the constraints imposed on the seawater phase and penalizing months when demand is not met by a penalty factor. The objective function of the model is defined as follows:

$$\max \quad Z = \sum_{c\in\mathcal{C}} \sum_{i\in J_c} \sum_{t\le M+2} \beta_{cit} x_{cit} + \sum_{c\in\mathcal{C}} \sum_{i\in J_c} \sum_{(s,t,t')\in\mathcal{F}_J} \beta_{cist} w_{cistt'} - \alpha_p \cdot \sum_t HG_t.$$

The first two terms of the objective function represent the total harvested biomass in the horizon. The term $\sum_t HG_t$ represents the total amount of demand that is not met in the horizon. The penalization cost of each unit of unsatisfied demand is α_p. We used a value of $\alpha_p = 50{,}000$ that was determined in conjunction with the company planners. We conducted some experiments with a few other values of α_p, but no significant difference was found.

4.4. Results

We now examine the results of the numerical experiments that were conducted to evaluate the seawater planning model just formulated in the preceding paragraphs. Our analysis will focus on comparison of the decisions suggested by the model with those made by the Multiexport Foods planners.

A Dell Inspiron 6400 notebook computer with an Intel® Core™2 Duo Processor T5600 (1.83 GHz) and 3 GB of RAM is used for all the experiments. The computer's operating system was Windows Vista (32-bit). The algebraic modeling language used was GAMS v2.0.36.7 and the optimization software was the MIP solver of CPLEX v10.2.0 with a stop criterion defined by a relative MIP gap tolerance of 3%. The optimization model had around 560,000 variables of which only 180 were continuous and the rest binary.

4.4.1. Evaluation methodology and real-world instance

The evaluation of the model was performed on a real-world instance consisting of the decisions made by the Multiexport planners over a 47-month horizon. The known decision data included cage harvest dates, projected future center stocking dates, and the number and average weight of individuals actually stocked per cage or projected for future stocking operations.

The seawater model generates plans for 60 months, the period recommended by the planning team considering the time considerations of the production chain. The biomass harvest figures for the 47 months of company information were used in the model to represent the demand data for that time period, and averages of the same numbers were used to fill out the remaining 13 months of the model-generated plan.

Although the planners suggest future stocking operations only in terms of modules without assigning them to a particular center, the seawater planning model defines exactly when to stock at each center. The comparisons relating to future stockings will therefore refer either to modules or centers, whichever is appropriate.

4.4.2. Numerical results

The planning parameters chosen for comparison were fundamental factors in seawater planning: the number of centers to be stocked in future operations, the length of harvest periods, average harvest weights, the total biomass harvest, and planning time.

1. *Number of centers to be stocked*: Since it is costly to open a center and maintain it in operation, good planning will keep the number of centers the operator uses as few as possible. This number is estimated in terms of the number of modules stocked. Under the company plan, 76 modules were stocked to satisfy demand for the 47 months, whereas the model called for only 70 modules. In other words, the planners used the equivalent of three to six additional centers (depending on whether they were single or double). Therefore, the application of the model would have generated significant savings for the farm.
2. *Length of harvest periods*: For existing stocks, the company planning decisions complied with the three-month harvest limit at only 7 of 21 centers. Many of them in fact had harvest periods of at least five months, extending in some cases up to seven months. Regarding future stocking, a similar contrast was observed. The company plan failed to meet the harvest restriction at more than half of the centers, and at the great majority of them the harvest period was at least four months, in some cases stretching to six months—more than double the maximum time permitted.

   The three-month rule was intended to avoid operating centers with small numbers of stocked cages and thus reduce the fixed cost burden of monthly center maintenance. The additional expense of an extra month for maintenance of the number of cages can exceed the added profit earned due to the increase in average weight, resulting in a financial loss for that month.
3. *Average harvest weights*: For existing stocks, the company plan fell short of the minimum weight constraint by a third of the centers, and at a single center it exceeded the maximum weight. More serious concern, however, was that at some centers the salmon were harvested too early, with average cage weights at the time of less than 3.0 kg. This is unsatisfactory in terms of return on inputs and will also result in future inventory imbalances, leading to insufficient supplies for meeting demand in the following months. In contrast, the model generated an average

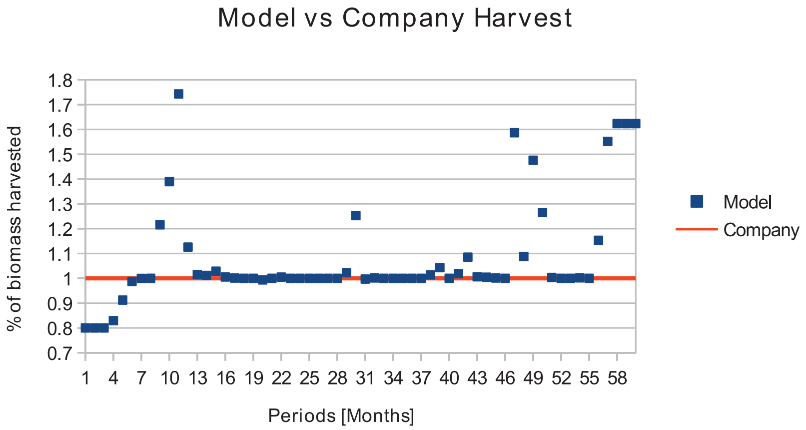


Fig. 1. Biomass obtained by the model as a relative percentage of company's harvest.

cage harvest of 4.296 kg with a standard deviation of 0.52, very close to the ideal weight of 4.6 kg.

For future stockings, given that the planning data are expressed in terms of modules or centers rather than cages, the harvest weights cannot be observed to the same degree of detail. Nevertheless, some useful comparisons can still be made. Although the company plan specified an average harvest weight of 4.036 kg with a standard deviation of 0.216, the proposed model yielded an average harvest weight of 4.517 kg with standard deviation of 0.480, the latter once again closer to the ideal figure.

4. *Total biomass harvest and satisfaction of demand*: As explained before, for purposes of evaluating the model's suggestions, the demand for the 60 periods was constructed in terms of the amount of harvest that was obtained each month using the company planning decisions. The model's and company's monthly harvest as a relative percentage of the demand defined in this manner is shown in Fig. 1. Because of the way demand was constructed, the company always generates a 100% harvest, whereas the model sometimes generates a harvest above (>100%), equal (=100%), or lower (<100%) than that of given by the demand.

   Notice that the harvest amounts generated by the model for the first months do not meet the minimum demand constraint. This behavior is explained because the model is designed to satisfy all the constraints on the periods of harvest and the fish weight ranges, and also the model allows a certain percentage of unsatisfied demand on each period. Therefore, the model does not have the "freedom" to achieve the same results as the company planners do in the short run by harvesting fish too early, which results in achieving less than the minimum harvest weight, and also the model focuses more in the long term, thus maximizing the biomass over the entire time horizon.

   For the rest of the planning horizon, the model always generates a harvest at least equal to the planners' biomass quantity, in a few months surpassing it by margins of up to 50% or more. The reasons for these results are that the model harvests fish with the correct weights (generally greater than company planners' weights). Moreover, since the model maximizes harvested

Table 4
Company planners versus optimization model results

	Company planners	Optimization model
Number of centers used	76	70
Centers with 3-month harvest period	7	All
Lenght of harvest period	3–7 months	3 months
Harvest weights	1.5–6 kg	3.2–5.3 kg
Harvested biomass (%)	100	103
Planning time	Days	Hours

biomass over the entire time horizon, it makes the fish stay longer in the cages resulting in their harvest with an extra weight (this effect can be noticed, e.g., in the last few periods).

