

Gap formation and its consequence in the evolution of SMBHs binaries in galaxy mergers

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Motivated by the theoretical and observational evidence that after a major merger of gas-rich galaxies a massive gaseous disk with a SMBH binary will be formed in the nuclear region of the remnant, we study the interaction between an unequal mass binary and an isothermal circumbinary disk. We focus our study in the transport of angular momentum from the binary to the disk and how this transport can result on the formation of a cavity or gap in the gaseous disk. We propose that, for comparable mass binaries, this exchange of angular momentum is driven by the gravitational interaction of the binary and a strong non-axisymmetric density perturbation that is formed in the disk as response to the presence of the gravitational field of the binary. We compare the efficiency of this gravitational torque on extract angular momentum from the binary with the efficiency of redistribution of the extracted angular momentum in the disk to derive a gap-opening criterion. We run a set of SPH simulations of binaries embedded in isothermal gaseous disks to test our gap-opening criterion. We find that our gap-opening criterion successfully predicts in which simulations a gap will form on the disk. We also run simulations of merging galaxies and we apply our criterion to the more real SMBHB-Disk systems that are formed in situ in these simulations. We find that, in the conditions of our galaxy mergers simulations, the formation of a circumbinary gap is unlikely.

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1 Introduction

In the context of hierarchical structure formation (White & Frenk 1991; Springel et al. 2005) the merger between galaxies is a common event. In these mergers, 60 to 90 % of the gas of the galaxies can accumulate in the central kiloparsec of the remnant as shown by simulations (Barnes & Hernquist 1992, 1996; Mihos & Hernquist 1996; Barnes 2002; Mayer et al. 2007, 2010) and observations of interacting galaxies such as ultra luminous infrared galaxies (Sanders & Mirabel 1996; Downes & Solomon 1998; Medling et al. 2014; Ueda et al. 2014).

In the early evolution of these mergers the massive black holes (MBHs), that are expected to live at the central region of every galaxy (Richstone et al. 1998; Magorrian et al. 1998; Gültekin et al. 2009), will follow the centers of each galactic core until they merge. Afterwards, the pair of MBHs will sink to the center and then they will form a binary. (Kazantzidis et al. 2005; Mayer et al. 2007; Hopkins & Quataert 2010; Chapon et al. 2013).

Even in a relative dry merger, for example the merger of two Sa galaxies of $10^{12} M_{\odot}$ with a gas mass fraction of the order of 1–5 % (Young et al. 1995), if the amount of gas that reach the central kiloparsec of the remnant is only the 60 %, the mass in gas of this central region will be of the order of $(1-5) \times 10^{10} M_{\odot}$, which is ten to hundred times greater

than the mass of the MBHs that lives at the center of these galaxies. For this reason it is natural to expect that not only the stars but also the nuclear gas that surrounds the MBH binary after a merger will have an important influence in its dynamical evolution.

Although stars and gas are good candidates to extract enough angular momentum from the MBH binary, in order to decrease its separation down to scale where the emission for gravitational waves can drive its final coalescence, the typical time scale in which this is achieved by stars and gas are very different.

For spherical stellar systems, the MBH binary will shrink its separation due to dynamical friction with the stellar background, increasing the binding energy of the binary until its orbital velocity begins to be greater than the typical velocity of the background stars (hard binary regime). Afterward, the binary, will begin to lose angular momentum by single stars scattering. This process kicks out stars of the systems depleting from stars the region on the phase space that interact with the binary (loss-cone) and subsequently stalling the decrease of its separation (Makino & Funato 2004; Berczik et al. 2005; Petro et al. 2011). The solution of this last parsec problem is to refill this zone with new stars but, for spherical stellar system, the refilling time is of the order of the relaxation time which is greater than the age of the universe.

For triaxial stellar system the existence of centro-philic orbits (Merrit & Poon 2004; Berczik et al. 2006) can keep

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the loss-cone full (Khan et al. 2011). This makes possible to the background of stars to drive the shrinking of the binary to coalescence in time scales of the order of 1 Gyr (Khan et al. 2011) to 10 Gyr (Berczik et al. 2006).

Even when the triaxiality is an expected characteristic of the merger remnant (Khan et al. 2012), and it solve the last parsec problem, the typical Gyr time scales that are obtained in this systems, for the coalescence of the MBH binary, are comparable or greater than the typical time that takes for a galaxy to be involve in two major mergers.

