Antiferromagnetic magnonic crystals

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(Received 17 August 2015; revised manuscript received 30 October 2015; published 17 December 2015)

We describe the features of magnonic crystals based upon antiferromagnetic elements. Our main results are that with a periodic modulation of either magnetic fields or system characteristics, such as the anisotropy, it is possible to tailor the spin-wave spectra of antiferromagnetic systems into a band-like organization that displays a segregation of allowed and forbidden bands. The main features of the band structure, such as bandwidths and band gaps, can be readily manipulated. Our results provide a natural link between two steadily growing fields of spintronics: antiferromagnetic spintronics and magnonics.

DOI: 10.1103/PhysRevB.92.224424 PACS number(s): 75.50.Ee, 75.30.Ds, 75.75.−c, 75.78.−n

I. INTRODUCTION

Small deviations in the local magnetization of a magnetic system can propagate coherently in the form of spin waves (SWs) whose associated quantum fields are known as magnons. Due to the lack of Joule heating associated with their transport they stand out as promising candidates for application in the context of information processing [1]. The field of solid-state physics concerning the manipulation, detection, and dynamics of SWs in a magnetic system has been dubbed magnonics [2]. The field of magnonics has grown into a well-established realm of magnetism and opened new paths in the understanding of magnetization dynamics of complex structures. Most of the research in magnetism has focused on SWs propagating across systems with an overall ferromagnetic order. For example, one of the most studied systems is ytrium-iron-garnet [2], which, being ferrimagnetic, displays a net magnetic moment in each unit cell. A central theme in the field of magnonics is the implementation of magnonic crystals, the SW analog of photonic and plasmonic crystals, structures with magnetic properties spatially modulated in a periodic fashion. In a magnonic crystal the SW spectrum is organized in the form of bands with associated band gaps that can be tailored by proper adjustment of the magnonic crystal properties. A variety of magnonic devices has been proposed, which profit from SWs as information carriers [3,4], signal filters, phase shifters, isolators, and signal processing elements [2].

In this work we propose using antiferromagnets (AFs) as the basic background material in magnonics devices [5]. To highlight the potential of the use of antiferromagnetic materials we illustrate two examples of magnonic crystals that can be implemented using antiferromagnetic elements. Among the main results we highlight the possibility of tailoring the magnonic bands by the use of modulations in the magnetic field or the anisotropy of the elements or simply by manipulation of the geometry of the system. Our results point towards an effective engineering of the magnonic bands.

Antiferromagnetic-based spintronics is a rapidly developing new field due to its promising and unique properties for future spintronic devices in magnetism. Despite their lack of macroscopic magnetization, AFs interact with spin-polarized currents and can give rise to spintronic effects such as magnetoresistance and spin transfer torques [6–9], the piezospintronic effect [10], and skyrmion textures [11]. Antiferromagnetic systems present a number of advantages over ferromagnetic ones regarding potential applications. The lack of stray fields, rapid frequency switching in the terahertz range, and diverse functionalities to be integrated with ferromagnets are among the best qualities of AFs [5,7]. The first one relates to the already mentioned lack of net magnetization. Due to this, they do not create magnetic fields, which renders local all the interactions involved in their manipulation. The second is that the typical time scale associated with changes in the magnetic structure is several orders of magnitude shorter than the one associated with ferromagnetic systems [12]. This opens the possibility of implemening high-speed effects operating in the terahertz range. Finally, antiferromagnetism is observed more often and under much softer conditions than ferromagnetism, being found even in semiconductors at room temperature [13]. This allows us to envision hybrid devices that display features of both electronic and spintronic characteristics. Among the large assortment of antiferromagnetic materials, we have in mind antiferromagnetic insulators with uniaxial anisotropy, such as NiO [14], MnF_2 [15,16], and FeF_2 [17]. SWs in AF have been studied since the dawn of quantum mechanics, both from the theoretical point of view [18] and from the experimental [19]. There is a great deal of knowledge accumulated over decades involving the spectra of SWs in a variety of AFs. This opens a window of opportunity for effective control of the SW degrees of freedom. In this paper, we work out, based on the phenomenological theory of spin dynamics in AFs developed by Hals *et al.* [9], how to account for the SW physics in spatially modulated AFs.