If we consider just the total harvested biomass during the first 47 months, the model biomass harvest is 3% higher than the planners' harvest, which would translate into greater profits for the company. An estimation of these benefits is described next. In 2007 and 2008 around 5000 tonnes of biomass, or equivalently 1,200,000 fish were harvested before they reached the recommended harvest weight. At this stage of development, if fish are held for one more month in seawater, there is a gain of about 300 g of biomass per fish when harvested. Assuming then that the fish are harvested one month before it is optimal, the total biomass loss is about 360 tonnes of biomass per year. The cost of keeping fish for one more month in the seawater to reach the extra 300 g of biomass is estimated to be \$2900 per tonne. Around 65% of the cost is related to feeding, vaccination, and operation of the cages, while the remaining part is incurred in the harvesting. On the other hand, the market price of fish per tonne of fish is estimated to be \$4000, hence the loss of harvesting fish a month in advance is about \$396,000 per year. Based on the solutions obtained with our model and the know-how of the planning time, the use of the model would help to reduce this loss in about 50%, or equivalently about \$200,000 per year.

5. *Planning time*: The model's execution time for realistically sized instances of the complete 60-month horizon is 1.8 hours with average gap of 2.52%. This is to be compared with the one or two afternoons required by the company planners to carry out a single (local) modification to the 47-month plan using manual methods.

We present in Table 4 a summary with the results of all the factors discussed above.

The numerical evaluation experiments thus demonstrated that the proposed seawater planning model reduces the number of centers used while satisfying the various constraints conducive to a more orderly planning of production, such as the restrictions on maximum harvest cycle length and permitted average harvest weight range. The model also showed its ability to maximize biomass, the main objective of the company's production planning. Moreover, it generates a plan more quickly than the company's manual methods, leaving more time for evaluating different planning scenarios, which in turn should result in more robust decisions. The model thus proved itself to be a useful tool that markedly strengthens seawater planning.

5. Freshwater planning model

In this section, we present the various elements that lie behind the development of our optimization model for planning the freshwater phase.

5.1. Description of the freshwater phase

This link in the salmon production chain begins with the selection of reproducers for fertilization and generation of eggs and ends with the completion of the estuary stage when the fish are ready for transfer to seawater. The entire cycle has a duration of 8–13 months.

The planning problem for this phase consists of the monthly decisions on fish cultivation for the hatchery, smoltification, and estuary stages. The planners have a measure of freedom regarding hatchery egg stocking but the results must fulfill the seawater requirements for fish from the estuary.

Multiexport Foods maintains centers at rivers or lakes in southern Chile for each of the freshwater stages, where the salmon are cultivated in ponds or cages. The fish move through the different freshwater stages as they reach the necessary minimum weight for each one. Because of the country's climate, stocking is feasible all year round. Four different families or types of salmon are raised at the company's ponds, depending on seasonal availability.

Fish growth in this phase is estimated by a weight projection model based on cultivation time. Assuming feed conditions are such that they permit maximum growth, the most important variable in the growth process is temperature. At each freshwater stage, the salmon are separated into three or four weight ranges so that the individuals in any given cage will be of similar weight. This avoids putting the smaller fish at a disadvantage, which would tend to be the case in a more dispersed environment where the mortality of small individuals is observed to be higher, see Bjørnsson (1994). Separation by weight also means that fish are transferred through the successive stages in more homogeneous groups, thus securing greater population survival rates.

Fish mortality is a natural phenomenon, however, and must be considered in the production chain planning process. For each freshwater center, it is estimated that depending on the information available, freshwater stages may be described by family and weight ranges. Further details on each of the freshwater stages at Multiexport as of 2008 are given below.

Hatchery: The company operates four hatchery centers, all located near rivers. Each has a fixed incubation capacity, and together these limits define the maximum number of fish that can be stocked in a single cultivation period. Stocking of less than 30–40% of incubation capacity is avoided.

The fish are introduced into the hatchery ponds as alevins and kept there from two to four months until they reach the parr or fry state. At this point, they weigh between 15 and 20 g and are ready for being transferred to the smoltification stage in rivers or lakes. Note that alevins above 30 g are not permitted to remain in the hatchery stage and must therefore be transferred before that size is reached.

The more relevant health measure with the maximum permitted density at the hatchery stage is 25 kg/m^3, and fish of different families cannot be kept in the same pond. In addition, only two different families may be cultivated at the same time at a single center and no new stocking of a family already in cultivation can be introduced until the passage of a fallow period.

The families are transferred to the next stage, smoltification by trucks that are capable of transporting only one family at a time with a maximum density of 30 kg/m^3. The weight grading is maintained at the smoltification site so that fish of various weights could be kept separate.

Smoltification: The company has three smoltification centers situated at lakes or rivers. Each has a fixed capacity defined by the number of cages. The fish arrive from the hatchery as parr or fry and remain in the smoltification stage for four to six months until they reach the presmolt state. Once they exceed 60 g, they can be transferred to the estuary. Ideally, no fish weigh at this stage beyond 95 g.

The maximum permitted density is 10 kg/m^3, which means the introduction of 100,000 individuals per cage. The families are kept in separate cages and not more than two are cultivated at the same time in a given center.

The families are transferred to the next stage, i.e., estuary, by trucks that are capable of carrying only one family at a time with a maximum density of 50 kg/m^3. Like the previous stage, the weight grading is maintained at the estuary.

Estuary: The presmolts must be maintained in a highly saline environment, so the individuals are prepared for transfer to the sea sites. They are kept for one to three months at the three estuary centers, generally located at river mouths. Once they become smolts and their weight has reach approximately 100 g, or their cultivation time reaches 14 months, they can be transferred to the seawater phase.

At this stage, the company has the option to acquire fish from an external provider in case the internal production of smolts is not enough to satisfy the demand from seawater. We assume that all fish acquired in this way are of fixed age and size class (a', i') and that there is a maximum amount of fish that can be acquired by the outside provider.

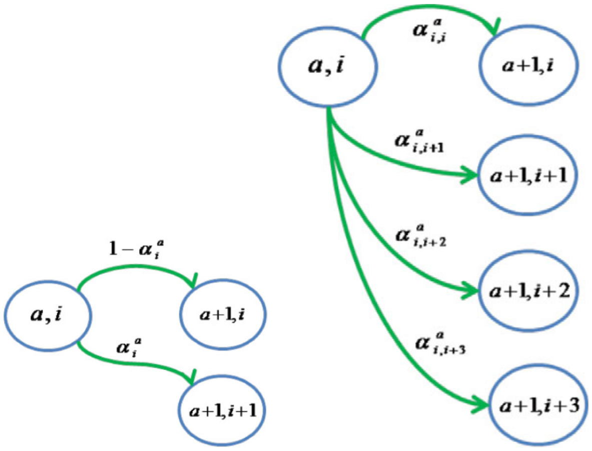
Like the smoltification stage, each estuary center has a fixed capacity determined by the number of cages, which may be of varying storage volumes. The health measures and transfer considerations are the same as for smoltification.

The objective of freshwater planning is to minimize costs, taking into account the most important cost items as follows: eggs for hatchery stocking – fish cultivation and handling at each center in each freshwater stage – cage operation and maintenance – truck or boat rentals.

5.2. Planning support models

Like the seawater phase, good planning of freshwater operations necessitates support models that can estimate fish growth during cultivation. This information is essential to guide decision making on transfers and facility use based on the fish population's weight distribution, and the possibility to accelerate or delay fish growth is due to different temperature conditions offered by the centers.

