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Optimizing flotation bank performance through froth depth profiling: Revisited

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ABSTRACT

This communication revisits previously reported results on froth depth profiling along a rougher flotation bank. The optimization problem is reformulated as to maximize the overall bank Cu recovery subject to a lower bound constraint on the overall Cu concentrate grade. This formulation differs from that originally proposed in Maldonado et al. (2007) where the sum of the squared Cu tailing grade of each cell group was minimized for a given target bank Cu concentrate grade. A semi-empirical steady-state mathematical model of a bank of cells previously validated using industrial flotation data from Los Pelambres mine in Chile was used to simulate the process. In order to improve resolution a genetic algorithm was implemented to search for the optimal froth depth profile as opposed to the discrete dynamic programming technique originally implemented. Results show that optimal froth depth profiling resulting from solving the reformulated optimization problem produces an increase in the overall bank recovery for a given target Cu concentrate grade compared to that obtained when solving the original formulation. Moreover, the resulting optimal mass-pull profile tends to be more balance in the first cells which partially agrees with recent observations.

1. Introduction

Flotation banks also known as rows or lines are serial arrangement of cells where the tail stream of one cell is the feed stream to the next cell down the bank. Bank optimization consists of selecting the best operating conditions in each flotation cell such that the overall bank metallurgical performance is optimized. Bank optimization has received increasing attention as flotation stages such as roughing, cleaning and scavenging are all made up of bank of cells and their optimization is therefore an important step toward the optimization of a whole flotation plant. In addition, optimization of the roughing stage is particularly important as this stage is commonly encountered in an open circuit configuration, i.e., rougher tails being reported to the final tailing, therefore any performance losses at this stage cannot be compensated elsewhere. Moreover, current trend of having short banks of larger cells demands for better operational practices as any poor performance in any of these large cells will have a significant detrimental effect on the overall bank performance. Several strategies have been devised to improve bank performance such as: gas profiling (Cooper et al., 2004; Aslan et al., 2010; Smith et al., 2010), froth-depth profiling (Maldonado et al., 2007; Bergh and Yianatos, 2013), peak-air recovery profiling (Hadler and Cilliers 2009; Hadler et al., 2012) and froth velocity (mass pull) profiling (Supomo et al., 2008; Figueroa et al., 2009). Recently, a balance recovery profiling has found to be optimal in the sense of maximizing the separation efficiency for a given target overall bank recovery for the case of having only true floating materials involved (Maldonado et al., 2011). This has been recently confirmed using JKSimFloat simulations (Singh and Finch, 2014) and through an optimization campaign conducted at a talc operation in Timmins, Canada where improvements in both grade and yield were achieved as recovery down the bank moves toward a balance profile (Blonde et al., 2013). For the more general case involving gangue entrainment, Singh and Finch (2014) have recently suggested, using a mathematical analysis and simulations, that a balanced mass-pull profile would be the optimal policy. This communication revisits results on froth depth profiling along a simulated rougher Cu flotation bank described in (Maldonado et al., 2007) and its connection to mass-pull profiling. Specifically, the

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objective function originally proposed in Maldonado et al. (2007) consisting of minimizing the sum of the squared Cu tailing grades down the bank for a given target Cu concentrate grade is recast as to maximizing the overall Cu recovery while satisfying a constraint on the minimum allowed Cu concentrate grade. To improve resolution a genetic algorithm search method was used instead of the discrete dynamic programming originally implemented. The article is organized as follows: next section briefly describes the flotation bank under study, Section 3 details the genetic algorithm parameters utilized. Then, simulation results are presented in Section 4 and finally discussion and concluding remarks are provided in Sections 5 and 6 respectively.

