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# PERVASIVE FACTORS IN STOCK RETURNS: 1984-1999

Jorge Gregoire C. *
Víctor Zapata Z.*
Salvador Zurita L.**
*Universidad Adolfo Ibáñez y Universidad de Chile*

**SUMMARY**

The purpose of this paper is twofold: on the one hand, to empirically determine the number of systematic risk factors modeled in the APT as developed by Ross (1976 a and b), to be observed in the Chilean stock market during the period 1984-1999; and, on the other, to investigate which would have been a risk premium other than zero.
The main results are (a) in the full period there are five pervasive risk factors that make it possible to replicate the covariance matrix of 44 security returns (b) only one of such factors showed statistically significant risk premia in the full period; and (c) for the sub-period 1991-1999 we find 11 factors for a larger sample of 79 securities, where again only one of them (but not necessarily the same one) features a significant risk premium.
The paper is organized as follows: Section I briefly reviews the APT and the different methods for estimating it; Section II describes the sample and determines the number of factors and which of such factors are priced. Finally, Section III sets forth the main conclusions arrived at.

* Universidad de Chile.
** Universidad Adolfo Ibáñez.

## I. The APT Model

Ross (1976 a) makes the assumption that individuals have homogeneous beliefs as to the linear stochastic process that generates returns and which would be governed by *"k"* common factors, where k is much smaller than the total number of securities, n:

$$\tilde{R}_i = E_i + b_{i1}\tilde{\delta}_1 + \ldots + b_{ik}\tilde{\delta}_k + \tilde{\varepsilon}_i\,; \quad i = 1,\ldots,n \tag{1}$$

Where $\tilde{R}_i$ is the random return of the i-th asset; $E_i$ is the expected return of the i-th asset; $\tilde{\delta}_j$ is the j-th mean zero (systematic) common factor that affects returns; $b_{ij}$ quantifies the sensitivity coefficients of the return of the i-th asset to the movements of common factor $\tilde{\delta}_j$; and $\tilde{\varepsilon}_i$ is the noise term or idiosyncratic risk component.

Furthermore, the model assumes that common factors are completely independent between themselves, $E\left[\tilde{\delta}_i\tilde{\delta}_j\right] = 0$, not correlated with the noise term $E\left[\tilde{\varepsilon}_i\tilde{\delta}_j\right] = 0$, and also idiosyncratic risk is independent between securities, $E\left[\tilde{\varepsilon}_i\tilde{\varepsilon}_j\right] = 0$. If the latter assumption is not fulfilled, the interdependence between idiosyncratic components would clearly indicate the presence of additional common factors.

The theory states nothing as to the identity of risk factors[1]; however, if only a few systematic components of risk exist, it should be expected that they are related to key macroeconomic variables, such as GDP, the interest rate structure or inflation (see Chen, Roll and Ross (1986)).

If no arbitrage opportunities exist, each investment portfolio formed on the basis of "n" assets that meeting the conditions that they do not use wealth and have no risk (neither systematic nor idiosyncratic) must also have a return equal to zero on average. This implies that there will be $k$+1 constants, $\lambda_{o,}\lambda_1,\ldots,\lambda_k$ , such that for every I it is fulfilled that

$$E_i = \lambda_0 + \lambda_1 b_{i1} + \ldots + \lambda_k b_{ik} \tag{2}$$

[1] Roll and Ross (1980) state that equivalently CAPM does not shed any light that could explain any particular beta either.

if there is a risk-free asset with a return, $E_0$, then $b_{0j} = 0$, for every *j*, and $E_0 = \lambda_0$, then (2) may be written as

$$E_i - E_0 = \lambda_1 b_{i1} + \ldots + \lambda_k b_{ik} \qquad (2')$$

where $E_0$ represents the rate of return common to all beta zero assets, i.e., assets with $b_{0j} = 0$, for every *j,* and in particular, the risk-free rate of return if such asset does exist. In forming investment portfolios that have unit systematic risk for one factor and zero for the rest of the factors, each $\lambda_j$ may be interpreted as an excess return or premium for market risk for investment portfolios that only have the common factor j of systematic risk, $\lambda_j = E^j - E_0$. This is the central equation of the APT; it is accurate only in the case of a large economy (infinite number of securities), and is to be interpreted as an approximation in the case of finite economies (see Ross (1976 a and b) and Grinblatt and Titman (1983)).

### *A. Factor determination*

There are three methods for determining common factors: Sensitivity coefficients[2], consisting in an algorithmic analysis of the estimated covariance matrix of the securities (see Roll and Ross (1980), Chen (1983), Lehman and Modest (1988)); Macroeconomic Variables, where the researcher, based solely on his judgment, chooses factors[3] and then estimates sensitivity coefficients and verifies if they explain the cross section of the returns (see Chan, Chen and Hsieh (1985), Chen, Roll and Ross (op cit.) and Chen and Jordan (1993)); and Firm Characteristics, which resembles the previously mentioned method, though resorting to empirical regularities (anomalies) present in the returns, as for instance the size effect (see Huberman and Kandel (1987) and Chen (1993)).