Growth model: The growth model is derived from population dynamics theory and describes how an individual grows through discrete weight classes while in cultivation. The approach assumes that growth occurs according to a Markovian process in which each state represents a weight class in a cultivation period, and transition probabilities indicate the likelihood a fish will move to the next weight class after a cultivation period. These probabilities will later be included as parameters in the freshwater planning model.



(a) Transfers in original model (b) Transfers in extended model

Fig. 2. Transfers between weight classes.

Our growth model formulation extends the one presented in Forsberg (1999), allowing the fish to be moved from one weight class to any of the next three rather than just to the immediate weight class. This variation in the original specification enables the model to capture the growth of the fish in their first three months of development over predefined weight classes. Figure 2 shows that fish of age a, which belong to weigh class i, the possible weight class transition after one period of culture.

On the basis of the projection curve for the average weight of the fish population, the valid freshwater weight range is discretized into I weight classes. The population weight class is assumed to be a normally distributed random variable whose mean is determined by the average weight projection model (see Forsberg, 1999). We discussed this assumption with the freshwater planning team and they agreed that this was a reasonable modeling assumption; however, we were not able to validate this empirically. In the literature, we found that Løland et al. (2011) model the distribution of the weight of a specific weight class as a beta distribution and estimate the parameters based on real data from a Norwegian fishery. The choice of the beta distribution seems to be arbitrary and only motivated by the tractability that this distribution provides.

Let $a \in \{1, \ldots, A\}$ be the cultivation periods (or age), and $i \in \{1, \ldots, I\}$ be the discrete weight classes. Class i is defined as the fish whose weight is $w \in [w_i^{min}, w_i^{max}]$. A fish whose cultivation time is a and whose weight falls within weight class i is said to be in state (a, i) of the Markov chain. Thus, we define $\varphi(w, a)$ as a normally distributed random variable with mean $\overline{w}(a)$ and variance $\sigma(a)$. The probability π_i^a of belonging to class i with cultivation time a is given by

$$\pi_i^a = \int_{w_i^{min}}^{w_i^{max}} \varphi(w, a)\, dw \quad \forall a, i \quad \text{such that} \sum_i \pi_i^a = 1 \quad \forall a.$$

We can now specify the weight class transition probabilities in the chain, which will be incorporated into the freshwater planning model. α_{ij}^a= probability that fish with cultivation time a and weight class i move to weight class j in cultivation period $a+1$. Extending the original transfer model, since individuals may be transferred to any of the following three weight classes,

$$\sum_{j=i}^{i+3} \alpha_{ij}^{a} = 1 \quad \forall a, i \text{ where } \alpha_{ij}^{a} \geq 0. \tag{1}$$

The flow conservation equations are

$$\pi_1^{a+1} = \alpha_{01}^{a}\pi_0^{a} + \alpha_{11}^{a}\pi_1^{a} \quad \forall a,$$

$$\pi_2^{a+1} = \alpha_{02}^{a}\pi_0^{a} + \alpha_{12}^{a}\pi_1^{a} + \alpha_{22}^{a}\pi_2^{a} \quad \forall a,$$

$$\pi_3^{a+1} = \alpha_{03}^{a}\pi_0^{a} + \alpha_{13}^{a}\pi_1^{a} + \alpha_{23}^{a}\pi_2^{a} + \alpha_{33}^{a}\pi_3^{a} \quad \forall a,$$

$$\pi_i^{a+1} = \sum_{j=i-3}^{i} \alpha_{ji}^{a}\pi_j^{a} \quad \forall a, i > 3.$$

The solution to this system of equations gives the transition probabilities between the weight classes by age or cultivation time at a freshwater center depending on the temperature conditions.

Mortality model: In the absence of a predictive mortality model for the freshwater phase, we resorted using the historical mortality data collected by the production centers. This parameter is thus determined by the available information and can be indexed by center, month of the year, fish family, weight class, etc. The actual metric used is average mortality per center.

5.3. Optimization model

The freshwater planning model was designed using the techniques of mixed-integer linear programming. The formulation has a multiproduct network flow structure in which the products are defined as the various cultivated species or families distributed by weight class and age (cultivation time). It includes flow transformations associated with fish mortality and growth plus various constraints reflecting production center capacities.

As stated earlier, the ultimate objective of the model is to satisfy the fish quantity requirements of the seawater phase at the minimum cost for each period of the time horizon. For each period, the model must determine what to maintain at each center in each stage, what is to be transferred between stages, the number of cages per weight class, and the transport capacity to be used. It must also update the flows for growth and mortality and ensure the constraints applicable to the freshwater operations are satisfied.

In this context, we describe the decisions, the constraints, and the objective function of the seawater optimization model. See Tables 5 and 6 for a description of the index sets and parameters of the model. Definitions of the decision variables can be found in Table 7.

Table 5
Index sets for the variables and parameters of the model

Index sets	
$\mathcal{T} = \{0, \ldots, T\}$	Time horizon
$\mathrm{CP} = \{P_1, P_2, P_3, P_4\}$	Hatchery centers
$\mathrm{CS} = \{S_1, S_2, S_3\}$	Smoltification centers
$\mathrm{CE} = \{E_1, E_2, E_3\}$	Estuary center
$\mathrm{C} = \{CP \cup CS \cup CE\}$	All the centers
$\mathcal{F} = \{F_1, F_2, F_3, F_4\}$	Salmon species or families
$\mathrm{Age} = \{0, \ldots, A\}$	Fish cultivation time or age
$\mathrm{Cl} = \{0, \ldots, I\}$	Weight classes
$\mathcal{J} = \{1, \ldots, J\}$	Types of cages
i_P, i_S, and i_E	Minimum acceptable weight class for transfer from hatchery to smoltification, from smoltification to estuary, and from estuary to seawater, respectively
Ageclass[i]	$\{a \in Age \mid \exists$ fish of age a in weight class $i\}$
NextClass[a, i]	$\{k \in Cl \mid$ if from class k at age $a-1$, fish move to class i at age $a\}$

Table 6
List of all the parameters of the model

Parameters	
W^i	Average weight in kilograms of class i
$Exist_{fc}^{ai}$	Quantity of fish of family f, age a and weight class i at freshwater center c at the biginning of time horizon
CAP_j	Capacity in kilograms of a type j cage
$NCAP_j$	Maximum number of fish in a type j cage
JC_{cj}	Number of type j cages available in center c
$MinCapStock_p$	Minimum number of eggs that may be stocked into hatchery p in a stocking operation
$MaxCapStock_p$	Maximum number of eggs that may be stocked into hatchery p in a stocking operation
$MaxOutsourced$	Maximum number of outsourced fish that can be introduced into an estuary center
$MaxCap_c$	Maximum capacity in kilograms of center c
$CapTruck_{PS}$	Maximum truck capacity in kilograms to transport fish from hatchery to smoltification stage
$CapTruck_{SE}$	Maximum truck capacity in kilograms to transport fish from smoltification to estuary stage
$CapTruck_E$	Maximum truck capacity in kilograms to transport fish from estuary stage to seawater phase
μ_c	Survival rate for fish at center c
α_{mc}^{aik}	Percentage of fish of age a in weight class i at center c that will move to weight class k in the following period at age $a+1$ given that they were stocked in period m
β_f^t	Equal 1 if it is possible to stock fish of family f in period t, 0 otherwise
γ_f^t	Equal 1 if it is possible to outsource purchases of family f in period t, 0 otherwise
D_t	Demand for fish in period t originated by seawater phase
$Cost_c^i$	Cost of maintaining an individual of class i at center c per period
$Cstock$	Cost per egg for hatchery stocking
$Cfam$	Operating cost per family in cultivation
$Cout$	Outsourcing cost per presmolt for introduction into estuary
$Ccage_j$	Cost of maintaining a type j cage in operation per period
$Ctruck_{cl}$	Cost per truck transfer of fish from center c to center l or to seawater phase