2. Rougher flotation modeling

A static model of a bank of nine self-aerated Wemco 4500 ft^3 flotation cells previously validated using reconciled data obtained from several sampling campaigns at Los Pelambres Mine in Chile was used (Maldonado et al., 2007). The mathematical model is included in Appendix. Samples were taken at the feed, concentrate and tails of each group of cells as pointed out in Fig. 1 using dots. Mass balance equations for copper, iron and gangue were implemented. The bank of cells is arranged in a 1-2-2-2-2 configuration (i.e., 1 single cell followed by 4 pairs of 2 cells) as shown in Fig. 1. Froth depth was measured in the last cell of each group of cells, i.e., in cell 1, 3, 5, 7 and 9.

3. Optimization using genetic algorithm search method

Advances on computer technology have allowed the application of new optimization algorithms. Genetic algorithm (GA) is a metaheuristic technique inspired by the Darwin's theory of evolution by natural selection and uses a direct analogy of its mechanisms such as selection, reproduction, crossover and mutation. GA technique is a population based approach that provides best population of parameters after every generation based on a predefined objective function (Goldberg, 1989). Due to their capability to explore a large searching space GAs have been successfully applied in many fields. Among applications of GA to mineral processing we have: optimization of comminution processes (Svendensten and Evertsson, 2005; Farzanegan et al., 2009; Wang et al., 2010), coal preparation plants (Gupta et al., 2007) and flotation (Guria et al., 2005). More recently, Ghobadi et al. (2011) proposed a GA oriented process-based rules to find the optimum flotation circuit configuration for two and four-stage flotation circuits. In this communication GAs are used in conjunction with a mathematical model of a rougher flotation bank to determine the froth depth profile that optimizes the overall bank metallurgical performance. The MatLab© optimization toolbox was used in order to run the genetic algorithm searching method. The GA parameters chosen for simulation in MatLab are described below:

(a) *Variable coding*: In order to perform the Genetic Algorithm and use the operators the independent variables located into the chromosomes of every individual are codified into a Double vector population type.

(b) *Fitness function*: the overall Cu bank recovery is selected as the objective function and its maximization is represented as the minimization of the negative overall Cu recovery, i.e.

$$\text{minimize} - R_{Cu}$$

The independent variables selected for this purpose are the froth heights (h_{Fi}). In this particular case the chromosomes of each individual correspond to the froth depth in each group of cells as shown in Fig. 2.

(c) *Constraints*: the overall Cu concentrate grade cannot be lower than a certain minimum concentrate grade G_{min}, i.e. $G_{Cu} - G_{min} \geqslant 0$

There are also physical constrains on the value of the froth depth which were determined according to Los Pelambres plant operational practice, i.e., from 10 to 400 [mm]. This constraint is mathematically expressed as follows:

$$10 \leqslant h_F \leqslant 400[\text{mm}]$$

(d) *Initializing population*: The common method is to create randomly solutions (froth depth profiles). The default population type (double vector) and size (20) were selected while respecting the operational range described above.

(e) *Parent Selection*: The roulette wheel method has been chosen for the selection step. Roulette selection chooses parents by simulating a roulette wheel, in which the area of the section of the wheel corresponding to an individual is proportional to the individual's expectation (fitness). The algorithm uses a random number to select one of the sections with a probability equal to its area. This elitist strategy is used to keep the best individuals from the current generation to the next generation.

(f) *Reproduction operators* (crossover and mutation): Offspring's are created by the action of the two operators, crossover and mutation. The crossover and mutation operators are the two most important space exploration operators.
Crossover: this option specifies how the genetic algorithm combines two individuals, or parents, to form a crossover child for the next generation. The crossover operator can generate unfeasible chromosomes as in the initialisation step. Arithmetic crossover was selected from the option menu. This function creates children that are the weighted arithmetic mean of two parents. If a chromosome violates this constraint, it is repaired with permutations from parent chromosomes. As a result children are always feasible with respect to linear constraints and bounds.
Mutation: The mutation operator enables to introduce unexplored search space to the population. Mutation options specify how the genetic algorithm makes small random changes in the individuals in the population to create mutation children.