The method that we will use is the Maximum Likelihood Factor Analysis[4], which is an algorithmic approach and is particularly useful if a strict factor structure, as the one modelled by Ross (1976 b), is assumed. In this structure, the idiosyncratic components of the returns of assets are assumed not to be correlated between themselves and their

[2] Also known as "factor loadings" in statistical literature.

[3] For instance, unexpected shocks in the intertemporal structure of the interest rate, in the premium for credit risk of the bonds, in inflation expectations, and in the rate of growth of industrial production or the price of oil.

[4] This method was introduced by Gehr (1975) and expanded on at a later date by Roll and Ross (1980).

variances are uniformly bounded by some finite values when the number of assets tends to infinity. An alternative method, to be recommended in the case of approximate factor structures such as those modeled by Chamberlain and Rothschild (1983)[5], would be the Asymptotic Analysis of Principal Components. Lehman and Modest (1985) compare different methods used in estimating sensitivity coefficients and conclude that the best one is Maximum Likelihood Factor Analysis resorting to as many securities as possible. A limitation to this method is that —considering that it is computationally more expensive— there has been, as stated by Huberman (1994), a trend to use it for subsets of securities; however, in our case, given the (small) number of securities the sample, this restriction is in no way relevant.

In what respects the number of priced factors in the USA, Roll and Ross (1980) find at least three and, probably, four such factors, while Huang and Jo (1995) record that the number of factors priced is one and at the most two, in the case of a later period. With data for shares traded on the Chilean stock exchange, Gregoire and Zurita (1987) report five factors during the period 1975-1987, though only one of them is priced.

### *B. Model Estimation through Factor Analysis*

In principle, the Arbitrage Pricing Theory (APT) involves three hypothesis that can be contrasted against the data (a) assuming the returns generating process (1), the valuation equation (2) can be proved; (b) in the equation (2') the intercept is equal to the risk-free rate of return or the zero beta rate of return; and (c) no variable unrelated to the model (standard deviation, size, etc.) should explain the expected returns in cross-section.

Factor analysis aims at determining whether the interrelationships between a set of observed variables are explained in terms of a small number of underlying variables (non observable variables or factors); it resembles a multiple regression except that the variables observed are regressed against non observable factors. On the basis of the variance-

[5] These authors developed the concept of the approximate factor structure, where idiosyncratic components of the returns can be weakly related among themselves. They show that both factor structures (strict and approximate) are asymptotically equivalent, where both methods yield consistent estimates of sensitivity coefficients when the number of securities is large enough. Subsequently, Grinblatt and Titman (1985) confirm this result showing that the economy of Ross and the economy of Chamberlain and Rothschild are equivalent in that investors may replicate one economy from the other simply by rearranging their investment portfolios.

covariance matrix of the returns, factor analysis enables us to determine both the number of factors as well as the sensitivity coefficients $b_{ij}$.

More specifically, since the estimating process standardizes the factors leaving them with unit variances (see J Shanken (1987)), the factor process (1) implies that variance $\sigma_i^2$ of the return of stock i is given by

$$\sigma_i^2 = \sum_{j=1}^{k} b_{ij}^2 + \phi_i \qquad (3)$$

where $\phi_i$ is the variance of $\tilde{\varepsilon}_i$ (noise term). The first term to the right is known as the "commonality" of the return and it represents the variance shared with other variables through the common factors; the second term , $\phi_i$, is the so-called specific or unique variance.

Furthermore, the covariance of the returns between the i-th and j-th stock is given by its relationships with the common factors (strict factor structure)

$$\sigma_{ij} = \sum_{l=1}^{k} b_{il} b_{jl} \qquad (4)$$

Equations (3) and (4) may be summed up as follows:

$$V = BB^T + D \qquad (5)$$

where V is the population covariance matrix, $B = [b_{ij}]$ is a $N \times k$ matrix of the sensitivity coefficients, $B^T$ is the transposed matrix B and D is a diagonal matrix of the variances that are proper to the returns (where its i-th diagonal element is $\phi_i$, which represents the variance that is proper to the stock i) under a strict factor structure.