Table 7
List of all the variables of the model

Flow variables	
PP^{ait}_{fp}, EE^{ait}_{fe}, and SS^{ait}_{fs}	Quantity of fish of family f, age a, and weight class i maintained from period t to the following period $t+1$ in hatchery, smoltification, and estuary, respectively. Hatchery stockings are assumed to begin at age 0 and weight class 0.
PS^{ait}_{fps}, SE^{ait}_{fse}, and E^{ait}_{fe}	Quantity of fish of family f, age a, and weight class i transferred in period t from hatchery to smoltification, smoltification to estuary, and estuary to seawater, respectively.
$EMAQ^{t}_{fe}$	Outsourced estuary purchases. We assumed they are of age a' and size i'.
Variables for the number of cages used in each stage	
$NCage^{it}_{fcj}$	Number of type j cages used for fish of weight class i and family f in center c in period t.
Variables for the number of trucks for transfers between stages	
$NTruck^{t}_{fcl}$	Number of trucks used for transferring fish of family f from center c to center l in period t.
Binary variables	
FC^{t}_{fc}	1 if there are fish of family f in center c in period t, 0 otherwise.
YP^{t}_{fp}	1 if fish of family f are stocked into hatchery p in period t, 0 otherwise.

The constraints of the model are described below.

1. *Define the number of cages used at a center by fish family and weight class in each period.*
   The number of ponds or cages required to hold the fish depends on the maximum weight that ponds can hold in the case of hatchery and the maximum number of fish in the case of cages for smoltification and estuary stages.
   Hatchery

$$\sum_{a\in \text{Ageclass}[i]} W^{i} PP^{ait}_{fp} \le \sum_{j\in J} CAP_{j} NCage^{it}_{fpj} \quad \forall p \in \text{CP}, i \in Cl, t \in \mathcal{T}, f \in \mathcal{F}$$

   Smoltification

$$\sum_{a\in \text{Ageclass}[i]} SS^{ait}_{fs} \le \sum_{j\in J} NCAP_{j} NCage^{it}_{fsj} \quad \forall s \in \text{CS}, i \in Cl, t \in \mathcal{T}, f \in \mathcal{F}$$

   Estuary

$$\sum_{a\in \text{Ageclass}[i]} EE^{ait}_{fe} \le \sum_{j\in J} NCAP_{j} NCage^{it}_{fej} \quad \forall e \in \text{CE}, i \in Cl, t \in \mathcal{T}, f \in \mathcal{F}$$

2. *Ensures compliance with the number of cages available at each center.*
   The total number of ponds or cages needed to hold fish must be smaller than the total number of ponds or cages available in each center in every period.

$$\sum_{i\in \text{Cl}} \sum_{f\in\mathcal{F}} NCage^{it}_{fcj} \le JC_{cj} \quad \forall c \in \mathcal{C}, t \in \mathcal{T}, j \in \mathcal{J}$$

3. *Conservation of flow at each center and stage.*
   Hatchery
   The number of fish of age a and weight class i in a given period $t+1$ is defined as the number of fish of age $a-1$ that stayed in class i or reached this class from lower weight classes in one period, adjusted by mortality, minus the fish that are transferred to any center in the smolitification stage.

$$PP_{fp}^{ait+1} = \sum_{k \in \mathrm{NextClass}[a,i]} \mu_p \alpha_{mp}^{a-1ki} PP_{fp}^{a-1kt} - \sum_{s \in CS} PS_{fps}^{ait+1} \quad \forall p \in \mathrm{CP}.$$

   Smoltification
   The same as before, but we include the fish transferred from the hatchery stage minus the fish transferred to the estuary stage.

$$SS_{fs}^{ait+1} = \sum_{k \in \mathrm{NextClass}[a,i]} \mu_s \alpha_{ms}^{a-1ki} SS_{fs}^{a-1kt} + \sum_{p \in CP} PS_{fps}^{ait+1} - \sum_{e \in CE} SE_{fse}^{ait+1} \quad \forall s \in \mathrm{CS}.$$

   Estuary
   The same idea as in the previous stages, but we add the fish coming from smoltification minus the fish transferred to the seawater phase

$$EE_{fe}^{ait+1} = \sum_{k \in \mathrm{NextClass}[a,i]} \mu_e \alpha_{me}^{a-1ki} EE_{fe}^{a-1kt} + \sum_{s \in CS} SE_{fse}^{ait+1} - E_{fe}^{ait+1} \quad \forall e \in \mathrm{CE}.$$

   When we allow outsourced purchases, the above constraint becomes

$$EE_{fe}^{a'i't+1} = EMAQ_{fe}^{t+1} + \sum_{k \in \mathrm{NextClass}[a',i']} \mu_e \alpha_{me}^{a'-1ki'} EE_{fe}^{a'-1kt} + \sum_{s \in CS} SE_{fse}^{a'i't+1} - E_{fe}^{a'i't+1}$$

$$\forall i \in \mathrm{Cl} \setminus \{0\},\, a \in \mathrm{AgeClass[i]},\, t \in \mathcal{T} \setminus \{T\},\, f \in \mathcal{F},$$

   where m is the period (month) in which the fish were stocked.

4. *Minimum and maximum cultivation time in each stage by weight class.*
   Hatchery
   We do not allow ourselves to keep fish of weight classes larger than $i_p + 1$ in hatchery.

$$PP_{fp}^{ait} = 0 \quad \forall a \in \mathrm{Age},\, i \in \{\mathrm{Cl} \mid i > i_p + 1\},\, p \in \mathrm{CP},\, t \in \mathcal{T},\, f \in \mathcal{F}.$$

   Smoltification
   We do not allow ourselves to keep fish of weight classes smaller than i_p or larger than $i_s + 1$ in smoltification.

$$SS_{fs}^{ait} = 0 \quad \forall a \in \mathrm{Age},\, i \in \{\mathrm{Cl} \mid i < i_p \wedge i > i_s + 1\},\, s \in \mathrm{CS},\, t \in \mathcal{T},\, f \in \mathcal{F}.$$

Estuary
We do not allow ourselves to keep fish of weight class smaller than i_s or fish that have been in the freshwater phase for A periods in estuary.

$$EE_{fe}^{ait} = 0 \quad \forall a \in \text{Age}, i \in \{i \in \text{Cl} \mid i < i_s\}, e \in \text{CE}, t \in \mathcal{T}, f \in \mathcal{F},$$

$$EE_{fe}^{Ait} = 0 \quad \forall i \in \{i \in \text{Cl} \mid i \geq i_s\}, e \in \text{CE}, t \in \mathcal{T}, f \in \mathcal{F}.$$

5. *Only what was available at the beginning of a period may be transferred to seawater.*
Transfers to seawater are limited by the quantity of fish available in estuary at the beginning of a period after adjusting for mortality and growth but before receiving the transfers from smoltification.

$$E_{fe}^{ait+1} \leq \sum_{k \in \text{NextClass}[a,i]} \mu_e \alpha_{me}^{a-1ki} EE_{fe}^{a-1kt} \quad \forall e \in \text{CE},$$

$$\forall i \in \text{Cl} \setminus \{0\}, a \in \text{AgeClass[i]}, t \in \mathcal{T} \setminus \{T\}, f \in \mathcal{F}.$$

6. *The families that can be stocked are those that are valid for the time of year.*
Which family is available to be stocked into hatchery depends on the season of the year. This is captured by the parameter β_f^t that takes value 1 when the family is available, 0 otherwise.

$$YP_{fp}^t \leq \beta_f^t \quad \forall p \in \text{CP}, t \in \mathcal{T}, f \in \mathcal{F}.$$