(g) *Stopping criteria*: The stopping criterion used in this case was the pre-defined number of generations. All others stoppings criteria where disabled in order to accomplish the proposed specification (see Table 1).

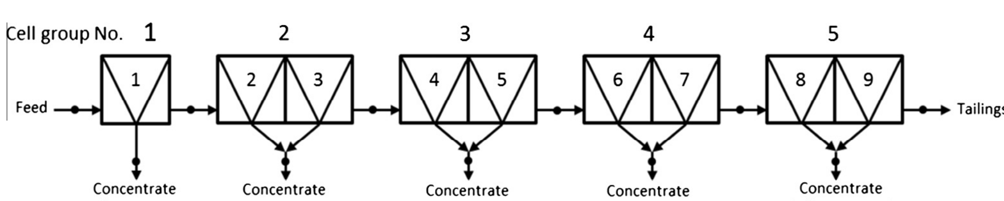

Fig. 1. Flotation bank configuration arrangement.

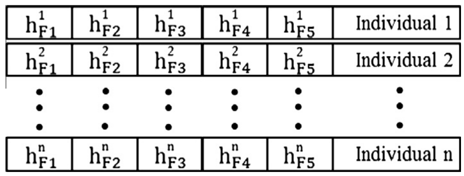

Fig. 2. Each individual is composed by 5 chromosomes. Each chromosome corresponds to the froth depth of each cell group identified by a number *i* from 1 to 5.

Table 1
Summarizes the GA settings parameters used in MatLab© optimization toolbox.

Search parameters settings used in genetic algorithm toolbox	
Parameter	Setting
Population size	20
Initial range	10–400 mm
Population type	Double vector
Fitness scaling	Rank
Selection function	Roulette
Reproduction elite count and crossover fraction	2; 0.8
Mutation function	Constraint dependent
Crossover function	Arithmetic
Migration direction, fraction and interval	Forward; 0.2; 20
Stopping criteria	Generations; 500

4. Simulation results

During simulations, feedrate was set to 1252 tonnes per hour (dry solids) at 39% solids by weight and Cu and Fe feed grades were set to 1.03% and 2.12% respectively. Table 2 compares the optimal froth depth profiles and their resulting overall Cu recovery and concentrate grade for two formulations of the optimization problem:

- *OPT1*: corresponds to the original formulation proposed in Maldonado et al. (2007), i.e., minimization of the sum of the squared Cu tailings of each cell group for a given overall Cu concentrate grade (16%, 18% and 20%).
- *OPT2*: corresponds to the reformulated problem proposed in this communication, i.e., maximization of the overall Cu recovery while satisfying a minimum Cu concentrate grade (16%, 18% and 20%).

From Table 2 it can be observed that the inequality constraint imposed on the Cu concentrate grade in the formulation OPT2 turns into an active equality constraint, i.e., since there is a trade-off between recovery and grade, maximizing the overall Cu recovery pushes the overall concentrate grade to its lower bound, as expected. It can also be observed that the higher resolution of the implemented GA compared to the original discrete dynamic programming method used makes the overall Cu concentrate

Table 2
Optimal froth depth profiles and metallurgical assessment variables for three scenarios of concentrate grade constraints (16%, 18% and 20%).

Scenario	Optimization formulation	Froth depth (mm) in cell group					Overall bank Cu	
		No. 1	No. 2	No. 3	No. 4	No. 5	Recovery (%)	Concentrate grade (%)
1	OPT1	33.8	40.8	191.6	390.8	397.7	92.34	15.8
	OPT2	174.6	40.1	23.7	370.2	400.0	93.08	16.0
2	OPT1	51.5	75.7	218.8	383.9	380.8	91.48	17.96
	OPT2	138.8	58.8	44.9	374.1	400.0	92.12	18.0
3	OPT1	71.2	155.7	169.5	393.1	383.2	90.95	19.48
	OPT2	147.1	57.4	146.6	392.2	400.0	91.19	20.0

