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# **EXPERIMENTAL ASSESSMENT OF HANG UP AND SECONDARY FRAGMENTATION FOR BLOCK CAVING**

**TESIS PARA OPTAR AL GRADO DE MAGÍSTER EN MINERIA**

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## Resumen

En minería es cada vez más recurrente el desarrollo de operaciones a mayores profundidades. En particular, en la minería subterránea masiva explotada por el método de Block/Panel caving esto genera como consecuencia grandes alturas de columna de material quebrado, desarrollando altos esfuerzos producto de la sobrecarga. Es así que surge la presente investigación, con el propósito de estudiar el impacto de mayores esfuerzos en la extracción y fragmentación del mineral

El estudio se centra en dos áreas de interés: la fragmentación secundaria y los eventos de colgadura. Estos tópicos son analizados a través del modelamiento físico, para el cual es utilizado un modelo de flujo confinado ubicado en el Laboratorio de Block Caving de la Universidad de Chile. Este modelo emula la extracción desde una batea a través de un sistema LHD. Los experimentos desarrollados consideran mineral el cual fue previamente preparado para representar dos curvas de fragmentación primaria. Dentro de las variables estudiadas se encuentran: el tamaño de los fragmentos, la carga vertical y la humedad; para las cuales se considera la extracción desde un punto de extracción y desde ambos puntos de la batea.

Al evaluar los eventos de colgaduras, los resultados experimentales muestran que una mayor relación entre la abertura de extracción y el tamaño medio de partícula mejora la capacidad de flujo del material, así como la extracción desde ambos puntos de extracción también favorece la aparición de una menor cantidad de eventos, disminuyendo los eventos de colgadura en entre un 20 y 30 por ciento al compararlos con la extracción desde un solo punto de la batea. Mientras que al aumentar la carga vertical y la humedad, incrementa el número de colgaduras generadas. También se observa que la altura de las colgaduras incrementa con la carga vertical en mineral seco. A través de los resultados obtenidos, fue posible desarrollar un gráfico para estimar la frecuencia de colgaduras el cual es validado con datos de terreno de diversas operaciones mineras.

Por otra parte, es presentado un modelo de conminución con base experimental, el cual permite estimar la fragmentación secundaria a diferentes alturas de columna extraída. A diferencia de otros modelos de predicción de fragmentación secundaria, este modelo considera el esfuerzo vertical asociado a la sobrecarga de material quebrado. Finalmente, el modelo propuesto es comparado con la fragmentación generada en operaciones de block caving mostrando la capacidad del modelo planteado para modelar las curvas de fragmentación.

Los resultados obtenidos en cuanto a colgaduras y predicción de la fragmentación secundaria, tienen aplicación en el diseño y planificación en minería de caving, a través del gráfico de estimación de colgaduras y el modelo de conminución presentados.

## **Abstract**

Future mining proceeds towards mining of low grade deposits at deeper levels. In particular, in massive underground mining which are developed by block/panel caving method, deeper level implies higher columns of broken material. In Consequence, higher loads on production levels would be exist due to overloading of these columns. Therefore, this research aims to study the impacts of higher loads on mine productivity and fragmentation.

The analysis focuses on two main topics: secondary fragmentation and hang up events. A confined physical model is implemented to study these topics, in the Block Caving Laboratory of the University of Chile. This model simulates the ore extraction of Load Haul Dump (LHD) system in a drawbell. Two primary size distribution curves of crushed ore are considered in the experiments. During the confined gravity flow experiments various parameters are considered including: fragment size, vertical load and humidity content; for these variables two extraction strategies are implemented, extraction only by one drawpoint and extraction by both drawpoints of the drawbell.

In relation with hang ups event, the experimental results shown that a high ratio between the apex and the mean fragment size implies a better flowability of the granular material. Moreover, the extraction from both drawpoint (one by one) decreases the hang up occurrence between a 20 and 30 percent comparing with the extraction from just one drawpoint. In the other hand, the vertical load and humidity increase the number of hang ups. In experiment is also observed that the hang up height increases due to the vertical load in dry ore material. A hang up prediction graph is developed based on results and it is validated with mine data.