7. *A family can be stocked into a hatchery center only if no fish of that family are in cultivation.*
To avoid fish of the same family that started cultivation at different time getting mixed in hatchery, we do not allow ourselves to stock a new batch of a family if this family is already stocked in the center.

$$YP_{fp}^t \leq 1 - FC_{fp}^{t-1} \quad \forall p \in \text{CP}, t \in \mathcal{T} \setminus \{0\}, f \in \mathcal{F}.$$

8. *Upper and lower bounds on stockings in hatchery.*
The quantity of fish in a new batch of a specific family has a minimum and maximum depending on each center in hatchery.

$$MinCapStock_p YP_{fp}^t \leq PP_{fp}^{00t} \leq MaxCapStock_p YP_{fp}^t \quad \forall p \in \text{CP}, t \in \mathcal{T}, f \in \mathcal{F}.$$

9. *Total maximum stocking.*
New stocks of fish across all families include a maximum quantity of fish in each center in hatchery.

$$\sum_{f \in F} PP_{fp}^{00t} \leq MaxCapStock_p \quad \forall p \in \text{CP}, t \in \mathcal{T}.$$

10. *Maximum and minimum numbers of outsourced fish that can be introduced by time of year.* In estuary, we allow ourselves to buy fish from an outside provider. The family that can be bought depends on the season of the year, captured by the γ_f^t parameter, and there is also a maximum amount that can be bought.

$$EMAQ_{fe}^t \leq MaxOutsourced\gamma_f^t \quad \forall e \in \mathrm{CE}, t \in \mathcal{T}, f \in \mathcal{F}.$$

11. *No more than two families may be cultivated at a single center.* For sanitary reasons no more than two families can be cultivated in a single center simultaneously regardless of the stage.

$$\sum_{f \in \mathcal{F}.} FC_{fc}^t \leq 2 \quad \forall c \in \mathcal{C}, t \in \mathcal{T}.$$

12. *Defines for each stage whether a given family is in cultivation.* Let the variable FC_{fc}^t take the value 1 if there is fish of family f in center c in period t.
Hatchery

$$\sum_{i \in Cl} \sum_{a \in \mathrm{Ageclass}[i]} W^i PP_{fp}^{ait} \leq MaxCap_p FC_{fp}^t \quad \forall p \in \mathrm{CP}, t \in \mathcal{T}, f \in \mathcal{F}.$$

Smoltification

$$\sum_{i \in Cl} \sum_{a \in \mathrm{Ageclass}[i]} W^i SS_{fs}^{ait} \leq MaxCap_s FC_{fs}^t \quad \forall s \in \mathrm{CS}, t \in \mathcal{T}, f \in \mathcal{F}.$$

Estuary

$$\sum_{i \in Cl} \sum_{a \in \mathrm{Ageclass}[i]} W^i EE_{fe}^{ait} \leq MaxCap_e FC_{fe}^t \quad \forall e \in \mathrm{CE}, t \in \mathcal{T}, f \in \mathcal{F}.$$

13. *Defines the number of trucks to be used for transfers between consecutive stages.* The number of trucks needed to transfer fish from one stage to another depends on the weight of the fish being transferred as well as the specific capacity (weight to be carried) of the truck, which varies depending on the stage.
Hatchery

$$\sum_{i \in Cl} \sum_{a \in \mathrm{Ageclass}[i]} \frac{W^i PS_{fps}^{ait}}{CapTruck_{PS}} \leq NTruck_{fps}^t \quad \forall p \in \mathrm{CP},\ s \in \mathrm{CS}, t \in \mathcal{T}, f \in \mathcal{F}.$$

Smoltification

$$\sum_{i \in Cl} \sum_{a \in \mathrm{Ageclass}[i]} \frac{W^i SE_{fse}^{ait}}{CapTruck_{SE}} \leq NTruck_{fse}^t \quad \forall s \in \mathrm{CS},\ e \in \mathrm{CE}, t \in \mathcal{T}, f \in \mathcal{F}.$$

Estuary

$$\sum_{i \in Cl} \sum_{a \in \text{Ageclass}[i]} \frac{W^i E_{fe}^{ait}}{CapTruck_E} \leq NTruck_{fe}^t \quad \forall e \in \text{CE}, t \in \mathcal{T}, f \in \mathcal{F}.$$

14. *Seawater phase demand must be satisfied in each period.*
In every period, the amount of fish transferred from estuary to seawater must be at least the demand from the latter.

$$\sum_{e \in CE, f \in \mathcal{F}} \quad \sum_{i, a \in \text{Ageclass}[i]} E_{fe}^{ait} \geq D_t \qquad \forall t \in \mathcal{T}.$$

15. *Stocks in centers at the start of the planning horizon.*
Every stage has a initial stock of fish at the beginning of the planning horizon. The initial stock is defined by family, age, and weight class in every center.
Hatchery

$$PP_{fp}^{ai0} = Exist_{fp}^{ai} \quad \forall p \in \text{CP}, i \in \text{Cl} \setminus \{0\}, a \in \text{AgeClass[i]} \setminus \{0\}, f \in \mathcal{F}.$$

Smoltification

$$SS_{fs}^{ai0} = Exist_{fs}^{ai} \quad \forall s \in \text{CS}, i \in \text{Cl} \setminus \{0\}, a \in \text{AgeClass[i]} \setminus \{0\}, f \in \mathcal{F}.$$

Estuary

$$EE_{fe}^{ai0} = Exist_{fe}^{ai} \quad \forall e \in \text{CE}, i \in \text{Cl} \setminus \{0\}, a \in \text{AgeClass[i]} \setminus \{0\}, f \in \mathcal{F}.$$

16. *No outsourced purchases at the start of the horizon.*
Outsourced purchases are not allowed in the initial period.

$$EMAQ_{fe}^0 = 0 \quad \forall e \in \text{CE}, f \in \mathcal{F}.$$

17. *Nature of the variables.*

$$PP_{fp}^{ait}, \quad PS_{fps}^{ait}, \quad SS_{fs}^{ait}, \quad SE_{fse}^{ait}, \quad EE_{fe}^{ait}, \quad E_{fe}^{ait} \quad EMAQ_{fe}^t \quad \in \mathbb{R}^+ \cup \{0\}$$

$$NCage_{fcj}^{it}, \quad NTruck_{fcl}^t \quad \in \mathbb{Z}^+ \cup \{0\}$$

$$FC_{fc}^t, \quad YP_{fp}^t, \quad \in \{0, 1\}$$

$$\forall t \in \mathcal{T}, p \in \text{CP}, s \in \text{CS}, e \in \text{CE}, c, l \in \text{C}$$

$$i \in \text{Cl}, a \in \text{AgeClass[i]}, f \in \mathcal{F}, j \in \mathcal{J}.$$

The objective function of the optimization problem includes minimizing the fixed cost of having a family stocked in a center, the variable cost of new batches of fish, a variable cost (e.g. feeding, vaccination, etc.) for cultivating fish depending on their weight class, the cost per cage or pond used, the cost of buying fish from an external provider. All these costs are for every stage and we also add the cost of transfers, and trucks needed for transfers between stages.