When it is taken into account the effect of the gas over the evolution of a binary it is found that, for simplify models of the typical environment on the nuclear region of strongly interacting galaxies, the coalescence of the MBH binary is achieve on a timescale of the order of 2.5 Myr (Escala et al. 2004, 2005) or 16 Myr (Dotti et al. 2006) which is between a hundred to thousand times shorter than the time scales found for triaxial stellar systems.

But the gas can also have problems in bring the binary to coalescence because the existence of gas inside the binary orbits is the result of the efficient redistribution of the angular momentum extracted from the binary. If this dissipation is not quick enough the tidal torque exerted by the binary will clear a cavity or gap (Artymowicz & Lubow 1996; del Valle & Escala 2012, 2014) with a size of the order of 2 times the binary separation (Artymowicz & Lubow 1996; Farris et al. 2014).

In this proceeding we review our derivation of a gap-opening criterion for comparable mass binaries and we present some preliminary result of the prediction of this criterion in simulations of galaxy mergers.

2 Gap-opening criterion

2.1 Analytic derivation

In this section we will briefly explain how we derive a gap-opening criterion in del Valle & Escala (2012, 2014) (hereafter dVE12 and dVE14 respectively). When a binary is embedded in a gaseous disk it produces a gravitational torque τ onto the gas (Lin & Papaloizou 1979; Goldreich & Tremaine 1980). The time scale in which this gravitational torque can open a gap of radius Δr is $\Delta t_{\text{open}} = \tau / \Delta L$, where ΔL is the angular momentum needed to open the gap, which can be expressed as $\Delta L \approx \rho (\Delta r)^2 h r v$. This gap opening process is opposed by the viscous diffusion of the gas which tend to fill the gap in a timescale $\Delta t_{\text{close}} = (\Delta r)^2 / \nu$ with ν the turbulent viscosity of the gas that can be parametrised assuming the standard α -prescription $\nu = \alpha_{\text{ss}} c_s h$ of Shakura & Sunyaev (1973) where α_{ss} is the dimensionless viscosity parameter and c_s is the sound speed of the gas.

A gap will form if the gap-opening time scale Δt_{open} is smaller than the gap-closing time scales Δt_{close} . However to evaluate Δt_{open} we need to estimate the gravitational torque produced by the binary onto the disk.

Typically, the torques between the binary and the disk are computing assuming that the gravitational potential produced by the secondary is a perturbation to the axisymmetric gravitational potential of the primary-disk system. From this approximation studies found that the sum of the torques, arising from the inner and outer Lindblad and co-rotation resonances, drive the interaction between the secondary and the disk. This approach leads to predictions of the gap structure that are consistent with simulations within the same regime of validity $q \ll 1$ and $M_{\text{disk}}/M_{\text{primary}} \ll 1$ (Ivanov et al. 1999; Armitage & Natarajan 2002; Nelson & Papaloizou 2003; Haiman et al. 2009; Baruteau & Masset 2012).

However, for the case of a comparable mass ratio binary ($q \sim 1$) this approximation is invalid because the secondary can not be considered as a perturbation to the axisymmetric gravitational potential of the primary-disk system. For this reason we consider the tidal nature of the binary-disk interaction to model the gravitational torques between the binary and the disk. This tidal-torque approach is motivated by the work of Escala et al. (2004, 2005) where they found that the exchange of angular momentum between an equal mass binary and a disk is driven by the gravitational interaction between a strong non-axisymmetric density perturbation on the disk and the equal mass binary.

When the binary is embedded in the disk the shape and size of the strong non-axisymmetric density perturbation is determined by the dominant gravitational potential of the binary, whose typical scale length is the binary separation (see Fig. 1). Therefore, we can assume that the torque produced by the non-axisymmetric density perturbation onto the binary can be written as $\tau_{\text{ellip}} = -a^2 \mu \rho G K_q$ where ρ is the density of the perturbation, a is the binary separation which determines the scale length of the non-axisymmetric density perturbation, $\mu = m_1 m_2 / (m_1 + m_2)$ is the reduced mass of the binary, and K_q is a parameter that depends on the geometry of the density perturbation which can depend on the mass ratio of the binary (see dVE12 for a detail derivation). However, when a is greater than the disk's vertical scale h the density perturbation is truncated by the disk and therefore its size depends on a and h . Therefore when $a > h$, the gravitational torque produced by the strong non-axisymmetric density perturbation can be written as $\tau_{\text{spiral}} = -a h \mu \rho G K_q$.