The structure of this paper is the following. In Sec. II, the phenomenological model for AFs is presented and the basic physics of SWs in AFs is discussed. An enquiry into antiferromagnetic magnonic bands, in Sec. III, is considered under two scenarios, modulating periodically either the anisotropy or the external magnetic field.

II. PHENOMENOLOGICAL THEORY

We start off our discussion by stating the basic features of our model. We study the dynamics of the staggered

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magnetization field in a spatially modulated AF [9,20,21]. In terms of the microscopic exchange energy J, lattice constant ℓ, coordination number z, and uniaxial anisotropy D, the free energy density F for this system can be expressed [9] as a functional of the staggered magnetization and magnetization fields, $\mathbf{n}$ and $\mathbf{m}$, respectively,

$$F = \left[\frac{a}{2}\mathbf{m}^2 + \frac{A}{2}\sum_i (\partial_i \mathbf{n})^2 - \frac{K_z}{2}(\mathbf{n}\cdot\hat{z})^2 - \mathbf{H}\cdot\mathbf{m}\right], \quad (1)$$

where $a = 4zJS^2/\ell^3$, the homogeneous exchange energy; $A = zJS^2/2\ell$, the exchange stiffness; $K_z = 2DS^2/\ell^3$, the anisotropy (easy axis); and $\mathbf{H} = g\mu_B\mathbf{B}/\ell^3$, the external magnetic field. These parameters have spatial dependencies whose specific form is specified later on. In F the fields are further constrained to obey $\mathbf{n}\cdot\mathbf{m} = \mathbf{0}$ and $\mathbf{n}^2 = 1$ at every instant and everywhere within the system. The equations of motion can be obtained from a variation of the action where the constraints are enforced with the aid of suitable Lagrange's multipliers. The resulting dynamics is ruled by

$$\dot{\mathbf{n}} = \gamma \mathbf{f}_m \times \mathbf{n}, \quad (2)$$

$$\dot{\mathbf{m}} = \gamma(\mathbf{f}_n \times \mathbf{n} + \mathbf{f}_m \times \mathbf{m}), \quad (3)$$

where $\mathbf{f}_m = -a\mathbf{m} + \mathbf{n}\times(\mathbf{H}\times\mathbf{n})$, $\mathbf{f}_n = A\mathbf{n}\times(\nabla^2\mathbf{n}\times\mathbf{n}) + K_z(\mathbf{n}\cdot\hat{z})\mathbf{n}\times(\hat{z}\times\mathbf{n}) - (\mathbf{n}\cdot\mathbf{H})\mathbf{m}$, and γ is the effective gyromagnetic ratio.

To determine the SWs we consider small variations around the staggered magnetization (or Néel) vector and the canting field. The canting field $\mathbf{m}$ is small with respect to the local magnetic moment, which allows us to keep only the first-order term. Considering $\mathbf{n} = \mathbf{n}_0 + \delta\mathbf{n}(\mathbf{x},t)$ and $\mathbf{m} = \mathbf{m}(\mathbf{x},t)$ we expand up to first order the equations of motion [Eqs. (2) and (3)] as

$$\frac{\delta\dot{\mathbf{n}}}{\gamma} = -a\mathbf{m}\times\mathbf{n}_0 - (\mathbf{n}_0\cdot\mathbf{H})\delta\mathbf{n}\times\mathbf{n}_0, \quad (4)$$

$$\frac{\dot{\mathbf{m}}}{\gamma} = A\nabla^2\delta\mathbf{n}\times\mathbf{n}_0 - K_z(\mathbf{n}_0\cdot\hat{z})^2\delta\mathbf{n}\times\mathbf{n}_0 - (\mathbf{n}_0\cdot\mathbf{H})\mathbf{m}\times\mathbf{n}_0. \quad (5)$$