Unfortunately, the sensitivity coefficients matrix B is not determined solely by equations (1) and (5). This is easy to ascertain, if we assume that M is an orthogonal matrix of order $k \times k$ , such that $MM^T = I$, then (5) implies

$$V = BMM^T B^T + D \qquad (6)$$

or else,

$$V = (BM)(BM)^T + D \qquad (6')$$

This latter equation implies that if the factors $\tilde{\delta}$ with their respective sensitivity coefficients $B$ provide an explanation for the observed covariance of the returns, then the factors $M^T\tilde{\delta}$ will also do so, with the respective sensitivity coefficients $BM$, for any orthogonal matrix M. For instance, clearly there will be no difference in equation (1) if the first two factors are interchanged in places, or if the respective j-th sensitivity coefficients and the j-ith coefficient are downscaled by the same constant $g$ given that $bz_{ij}\tilde{\delta}_j = gb_{ij}\left(\frac{1}{g}\tilde{\delta}_j\right)$,the distribution of the returns, will remain unaltered.

This indetermination problem in the solution makes it necessary to impose (arbitrary) restrictions to the parameters of the model, to ensure a unique solution. An (interpretable) solution consists in transforming sensitivity coefficients $B' = BM$, a process known as factor rotation; this is the alternative that we follow in this paper.[6]

In order to detect the number of factors present in the variance-covariance matrix V a likelihood ratio test is performed. Intuitively, the test that *"k"* is a sufficient number of factors would be equivalent to testing whether the idiosyncratic covariance matrix is diagonal under a structure of *"k"* factors.

Once the expected returns $E_i$ and the sensitivity coefficients B have been estimated through factor analysis, we can prove the valuation equation (2), resorting to cross-section regressions of the form

$$\hat{E}_i = E_0 + \lambda_1 \hat{b}_{i1} + \ldots + \lambda_k \hat{b}_{ik} \qquad (7)$$

The parameters $E_0$ and $\lambda_1, \ldots, \lambda_k$ are estimated by linear regression. Given that the factor analysis estimation procedure used is a maximum likelihood procedure, in a normal multivaried world the estimates are asymptotically consistent.

Several studies provide evidence of instability in the number of factors present in the stock returns, see Kryzinowsky and To (1988), Dhrymes, Friend, Gultekin and Gultekin (1985), Cho and Taylor (1987), Gultekin and Gultekin (1987) and Lehman and Modest (1988). Specifically, the number of factors with a premium seems to vary with the number of assets and with the length of the time series. By contrast, Brown (1989), Trzcinka (1986) and

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[6] Another possibility is to require that the matrix G defined by $G = B^T D^{-1} B$ be diagonal, with its elements arranged in a descending order of magnitude. This restriction drives the factors in a manner such that the first makes the maximum contribution to the common variance of the returns, the second factor also makes its maximum contribution on condition that it is not correlated to the first one, and so forth.

Shukla and Trzcinka (1990) find that a dominant factor explains a greater proportion of the variability in returns of assets and this result is robust across different sample sizes.

In addition, Cho and Taylor (1987) and Gultekin and Gultekin (1987) also arrive at the conclusion that the empirical tests of the APT model with data of returns of assets are quite sensitive to the anomalies observed in January and in small firms. Connor and Korajczyk (1993) find only one or two significant factors in months other than January but from three to six factors of returns in January.

An unstable number of factors makes the application of APT difficult. For instance, estimation and inference procedures based on maximum likelihood factor analysis requires that the structure of the factors be stable across several subsets of the universe of stocks and across several periods of time for the same sample of assets; however, unstable factors do not necessarily render the APT invalid.

On the other hand, the ATP as developed by Ross is not restricted to a specific frequency of returns. Accordingly, it has been estimated on the basis of daily returns [Dhrymes, Friend and Gultekin(1984), Dhrymes, Friend, Gultekin and Gultekin (1985), Shanken (1987)], weekly returns [Lehman and Modest (1988), Brown (1989) and Shukla and Trzcinka (1990)] and monthly returns [Roll (1988), Connor and Korajczyk (1988), Huang and Jo (1992)]. In this respect, Huang and Jo (1995) find that the ratios for the total variance explained, the number of factors and the number of priced factors are stable across the data frequencies (daily, monthly weekly) adjusting the estimation of matrix V by non-synchronous trading in the case of daily returns. The results obtained by Huang and Jo also show that the number of factors is equal to one or at the most two.

Finally, and with respect to the number of factors and size of the sample, Roll and Ross (1984) contend that it can be expected that there exist as many factors as sets of assets and that all of them may be detected with sufficiently powerful tests, and hence the number of factors should increase with the size of the sample. But almost all of them are diversifiable, and hence the number of non-priced factors in these groups is greater than the number of non-priced factors for small groups of stock or samples of a smaller time series. As a consequence, the number of real or priced factors in both groups is similar.

## II. The sample and the estimation of the model

The sample consists of the weekly stock returns of 44 securities traded on the Santiago Stock Exchange (Bolsa de Comercio de Santiago) between January 1984[7] and March 1999, and 79 securities in the sub-period 1991-1999. The results of the first sample are shown in the next section and the results for the broader sample of securities, but for a shorter period, are shown in the section that follows.