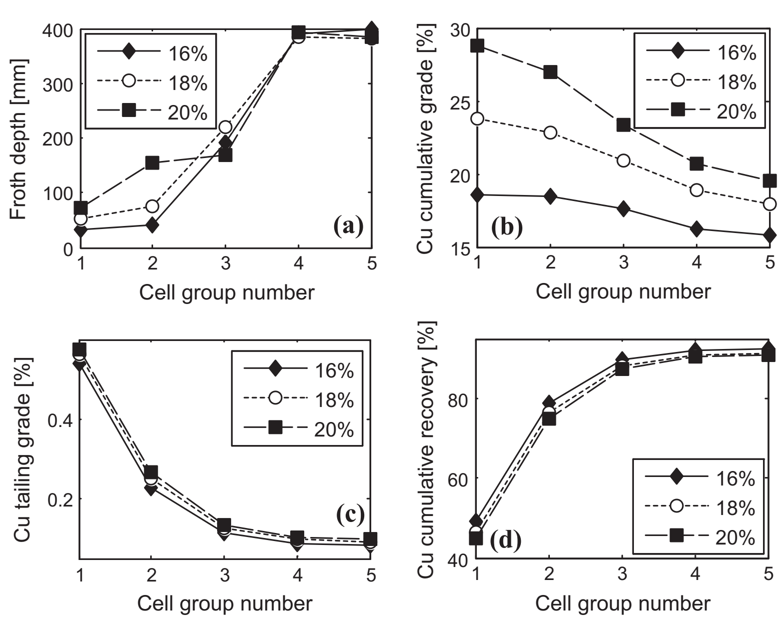

Fig. 3. Optimal profiles resulting from solving optimization problem OPT1 for different Cu concentrate grade. (a) Froth depth. (b) Cu cumulative grade. (c) Cu tailing grade. (d) Cu cumulative recovery Maldonado et al. (2007).

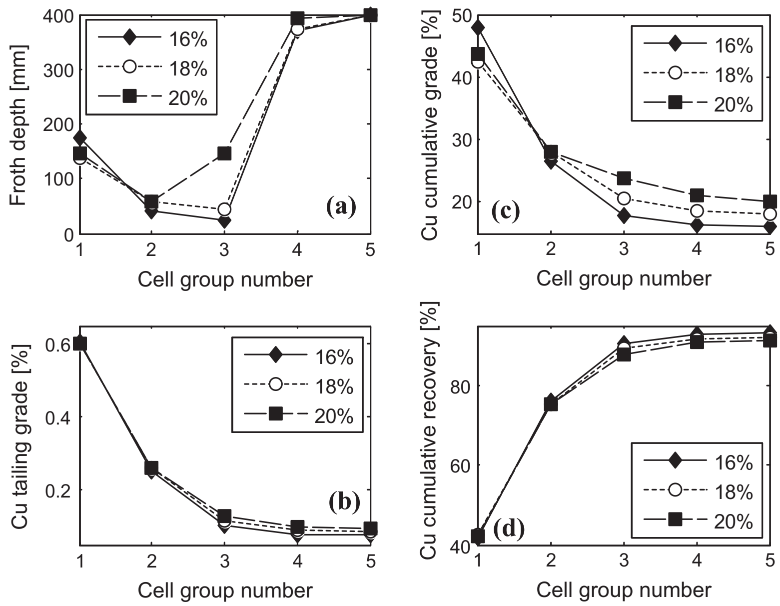

Fig. 4. Optimal profiles resulting from solving optimization problem OPT2 for different Cu concentrate grade. (a) Froth depth. (b) Cu cumulative grade. (c) Cu tailing grade. (d) Cu cumulative recovery.

grade perfectly met the lower bound constraint (16%, 18% and 20%). In each scenario, the solution to the new problem formulation OPT2 provides better metallurgical performance than that obtained when solving OPT1, i.e., higher recovery and grade.