With this two expression for τ it is straightforward to find that the binary will open a gap in the disk if

$$\frac{\Delta t_{\text{open}}}{\Delta t_{\text{close}}} = \frac{1}{f_q} \left(\frac{c_s}{v} \right)^3 \left(\frac{v}{v_{\text{bin}}} \right)^2 \leq 1 \quad \text{if } a < h, \quad (1)$$

$$= \frac{1}{f_q} \left(\frac{c_s}{v} \right) \left(\frac{v}{v_{\text{bin}}} \right)^2 \left(\frac{H_{\text{disk}}}{a} \right) \leq 1 \quad \text{if } a > h, \quad (2)$$

where $f_q = 2K_q/\alpha_{\text{ss}}$, v the rotational speed of the binary/density-perturbation system and v_{bin} the Keplerian velocity of the binary. In this form the gap-opening criterion depends on the relative strength between the gravitational and viscous torques (K_q/α_{ss}), the thermal and rotational support of the disk (c_s/v) and the relative strength between the total mass of the system and the mass of the binary (v/v_{bin}).

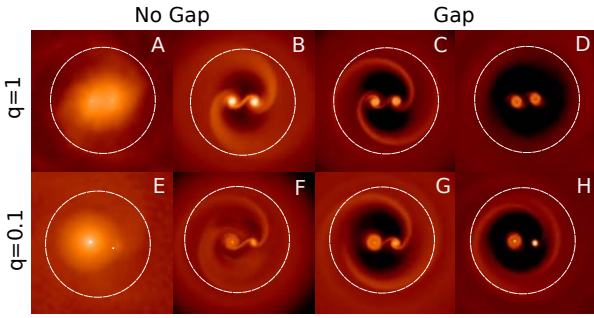


Fig. 1 Surface density of eight simulations. The white dashed line enclose the region $r < 2a$ of the disk, with a the binary separation. The figures at the top are simulations with $q = 1$. The bottom figures are simulations with $q = 0.1$. From left to right, the figures show 1A and 1E: binary that is far from the gap-forming regime; 1B and 1F: binary with parameters in the vicinity of the gap-forming regime that does not form a gap on the disk; 1C and 1G: binary that begins to excavate a gap on the disk; and 1D and 1H: binary that excavate a gap on the disk. In all the cases, the typical size of the strong non-axisymmetric density perturbation in the plane of the disk is determined by the binary separation.

2.2 Numerical Test

We run SPH simulations using the code Gadget-2 to study the binary-disk interaction and test the generalization of our gap-opening criterion. All our simulations consist of a coplanar comparable mass binary of mass ratio q , initial separation a_0 , and mass M_{bin} embedded on an isothermal and stable ($Q > 1$) gas disk of radius R_{disk} and mass M_{disk} . In these simulations we use a natural system of units where [mass] = 1, [distance] = 1, and $G = 1$. In these units, we set the initial radius of the disk as $R_{\text{disk}} = 30$ and the mass of the disk as $M_{\text{disk}} = 30$ for all the runs. The disk of gas is initialized with the same density profile that we used in dVE12: a surface density that is constant for $R < R_c$ and $\propto R^{-1}$ for $R > R_c$, where $R_c = 0.1R_{\text{disk}} = 3$ and a vertical density profile that has the functional form $\cosh(z/H_d)$ where H_d is constant for $R < R_c$ and $\propto R$ for $R > R_c$. With this set up the mass of the disk in the inner region ($R < R_c$) is $M_{\text{gas}}(< R_c) = 1$.

The parameter space that we explore with our numerical simulations is determined by the variation of four parameters a_0 , M_{bin} , h_c and q , in the ranges, $a_0 \in [2, 6]$, $M_{\text{bin}} \in [1, 33]$, $h_c \in [0.8, 3]$ and $q \in [0.1, 1]$. We run 16 simulations with different combinations of these parameters.