To solve these equations let us look for monochromatic waves in the form

$$\delta\mathbf{n}(\mathbf{x},t) = (\boldsymbol{\epsilon}_1 n_1 + \boldsymbol{\epsilon}_2 n_2)e^{i\mathbf{k}\cdot\mathbf{x} - i\omega t}, \quad (6)$$

$$\mathbf{m}(\mathbf{x},t) = (\boldsymbol{\epsilon}_1 m_1 + \boldsymbol{\epsilon}_2 m_2)e^{i\mathbf{k}\cdot\mathbf{x} - i\omega t}. \quad (7)$$

In this representation we have used the constraints to express both fluctuating fields in the plane perpendicular to $\mathbf{n}_0$, which is spanned by the mutually orthogonal (but otherwise arbitrary) vectors $\boldsymbol{\epsilon}_1$ and $\boldsymbol{\epsilon}_2$. In this expression $n_{1,2}$ and $m_{1,2}$ are complex coefficients. Starting from Eqs. (4)–(7) in a direct way we can assess the problem of SWs in a homogeneous AF. The results are, naturally, consistent with the well-known dispersion relation [21]

$$(\omega \pm \gamma H)^2 = a\gamma^2(Ak^2 + K_z). \quad (8)$$

As shown in Fig. 1 there are two independent solutions. One has a phase difference between n_1 and n_2 equal to $\pi/2$, while the other has a phase difference equal to $-\pi/2$. These solutions correspond, therefore, to SWs circularly polarized to the left and to the right. In the absence of a magnetic field both branches are degenerated. This degeneracy is split by the magnetic field, which shifts the dispersion relation of the right-polarized waves upward and the dispersion relation of the left-polarized waves downward by an equal amount, γH. An important aspect of the SW spectra that is encoded by Eqs. (4) and (5) is that oscillations in the staggered magnetization order parameter $\delta\mathbf{n}$ are linked to oscillations in the magnetization. In fact, a quick look at Eqs. (4) and (5) allows us to write

$$\mathbf{m} = -\frac{1}{a}\left(\frac{1}{\gamma}\mathbf{n}_0\times\delta\dot{\mathbf{n}} + (\mathbf{n}_0\cdot\mathbf{H})\delta\mathbf{n}\right). \quad (9)$$

This simple result is of great importance since it provides a way to excite and measure antiferromagnetic SWs by coupling them to the oscillation in the magnetization field that they carry. This coupling to the magnetic degrees of freedom has been used for decades to characterize the SW spectra of AFs [22]. In this way it is possible to use magnetization probes, such as Faraday's and Kerr's effects or Brillouin light scattering [2], of widespread use in the field of magnonics, to control and study antiferromanetic SWs.

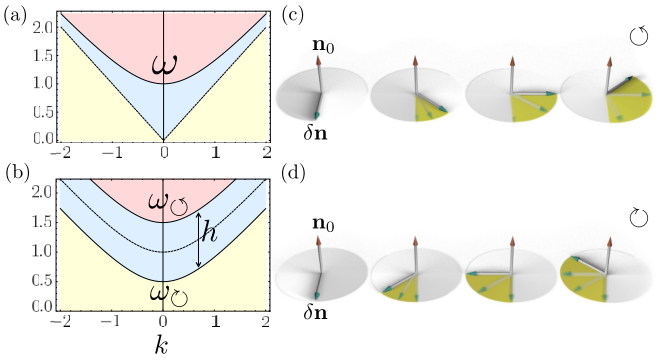

FIG. 1. (Color online) (a) Spin-wave dispersion relation for the homogeneous antiferromagnet. Without anisotropy the relation is dispersionless (dashed line). The addition of anisotropy raises a gap and changes the dispersion relation to the Klein-Gordon form (solid line). In both cases the dispersion relation is doubly degenerated, reflecting the two possible polarizations of the spin wave. (b) The addition of a homogeneous magnetic field splits the degeneracy and creates different dispersion relations ($\omega_{\circlearrowleft}$ and $\omega_{\circlearrowright}$) for the two opposite polarizations. (c, d) Illustration of the two polarizations for the spin wave. The disturbance is perpendicular to the equilibrium staggered magnetization vector ($\mathbf{n}_0$) and precesses in a clockwise or counterclockwise sense.