On fragmentation analysis, a fragmentation model is developed which enables to estimate the secondary fragmentation for various extraction heights based on the experimental results. One of the characteristics of this model is considering the overload of the broken column. Finally, the comminution model is compared with the fragmentation curves from block caving operations. The comparison of the developed comminution model with mine fragmentation curves shows the ability of this model to predict the secondary fragmentation.

The hang up prediction graph and the comminution model presented in this project could be very applicable in mine planning and design of caving operations.

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## Introduction on work thesis

This research was developed in Block caving Laboratory, University of Chile, and it patronized by the Advance Mining Technology Center (AMTC) through the "Gravity flow technologies and fundamentals" project.

In the late years caving mining environments have been changing. This changes includes greater depths, lower average grade deposits, demand for increased productivity, escalating mining cost (capital and operating), harder and heterogeneous rock masses and higher stress (Flores G, 2014<sup>1</sup>). In Chile we have some examples of new project under these new challenges such as: Chuquicamata Underground project, Nuevo Nivel Mina at El Teniente and next Nuevo Nivel Profundo.

Following these challenges, physical modelling was implementing to carry out this investigation where fragmentation and hang ups phenomenon in Block Caving mining were analysed. The motivation of this work was born to increasing the knowledge on the behaviour and phenomena related to broken ore extraction under high stress field in Block/Panel Caving mining. The mayor stresses are due to the deeper condition on underground mining, this imply higher overload in the extraction level owing to the broken column in caving operations. These stresses condition impact on the gravity flow, but also has generated collapse of production areas (DOZ case, Indonesia – FreePort<sup>2</sup>), in this operation overload of the broken column implied loss between 15 and 30 percent of theoretical reserves in the production zones.

Furthermore, deeper condition implies higher broken ore column, consequently, ore is draw under high vertical loads. There is a lack of understanding of vertical load effect on fragmentation and ore flow. Fragmentation is a key parameter in Block Caving mining, due to its relevance on (Laubscher, 1994<sup>3</sup>): drawpoint spacing, dilution entry into the draw column, draw control, draw productivity and secondary blasting (Hang ups and oversizes). All aforementioned factors have an economical implication on the mine business. In response thereto, physical modelling under confinement condition come out as an advantageous tool.

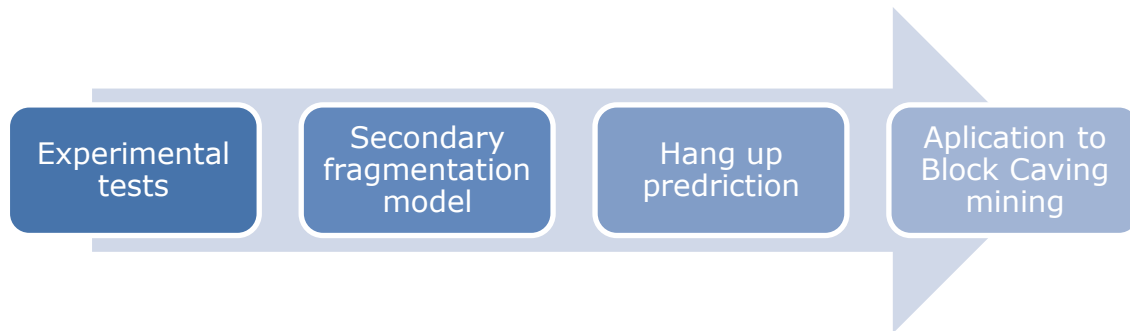
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<sup>1</sup> Flores, G 2014. Future Challenges and why cave mining must change. Caving 2014. p . 23-52.

<sup>2</sup> Sahupala H, Brannon C, Annavarapu S and Osborne K. Recovery of extraction level pillars in the Deep Ore Zone (DOZ) block cave, PT Freeport Indonesia. In: Massmine 2008. Lulea, Sweden. p. 191 - 202.

<sup>3</sup> Laubscher, 1994. The state of the art. Journal of the South African Institute of Mining and Metallurgy.