### *A. The period 1984-1999*

During this 775 week period, the stock of 91 firms were listed on the exchange throughout the complete period, and 44 of such firms with an average presence of at least 15% on the stock exchange were included.[8] The choice of the weekly frequency represents a balance between the advisability of having a series as long as possible in order to obtain more precise estimates of the variance-covariance matrix and the need to minimize problems brought about by the lack of stationarity in the series as well as by nonsynchronous trading in the series (all the more acute with daily data), affecting the consistency and bias of the estimators.

The estimation of the factor model involves the following steps: (a) estimating the covariance matrix for the group of stock under study, on the basis of the sample in time series of the returns of the stock; (b) estimating the number of factors $k$ and sensitivity coefficients $b_{ij}$ through maximum likelihood factor analysis, for the covariance matrix calculated previously, (c) using sensitivity coefficients to explain the variation in cross section of the individual returns of the stocks, which also permits (d) estimating the risk premia associated to the factors estimated.

In order to minimize the problem of spurious correlation between the returns of the stock and the risk measures and to be able to prove the real strength of the results, most empirical researches divide the time interval by two, using the first part for determining the sensitivity coefficients and the second for determining the risk premia. One alternative is

[7] The intention was to begin the sample in January 1981, but the number of firms present in the complete period went down considerably.

[8] Stock exchange presence is reported by the Santiago Stock Exchange (Bolsa de Comercio de Santiago), and it corresponds to the quotient between the number of days when the stock was traded and the number of days that the stock exchange was open during the period. Some of the 91 stock experienced transitory unlistings, trading suspensions or loss of their data.

to use the data of the returns in a net-like manner, as in Chen (1983) and Gregoire and Zurita (1987). In keeping with this in order to calculate the covariance matrix of the stock returns and estimate the sensitivity coefficients the data 1,3,5,7,9… of the net are used, while observations 2,4,6,8… of the net are used for calculating the risk premia and their statistical significance. The latter alternative is preferable because it minimizes the non-stationarity problems that may occur in the time interval of the research

Due to the (large) size of the samples and to computer-related limitations, in several of the studies that use sensitivity coefficients, the stock were divided into sub-groups. This was not necessary in our study, and therefore maximum likelihood factor analysis[9] is applied to the complete sample.

Since the theory is silent with respect to the number of factors, some criterion is needed for determining how many factors are suggested by the data. In this paper we use the Chi square test $\left(\chi^2\right)$ for large samples associated with maximum likelihood solutions.[10] Since the theory does not give any prior hypothesis as to the number of factors, a sequential procedure is followed in the determination, and which consists in giving *"k"* different values; first, we start with some small value (usually 1), then we estimate the parameters using the factor model with the maximum likelihood method. If the *U*-statistic test is not significant, the model with this number of factors is accepted, otherwise *k* is increased by one and the process is repeated, until an acceptable solution is found. If at some stage the degrees of freedom end ($v = 0$) the factor model with the assumption of the linear relationship between the variables is debatable.

Table 1 reports the results of sequentially applying test $\chi^2$ to our sample of returns, which allows to successively reject the hypotheses that at the most 1, 2, 3 and 4 factors generate

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[9] Other alternatives include of generalized least squares factor analysis, unbalanced least squares factor analysis and other approximate methods; but the maximum likelihood method has the advantage that its statistical properties are better known.

[10] This test is based on the fact that if k factors are enough to describe the returns, the statistic $U = p'Min\left(F\right)$ is distributed asymptotically as a $\chi^2$ with *v* degrees of freedom,

where $p' = p - 1 - \frac{1}{6}(2n+5) - \frac{2}{3}k$, *p* the number of observations in the time series, *n* the number of returns (observable variables), *k* represents the number of factors, F is the likelihood function, and

$$v = \frac{1}{2}\left(n-k\right)^2 - \frac{1}{2}\left(n+k\right)$$

the weekly returns, with different levels of significance (p-values of 0,000 and 0,00035, 0,012 and 0,058, respectively). On the other hand, if we continue increasing the number of factors, we have that the hypothesis that at the most 5 or 6 factors generate the stock returns can no longer be rejected according to their p-values of 0,151 and 0,328, if we consider a critical value of significance at 10%. Ultimately, we obtain as a result that five factors are enough to explain the returns.[11] Similarly, the fact that we have not found "many" factors suggests that the initial assumption of a strict factor structure is reasonable.