III. ANTIFERROMAGNETIC SPIN-WAVE BANDS

Now we are ready to present the main result of this work, the study of different ways in which these antiferromagnetic SWs can be manipulated with the aid of periodic manipulation of the system parameters. We see how this manipulation gives rise to magnonic bands that can be tailored with precision. There are basically two essentially different ways to control antiferromagnetic SWs. Starting from Eqs. (2) and (3) we note the possibilities of modulating the system parameters

(exchange constants, anisotropy, etc.) or the magnetic field. Both parameters can be modulated to generate magnonic crystals and in what follows we study the peculiarities of each particular modulation.

Eliminating **m** from the equations of motion we are led to the wave equation

$$\delta\ddot{\mathbf{n}} = \nabla^2\delta\mathbf{n} - \kappa\ \delta\mathbf{n} + 2h\ \delta\dot{\mathbf{n}} \times \mathbf{n}_0 + h^2\delta\mathbf{n}, \qquad (10)$$

where we have set $\tau = 1/\sqrt{K_z a\gamma^2}$ as the unit of time and $\lambda = \sqrt{A/K_z}$, the domain wall width, as the unit of length. In the last equation κ and h, the dimensionless anisotropy coefficient and dimensionless magnetic field [$h = \gamma\tau(\mathbf{n}_0 \cdot \mathbf{H})$], respectively, are regarded as periodically modulated. For common AFs [15–17,23] the value of τ and λ lie in the range of picoseconds and a few nanometers, respectively.

The solutions to the wave equation under periodic modulation can be expressed in the form of Bloch wave functions $\delta\mathbf{n}(\mathbf{x},t) = e^{i\mathbf{k}\cdot\mathbf{x}-\omega t}(\boldsymbol{\epsilon}_1 n_1(\mathbf{x}) + \boldsymbol{\epsilon}_1 n_2(\mathbf{x}))$, where $n_1(\mathbf{x})$ and $n_2(\mathbf{x})$ are periodic functions with the same period as the spatial modulation. The equation of motion, within the Bloch's representation, unfolds into two coupled equations for $n_1(\mathbf{x})$ and $n_2(\mathbf{x})$ that can only be fulfilled by choosing a $\pm\pi/2$ phase shift between them. The waves are, therefore, circularly polarized as in the homogeneous case. Due to the magnetic field there is a splitting between the two circular polarizations. For right-polarized waves we have $\delta\dot{\mathbf{n}} \times \mathbf{n}_0 = \omega\delta\mathbf{n}$, while for left-polarized waves we have $\delta\dot{\mathbf{n}} \times \mathbf{n}_0 = -\omega\delta\mathbf{n}$. The equation of motion becomes

$$-\omega^2\delta\mathbf{n} = \nabla^2\delta\mathbf{n} - \kappa\delta\mathbf{n} \pm 2\omega\ h\delta\mathbf{n} + h^2\delta\mathbf{n}, \qquad (11)$$

where the $\pm$ sign is fixed by the polarization of the SW. This equation, with κ and h regarded as periodic functions of space, is the main tool to describe an SW crystal within an AF. Let us give two examples of antiferromagnetic magnonic crystals that use this equation as the starting point.

A. Periodically modulated anisotropy

We begin our discussion by considering the problem in the absence of a magnetic field. We focus on a one-dimensional array with spatially modulated anisotropy as illustrated in Fig. 2. When modulating the spatial anisotropy one can expect that the exchange parameters, a and A, will also be modified. Our theory is capable of handling those modulations in a straightforward manner. However, to avoid clumping our discussion with far too many parameters we focus on a model problem where only the anisotropy is modulated. We have, then, a series of slabs of width β with different anisotropies arranged with period α. SWs that propagate along the direction transverse to the slabs experience a periodic modulation of the anisotropy parameter, thereby giving rise to magnonic bands. This situation can be modeled by the equation

$$\delta\ddot{\mathbf{n}} = \nabla^2\delta\mathbf{n} - \kappa(y)\delta\mathbf{n}, \qquad (12)$$

which shows complete degeneracy for the different polarizations. Searching for plane waves along the x direction, with wave number k_x, we find

$$\left(\omega^2 - k_x^2\right)\delta n_\pm = -\frac{d^2}{dy^2}\delta n_\pm + \kappa(y)\delta n_\pm. \qquad (13)$$