In this context, it proposed as objectives develop a comminution model for granular material and be able of estimate the hang up frequency in caving mining. Both results based on experimental data through a scaled physical model under high vertical load, and then, analysed with mine data. The general methodology of work is illustrated:



### **General scheme of work**

The following technical papers have resulted from the research presented in this thesis:

1. Title: "*Experimental quantification of hang up for block caving applications*", paper elaborated for the *International Journal of Rock Mechanics and Mining Sciences*.

This article analysed the hang-ups occurrence and the capability for their prediction. Draft was send to the journal, to date comments were received.

2. Title: "*Experimental and comminution model framework for secondary fragmentation quantification for block caving*", paper elaborated for the proceeding Massmine 2016.

A comminution model used in mill comminution processes is presented, this model is adjusted and calibrated for secondary fragmentation prediction, to date the abstract of the paper was accepted to submit the article.

3. Title: "*Use of experiments to quantify the flow-ability of caved rock for block caving*", paper presented in Caving 2014, pp 299 – 306.

The article is a summary of the preliminary results of the investigation, related to the flow ability of the granular material under confinement test.



# **Paper 1: Experimental quantification of hang up for block caving applications**

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## **Abstract**

Block/Panel caving is an underground mining method in which ore production is highly affected by interferences associated with the caving processes; especially those related to gravity flow, such as, hang-ups and oversize rocks. Despite its importance to production rate estimates, a review of current methods indicates a lack of quantitative methods to estimate hang-up frequency and height.

This paper presents the results of an experimental program aimed to study the influence of vertical load, fragmentation, drawing strategy and humidity on hang-ups when extracting from a drawbell. The study shows the effect of vertical load on hang-ups frequency and hang-ups height and indicates for the possibility of decreasing hang-up events by controlling the draw strategy. The influence of humidity on hang-up height and frequency has also been analysed. The experimental data was compared to a database created from data from caving mines showing good correlation when the results were scaled-up proving the value in conducting confined flow tests for mine design applications using a physical model in a laboratory setting.

Key word: Hang up, gravity flow, caving, confined flow.

## **1. Introduction**

Currently, the most economical underground mining method applied to large, deep, massive deposits is block caving. Although the method was first applied in cave-able deposits, due to its advantages, block caving became interesting in hard rock mining. The method consists of rock fragmentation being induced through the caving process with material loaded at drawpoints, which are constructed below the caved area. Various feasibility studies of block caving operations indicate that coarse fragmentation and interrupted caving are the

main risks that can make the method unfeasible, especially in hard rock and poorly jointed deposits. Operationally, one of the most challenging aspects is the handling of oversize rocks, which have a high impact on the production rate of a drawpoint. According to Laubscher [1], the main parameters affecting the draw rate are fragmentation, methods of draw, percent of hang-up and secondary breakage. Therefore, practical hang-up estimation would assist in production planning and scheduling as well as equipment selection for the mitigation of hang-ups. Thus, the prediction of the hang-up frequency for a given geotechnical design or environment is a key to determining the production rate of a cave operation [2].

A hang-up is an interlocking arch of fragments that lies across the top of the drawpoint blocking the flow of the material. There are multiple parameters that influence hangs-up. Kvapil [3] listed thirteen parameters that influence hang-up occurrence: particle size distribution, max size ( $d_{100}$ ), shape of fragments, surface roughness and friction between particles, fragment strength, presence of fine material and moisture content, compressibility and compaction, extraction point geometry, magnitude, distribution and direction of external loads and extraction rate. However, there is a need to quantify the influence of these parameters on hang-up frequency. In general, hang-up frequency could be defined as the tons of material extracted from one drawpoint between interruptions of the flow. Hang-up frequency usually increases during the extraction of a drawpoint due to the decrease in the percentage of coarse fragments in relation to the secondary fragmentation [1].