Fig. 3 shows the optimal profiles (froth depth, cumulative Cu grade, Cu tailing grade and cumulative Cu recovery down the bank) obtained from solving the original problem formulation OPT1 for three target Cu concentrate grades, namely, 16%, 18% and 20% (Maldonado et al., 2007). It can be observed that an increasing froth depth profile is obtained, i.e., shallow froths in the first cells and deeper froth depths down the bank. Shallow froths increase recovery and therefore are in line with the objective of minimizing Cu tailing grade in each cell group. However as the final concentrate grade must meet a specific value froth depth are subsequently increasing down the bank. Thus, the cumulative concentrate grade reduces monotonically to its target final value as shown in Fig. 3(b).

Fig. 4 shows the optimal profiles obtained from solving the reformulated problem OPT2. It can be observed that the optimal froth depth profile is not increasing anymore but actually decreases from the first cell down to the third cell group and then increases rapidly. A deeper froth depth in the first cell produces a higher Cu concentrate grade as shown in Fig. 4(b). Subsequently, froth depth in cell groups 2 and 3 is reduced to maximize recovery. Finally, froth depth in the last two cell groups is increased to reduce concentrate degradation due to gangue entrainment.

Fig. 5 shows the concentrate mass-pull profile generated when implementing the optimal froth depth profiles resulting from solving OPT1 and OPT2. Due to the uneven distribution of cells in the bank, i.e., the first cell group consists of a single cell whereas the others are made of two, the mass-pull of a given cell in a cell group having two cells is assumed to be half of the mass-pull of the cell group. It can be observed that the optimal froth depth profile

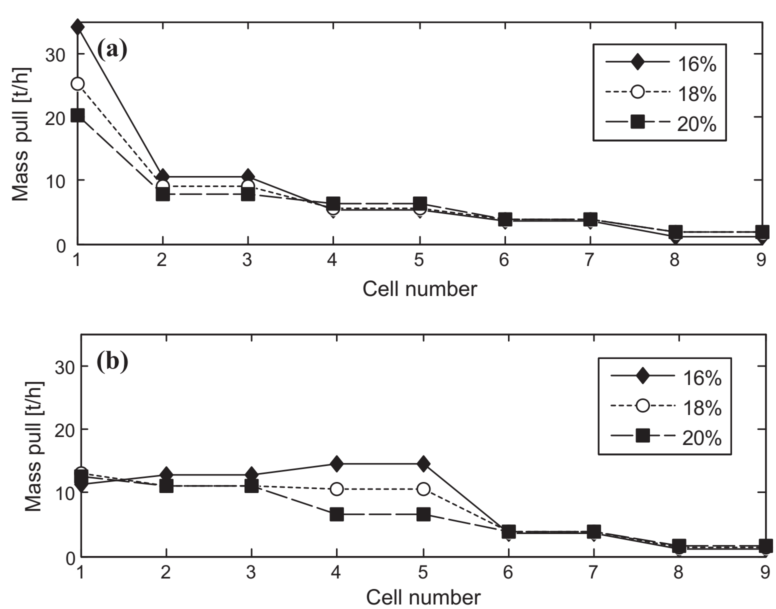

Fig. 5. Optimal mass-pull profiles. (a) OPT1. (b) OPT2.

obtained from solving OPT2 suggests a more balanced mass-pull profile compared to that obtained from solving OPT1.