To determine if a gap is formed in a certain time t in one of our SPH simulations we seek two characteristics: (i) a density peak in the perimeter of the gap whose maximum has to be greater than 0.015 (in internal units of the code) and (ii) that the semi-major axis a of the binary does not decrease by more than 10%. With these two characteristics we define a numerical threshold to determine in which simulations a gap is formed. We define as a disk with a gap all disks where condition (i) and (ii) are fulfilled. We will call *opened* simulations to every simulation where a gap is form and *closed* simulations to every simulation without a

gap. The times in which we analyze our simulations are the times in which the binary completes 2, 3, 5, 7, 10, and 15 orbits. For more details on this numerical conditions and their justifications as trait of the gap formation, we refer the reader to dVE12 section §2.

In Fig. 2, we plot with open circles the *opened* simulations and with filled circles the *closed* simulations. In this plot, the horizontal axis is $(v_{\text{bin}}/v)^2$ and the vertical axis is $(c_s/v)^3$. Each point in the plot corresponds to a given time in which we analyze a simulation. We use the secondary's orbital speed as the speed of the binary/density-perturbation system (v) because the strong non-axisymmetric density perturbation is formed by the gas that tends to follow the gravitational potential of the binary and therefore corotates with it.

We can see, from Fig. 2, that the group of *opened* simulations and the group of *closed* simulations populate two different regions of parameter space. The region that is populated by the *opened* simulations is the region for which the opening time of a gap is shorter than the closing time ($\Delta t_{\text{open}} < \Delta t_{\text{close}}$) and the region populated by the *closed* simulations is the region for which the closing time of a gap is shorter than the opening time ($\Delta t_{\text{open}} > \Delta t_{\text{close}}$). Therefore the threshold between these two regions is where the closing time of a gap is equal to the opening time of a gap. We can find the expected shape of this interface evaluating our gap-opening criterion (Eqs. 1 and 2) for the limit case $\Delta t_{\text{open}} = \Delta t_{\text{close}}$. For this limit, in the parameter space $((v_{\text{bin}}/v)^2, (c_s/v)^3)$, the ellipsoidal ($h > a$, Eq. 1) gap-opening criterion predicts that the interface between these two set of simulations has a linear shape of slope $m_q = f_q = 2K_q / \alpha_{\text{ss}}$.

Figure 2 shows that, although the line separates fairly well the distribution of *closed* simulations from the distribution of *opened* simulations, there are some simulations that are inconsistent with this threshold line. In comparison the purple dashed lines which represents the thresholds between the *closed* and *opened* simulations, that is predicted by the flat-spiral gap-opening criterion ($h < a$, Eq. 2), separate in a better way the *opened* simulations from the *closed* simulations. This is consistent with the fact that in all our simulations $h < a$.

3 Galaxy mergers

We simulate merging galaxies to evaluate with our gap-opening criterion how likely a SMBH binary open a gap on the massive nuclear gaseous disk of a merger remnant.

The simulations consists of the merger of two equal-mass disk galaxies that collide on a parabolic orbit with pericentric distance $R_{\text{min}} = 8$ kpc. We run four mergers with different orbital parameters that correspond to the orbital parameters DIRECT, RETROGRADE, POLAR and INCLINED defined by Barnes (2002) (see Table 1 of Barnes 2002).

The galaxies are initialized using the code GalactICS: in particular we use the ‘‘Milky Way model’’ (see Kuijken