To-date, attempts have been made to quantify hang-up frequency for coarse, caved rock through the collection of data from mines, controlled experiments and numerical modelling [2-13], mine data has been extensively used to predict hang-up frequency [4-7]. Hadjgeorgiou [8], who summarized experiments and numerical modelling under low confined conditions, showed the influence of the ratio of the particle size to the ore pass diameter on flow conditions. Orellana [9] quantified the hang-up frequency in a physical model for different types of granular material showing the influence of density, strength, friction properties and shape of the fragments on hang-up frequency. However, the results obtained during low confined experimental conditions could not be extrapolated to full-scale caving operations as the models could not emulate the overload of the in-situ column of caved ore, which induces fragmentation and compaction. Recently, the results of the confined flow experiments using a circular exit point with vertical pressures from 1-6 MPa demonstrated that the mean vertical pressure and the fragment size influence the flow-ability and hang-up frequency [2, 10, 11]. The applicability of these latest experimental results to material flow at the full-size

cave mines is not clear as the influence of the geometry of the drawbell and type of loading machine was not investigated.

There are two numerical models that can be used to predict hang-up frequency: Block Cave Fragmentation (BCF) and Core2Frag [12, 13]. These models have been developed to predict the fragmentation and hang-up frequency. Indeed, hang-up frequency prediction is accomplished according to the rule-based approaches. For example, BCF determines the probability of hang-ups occurrences in a drawbell using the secondary fragmentation results as an input to the model. BCF applies two methods to estimate the hang-up frequency: Ore Pass Rules and Robin Kear Rules. Core2Frag hang-up module likewise considers a different Robin Kear Rule-Based Approach and uses secondary fragmentation as an input [13]. Both Ore Pass and Robin Kear rules consider the diameter of fragments and drawbell and define the hang up frequency based on different assumptions [12]. Even though good agreement was obtained with Robin Kear method and small physical models of drawpoints [6], this method seemed to overestimate the number of hang-ups. However, it appears that the Ore Pass rule maybe under estimating the number of hang ups. Besides, both rules have the deficiency of considering the role of vertical loads, moisture, fine material and draw strategy on hang-up frequency. While useful, rules-based methods to predict hang-ups require further development to be considered a validated approach.

In this paper, the results of the experiments using a scaled, confined physical model to investigate the flow mechanisms during ore extraction in block caving method are described. The experiments were conducted using granular material, which was drawn from one or two drawpoints of the same drawbell. Hang-up frequency and height of hang-ups were quantified for different mean vertical pressures while extracting from one or two drawpoints. In addition, the influence of fragmentation and moisture on hang-up occurrence was also analysed, showing the importance of considering confined conditions on hang-ups occurrence.

## **2. Similitude analyses**

The use of scaled models for engineering applications requires a provision for the conditions of similitude that depend on the problem to be solved. Castro (2006) proposed six criteria to achieve kinematic similitude in a large physical model to study free flow in granular materials for block caving [14]. The conditions of similitude include: geometrical similitude (shape and size of particles, geometry of drawpoints), friction angle (residual friction angle and boundary friction angle), bulk density (related to size distribution), and time

(draw rate). However, dynamic similitude refers to the scaling of the most important forces within the model.

In the gravity flow of caved rock in block caving, the main forces are vertical loads, friction and cohesion. In summary, a reduced system should preserve the geometry, velocities and the acting forces of the system under study (prototype). Of course, there are distortions that are likely to occur due to the presence of spurious forces that may affect the scaled system. Table 1 lists the scale factors that needs to be considered, which are scaled in accordance with the geometric scale factor ( $1:\lambda_L$ ).

**Table 1: Similitude analysis variables scaling parameters**

Variable	Scale Factor
Length	$\lambda_L$
Area	$\lambda_L^2$
Volume	$\lambda_L^3$
Velocity	$2\lambda_L^{1/2}$
Time	$\lambda_L^{1/2}$
Weight	$\lambda_L^3$
Density	1
Friction Angle	1

As noted in Table 1, the introduction of vertical forces also requires the scaling of the strength of the rock mass to achieve dynamic similitude. To overcome these limitations, Castro et al. (2014) presented the design and results of an experimental setup to investigate confined flow [2]. In this case, a vertical pressure was applied through a cylindrical press during the flow experiments. The experiments showed the influence of the load on fragmentation, compaction and hang up frequency. However, it should be noted that these experiments required a special setup, which may not be available to all physical modelers.