Hatchery cost:

$$Cost_P = \sum_{t,f,p} \left(CfamFC_{fp}^{t} + CstockPP_{fp}^{00t} + \sum_{i \geq 1} Cost_p^i \sum_{a \in \text{Ageclass}[i]} PP_{fp}^{ait} + \sum_j Ccage_j NCage_{fpj}^{it} \right).$$

Smoltification cost:

$$Cost_S = \sum_{t,f,s} \left(CfamFC_{fs}^{t} + \sum_{i \in Cl} Cost_s^i \sum_{a \in \text{Ageclass}[i]} SS_{fs}^{ait} + \sum_j Ccage_j NCage_{fsj}^{it} \right).$$

Estuary cost:

$$Cost_E = \sum_{t,f,e} \left(CfamFC_{fe}^{t} + CoutEMAQ_{fe}^{t} + \sum_{i \in Cl} Cost_e^i \sum_{a \in \text{Ageclass}[i]} EE_{fe}^{ait} + \sum_j Ccage_j NCage_{fej}^{it} \right).$$

Transport costs:

$$\begin{aligned} Cost_{TT} = \sum_{t,f} &\left(\sum_p \sum_s Ctruck_{ps} NTruck_{fps}^{t} \right) + \left(\sum_s \sum_e Ctruck_{se} NTruck_{fse}^{t} \right) \\ &+ \left(\sum_e Ctruck_e NTruck_{fe}^{t} \right). \end{aligned}$$

Thus, the objective function is given by next expression:

$$\min Z = Cost_P + Cost_S + Cost_E + Cost_{TT}.$$

5.4. Results

In this section, we set out the main results of the evaluation of our freshwater planning model. Since Multiexport Foods is not engaged in formal planning of its freshwater operations, comparison with the company planners' decisions is difficult, and with the emergence of the industry health crisis the model could only be partially validated.

The model has approximately 300,000 variables, about 4000 of which are integers. It was written in *OPL* v6.0.1 and solved by *CPLEX* v11.1 on a PC with an Intel® Core™2 Quad Processor Q9400 (2.66 GHz) and 3 GB of RAM. The operating system was Windows XP version 2002 (32-bit). A solution with a relative gap of 1.34% was obtained in little more than 1 hour, and after 5 hours the relative gap had not come down below 0.95%. In light of this and given that a reasonable feasible solution is considered to be a good one, it was decided that solutions with a residual gap of less than 2% would be acceptable.

5.4.1. *Evaluation methodology and real-world instance*

The evaluation of the model was based on recommendations gleaned from the salmon-farming literature and performed on a real-world instance consisting of the actual freshwater operations of Multiexport Foods in 2008. The planning horizon was 25 months, and the demand for fish originating in the seawater phase was reported on a monthly basis. Initial stocks were assumed for each center at the beginning of the horizon and were indexed by cultivation time and weight.

The growth model was calibrated with the annual temperature profiles for each center and the mortality figures of each center were obtained from company records. The costs and capacities used for the various facilities are approximations complemented by market data. All the data needed for the model were approved by the experts in the freshwater planning team and reflect the true tradeoffs in the operations of the this phase.

5.4.2. *Numerical results*

An analysis of the main results for the planning model validation is presented below.

1. *Number of fish in cultivation*: The quantity of fish by family at each center over the planning horizon is shown in Fig. 3. The effects of mortality are observed in these trends, which reduce the numbers of fish in consecutive periods, and the effects of transfers, which can be made in multiple periods as individuals reach the required weight for introduction to the following stage. In hatchery $P1$, the mortality effect is clearly visible given that families are stocked in a single period, their numbers then decline slowly from the initial level in the rest of the horizon. Quantities of fish drop sharply with transfer to the smoltification stage until none are left in the hatchery. In the smoltification $S1$ and estuary $E1$ stages, the impact of mortality is less noticeable because incoming transfers may be made over various periods. Thus, a family that starts in $S1$ or $E1$ at a certain quantity will experience increases later as further transfers continue, the numbers will begin to decrease only once the transfers end while mortality continues. Significant decreases in these stages indicate outgoing transfers, either from smoltification to estuary or from the latter to the seawater phase.
2. *Distribution of fish at each center*: The quantity of fish cultivated over the planning horizon distributed by cultivation time (age) is plotted in Fig. 4. As can be seen, the curves obtained are bell-shaped located in specific age ranges directly related to the permitted weights in each stage. Despite the effect of initial stocks, the bulk of the population was found to be concentrated at a certain fish age for each of the stages. Smaller proportions were present at shorter and longer cultivation time in each stage depending on the amount of transfers between freshwater stages or to the seawater phase that were to be advanced or delayed.
3. *Satisfaction of demand*: The satisfaction of demand is measured by the quantity of fish sent to the seawater phase in each period of the planning horizon. The model satisfies exactly the demand in almost all period as shown in Fig. 5, except for two periods where initial stocks that

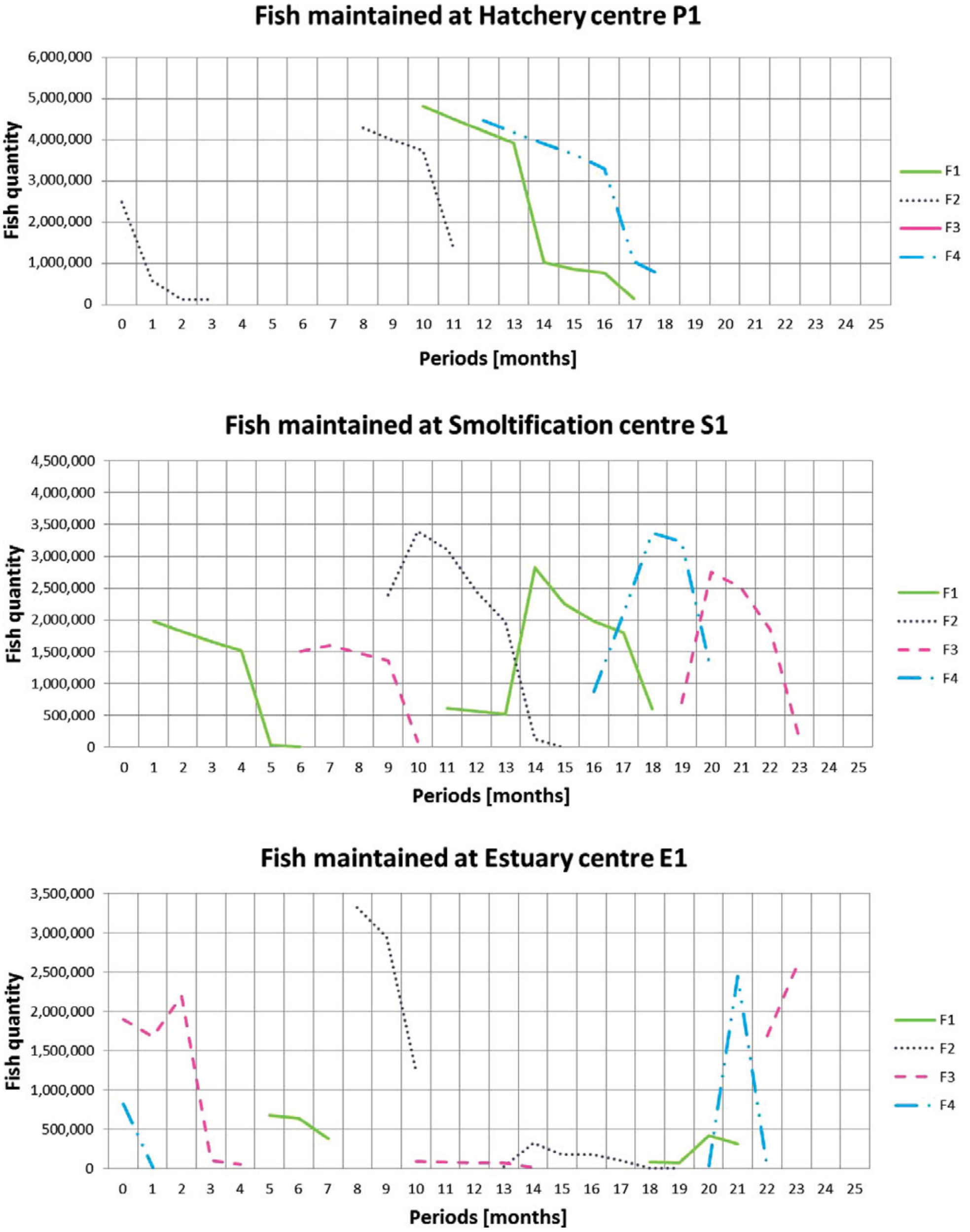


Fig. 3. Quantity of fish per family in $P1$, $S1$, and $E1$ over the complete planning horizon.

were not aligned with demand and which could not be held back in the freshwater phase due to their accumulated cultivation time. Note that since there is no benefit or penalty for extra deliveries and additional output incurs extra costs, no incentive, which is more than the amount demanded, could be delivered.

