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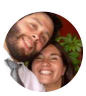

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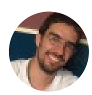

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Technical Note

Multifractal analysis in mining microseismicity and its application to seismic hazard in mine

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1. Introduction

During the last decades several authors have shown that the spatial distribution of earthquakes follows fractal and multifractal laws [1–3] and the most interesting behaviour is the decrease of the fractal dimension value before the occurrence of large earthquakes and their main aftershocks [4,5]. The monofractal analysis is not always enough to characterize the complex dynamic in a specific system. In this sense, the multifractal behaviour is better to describe the nonlinear dynamic in many systems, like rain and clouds [6], fractures [7], solar wind [8] and earthquakes [9].

This multifractal tool is essentially used for the characterisation of spatial scaling properties, in particular the method that is mostly used is the correlation dimension [10]. With this method the moments q are calculated using the linear relation between the correlation-integral partitioning, $\log_{10}(C(\varepsilon))$, and the radius of the sphere, $\log_{10}(\varepsilon)$. The slope that involves this relationship is the D_q value, which corresponds to the called spectrum of fractal dimensions.

Studies with multifractal tools are widely applied to natural and induced seismicity [11,12], in particular, the recognition of a pattern in data sets of induced seismicity in Creighton mine (considering microearthquakes) shows that the spatial clustering of seismicity is more evident before large events [12].

The purpose of this paper is to verify the relationship between the decrease of the value of D_q (the multifractal spectrum dimension) and the occurrence of mining-induced microseismicity, applying a multifractal analysis to over 55.920 micro-earthquakes recorded between January 2006 and January 2009, in Creighton mine, Canada.

The events with magnitudes $M_w \geq 1.0$ are considered as relevant seismic events, due to their ability to generate a violent expulsion of rock mass inside the mine, known as rockburst.

In order to improve the resolution, the complete data set was separated in six time periods. The multifractal analysis was carried out in each time period, each one contained 9.320 data points. A moving window was used, containing a constant number of events in order to guarantee the precise estimation of the fractal dimension. After different trials, we chose 200 data points for the number of the data points in each window. Two consecutive windows were shifted by 20 points, opening each time and looking for a change in the parameter D_q.

The multifractal analysis of each time period shows that there is a systematic decrease of the fractal dimension (D_q) with time before the occurrence of a relevant earthquake, like in studies with natural seismicity. This methodology was repeated for other two cut-off magnitudes, $M_w \geq 1.5$ and $M_w \geq 2.0$. The performance of this analysis was evaluated using the Receiver Operating

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Characteristic (ROC) analysis [13]. Recent research [14,15] has applied this method to evaluate the performance of seismicity indicators and in these researches were obtained satisfactory results.

2. Data and methods

2.1. Creighton mine data

Creighton mine is located in the southern edge of the Subdury Igneous complex, of Northern Ontario, Canada.

We have considered the deepest microseismicity in Creighton mine, between 1.828 and 2.377 m of depth, this means between the 6.600 and 7.800 levels, including blasting events. The time period used corresponds to January 2006 and January 2009, when 55.920 micro- earthquakes were recorded in the study area, with magnitudes between -1.5 and 3.7.

In order to improve the resolution, the complete dataset was separated in time periods.

2.2. Multifractal analysis

The method of Correlation–Integral Partioning was applied in the multifractal analysis [16]. This method consists of counting the number of data points inside a sphere of radius ε around each event x_i, excluding the point itself. This number is $\tilde{N}_i$. The space is divided in possibly overlapping spheres of radius ε and $\tilde{N}_i$ is calculated for each ε

$$\tilde{N}_i(\varepsilon) = \sum_{|j-i|\geq 1} \Theta(\varepsilon - \| x_j - x_i \|). \tag{1}$$

Using this value of $\tilde{N}_i$ the generalized Rényi entropy, $H_q(\varepsilon)$, was calculated

$$H_q(\varepsilon) = \frac{1}{1-q} \log_{10}\left(\frac{1}{N} \sum_{i=1}^{N} \tilde{p}_i^{(q-1)}(\varepsilon) \right), \tag{2}$$

where $\tilde{p}_i$ is the probability to have a point inside the i-sphere and q is the index of the inhomogeneity inside the fractal.