**Table 1.** $\chi^2$ Test of the Hypothesis that *"k"* Factors Generate Weekly Stock Returns in the Sample Studied.

| Number of factors (k) | $\chi^2$ Value | Degrees of freedom | p- values |
|---|---|---|---|
| 1 | 1123.895 | 902 | .000 |
| 2 | 1006.380 | 859 | .0003 |
| 3 | 910.488 | 817 | .012 |
| 4 | 838.797 | 776 | .058 |
| 5 | 775.631 | 736 | .151 [a] |
| 6 | 713.094 | 697 | .328 [a] |

[a] not significant at level of 10%

Having estimated both the number of factors and the sensitivity coefficients from data 1,3,5… in the net of returns, ordinary least squares regressions are run in order to estimate the price equation (2) with data 2,4,6… of the net of returns, performing 387 regressions in cross section. This results in an estimation of $\lambda_0, \lambda_1, \ldots, \lambda_5$ for each one of the 387

[11] It is to be borne in mind that if the *p*-value is larger than the critical value the only effect is that some redundant factor is being included and which may be analyzed in the following stage of the test, as it operates as a confirming part and will not bring about any major problems in the tests.

regressions, obtaining time series for each λ. With these, we test the null hypothesis that the risk premium is zero, through a means t-test.

Results are reported in Table 2. This table shows that the expected rate of return of the risk-free asset $\lambda_0$ is statistically different from zero at 1%, and that only one of the common factors (factor 3) has a statistically significant risk premium at 10%. In what respects its amount, 0.65% weekly is equivalent to an average monthly nominal rate of 2.81%. On the other hand, the average monthly inflation rate in the period was 1.1/%, with which the intercept would be consistent with an average monthly indexed (to the CPI) rate of interest of 2.43%. This rate does not seem far from the cost of debt in the period, though it is higher than the real interest rate on term deposits in the banking system in the period. However, it is worth noting that the estimation of $\lambda_0$ is quite noisy, since it stems from simultaneously estimating risk premia for a period in which inflation has featured instability from one year to another (with the only exception of the last four years, when it has displayed a systematically decreasing trend).

**Table 2. t-Test of significance for risk premia obtained using the Fama and MacBeth procedure through the regression**

$$R_t = \hat{\lambda}_0 + \hat{\lambda}_{1t}\hat{b}_1 + \hat{\lambda}_{2t}\hat{b}_2 + \hat{\lambda}_{3t}\hat{b}_3 + \hat{\lambda}_{4t}\hat{b}_4 + \hat{\lambda}_{5t}\hat{b}_5 + \hat{\xi}_t$$

| | Sampling average | t-test | Probability |
|---|---|---|---|
| $\lambda_0$ | 0.006482 | 3,841645 | 0,0001 [a] |
| $\lambda_1$ | 0.001999 | 0.487798 | 0.6260 |
| $\lambda_2$ | -0.000100 | -0.042789 | 0.9659 |
| $\lambda_3$ | 0.012344 | 1.734229 | 0.0837 [b] |
| $\lambda_4$ | 0.009234 | 1.521045 | 0.1291 |
| $\lambda_5$ | 0.002083 | 0.249320 | 0.8032 |

[a] significant at 99% of confidence.

[b] significant at 90% of confidence.

As a final test for the model, we use the standard deviation of the individual returns as an explanatory variable additional to the factors, which by null hypothesis should not have any explanatory strength. This due to the fact that even in a finite economy but with an idiosyncratic vector with sufficient independence, the diversifiable component should be eliminated by the formation of investment portfolios and the non-diversifiable part depends only on sensitivity coefficients .

According to Roll and Ross (1980) and Chen (1983) the use of the standard deviation of the returns as a variable that could explain the returns of the stock is a particularly good alternative in the attempt to reject the APT, because there exists a well documented high positive correlation between the standard deviation and the mean sampling returns.[12]

The ordinary least squares regression was estimated with the mean returns as a dependent variable calculated with data 2,4,6… from the net of returns, against sensitivity coefficients estimated in the factor analysis and the standard deviation calculated using data 1,3,5… from the net of returns

$$\overline{R}_i = \hat{\lambda}_0 + \hat{\lambda}_1 \hat{b}_{i1} + \hat{\lambda}_2 \hat{b}_{i2} + \hat{\lambda}_3 \hat{b}_{i3} + \hat{\lambda}_4 \hat{b}_{i4} + \hat{\lambda}_5 \hat{b}_{i5} + \hat{\lambda}_6 s_i + \hat{\xi}_i$$

In this case we use an ordinary least squares regression because there is no other adequate alternative because the standard deviation is not a sensitivity coefficients and therefore it is not calculated through factor analysis, thus preventing the use of any other type of regression.

According to Roll and Ross there is the possibility of a spurious effect of the standard deviation in the returns brought about by a possible bias in the distribution of the individual returns. In line with this, a positive bias may create a positive dependence between the sampling mean and the sampling standard deviation and on the other hand a negative bias would create a negative dependence between the sampling mean and the standard deviation.