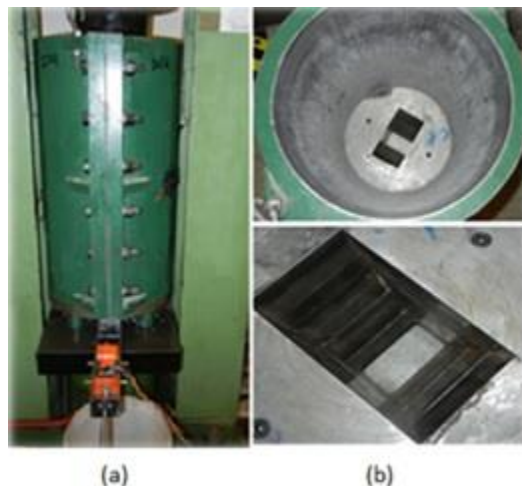
### 3. Definition model condition

The experimental model of confined flow was implemented in the laboratory to investigate the impact of mean vertical pressures, moisture and draw policies on hang-up occurrence. This model was represented by three main sections. The first section was a "Physical model" which contained the steel-based container with granular material under high stress condition, including the

drawbell located at the bottom of the model. The second section was "Loading system" which involved LHD system to draw material at the drawpoints. Moreover, this model comprised a press as a "Hydraulic press machine" to apply the vertical pressure during the experiments.

### 3.1 Physical model

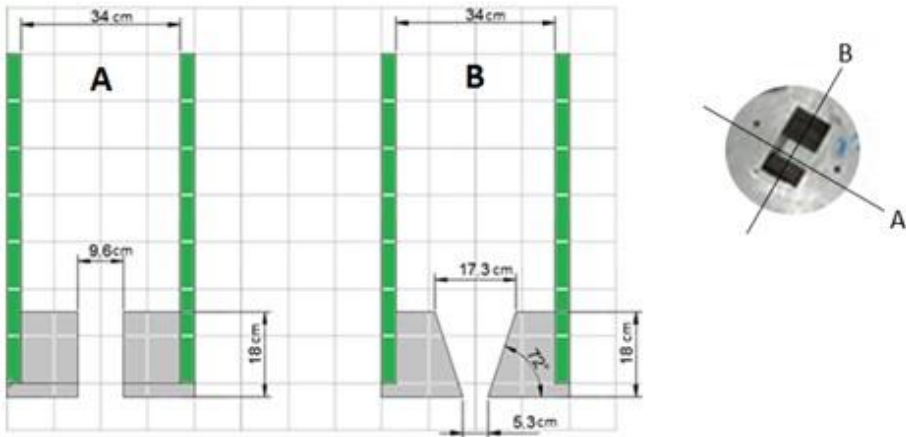
The physical model consisted of a steel cylinder, with a hydraulic press machine which was filled using broken rock (60 kg of crushed ore) (Figure 1). Fuenzalida [10] introduced this geometrical model to analyse the effects of vertical pressures on gravity flow. However, the base of Fuenzalida model consisted of a circularly-shaped hole to draw material. In this research, for a practical application in cave mining, a drawbell was designed with a rectangular opening of 53 mm x 96 mm at the bottom of the model (Figure 1.b). The dimensions of the opening were defined to emulate a drawbell designed at Andina copper mine, Chile, with a scale of 1:75 (in all three x,y,z dimensions). The main reason of selecting Andina mine was its regular geometry of drawbell and the simplicity of constructing the scaled down model. However, the objective of the research did not resembling an exact situation, but a general condition. Hence, the model media was not the same as Andina's mine. Actually, two specific fragment sizes from the underground's Chuquicamata mine were selected to perform the experiments.



