# Application of linear stability via Lyapunov exponents in high dimensional electrical power systems

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ABSTRACT

Applications of linear stability via Lyapunov exponents in high dimensional test systems are presented. The stochastic disturbance model is given by a bounded Markov diffusion process, as it appears in the description of load or generation uncertainties of power systems, for example. For such systems, the Lyapunov exponents describe necessary and sufficient conditions for almost sure asymptotic stability. The present article reports results obtained by applying the proposed methodology to four-machine ten-bus, ten-machine thirty-nine-bus, and sixteen-machine sixty-eight-bus international test systems.

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## Introduction

At present, electric power systems are subjected to a variety of random disturbances that affect the behavior of the system, including small disturbance and transient analysis. Techniques have been reported that allow describing the indicated phenomena, with probabilistic analysis as one of the main contributions to the study of stability. This approach is oriented at the analysis of contingencies and safety/reliability, where the main argument is to assign a probability value to the occurrence of certain events that are of interest for the study that may be made. In this way, the probability that the system will be stable is calculated using the distribution functions of the elements that represent the random behavior of the system.

Within the context of transient stability, the random behavior of the systems has been approached considering different system scenarios and parameters which are associated with an occurrence probability [1–10]. In terms of quantitative assessment, reference [11] presents an index that allows determining the vulnerability when facing a voltage collapse, establishing that consumption has a random behavior. This stability index corresponds to the time for leaving a stable operation zone.

Stability studies of small probabilistic disturbances are proposed in references [12–20]. One of the main approaches is to assign a probability distribution of the real parts of the eigenvalues obtained from the linear equivalent model of the electric system, and then determine the probability that the real parts will be located in the left half-plane. In this context, applications are also presented in which the PSS (Power System Stabilizer) controllers are used to decrease the effect of the disturbances that affect the operation but occur in a single instant of time. Moreover, applications are included in which indicators are defined that allow the evaluation of the behavior of the electric system subjected to disturbances that are defined by the system's operator. However, one strategy for analyzing the conditions under which the disturbance is sustained in time has not been proposed.

To account for the random and permanent effect over time, Lyapunov exponents have been used as stability indices to analyze power systems [21]. Reference [22] gives a theoretical description of the calculation of Lyapunov exponents in structures, considering low dimension systems as applications. However, these studies are still in a theoretical stage and no numerical methods for estimating them are shown. With respect to the disturbance model, white noise has been considered traditionally to characterize the random and sustained in time effect that affects the permanent regime operation of electric systems [23]. That model is a stochastic process that has nonbounded trajectories and considers microscopic time scales in its dynamics. On the other hand, the phenomena of interest in stability correspond to load variations, power fluctuations in the generators, unforeseen line dropping out, etc., which have bounded magnitudes and take place in much greater time scales than those of Brownian motion.

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In brief, the specialized literature shows important progress in the study of stability considering random disturbances, but it is necessary to model those phenomena with dynamics sustained over time. In this paper a methodology is presented to generalize the analysis of small disturbances through the calculation of Lyapunov exponents, considering the presence random and self-sustained time disturbances. Results are given for the four- and ten-machine systems described in the international literature.

The paper is structured as follows: Section 'System model' gives the mathematical model used to represent an electric power system as a stochastic linear system. Section 'Numerical methods for estimating the Lyapunov exponent in stochastic linear systems' presents the numerical methods that were implemented in international test systems. Section 'Case studies' shows the results obtained for the cases analyzed and the analyses made. Finally, Section 'Conclusions' includes the conclusions and future work.

## System model

### Previous mathematical concepts

The general model for studying a linear system subjected to stochastic disturbances is detailed in this section.

To begin, let us consider an electric power system represented by a nonlinear system of the form

$$\dot{y} = f(y, p) \tag{1}$$

The system is defined in the state space $\mathbb{R}^N$, where $N_1 = 2n$ corresponds to the relative angles of the rotors $(\delta_1, \ldots, \delta_n)$ and velocities $(\omega_1, \ldots, \omega_n)$, and $N_2$ corresponds to the rest of the state variables $(N_1 + N_2 = N)$. The vector $p \in \mathbb{R}$ corresponds to the variables that can be fitted to guarantee the optimum operation of the system.