[12] Dhrymes, Friend, Gultekin and Gultekin (1985) also used residual or specific standard deviation ($\phi_j$) as another possibility to attempt rejecting the APT model, which should not be significant either if the APT were valid since the effect of this variable should be eliminated by the formation of investment portfolios. Due to its simplicity, to test the validity of the APT we use the standard deviation of the returns.

A procedure that may be of help in solving this statistical problem, brought about by the possibility of a bias in the distribution of the returns, is that of simply estimating each parameter for a set of different observations. Thus the sampling dependence between the estimation of factor loading, mean returns and standard deviation is eliminated if the time series from which these parameters are calculated are not correlated temporally.

However, intertemporal dependence persists in changes of absolute prices or in squared changes. This implies that the standard deviation of the returns and the sensitivity coefficients are estimated from adjoining days will maintain some sampling dependence. In order to overcome this problem, the parameters of different sets of observations isolated at least one day must be calculated. Thus of the 775 weeks that we have for estimating the different parameters we will use only 389 weeks, using 130 weeks for estimating each one of the parameters.

Thus, the procedure involves the use of observations 1,7,13…for determining the mean returns, observations 3,9,15… for estimating the sensitivity coefficients and finally observations 5,11,17 for calculating the standard deviation.

The results of this test are shown in Table 3, in which we can see that the standard deviation does not affect the returns of the stocks (t = 0,788218), whereas the risk-free asset appears to be significant at 1% (t= 3,535451), and the remaining factors are not priced.

**Table 3. Significance Test of Standard Deviation using Different Sets of Observations Separated by Isolated Days for Calculating the Parameters**

| Factor | Risk premium | Value of t-test | Probability |
|---|---|---|---|
| $\lambda_0$ | 0.008594 | 3.535451 | 0.0011 [a] |
| $\lambda_1$ | -0.004579 | -0.765535 | 0.4488 |
| $\lambda_2$ | -0.001269 | -0.200849 | 0.8419 |
| $\lambda_3$ | 0.000634 | 0.100035 | 0.9209 |
| $\lambda_4$ | 0.000353 | 0.052937 | 0.9581 |
| $\lambda_5$ | 0.000031 | 0.004654 | 0.9963 |
| $s_j$ | 0.011926 | 0.788218 | 0.4356 |

[a] Significant at 99% of confidence.

### B. Period 1991-1999

In this section we study the number of factors and the price equation of APT in the sub-period 1991-1999. Since several stocks begin to be listed after 1984, in this period the number of securities in the sample increases substantially, and additionally they have specific characteristics that question the stationarity of the longer series.[13] Applying the

[13] In this period there was an increase in foreign investment in the Chilean stock market; for the first time ADRs were issued on the New York Stock Exchange, with the ensuing arbitrage between that stock exchange and domestic stock exchanges, and pension funds actually begin to invest in Chilean stock. All these factors had a significant bearing on the depth and liquidity of the market.

criterion of requiring stock exchange presence 15% or greater (as in the previous section), we obtain a sample of 79 stock and 420 weekly returns.

Table 4 reports the result of the sequential tests for determining the number of factors, and we find that 11 factors are enough to explain the returns at a 10% level of significance.

**Table 4.** $\chi^2$ Test for determining the number of factors needed for sample of stocks in the period 1991-1999.

| Number of Factors | $\chi^2$ | Degrees of Freedom | Significance |
|---|---|---|---|
| 8 | 2669.759 | 2477 | .004 |
| 9 | 2560.615 | 2406 | .014 |
| 10 | 2457.084 | 2336 | .040 |
| 11 | 2353.637 | 2267 | .100 |
| 12 | 2257.146 | 2199 | .190 |

Therefore, our results are consistent with those in Dhrymes, Friend, Gultekin and Gultekin (1985), in that by increasing the sample of stock for which the factor analysis is performed, we tend to need a greater number of factors to replicate the covariance matrix of the returns.

As a next step the Fama and MacBeth (1973) methodology is used for determining the number of priced factors in the sampling period. The results are shown in Table 4, which shows that the risk-free rate is significant (t = 2,403800), whereas factor 8 is priced at a 10% of significance (t= -1,843154), and the rest of the factors are not priced in the sample.

As to the profitability level of a zero-risk asset, it appears higher than in the complete sample, which is consistent with a very active role in monetary policy played by the Chilean Central Bank in the nineties. However, the estimation is noisier (as reflected by the fact that the lest is lower, although the coefficient estimated is higher), for which reason a confidence interval that contains the true value would be broader..