**Figure 1: Experimental model: (a) cylindrical model in a press machine which changes the vertical pressure,  $\sigma_v$ , and (b) drawbell, located at the bottom, center of the model.**

The detailed geometry of the drawbell is displayed in Figure 2 **iError! No se encuentra el origen de la referencia.**, including two different sections of the physical model. Drawbell was located in the centre of the cylinder with rectangular openings for two drawpoints. This model enabled analysing the flow-ability of granular material subject to the interaction between drawpoints.

However, the interaction between drawbells was not in the scope of this research. Scaled and actual drawbell dimensions are indicated in Table 2.



**Figure 2: Physical model dimension.**

**Table 2: Mine infrastructure dimensions**

Item	Scaled model Value	Actual size
Drawbell angle (A section)	90°	90°
Drawbell angle (B section)	72°	72°
Drawbell height	180 mm	13.5 m
Drawbell apex	53 mm	4 m
Gallery height	60 mm	4.5 m
Gallery width	60 mm	4.5 m

**3.2 loading system**

In this physical model, an extraction system was built to replicate the LHD extraction, as is the case of mine site conditions. The material extraction from the drawbell was carried out utilizing two LHD systems (scaled from a bucket of 14 yd<sup>3</sup>). The LHD extraction system was developed to replicate the draw system of the production level in a Block/Panel caving operation (see Figure 3). The system had two motors: stepper and servo. Stepper motor provided horizontal movement of the extraction system and the servo motor controlled the bucket movement during loading and dumping of the fragmented material (between 50 and 60 grams of material).

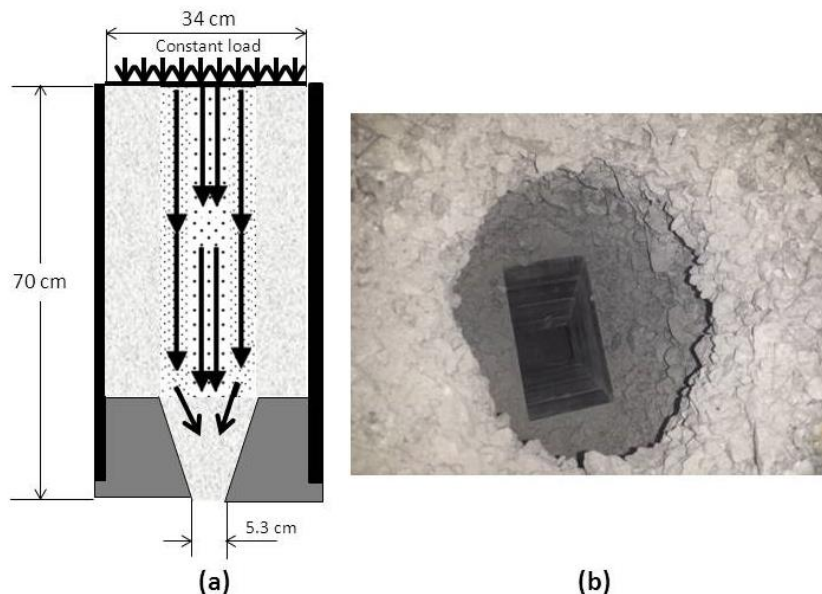


**Figure 3: LHD extraction system.**

### 3.3 Boundary condition

The cylindrical geometry of the model was selected to prevent the boundary effect related to arching and stress concentration in the corners. Moreover, the design of drawpoints and LHD system allowed for the study of hang-up occurrence in the drawbell, taking into account the boundary conditions related to drawpoint geometry and conditions emulating the mine site conditions.