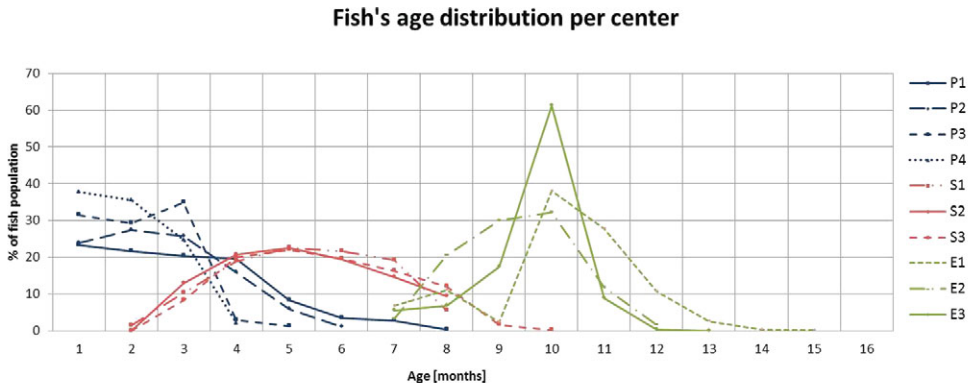


Fig. 4. Percentage of fish at each center distributed by cultivation time (age).

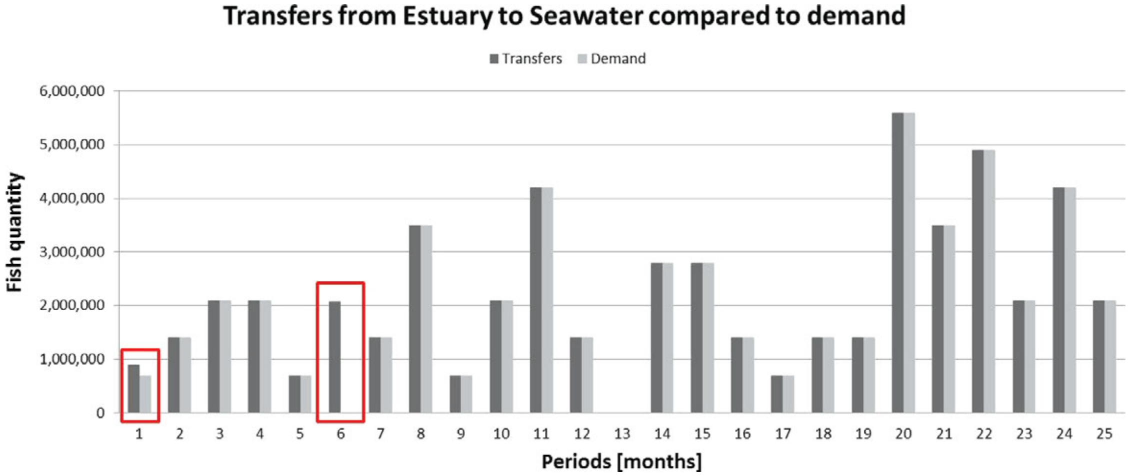


Fig. 5. Proportion of the seawater demand being satisfied.

The model fulfills the seawater phase demand, which ensures that the requirements are at least equaled. The model is able to satisfy demand either by sending fish that came from hatchery or by buying fish from an outside provider in the estuary stage. This flexibility is crucial for obtaining a feasible production plan and, in particular, for maintaining output given that the various phases are linked in a production chain, and failing to meet planning output levels will cause significant delays up and down the line. Under the demand instances we tested, the model was always feasible. However, if demand increases considerably, the model may be infeasible since stock from hatchery and outside purchases are limited, and not enough fish will be produced to meet demand. In practice, operators often prefer to advance the production process by disregarding the minimum transfer and/or harvest weights in order to ensure planned output is achieved. This can lead to major losses as the fish obtained are underweight (less biomass) and mortality may also increase. Resorting to such measures might fulfill requirements in the short term, but the shortfall of fish is merely postponed and a deficit will emerge in the medium term.

4. *Total freshwater cultivation time*: The total freshwater cultivation time can be estimated from the ages of the fish transferred from the estuary stage to the seawater phase from the estuary stage. In the results of the planning model, close to 50% of the transferred fish spent 9–11 months in the freshwater stages. The company data, in contrast, indicate that fish remain in the freshwater phase for 12 months on average. The difference is attributable to the more timely decisions generated by the planning model, which depends on the growth model that can estimate better when the fish have reached the required weight characteristics for transfer. In practice, since the growth rates can be managed by choosing the right center where to grow the fish, there is a certain time window for holding fish back longer than actually required in the freshwater phase. In the hatchery stage, the results of the model indicate that more than 70% of the transfers were in cultivation for four months or less, whereas 20% were there for five months or more. In smoltification, the fish stay for four to six months. These are consistent with the average cultivation time estimate for each stage.
   It should be noted that in order to apply the planning model, careful estimates of the parameters must be made and the growth and mortality model must be calibrated using actual data so that operations are effectively modeled, and the results generated are consistent with real-world management.
   The results of the planning model indicate that the formulation is capable of turning out a production plan that conforms to established farming production standards. In particular, the model generates correct cultivation times for the different freshwater stages so that the fish reach the required weights. This is highly important given that growth and mortality are directly related to cultivation conditions, and if these are not appropriate to the state of development of the fish, poor growth and increased mortality are likely to take place.

Finally, the model also enables a more rigorous control and management of the production process in that existing stocks are known at every moment of the planning horizon and health measures can be incorporated into the planning to help prevent diseases and their propagation. The health measures included in the model were related to restricting the number of families in cultivation per center and stocking a family only at centers where previously it was not present.

6. Conclusions

This paper has presented two models for planning production in a salmon farming production chain. The first model focuses on the seawater phase and seeks to maximize harvested biomass over the planning horizon time. The input for this model is the processing plant's biomass demand, and it decides the schedule of the stocking and harvesting cycles. The second model receives input on the seawater stocking plan from the first model and uses it for planning the freshwater phase operations, so that its production supplies the seawater phase fish requirements at minimum cost. Both the models take into account mortality data and the results of growth projection models while also satisfying various economic and health constraints peculiar to salmon culture.

The seawater planning process was modeled using mixed-integer linear programming, which proved to be reasonably appropriate for the problem addressed. The methodology used was found to function very efficiently in realistically sized instances, improving the quality of seawater stocking schedules while automating the planning process and notably reducing the time required to do it. Furthermore, small modifications that a planning team working manually would take some two days to complete and can be executed by the model in less than 2 hours, the solution generated has the added advantage of incorporating the changes globally rather than just locally, which is similar to the manual approach.