5. Discussion

In the original article of Maldonado et al. (2007) the optimization problem was formulated as to minimize the sum of the squared Cu tailing grade in each cell group for a given target concentrate grade. The origin of this formulation can be traced back to the single unit case where for a given feed and target concentrate grade minimizing the tailing grade is equivalent to maximizing recovery. Results show that this rational is lost when optimizing a bank, i.e., minimizing the tailing grades of each cell does not directly translate into the maximization of the overall bank recovery when a constraint on the overall concentrate grade is imposed. The solution to this problem produced increasing froth depth profiles i.e., shallow froths in the first cells and deeper froth depths down the bank, which causes the first cells to overpull producing an unnecessary detrimental effect on the concentrate grade that cannot be compensated by the subsequent cells down the bank. This solution agreed with the operating policy in place at that moment in Los Pelambres as reported in Maldonado et al. (2007). On the other hand, the resulting optimal froth depth profile that maximizes the overall recovery for a minimum concentrate grade produces a U-shape profile, i.e., grade is increase in the first cells by allowing the froth depth to become deeper, then froth depth reduces in the subsequent cells to increase Cu recovery and finally increasing again at the end of the bank to prevent concentrate grade reduction due to entrained particles. With this policy in place, an improvement in recovery and concentrate grade was observed as compared to the original formulation. Moreover, the resulting mass-pull profile tends to be more balance in the first banks and then drops at the end of the bank. This suggests that mass-pull in each cell can be used as the independent variable and a balance mass-pull profile as a guideline toward bank optimization. As more flotation plants are incorporating froth cameras, froth velocity control can be used to modify mass-pull profiling.

6. Concluding remarks

The problem of optimizing the operation of a bank of flotation cells using froth depth profiling has been revisited. The optimization problem has been recast as to maximizing the overall Cu bank recovery while satisfying a lower bound constraint on the overall Cu concentrate grade. The optimal froth depth to each cell group was determined using a genetic algorithm search method implemented in MatLab Optimization Toolbox. Results show that a U-shape froth depth profile, i.e., initially decreasing froth depth to subsequently increasing froth depth profile down the bank, turned out to be optimal in the sense of maximizing recovery while respecting a minimum concentrate grade. This optimal profile also provides a more balanced mass-pull profile which partially agrees with recent reports.

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Appendix A. Mathematical model of a flotation bank

Nomenclature

symbol	description
m	mass of solids in the cell
M	dry solid mass flowrate
G	grade
Q	volumetric flow rate of slurry
h_P	pulp level
h_F	froth depth
A_c	flotation cell cross-sectional area
ε_g	collection zone gas hold-up
k	overall flotation rate constant
H	height of flotation cells
subscripts	
i	components (Cu, Fe, Gangue)
j	cell number
F, C, T	feed, concentrate, tails respectively
α, β, θ	constants

Mass of component i in cell j

$$m_{ij} = \frac{M_{T_{j-1}} \cdot G_{T_{i(j-1)}}}{k_{ij} + \frac{Q_{T_j}}{A_c(1-\varepsilon_g)h_{P_j}}}$$

Volumetric flowrate of tails in cell j

$$Q_{T_j} = Q_{T_{j-1}} - Q_{C_j}$$

Froth depth in cell j

$$h_{F_j} = (H - h_{P_j}) \cdot 1000$$

Overall flotation rate constant for component i in cell j

$$k_{ij} = f(h_{F_j}, G_{T_{i(j-1)}}, Q_{T_{j-1}}, \theta)$$

Numerical values of the fitting parameters θ for each species in each flotation cell can be found in Maldonado (2006).

Volumetric flow rate of concentrate in cell j

$$Q_{C_j} = \alpha_j - \beta_j \cdot h_{F_j}$$

Concentrate mass flow rate of component i in the concentrate stream of cell j

$$M_{C_{ij}} = k_{ij} \cdot m_{ij}$$

Concentrate mass flow rate of component i in the tailing stream of cell j

$$M_{T_{ij}} = \frac{Q_{T_j}}{A_c(1-\varepsilon_g)h_{P_j}} \cdot m_{ij}$$

Concentrate grade of component i in cell j

$$G_{C_{ij}} = 100 \cdot \frac{M_{C_{ij}}}{\sum_{k=1}^{3} M_{C_{kj}}}$$

Tailing grade of component i in cell j

$$G_{T_{ij}} = 100 \cdot \frac{M_{T_{ij}}}{\sum_{k=1}^{3} M_{T_{kj}}}$$

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