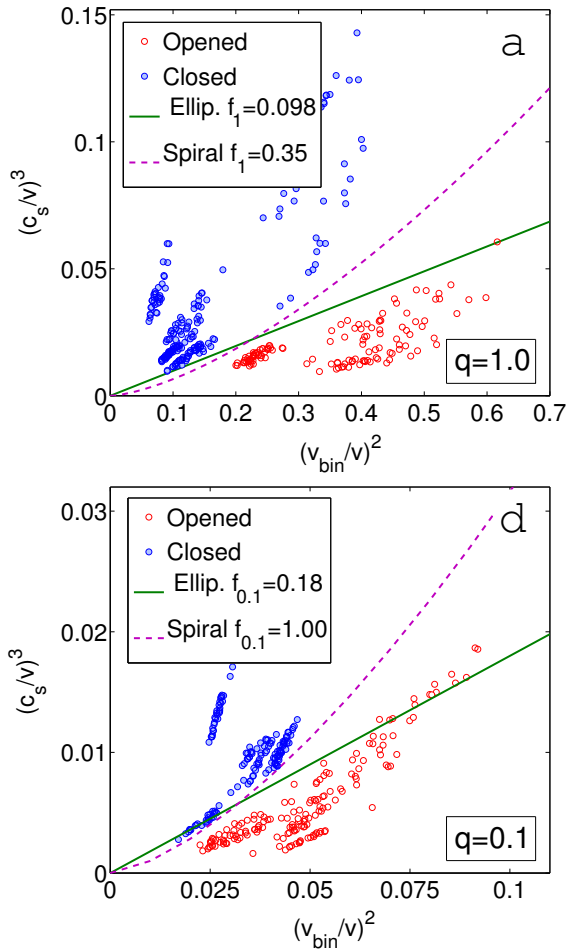


Fig. 2 Cubic ratio between the sound speed of the gas and the rotational velocity of the binary-disk system $(c_s/v)^3$ plotted against the quadratic ratio between the rotational velocity of the isolated binary and the rotational velocity of the binary-disk system $(v_{\text{bin}}/v)^2$. The red circles are simulations where the binary has opened a gap in the disk (*opened* simulations) and the blue filled circles are simulations where the disk does not have a gap (*closed* simulations). In all the figures, the green line is the threshold between the *opened* simulations and the *closed* simulations that is predicted by our ellipsoidal gap-opening criterion. The purple dashed curve is the threshold between the *opened* simulations and the *closed* simulations that is predicted by our flat-spiral gap-opening criterion ($h < a$) and the green line is the threshold predicted by our ellipsoidal gap-opening criterion ($h > a$). Below the purple dashed curve are the *opened* simulations and above the curve are the *closed* simulations.

& Dubinski 1995 for details). In each galaxy model, we include a gaseous disk with the same exponential profile as the stellar component (Kuijken & Dubinski 1995) and with a total gas mass corresponding to 10% of the total stellar disk mass. In our simulations, we use the following internal units: [Mass] = $5.8 \times 10^{11} M_{\odot}$, [Distance] = 1.2 kpc, and $G = 1$. The total number of particles is 420 000, 200 000 for sampling the gas, 120 000 for the dark matter halo, 80 000 for the disk component, and 20 000 for the bulge

We evolve the merging galaxies using the code Gadget-3 (Springel et al. 2001; Springel 2005). This code evolves

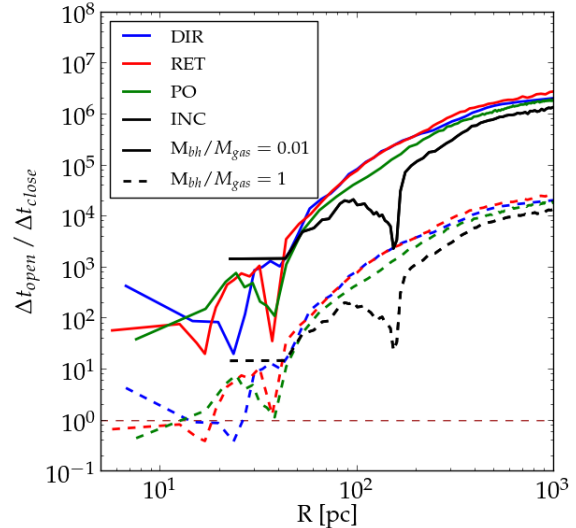


Fig. 3 Value of $\Delta t_{\text{open}}/\Delta t_{\text{close}}$ in function of the separation of the two SMBHs for our four galaxy merger simulations. The continuous lines correspond to the values computed from simulations where $M_{\text{SMBHB}}/M_{\text{gas}} \approx 0.01$. The dashed lines correspond to estimations of $\Delta t_{\text{open}}/\Delta t_{\text{close}}$ assuming $M_{\text{SMBHB}}/M_{\text{gas}} \approx 1$.

the system by computing gravitational forces with a hierarchical tree algorithm, and it represents fluids by means of smoothed particle hydrodynamics (SPH; e.g. Monaghan 1992). We used the recipes for cooling, stellar feedback and star formation included in the code Gadget-3. This recipe assumes that the star formation is self-regulated and is parameterized only by the star formation timescale t_0 . We use the typical value for this parameter, $t_0 = 2.1$, which provides a good fit to the Kennicutt law (Springel et al. 2001)