**Table 5.** t-Test for determining the statistical significance of risk premia for sample of stocks in the period from 1991 to 1999

| Factor | Premium | t-Test | Probability |
|---|---|---|---|
| Rf | 0.0113020 | 2.403800 | 0.0171 |
| Factor 1 | -0.008105 | -0.974631 | 0.3309 |
| Factor 2 | -0.008334 | -1.603739 | 0.1103 |
| Factor 3 | -0.013002 | -1.150789 | 0.2511 |
| Factor 4 | 0.003883 | 0.448153 | 0.6545 |
| Factor 5 | -0.00000753 | -0.001236 | 0.9990 |
| Factor 6 | -0.0098 | -1.082137 | 0.2804 |
| Factor 7 | -0.001933 | -0.269868 | 0.7875 |
| Factor 8 | -0.018374 | -1.843154 | 0.0667 |
| Factor 9 | -0.011283 | -1.564737 | 0.1192 |
| Factor 10 | -0.001570 | -0.251961 | 0.8013 |
| Factor 11 | 0.007091 | 1.001522 | 0.3177 |

The results obtained in this part of the research are consistent with prior results arrived at in other economies. For instance, Roll and Ross (1980) argued that although factor analysis calls for a greater number of factors to replicate the covariance matrix, when we increase the sample of stock with which we work, the number of factors tends to remain stable, with only the number of factors that are not priced increasing.[14] Additionally, the fact that in a previous study conducted by Gregoire and Zurita (1987) for the Chilean economy in the period 1975-1987, and in the two periods considered in this paper only one priced factor was to be found, is consistent with the observation made by Brown (1989), Trzcinka (1986) and Shukla and Trzcinka (1990), who report a dominant factor that explains most of the variability in the returns of the assets, regardless of sample size. Finally, in the aforementioned study carried out by Gregoire and Zurita (op cit.) monthly returns were used, while in this study weekly returns are used for a different sample of securities,

[14] However, the priced factor is not necessarily the same as the one found with the original sample of 44 securities, owing to the orthogonal rotation of factors performed in both cases.

obtaining in both cases only one priced factor; this result is consistent with Huang and Jo (1995), who found that the number of priced factors are stable across the frequency of the observations, and with the result of the same author that the number of priced factors in the economy of the USA is one or at the most two.

As a final exercise, we divided this broader sample of stocks into two sub-groups of 39 and 40 securities each, arranged alphabetically, with a view to compare our results with the results in the first part. In this case we obtain that the number of factors stabilizes at 8 and 5 factors, respectively, at levels of significance of 10%, that is, we find that the number of factors is more related to the number of securities in the sample than to the period itself. Furthermore, in the first group only the risk-free rate is significant (at 10%), and no risk factor is, while in the second group the risk-free rate keeps significant (at 1%) and factor 5 is priced (at 10%).

## III. Conclusions

In this paper we investigate the number of pervasive factors that would allow replicating the covariance matrix of Chilean securities in the periods 1984-1999 and 1991-1999, and which according to the APT theory could be priced in the capital market. The results show that only one factor is priced, both in the complete period as well as in the decade of the nineties, even though the number of factors detected through factor analysis increases with the number of securities in the sample, which goes up from 44 to 79 in the sub-period. These results obtained with weekly returns are consistent with research conducted in the USA and at earlier date in Chile for a previous period.

The intercept parameter, which in the APT represents the mean return of an asset free of systematic risk, is significant in all cases, and in the complete period is of an order of magnitude similar to the rate of interest on term deposits in the banking system. On the other hand, in the nineties, the estimated value of this rate in all likelihood represents the more active character of the monetary policy of the Chilean Central Bank, autonomous since the 1989 macroeconomic adjustment, and which affected the first part of the nineties.

Additionally, by including the variance in the APT price equation, this is not significant (as predicted by theory ).

Summing up, our results are consistent with the APT model, in which one factor is priced. An evident suggestion for forthcoming research work is to seek to identify such factor, correlating it with macroeconomic variables, and establishing whether it is the same variable which has explained the returns over time.

## IV. References


Brendan, M.J., 1971, "Capital asset pricing and the structure of security returns" unpublished manuscript, University of Columbia, May.

Brown, S., 1989, "The number of factors in security returns", Journal of Finance 44, 1247-1262.

Brown, S. and M. Weinstein, 1983, "A new approach to testing asset pricing models: The bilinear paradigm", Journal of Finance, June, 711-743.

Chamberlain, G. and M. Rothschild, 1983, "Arbitrage and mean-variance analysis on large asset markets", Econometrica 51, 1281-1304.

Chan, K. C., N. F. Chen and D. A. Hsieh, 1985, "An exploratory investigation of the firm size effect", Journal of Financial Economics, September, 451-471.

Chen, Nai-Fu, 1983, "Some empirical tests of the theory of arbitrage pricing", Journal of Finance 38, 1393-1413.

Chen N-F., R. Roll and S. Ross, 1986, "Economic forces and the stock market", Journal of Business, 383-403.

Chen S-J. and B. Jordan, 1993. "Some empirical tests in the arbitrage pricing theory: macrovariables vs. Derived factors", Journal of Banking and Finance 17, 65-89.