Let us also consider $y^* \in \mathbb{R}$ as a set point (operation point) of system (1) and the equivalent linear system at that point, given by

$$\Delta\dot{x} = A(p)\Delta x \tag{2}$$

The classical small disturbance stability study is based on getting the eigenvalues of the linear system of Eq. (2). If the real parts are negative, the system will be stable [28]. However, at the time of considering random and self-sustained in time disturbances, the classical approach stops being practical for the following reasons:

- The state matrix $A$ will be variant in time and for every instant $t$ there will be a different set of eigenvalues, making this approach impracticable due to the number of elements to be analyzed.
- On the other hand, there are examples in which the time-variant system has eigenvalues with negative real parts, but it is not stable [24]. To illustrate this idea, let us consider the following linear system:

$$\Delta\dot{x} = \begin{pmatrix} -1 - 9\cos^2 6t + 6\sin 12t & 12\cos^2 6t + \frac{9}{2}\sin 12t \\ -12\sin^2 6t + \frac{9}{2}\sin 12t & -(1 + 9\cos^2 6t + 6\sin 12t) \end{pmatrix} \Delta x$$

In this case, $x \in \mathbb{R}$ and for every $t \in \mathbb{R}$ the characteristic polynomial is $\lambda^2 + 11\lambda + 10 = 0$, which has roots $-1$ and $-10$. From the traditional standpoint, the system is stable. However, the system's fundamental matrix is given by

$$\begin{pmatrix} e^{2t}(\cos 6t + 2\sin 6t) & e^{-13t}(-2\cos 6t + \sin 6t) \\ e^{2t}(\cos 6t - 2\sin 6t) & e^{-13t}(2\cos 6t + 2\sin 6t) \end{pmatrix}$$

The components do not tend to zero as time increases to infinity, so the system is not stable [31]. This shows that when the

system is time-variant, the classical analysis of eigenvalues is not sufficient.

The linear system of Eq. (2), when subjected to random and sustained in time disturbances, can be described by the following model [26]:

$$\Delta\dot{x} = A(p, \xi_t)\Delta x \tag{3}$$

where

- $\xi_t$ is a random process that takes values in some set $U \subset \mathbb{R}^m$ and describes the way in which the disturbance affects the states of the system and
- $p$ is the set of control parameters (the gains of the machine controllers, for example).

Denoting the solution of Eq. (3) at time $\geqslant 0$ with an initial value $x_0 \in \mathbb{R}^d$ by $\phi(t, x, \xi_t)$, the exponential behavior is given by the Lyapunov exponents

$$\lambda(x, v) = \limsup_{t \to \infty} \frac{1}{t} \log(\|\phi(t, x, \xi_t(v))\|) \tag{4}$$

where $v$ is an element that represents a realization of the stochastic process that models random disturbance self-sustained in time.

In general, the stochastic system of Eq. (3) with ergodic disturbance $\xi_t$ can have a number $d$ of Lyapunov exponents. Under the conditions indicated in [25] there is a unique exponent with probability 1 given by the following expression:

$$\lambda(x, v) = \lim_{t \to \infty} \frac{1}{t} \log(\|\phi(t, x, \xi_t(v))\|) \tag{5}$$

According to [26], the stochastic linear system of Eq. (3) would be exponentially stable if and only if $\lambda < 0$.

### Electric power system disturbance models

Random and sustained in time variations that affect the operation of electric power systems can be found at the generation, transmission and distribution levels. Unconventional renewable energy sources, distributed generation, and sustained consumption increase introduce nondeterministic components that must be modeled and considered to evaluate the system's response in permanent and dynamic regime.

In this paper we present the results obtained when considering two sources of uncertainty in the operation of electric systems:

### Excitation system subjected to stochastic disturbances

To illustrate the model, let us consider that the currents in the generators in $dq$ coordinate axes vary as a function of time due to bounded disturbances around the system's stable operation point. In that case it is possible to represent the current components as follows:

$$I_{di} = I_{di}^{ss} \cdot (1 + \rho \cdot \sin(\xi_t))$$
$$I_{qi} = I_{qi}^{ss} \cdot (1 + \rho \cdot \sin(\xi_t)) \tag{6}$$
$$S_E(E_{fd})_i = S_E(E_{fd}) \cdot (1 + \rho \cdot \sin(\xi_t)), \quad i = 1, \ldots, n.$$

where

- $I_{di}$: current component along axis $d$ in machine $i$,
- $I_{qi}$: current component along axis $q$ in machine $i$,
- $S_E(E_{fd})_i$: represents the excitation system's saturation in machine $i$ [29],
- $ss$: the operating conditions in permanent deterministic regime,
- $n$: the number of machines in the system, and
- $\rho$: the size of the disturbance that affects the system.