The Grassberger and Procaccia method [10] and Eq. (2) were used in order to apply a generalized correlation function, to obtain the correlation function $C_q(\varepsilon)$

$$C_q(\varepsilon) = \left\{ \frac{1}{N} \sum_{j=1}^{N} \left[\frac{1}{N-1} * \sum_{i=1, i\neq j}^{N} H_q(\varepsilon - \| X_i - X_j \|) \right]^{(q-1)} \right\}^{1/(q-1)},$$

where N is the number of data points used to calculate $C_q(\varepsilon)$, and $\| X_i - X_j \|$ is the distance between two data points.

Finally, the Rényi multifractal spectrum dimension is given by

$$D_q = \lim_{\varepsilon \to 0} \frac{\log_{10} C_q(\varepsilon)}{\log_{10}(\varepsilon)}. \tag{3}$$

This result is valid in the linear range of $\log_{10}(C_q(\varepsilon))$ versus $\log_{10}(\varepsilon)$. After taking the linear range, the slope involved was calculated and the D_q value was obtained for a q used. As a result we plotted D_q versus q and obtained the fractal dimension spectrum for this data set. We introduced a window multifractal analysis using a moving window which contained 200 data points inside it. Two consecutive windows are shifted by 20 data points [4].

2.3. Receiver operating characteristic

We used the Receiver Operating Characteristic (ROC) in order to quantify the performance of the multifractal method [13]. We were interested in quantifying the occurrence of an induced seismic event, over the cut-off magnitudes, in the minimums in the fractal spectrum dimension. The matrix was constructed considering three cut-off of magnitude for data points in the complete catalogue: 1.0, 1.5 and 2.0, and it is defined like

$$M_c = \begin{pmatrix} TP & FP \\ FN & TN \end{pmatrix}, \tag{4}$$

where four scenarios are possible: in a minimum[1] value of the fractal spectrum dimension, an induced microseismic event with magnitude greater than the magnitude threshold happens (TP, True Positive); in a minimum value of the fractal spectrum dimension an induced seismic event does not happen (FP, False Positive); outside a minimum value of the fractal spectrum dimension an induced microseismic event occurs (FP, False Positive) and outside of a minimum value of the fractal spectrum dimension an induced microseismic event does not occur (TN, True Negative). These parameters count the number of microseismic induced events inside or not of a minimum.

Performance is measured by metrics

$$TPR = \frac{TP}{TP + FN}, \tag{5}$$

$$FPR = \frac{FP}{FP + TN}, \tag{6}$$

$$PSS = TPR - FPR. \tag{7}$$

where TPR is the True Positive Rate, FPR is the False Positive Rate and PSS is the Peirce Skill Score [17]. A PSS value equal to 1, 0 or -1 indicates that the method is perfect, random or wrong.

The perfect classifier, then, is produced when $TPR=1, 0$, $FPR=0, 0$ and $PSS=1, 0$.

3. Fractal analysis in mining-induced seismicity: Creighton mine

We calculate the fractal spectrum dimension D_q for each window with q between 2 and 20. The linear range of $\log_{10}(C_q(\varepsilon))$ versus $\log_{10}(\varepsilon)$ for this microseismic data set is when the ε value ranges between $\varepsilon_{\min} = 10^{1.3}$ m and $\varepsilon_{\max} = 10^{2.3}$ m, shown in Fig. 1. This condition is necessary to do the multifractal spectrum dimension.

The multifractal spectrum for the complete data set was divided in six time periods (see Section 2.1), these spectrums are shown in Figs. 2–7.