In all our simulations, more than 70% of the gas ends in the central kiloparsec of the merger remnant. However, the geometry of the gaseous nuclear region shows a dependence on the orbital parameters of the merger. In the four major mergers the ratio between the mass of the SMBH binary and the mass of the nuclear gaseous region is of the order of $M_{\text{SMBHB}}/M_{\text{gas}} \approx 0.01$

In Fig. 3 we show the value of $\Delta t_{\text{open}}/\Delta t_{\text{close}}$ in function of the separation of the SMBHs in our four mergers DIRECT (blue lines), RETROGRADE (red lines), POLAR (green lines) and INCLINED (black lines). The continuous lines show the value of $\Delta t_{\text{open}}/\Delta t_{\text{close}}$, in function of the binary separation, computed directly from our simulations (where $M_{\text{SMBHB}}/M_{\text{gas}} \approx 0.01$). The dashed lines correspond to estimations of $\Delta t_{\text{open}}/\Delta t_{\text{close}}$, in function of the binary separation, assuming that $M_{\text{SMBHB}}/M_{\text{gas}} \approx 1$. We compute $\Delta t_{\text{open}}/\Delta t_{\text{close}}$ assuming that $M_{\text{SMBHB}}/M_{\text{gas}} \approx 1$ because from our gap-opening criterion we expect that for higher values of M_{SMBHB} the formation of a gap is more likely to occur. In our simulation $M_{\text{SMBHB}}/M_{\text{gas}} \approx 1$ correspond to an upper bound for M_{SMBHB} because it correspond to a SMBH binary with a mass of $M_{\text{SMBHB}} \approx 10^{10} M_{\odot}$.

From Fig. 3 we find that the value of $\Delta t_{\text{open}}/\Delta t_{\text{close}}$ is always much greater than one and therefore the formation of a gap on the disk is unlikely. Also we find that even

for SMBH binaries with masses on the order of $10^{10} M_{\odot}$ ($M_{\text{SMBHB}}/M_{\text{gas}} \approx 1$) the formation of a gap on the disk may be difficult and therefore, for the conditions of our simulations, the SMBH binary will sink and finally merge in a timescale of the order of 10^7 yr, regardless of the geometry of the galaxy merger.

4 Conclusions

We derive an analytic criterion to determine when the gravitational torque exerted by a SMBH binary in a massive gaseous disk will produce the formation of a gap. We run 20 SPH simulations of binaries embedded on isothermal gaseous disks to test our gap-opening criterion and we find concordance between the predictions of our gap-opening criterion and the gaps formed in our simulations.

In disks where this gap or cavity does not form we expect that the extraction of angular momentum from the binary will ensure the coalescence of the binary in a timescale of the order of 10^7 yr (Escala et al. 2004, 2005; Dotti et al. 2006). In comparison, in systems where the gap is formed the coalescence timescale may be as long as 10^9 yr (Cuadra et al. 2009).

To estimate the likelihood of formation of a gap, and therefore of the coalescence of SMBHs, in galaxy mergers we run four simulations of major merger with different orbital parameter to apply our gap-opening criterion to the in situ disk-binary system formed in these simulations.

In the four simulations of major galaxy mergers that we run we found that more than 70% of the gas of the two galaxies ends in the central kilo parsec of the merger remnant and that the relation between the mass of the SMBH binary and the gas in this central region is $M_{\text{SMBHB}}/M_{\text{gas}} \approx 0.01$. We evaluate our gap-opening criterion for this four simulations and we find that is unlikely the formation of a gap even if we assume that $M_{\text{SMBHB}}/M_{\text{gas}} \approx 1$. Therefore, we may expect that in these conditions the two SMBHs will coalescence in a timescale of the order of 10^7 yr.

However, it is important to note that in these simulations the gas has a relatively smooth distribution and the nuclear region is composed mainly of high density hot gas. This happens because the density threshold for star formation in these simulations is low ($\sim 1 \text{ cm}^{-3}$) and the cooling function only permit the gas to reach temperatures down to 10^4 K, suppressing the formation of a more clumpy multi-phase ISM.

As a future work we will simulate the same galaxy mergers but with recipes that will permit the formation of a more clumpy and realistic ISM. These simulations will allow us to evaluate in a better way the likelihood of coalescence of SMBH binaries in the central region of interacting galaxies.

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