*Random variations in the system's consumption*

To represent the random and self-sustained in time variations that can be experienced by the consumption of an electric system, let us consider that the voltages in the load busbars have a time dynamic given by Eq. (7).

$$\theta_{Load^i} = \theta_{Load^i}^{ss} \cdot (1 + \rho \cdot \sin(\xi_t)) \tag{7}$$

$$V_{Load^i} = V_{Load^i} \cdot (1 + \rho \cdot \sin(\xi_t)), \quad i = 1, \ldots, n. \tag{8}$$

where

- $\theta_{Load^i}$: denotes the angle of the voltages in the load busbar $i$,
- $V_{Load^i}$: denotes the voltage angle in the load busbar $i$,
- $ss$: the operating conditions in permanent deterministic regime,
- $m$: the number of load busbars in the system, and
- $\rho$: the size of the disturbance that affects the system.

Considering the representations of the disturbances according to Eqs. (6)–(8), the electric system is described by the stochastic linear system of Eq. (3).

To model disturbance $\xi_t$ we use the results of Refs. [15,25,27], where it is stated that the Ornstein–Uhlenbeck process can be used to model random phenomena present in the electric power systems. Based on the above, we model $\xi_t$ according to

$$\xi_t = \rho \cdot \sin(\eta_t), \quad \rho \geqslant 0 \tag{9}$$

where

- $\xi_t$: corresponds to the solution of the following stochastic differential equation of the Ornstein–Uhlenbeck model:

$$d\eta_t = -\gamma \eta_t dt + \psi dW_t \tag{10}$$

Parameters $\gamma$ and $\psi$ are estimated from the real measurements of the phenomenon that it is desired to model. Here we specify $\gamma = 1$ and $\psi = 1$ as a particular case of what is reported in reference [15].

## Numerical methods for estimating the Lyapunov exponent in stochastic linear systems

This section summarizes three numericals methods reported in [26] that allow the estimation of the Lyapunov exponent for stochastic linear systems.

Let us consider the system of Eq. (3) and the following parameters:

- $\alpha$: number of initial conditions,
- $\beta$: number of realizations of disturbance $\xi_t$, which represents the random and self-sustained in time disturbances, and
- $T$: simulation time for calculating the Lyapunov exponent.

According to the definition given in Eq. (5), three numerical methods are presented in reference [26]:

*Method 1: Trajectory averages in the linear system*

For simulation time $T$, trajectory $i$ of disturbance $\xi_t$, and initial condition $x_0^j$ we have solution $x_n^j(i)$ of Eq. (3), from which the following calculation is made:

$$\lambda^j(i) = \frac{1}{T} \cdot \sum_{n=1}^{T} \log \|x_n^j(i)\| \tag{11}$$

Averaging the value obtained in Eq. (10) over the number of realizations and initial conditions $\alpha$ and $\beta$, respectively, we get

$$\lambda = \frac{1}{\alpha} \cdot \frac{1}{\beta} \sum_{j=1}^{\alpha} \sum_{i=1}^{\beta} \lambda^j(i) \tag{12}$$

For the stochastic linear system of Eq. (3), the Lyapunov exponent is obtained considering that $T \to \infty$. However, it is possible to get approximate results in a fixed time $T$ considering a large number of realizations of the disturbance and the initial conditions of the linear system [26]. Furthermore, due to the numerical errors of the first iterations, it is convenient to eliminate the initial time period of the simulation, so Eq. (11) is fitted as follows:

$$\lambda^j(i) = \frac{1}{T - T_1} \cdot \sum_{n=T_1}^{T} \log \|x_n^j(i)\| \tag{13}$$

where $T_1$ indicates the simulation period in which the numerical method presents numerical errors.