For each time period we note a decrease in the value of D_q when seismic event with magnitude greater than the threshold used occurred. In Figs. 2–7 we have marked with arrows some windows that contain several events greater than $M_w = 1.0$, in order to illustrate this behaviour.

In order to measure the performance of this method we choose the spectrum for $q=9$, because the decrease of D_9 value in the fractal spectrum is most clear and the existence of minimums is most evident. Thus, a minimum has been defined when the D_q value is between 0.7 and 1.0, in the fractal spectrum dimension for $q=9$, because the minimums are more evident in this range.

In order to quantify the efficacy of the multifractal spectrum

[1] We define a minimum when the value of D_q is between 0.7 and 1.0.

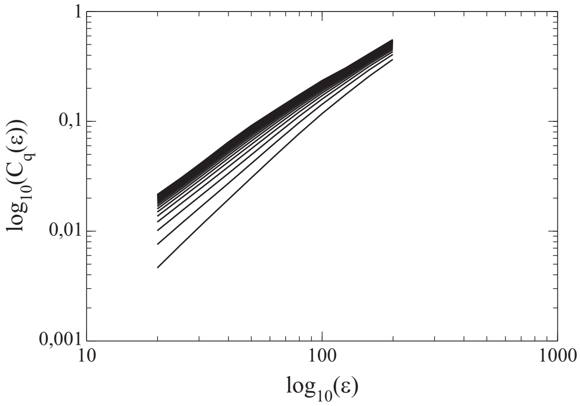

Fig. 1. Linear range for the complete microseismic data set, between $\log(\varepsilon) = 10^{1.3}$ and $\log(\varepsilon) = 10^{2.3}$.

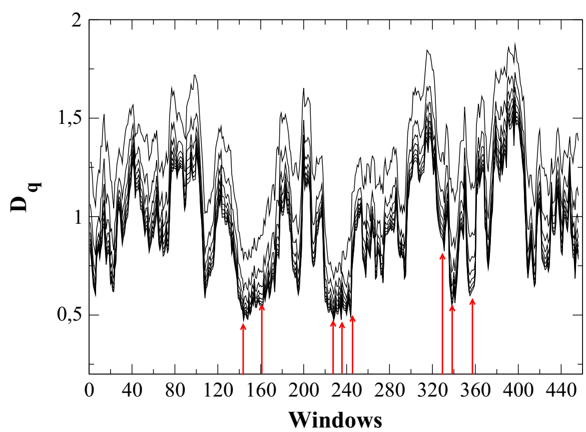

Fig. 2. Fractal spectrum dimension for the first time period, measured between January 1, 2006 and May 21, 2006.

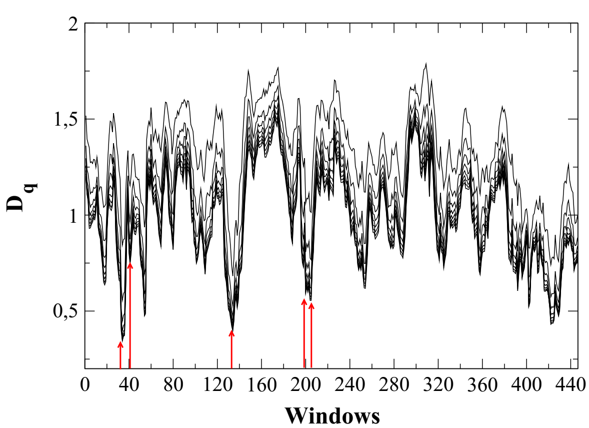

Fig. 3. Fractal spectrum dimension for the second time period, measured between May 5, 2006 and November 18, 2006.

dimensions to warn the proximity of a relevant seismic event in a mine, we have evaluated the decrease of the D_q value, using the Receiver Operating Characteristic. This analysis was done using three thresholds for magnitude $M_w = 1.0$, $M_w = 1.5$ and $M_w = 2.0$, i.e., we defined a relevant seismic event when the magnitude of the event is greater than one of these three magnitude thresholds.