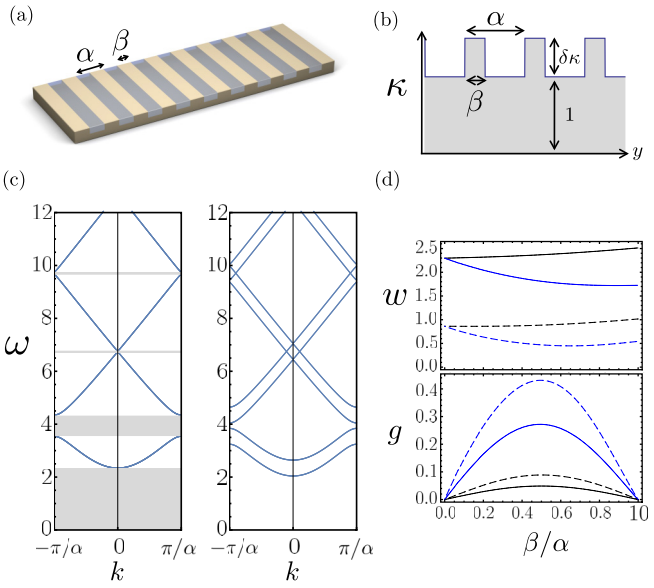

FIG. 2. (Color online) (a) Model system for a magnonic crystal, a heterostructure with changing anisotropy, illustrating the geometric features. (b) Simple effective potential that represents the effect of the modulated anisotropy. With the choice of units given in the text, the potential is characterized by a reference anisotropy equal to unity and deviations from it equal to $\delta\kappa$. (c) Left: Magnonic dispersion relation for $\alpha = 1.0$, $\beta = 0.5$, and $\delta\kappa = 10$. Bands are doubly degenerated to account for the different polarizations. Bands of forbidden frequencies are highlighted. Right: The same situation, under the action of a uniform magnetic field $h = 0.3$. The degeneracy between the different polarization states is broken. (d) Some features of the band structure are displayed as a function of β/α. Top: The bandwidth of the first bands is displayed for different values of the crystal; solid lines correspond to $\alpha = 1$ and dashed lines to $\alpha = 2$. Solid black and dashed (blue) lines correspond to $\delta\kappa = -0.5$ and $\delta\kappa = 3.0$, respectively. Bottom: With the same parameters we display the band gap between the first and the second bands.

In the physical problem at hand the anisotropy is modulated in a piecewise constant fashion. The background anisotropy of the system is chosen as the basis for the dimensionless anisotropy κ. There are slabs of width b distributed uniformly with period a, and within these the anisotropy is $1 + \delta\kappa$. This problem is equivalent to a Schrödinger equation with periodic piecewise constant potential, which leads us to the well-known Kronig-Penney model for electrons. Solutions of this model can be found in textbooks [24] and consist of the matching of solutions at each side of the structure and imposing Bloch-boundary conditions. In Fig. 2 we present the results for a variety of system parameters. A glance at Fig. 2 allows us to state the main results: (a) a double-degenerated band structure to account for different polarizations, (b) the appearance of forbidden energy bands, and (c) the appearance of bands of allowed energies with characteristic bandwidths. As shown in Fig. 2(d) these features can be controlled by the appropriate selection of the parameters of the magnonic crystal. The band structure can be further controlled by exposing the system to the effects of a magnetic, which that results in a splitting of the degeneracy of the bands. This is shown in Fig. 2(c).