*Method 2: Averages of projected trajectories*

For every trajectory of disturbance $\xi_t$, initial condition $x_0^j$ and simulation time $T$, the linear system of Eq. (3) is solved, getting the solution $x_n^j(i)$. Then the normalization $s_n^j(i) = \frac{x_n^j(i)}{\|x_n^j(i)\|}$ is made to calculate the expression of Eq. (13) (see [25]):

$$\lambda^j(i) = \frac{1}{T - T_i} \cdot \int_{T_1}^{T} s_t^j(i)^T A(\xi_t) s_t^j(i) dt \tag{14}$$

Finally, we get the average over that realizations and initial conditions according to Eq. (12).

*Method 3: Averages of trajectories in a sphere*

In this case we solve the nonlinear system of Eq. (15), which represents the dynamics of the system projected on a sphere, see [26].

$$\dot{s} = h(A(\xi_t), s), \quad s \in S^{d-1}$$
$$h(A, s) = (A - s^T A s \cdot I) s \tag{15}$$

where

- $I$ is the identity matrix, with dimensions given by matrix and
- $S^{d-1}$ indicates the sphere of dimension $d - 1$.

## Case studies

As an application of the numerical methods presented, the following test systems were considered as case studies:

- System I: Four-machine ten-bus system. The unilinear diagram is shown in Fig. 1, and it corresponds to the classical test system used for stability analysis. The system's operating parameters and conditions can be found in [29].
- System II: Ten-machine thirty-nine-bus system. The unilinear diagram is shown in Fig. 2, and it corresponds to one of the standard test systems used for stability analysis in multi-machine systems. The system's operating parameters and conditions can be found in [30].
  In both systems the disturbance that affects the dynamics of the machines is modeled according to Eq. (6). The results of applying the proposed numerical methods to the test systems are given below.
- System III: Sixteen-machine sixty-eight-bus system.
  This example also corresponds to one of the classical test systems used for stability analysis in multi-machine systems. The parameters can be found in [33].
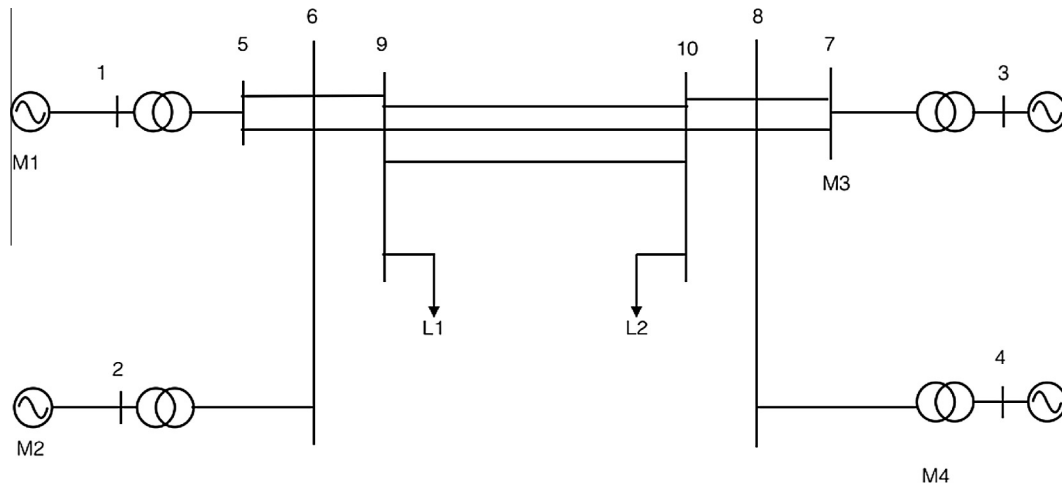
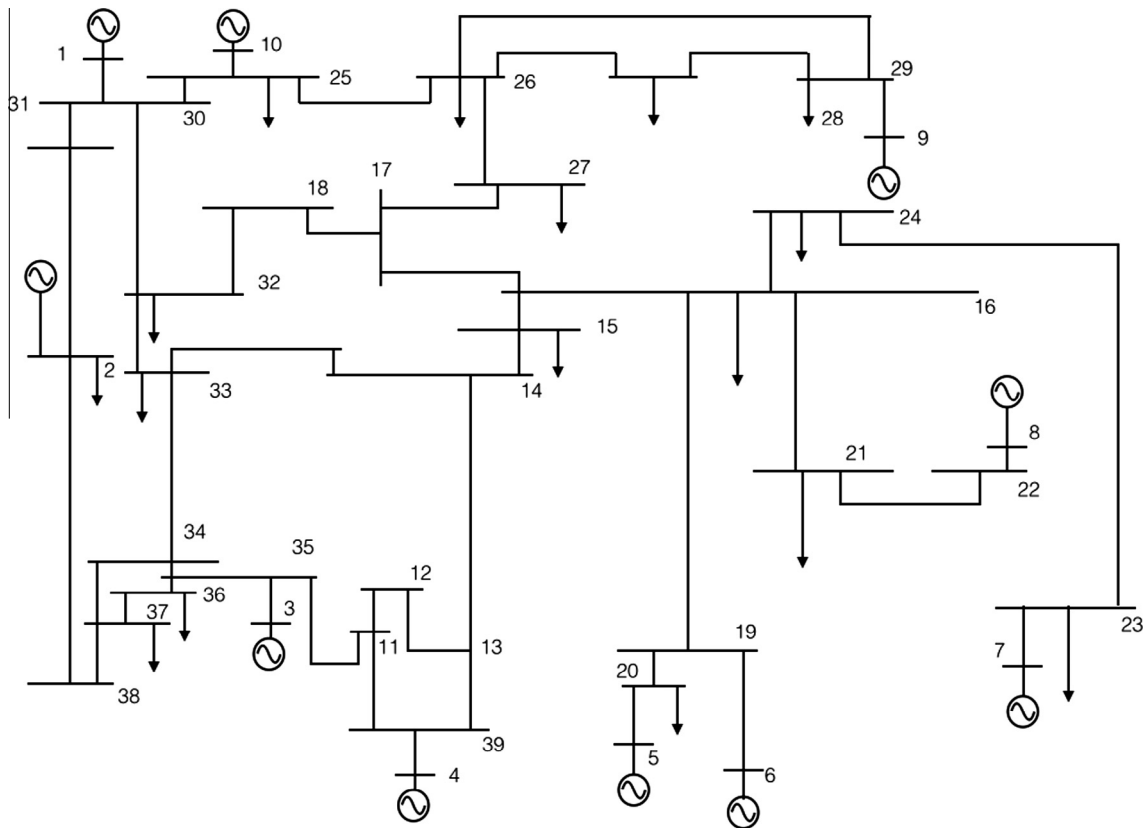**Fig. 1.** Four-machine ten-bus system.