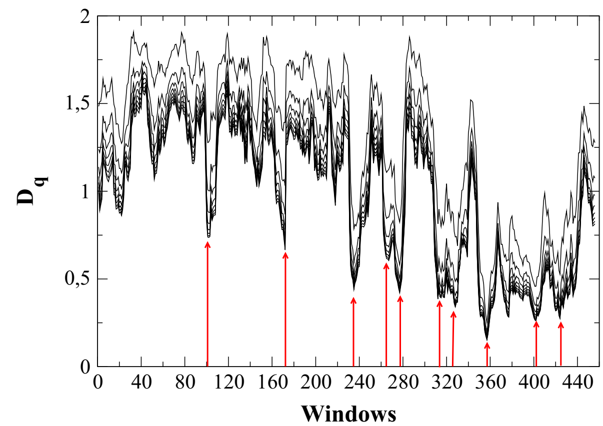

Fig. 4. Fractal spectrum dimension for the third time period, measured between November 18, 2006 and March 4, 2007.

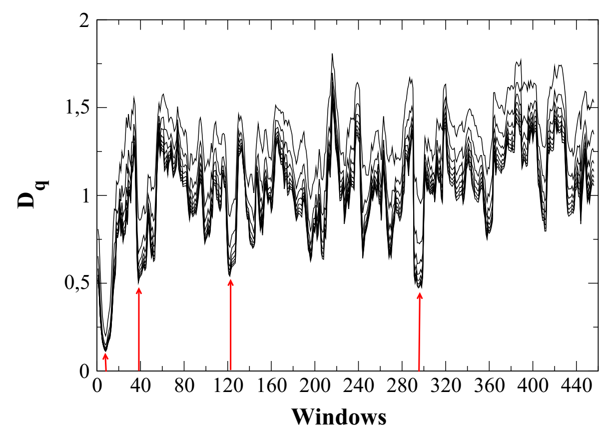

Fig. 5. Fractal spectrum dimension for the fourth time period, measured between March 4, 2007 and August 15, 2007.

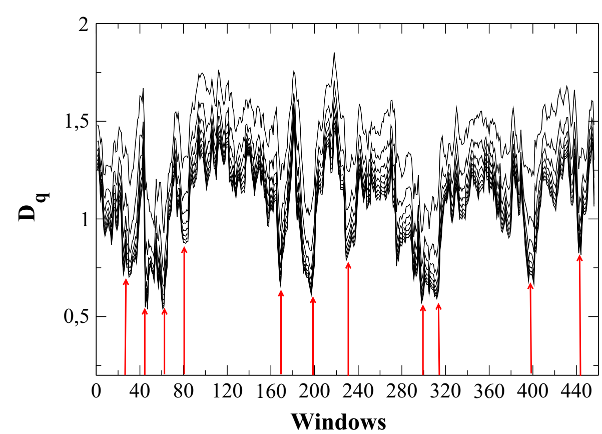

Fig. 6. Fractal spectrum dimension for the fifth time period, measured between August 15, 2007 and April 5, 2008.

A matrix for each Section of microseismic data was constructed under this constraint.

In Table 1 are the results of *TPR*, *FPR* and *PSS* for each section. These results give over 40% of right answers for time periods 1, 2, 3, 4 and 6. Table 2 shows the value of the metrics calculated along the whole time period. The best performance is for magnitude

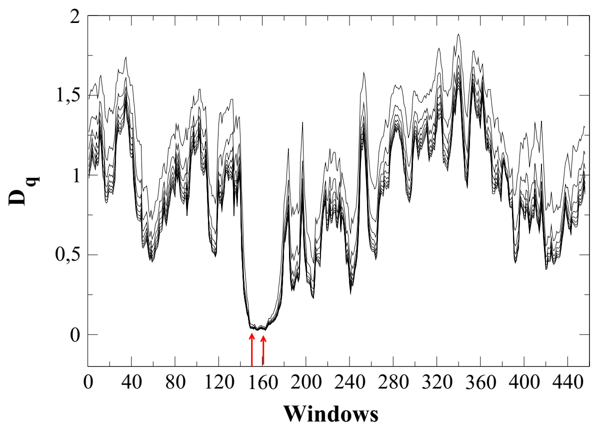

Fig. 7. Fractal spectrum dimension for the sixth time period, measured between April 5, 2008 and January 22, 2009.