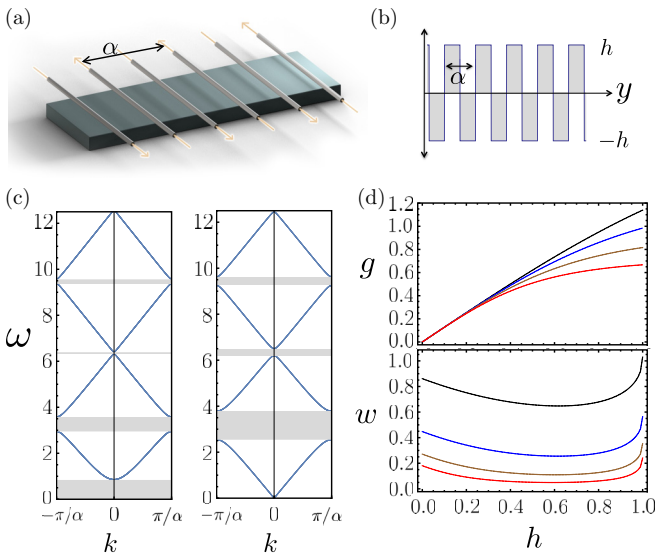

FIG. 3. (Color online) (a) Arrangement of wires on top of a two-dimensional antiferromagnetic sample. The magnetic field they generate form a magnonic crystal. (b) The system is characterized by a spatially modulated magnetic field that oscillates between two extrema $\pm h$ within a period α. (c) Left: Band structure for $\alpha = 1$, $\kappa = 1$, and $h = 0.5$. Right: Band structure for $h = \kappa = 1$. The lowest band minimum reaches 0, signaling the spin-flop instability. (d) As a function of the magnetic-field strength we display the band structure parameters. Top: Band gap for $\alpha = 2, 3, 4$, and 5. Bottom: Band width of the first band for the same parameters.

B. Field-mediated magnonic crystals

We now consider a magnonic crystal mediated by a magnetic field. In this system a two-dimensional AF is exposed to a periodically modulated external magnetic field. While the details of the generation of this magnetic field are irrelevant for the conclusions we are going to draw, we can picture the following arrangement: locate the antiferromagnetic system underneath a periodic array of wires as depicted in Fig. 3(a). The situation that we propose consists of having a current propagating across the wires. The magnitude of the current across each wire is constant, while the direction of the current is changed between consecutive wires. This type of arrangement has been proposed and studied experimentally in Ref. [25] for the design of current-controlled magnonic crystals. In this way the Oersted field generated by the array of wires acts on the AF in the form of a spatially periodic magnetic field that enters into the wave equation [Eqs. (4) and (5)]. To fix ideas on the nature of this equation we approximate the field by a piecewise constant behavior with values of $\pm h$. Assuming plane-wave behavior along the x direction we find

$$\left(\omega^2 - k_x^2 + \kappa + h^2\right)\delta n_\pm = -\frac{d^2}{dy^2}\delta n_\pm \pm 2h\omega\delta n_\pm. \quad (14)$$

This problem is solved following the standard procedure described in Ref. [24], in the same way as in our previous discussion. The main results are displayed in Fig. 3. The periodic magnetic field gives rise to a band structure with characteristic band gaps and bandwidths that are characterized in Fig. 3(d). As we increase the strength of the magnetic field, the lowest point in the first band decreases continuously from the value at zero field. This trend leads to an instability at $h = \sqrt{\kappa}$ when the lowest band touches the bottom of the axis. For fields greater than this critical value the ground state is distorted in what is known as the spin-flop transition [16,22].

IV. CONCLUSIONS

We have discussed the possibility of implementing magnonic crystals in the context of antiferromagnetic spintronics. We propose two complementary methods to achieve control over the magnonic degrees of freedom: first, by controlling the anisotropy properties of the SW system and, second, by exposing the AF to a periodically modulated magnetic field. In both cases we discuss quantitatively the properties of the resulting SW spectra and show how it led to a band-like structure of allowed and forbidden bands whose quantitative features can be tailored by proper adjustment of the parameters of the magnonic crystal. This proposal bridges together the two rapidly developing fields of magnonics and antiferromagnetic spintronics.

ACKNOWLEDGMENTS

The authors acknowledge funding from Proyecto Fondecyt No. 1150072, Center for the Development of Nanoscience and Nanotechnology CEDENNA FB0807, and by Anillo de Ciencia y Tecnonología ACT 1117.

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