The seawater model is able to produce larger biomass harvests for two reasons. First, the decisions it generates lead to better use of the cages, so that greater quantities of fish can be cultivated in a given period, and, second, its harvest solutions identify grow-out times that ensure the fish are at their optimal weight when sent to slaughter. These results are achieved through the imposition of harvest cycle length and average harvest weight constraints. In real-world situations, these restrictions are often disregarded in order to satisfy processing plant demand, sacrificing fish that are not in the optimal harvest weight range. Thus, use of the model promises economic benefits in the form of profits accruing to greater extracted biomass and lower maintenance costs at grow-out centers. The formulation was validated by the Multiexport planning team, which found that it was capable of modeling actual seawater operations and accurately representing stock management, generating decisions that met health standards and maximized the efficiency and effectiveness of seawater production.

The freshwater phase model was also created using mixed-integer linear programming, creating a multiproduct network flow structure with flow transformations that reflect fish mortality and growth in each freshwater cultivation period. The principal contribution of this model is that it plans the first phase of salmon growth and thus includes more than one consecutive stage. It also automates the planning process, a complex task that is difficult to perform manually given the number of decisions and options involved. In addition, the model can incorporate various health measures in culture management decisions, observing weight limits in each stage and cultivation period while also providing follow-up on the fish. The model was partially validated, performing well in regard to expected cultivation times and fish weights in each stage and generating total freshwater phase times that were consistent with real-world levels. It also satisfied seawater demand in each period and maintained in cultivation the right quantities of fish for meeting later seawater phase requirements. This last point is particularly significant given that an efficient operation requires that the production chain be balanced so that each stage or phase produces exactly what the next production in the chain needs, thereby avoiding wastage and potential losses. Nevertheless, more thorough evaluation is necessary in order to complete the model's validation.

The production planning might be modeled through just one model that captures the whole process and interaction between the two phases, seawater and freshwater. A unique model would be more efficient, coordinating and optimizing the operations in the whole chain. However, companies in the industry operate both phases separately. The reason for this is that the management and particular needs of each phase are very different, for example, in terms of health, feeding, and cultivation conditions. Therefore, modeling the operations of each stage separately yields fairly natural results, and justifies our approach.

The worldwide economic and salmon health crises of 2008–2010 have greatly affected the salmon farming industry, cutting production by half and exposing inefficient health practices for the prevention and control of ISA virus propagation. Industry reaction to these challenges has been relatively slow, and the decision to live with the virus as a means of maintaining industry activity has made disinfection and recovery of salmon farming areas even more difficult. In this context, integration of the two models could provide an important tool for supporting production planning by offering rapid evaluation of multiple planning scenarios and better decision response times when unexpected problems arise. In particular, the integrated modeling structure can measure the effect on planning of alterations in certain parameters, such as processing plant biomass requirements, growth and mortality rates, or cost factors. Moreover, it could incorporate and evaluate the various health measures the industry has recently adopted, especially those relating to the cultivation and management of fish in the freshwater and seawater phases. Unfortunately, at the time this work was developed, the industry was focused on surviving the twin crises facing it, and implementation of support tools for decision making was not an immediate priority. As a consequence, none of the models we proposed have been implemented yet. In 2012, the industry showed huge improvement by reaching production levels similar to those in 2008, and it is hoped that the two models can be implemented once the industry fully recovers in the medium term.

For future research, an interesting extension of the system presented here would be to deepen coordination between the two models by having them share information in both directions. Thus, the seawater model could be developed to take into account the fish available in the freshwater phase for future stockings in combination with outsourced purchases where necessary. Decisions on transfers between freshwater and seawater centers could be included directly, with the associated costs also incorporated into the planning process. Another variable that could be added is the availability of centers for production so that decisions to advance or delay transfers would be based on the production chain as a whole rather than the constituent links considered separately. In addition, given the nondeterministic nature of salmon farming, random parameters for mortality, the presence of diseases, and various economic factors could be introduced to address a range of eventualities, thereby generating a more robust planning.

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References

Begen, M., Puterman, M., 2003. Development of a catch allocation tool design for production planning at JS Mcmillan fisheries. *Infor-Information Systems and Operational Research* 41, 3, 235–244.

Bjørndal, T., 1988. Optimal harvesting of farmed fish. *Marine Resource Economics* 5, 2, 139–159.

Bjørndal, T., Herrero, I., Newman, A., Romero, C., Weintraub, A., 2012. Operations research in the natural resource industry. *International Transactions in Operational Research* 19, 1–2, 39–62.

Bjørnsson, B., 1994. Effects of stocking density on growth rate of halibut (hippoglossus hippoglossus l.) reared in large circular tanks for three years. *Aquaculture* 123, 3–4, 259–270.

Brett, J., 1979. Environmental factors and growth. In Hoar, W.S., Randall, D.J., Brett, J. (eds) *Fish Physiology,* Vol. 8, Academic Press, New York, pp. 599–675.

Cho, C.Y., Bureau, D.P., 1998. Development of bioenergetic models and the fish-prfeq software to estimate production, feeding ration and waste output in aquaculture. *Aquatic Living Resources* 11, 4, 199–210.

Cisternas, F., DelleDonne, D., Duran, G., Polgatiz, C., Weintraub, A., 2012. Optimizing salmon farm cage net management using integer programming. *Journal of the Operational Research Society* 64, 735–747.

Cluster del Salmon, 2008. *Cadena de Valor. Cluster del Salmón Chile*. Available at http://www.clustersalmon.cl/Cadenadevalor.htm (accessed September 2009).

Forsberg, O.I., 1996. Optimal stocking and harvesting of size-structured farmed fish: a multi-period linear programming approach. *Mathematics and Computers in Simulation* 42, 2–3, 299–305.

Forsberg, O.I., 1999. Optimal harvesting of farmed atlantic salmon at two cohort management strategies and different harvest operation restrictions. *Aquaculture Economics & Management* 3, 2, 143–158.

Gasca-Leyva, J.M.H., Veliov, V.M., 2008. Optimal harvesting time in a size-heterogeneous population. *Ecological Modelling* 210, 1–2, 161–168.

Jensson, P., Gunn, E., 2001. *Optimization of production planning in fish farming*. Technical report, University of Iceland, Reykjavik, Iceland.

Leung, P., Hochman, E., Rowland, L.W., Wyban, J.A., 1990. Modeling shrimp production and harvesting schedules. *Agricultural Systems* 32, 3, 233–249.

Leung, P., Shang, Y.C., 1989. Modeling prawn production management system: a dynamic Markov decision approach. *Agricultural Systems* 29, 1, 5–20.

Løland, A., Aldrin, M., Steinbakk, G., Huseby, R., Grøttum, J., Quinn, T., 2011. Prediction of biomass in norwegian fish farms. *Canadian Journal of Fisheries and Aquatic Sciences* 68, 8, 1420–1434.

Mangel, M., Stamps, J., 2001. Trade-offs between growth and mortality and the maintenance of individual variation in growth. *Evolutionary Ecology Research* 3, 583–593.

SalmonChile, 2009. *Informe Económico Salmonicultura 2007, SalmonChile A.G.* Available at http://engineering.purdue.edu/mark/puthesis (accessed September 2009).

Sparre, P., 1976. A Markovian decision process applied to optimisation of production planning in fish farming. *Meddelser Fra Danmarks Fiskeri Og Havundersøkelser* 7, 111–197.

Yu, R., Leung, P., 2005. Optimal harvesting strategies for a multi-cycle and multi-pond shrimp operation: a practical network model. *Mathematics and Computers in Simulation* 68, 4, 339–354.

Yu, R., Leung, P., Bienfang, P., 2006. Optimal production schedule in commercial shrimp culture. *Aquaculture* 254, 1–4, 426–441.