Table 1
Table with values of *TPR*, *FPR* and *PSS* performance metrics, for each section.

Section	M_w	TPR	FPR	PSS
	1.0	0.70	0.55	0.15
S_1	1.5	0.74	0.56	0.19
	2.0	0.81	0.61	0.20
	1.0	0.59	0.40	0.20
S_2	1.5	0.6	0.50	0.10
	2.0	0.67	0.53	0.14
	1.0	0.46	0.78	−0.32
S_3	1.5	0.44	0.51	−0.07
	2.0	0.52	0.46	0.10
	1.0	0.55	0.42	0.13
S_4	1.5	0.58	0.46	0.12
	2.0	0.59	0.49	0.10
	1.0	0.38	0.34	0.04
S_5	1.5	0.29	0.49	−0.20
	2.0	0.20	0.47	−0.26
	1.0	0.32	0.49	−0.17
S_6	1.5	0.59	0.81	−0.22
	2.0	0.78	0.67	0.11

Table 2
Table with values of *TPR*, *FPR* and *PSS* performance metrics, for complete data set.

M_w	TPR	FPR	PSS
1.0	0.55	0.55	−0.012
1.5	0.54	0.55	−0.012
2.0	0.58	0.54	0.04

$M_w = 2.0$.

4. Discussion and conclusion

It is interesting to note that the TPR value increases along with the threshold magnitude; this behaviour was presented in each time period, except in the fifth time period. Values for this index are greater than 0.5 for the time periods 1, 2, 4 and 6; the *TPR* index in the third time period is close to 0.4. The least value of *TPR* was presented in the fifth time period. The value of this seismic indicator evaluated in all time presented a 58% of microseismic events in the minimums when the value of D_q was decreased for threshold of magnitude $M_w = 2.0$; it means that the strategy used improves when the threshold magnitude is larger. This could be due to the fact that we are guessing that large seismic events are in the minimums because not many of them occur. Nevertheless, more than the 50% of these events are in the minimums, suggesting that this method is not random, also the value of *PSS* is greater than 0, except in the fifth time period, reaching values of 0.2, which supports the idea that this method is not random.

The value of *FPR* in each time period was around 0.5, except for the sixth time period, in which this value was greater than 0.5. This metric evaluated in all the time was 54%, for magnitudes greater than $M_w = 2.0$. Values of *PSS* and *FPR* imply that it is possible to use the D_q value as a seismic indicator for the occurrence of a relevant magnitude event, considering that this indicator improves when the magnitude threshold is larger. This fact could be due to the fact that the value of D_q decreases when a seismic event greater than a threshold is close, because the behavior of natural seismicity suggests that they make clusters before a significant seismic event [18] and this behaviour produces a decrease in the value of D_q. It is important to repeat this kind of studies in other data sets, in order to compare and verify the effectiveness of this strategy.

The D_q value is a measurement of singularities in a data set, we can interpret the variation in this index like a decrease in the inhomogeneities of the system before reaching a critical value, in this case, before the occurrence of a large seismic event. This means the system is preparing for a critical event forming clusters inside it.

This study suggests the existence of an universal behaviour in seismicity, either natural or induced, because the system seems to react in the same way for both sources. On the other hand, the fact that D_q decreases with time opens the possibility to generate a warning before the occurrence of a relevant event in a mine.

These results emphasize the relevance of applying multifractal analysis in the evolution of the spatio-temporal distribution of seismicity, natural or not, in relation to the occurrence of a relevant event for the system under study.

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