**Fig. 2.** Ten-machine thirty-nine-bus system.

In this case the disturbance models the variations that the system's load bus consumption can experience. Calculating the Lyapunov exponents it is possible to determine the maximum value by which the consumption can increase or decrease, without having to make a new economic dispatch to respond to the existing variations, which are inherent in the operation of the system.

*Four-machine ten-bus system*

Table 1 shows the values of the Lyapunov exponents for different disturbance sizes $\rho$ according to the disturbance model indicated in Eq. (6).

In the three methods presented, parameter $T_1$ (see Eq. (13) is fitted to eliminate the numerical errors of the first iterations. In our work its value was determined considering that in the absence of disturbance ($\rho = 0$) the calculated Lyapunov exponent would be the same as that of the real part closest to the origin of the deterministic linear system ($-0.0032$).

From the results indicated in Table 1, in the disturbance range of $0 \leqslant \rho \leqslant 0.2$ the three numerical methods showed similar behaviors. In fact, using Methods 1 and 2, the Lyapunov exponent obtained for $\rho = 0$ turns out to be the same as the real part closest to the origin of the deterministic system. With respect to Method 3, there is a slight difference due to numerical problems whose origin is found in the solution of the nonlinear associated system [26].

**Table 1**
Lyapunov exponents four-machine system.

| $\rho$ | Method 1 | Method 2 | Method 3 |
|---|---|---|---|
| 0 | −0.0032 | −0.0032 | −0.0019 |
| 0.1 | −0.0039 | −0.0054 | −0.0043 |
| 0.2 | −0.0040 | −0.0044 | −0.0038 |
| 0.3 | −0.0012 | 0.0002 | 0.0002 |
| 0.4 | 0.0044 | 0.0035 | 0.0038 |
| 0.5 | 0.0092 | 0.0099 | 0.0102 |

**Table 3**
Lyapunov exponents sixteen-machine system.

| $\rho$ | Method 3 |
|---|---|
| 0 | −0.0436 |
| 0.02 | −0.0440 |
| 0.04 | −0.0458 |
| 0.06 | −0.0453 |
| 0.08 | −0.0445 |
| 0.10 | −0.0440 |
| 0.12 | −0.0437 |
| 0.14 | −0.0437 |
| 0.16 | −0.0430 |

The results of Table 1 also indicate that for a disturbance size range of $0.2 < \rho < 0.3$ the system loses stability.