Cho, D. and W. Taylor, 1987, "The seasonal stability of the factor structure of stock returns", Journal of Finance 42, 1195-1211.

Connor G. and R. Korajczyk, 1988, "Risk and return in an equilibrium APT: application of a new test methodology", Journal of Financial Economics 21, 255-89.

----, 1993, "A test for the number of factors in an approximate factor model", Journal of Finance 48, 1263-1291.

Dhrymes P. J., I. Friend and N. B. Gultekin, 1984, "A critical reexamination of the empirical evidence on the arbitrage pricing theory", Journal of Finance 39, 323-345.

Dhrymes P. J., I. Friend, M. N. Gultekin and N. B. Gultekin, 1985, "New tests of the APT and their implications", Journal of Finance 40, 659-675.

Fama. E. and J. MacBeth, 1973, "Risk, return and equilibrium: Empirical tests", Journal of Political Economy 81, 607-636

Gehr, A., 1975, "Some tests of the Arbitrage Pricing Theory", Journal of the Midwest Finance Association, 91-105.

Gregoire Jorge and Zurita Salvador, 1987, "Teoría de Precios Por Arbitraje: Una Prueba Empírica", en Lecturas de Economía Financiera, Editorial de Economía y Administración, Universidad de Chile, 1995.

Grinblatt, M. and S. Titman, 1983, "Factor pricing in a finite economy", Journal of Financial Economics, 12, 497-507

----, 1985, "Approximate factor structures: Interpretations and implications for empirical tests, Journal of Finance 40, 2367-2373.

Gultekin, M. and N. Gultekin, 1987, "Stock return anomalies and the tests of the APT", Journal of Finance 42, 1213-1224.

Huang R. and H. Jo, 1992, "Transformed Securities and alternative factor structures", Journal of Finance 47, 397-405.

----, 1995, "Data frequency and the numbers of factors in stock returns", Journal of Banking & Finance 19, 987-1003.

Huberman, 1994, "Arbitrage Pricing Theory" en The New Palgrave Dictionary of Money and Finance, P. Newman, M. Milgate and J. Eatwell (eds.), The Macmillan Press Limited, London, The Stockton Press, New York, Volumen 1.

Huberman, G. and S. Kandel, 1987, "Mean variance spanning", Journal of Finance 42, 873-88.

Kryzanowsky, L. and M. To, 1983, "General factor models and the structure of security returns", Journal of Financial and Quantitative Analysis 18, 31-52.

Langetieg,T., 1978, "An application of a three factor performance index to measure stock holder gains from merger", Journal of Financial Economics 6, 365-383.

Lee, C. and J. Vinso, 1980, "Single vs. Simulation equation models in capital asset pricing: The role of firm related variables", Journal of Business Research, 65-80.

Lehman, B. and D. Modest, 1985, "The empirical foundations of the arbitrage pricing theory II: the optimal construction of basis portfolios", Department of Economics, Working Paper 292, Columbia University.

----, 1988. "The empirical foundations of the arbitrage pricing theory", Journal of Financial Economics 21, 213-254.

Kryzanowski, L. and M. To, 1983, "General factor models and the structure of security returns", Journal of Financial and Quantitative Analysis 18, 31-52.

Meyers, S,. 1973, "A re-examination of market and industry factors in stock price behaviors", Journal of Finance 28, 695-706.

Roll, R., 1988, "$R^2$", Journal of Finance 43, 541-566.

Roll R and S. Ross, 1980. "An empirical investigation of the arbitrage pricing theory", Journal of Finance 35,1073-1103.

----, 1984. "A Critical Reexamination of the Empirical Evidence on The Arbitrage Pricing Theory: A Reply", Journal of Finance 39, 347-350.

Ross, Stephen, 1976a, "Return, risk and arbitrage", en I. Friend and J. Bicksler (eds.), "Risk and return in finance", Cambridge, Massachusetts: Ballinger.

----, 1976b. "The Arbitrage Theory of Capital Asset Pricing", Journal of Economic Theory 13, 341 –360.

Schwarz G., 1978. "Estimating the dimension of a model", The Annals of Statistics 6, 461-464.

Shanken, J., 1987, "Nonsynchronous Data and the Covariance- Factor Structure of Returns", The Journal of Finance 42, June, 221-31.

Shukla R. and C. Trzcinka, 1990, "Sequential tests of the arbitrage pricing theory: a comparison of principal components and maximum likelihood factors", Journal of Finance 45, 1541-1564.

Trzcinka,C., 1986, " On the number of factors in the arbitrage pricing model", Journal of Finance 41, 347-368.

Tucker L. and C. Lewis, 1973. "A reliability coefficient for maximum likelihood factor analysis", Psychometria 38, 1-10.