It can also be stated safely that up to a disturbance value of $\rho = 0.2$ the system will be stable. The theorem that justifies this statement is found in [32].

Based on the control parameters $p$ of Eq. (3) it is possible to modify the value of the Lyapunov exponent as a function of the size of disturbance $\rho$, opening an interesting exploration field for controlling the stability of these systems. This idea will be developed and presented in a future publication.

*Ten-machine thirty-nine-bus system*

According to the tests carried out on the systems of Figs. 1 and 2, it is verified that Method 1 is the most efficient, as it requires less time and memory for the calculation process. Moreover, Methods 2 and 3 present discontinuities when initial conditions that are very close to the origin are evaluated [26].

Based on the arguments given above, Table 2 shows the values of the Lyapunov exponent calculated by Method 1, to analyze the stability of the ten-machine system. Different disturbance sizes were considered to determine the maximum disturbance size $\rho$ at which the system will be stable.

The same as for the four-machine system, parameter $T_1$ was fitted in such a way that the estimated Lyapunov exponent had the same value as the real part closest to the origin of the deterministic system.

Similarly to the four-machine system, it is possible to determine the maximum disturbance size at which the system is stable [32]. For the ten-machine system the value that ensures stability is $\rho = 0.3$.

*Sixteen-machine sixty-eight-bus system*

Table 3 shows the values obtained by calculating the Lyapunov exponents for different disturbance sizes. In this case, modeling the load bus voltage variations has been considered, representing the random behavior of the consumption existing in the system.

From the above it is possible to determine the maximum disturbance size $\rho$ at which the system will be stable [32]. For the sixteen-machine system the maximum voltage variation on the load busses corresponds to $\rho = 0.16$. Note that an important fact resulting from this analysis is that as the power systems become more complex (larger size), the maximum disturbance size that they can support without losing stability is increasingly smaller.

The results obtained from the implementation of the proposed numerical methods show that it is possible to estimate the Lyapunov exponent for systems with a large number of state variables. This allows quantifying and generalizing the classical linear analysis to the study of the behavior of a stochastic system. However, there are differences based mainly on the required computer time and memory for the calculation process, and Method 1 turns out to be the most efficient one.

With respect to the application in electric systems, we have a potential field for study and research, especially considering that the phenomena that affect the dynamics of current systems have a pronounced random nature. This can be seen, for example, in the analysis of networks with distributed generation and high wind penetration.

## Conclusions

The results obtained from the application of the proposed methodology to power systems with a large number of state variables are presented, showing the potential of the proposed numerical methods by allowing the evaluation of the stability of electric systems subjected to random and sustained in time disturbances.

The methodology consists in characterizing the random and self-sustained in time disturbances by means of a Markov-type stochastic process, ensuring the existence and uniqueness of the solution of the methods used. The analysis of stability consists in comparing the Lyapunov exponent obtained for the system without disturbance with the real part closest to the origin in the deterministic system.

The results of the international test systems show that the numerical methods implemented are coherent. Specifically, in relation to the precision of Methods 1 and 2, they give highly reliable results, but the accuracy of Method 3 is reduced due to numerical errors originating in the solution of the associated nonlinear system.

Also, from the tests and simulations made it is verified that Method 1 is the most effective, since it requires less computer simulation effort and time. This aspect is very important for systems that have a large number of state variables, as real systems generally do.

In the future we intend to develop a methodology to control the value of the Lyapunov exponent as a function of the parameters that can be tuned in the control systems of the machines, for example using the gains of the voltage regulators and power stabilizers, time constants, etc. Another development line has to do with the calibration of the disturbance model of Ornstein–Uhlenbeck with real measurements of the disturbances that affect the generators' excitation systems, which is being approached by the authors of the present paper.

**Table 2**
Lyapunov exponents ten-machine system.

| $\rho$ | Method 2 |
|---|---|
| 0 | −0.025406 |
| 0.1 | −0.026820 |
| 0.2 | −0.036681 |
| 0.3 | −0.021520 |
| 0.4 | 0.151920 |
| 0.45 | 0.28131 |

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