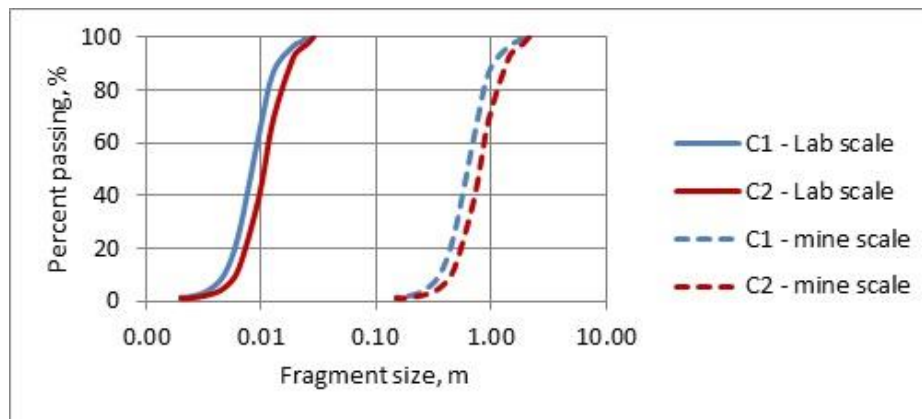
Considering the flow pattern within the cylinder, it was supposed that core flow pattern would be happened due to the wide angle of drawbell. Figure 4.a shows a schematic view of core flow pattern. The material statue at the end of experiments also confirmed this pattern (Figure 4.b). If the core is narrower than the width of cylinder ( $D$ ), as in Figure 4, the material near the top will cascade down the top surface into the flowing core and will be discharged before material at a lower level [15]. During the experiment the internal flow never reaches the wall of cylinder. The advantage of this flow pattern is that wall stresses peaks occur in the flowing core and the surrounding stagnant material shields the walls to some extent from these effects.



**Figure 4: (a) A schematic view of material core flow pattern; (b) Material state after the experiments**

### 3.4 Model media

The material used in the experiments was crushed sulphide ore with a high aspect ratio to represent the geometry of caved rock, sphericity of 0.58 and a roundness of 0.25, where sphericity measures the degree to which a particle approaches a spherical shape and roundness refers to the sharpness of the corners and edges of a grain. To evaluate the impact of size distribution on hang-up events, two different particle size distributions of the material were prepared and tested: one with the 80% passing size ( $d_{80}$ ) of 11.8 mm and the second with a  $d_{80}$  equals to 15.6 mm (see Figure 5: Curve 1 and Curve 2, respectively). Both samples had the same uniformity coefficient of 2 ( $Cu=d_{60}/d_{10}$ ). The particle size distributions were scaled from the predicted size distribution of the primary fragmentation curve of the underground's Chuquicamata mine project (Figure 5) [16]. Table 3 summarizes the characteristics of the samples, showing that the only difference between the samples is in the dimension of the particles.



**Figure 5: Comparison of particle size distribution of samples used in the experiments with real size distribution of mining material.**

**Table 3: Characteristics of the material used during the experiments**

Curve	Average size, $d_{50}$	$d_{80}$	Uniformity coefficient	Drawpoint width/ $d_{50}$ , initial	Drawpoint length/ $d_{50}$ , initial	Point load index	Initial humidity	Density	Internal friction angle
	mm	mm	Cu	$d_w/d_{50}$	$d_l/d_{50}$	$I_{s50}$ , MPa	% (solid by weight)	kg/m <sup>3</sup>	°
1	8.6	11.8	2.0	6.2	11.2	6.2	0.6	2,630	39
2	10.8	15.6	2.0	4.9	8.9				



### 3.5 Experimental program

A total of 33 laboratory tests of granular material under conditions of high confinement were conducted. Variables measured during tests included hang-up frequency, the height of hang-up and secondary fragmentation. The input variables were: size distribution, mean vertical pressure, draws strategy and humidity conditions.

During the experiments, different vertical pressures in dry and wet conditions were applied. The humidity conditions were created with 1.6% of water content (mass percent), which caused enough humidity to cover the surface of the fragments with moisture (4.3% degree of saturation). The hang-up frequencies and the heights of hang-ups were recorded while drawing material through one or two drawpoints. Table 4 summarizes the number of experiments carried out, for each the vertical pressures applied. In this table, the symbols "1D", "2D" and "1Dw" donate the extraction from one drawpoint, both drawpoints (in sequence) and one drawpoint applying wet material respectively. The bulk density and void ratio are also recorded for each test in Table 4. These parameters are found to vary significantly with applied different mean vertical pressures. This is mainly due to the rearrangement of the particles. It should be mentioned that bulk densities and void ratios were measured when the pressure was reached to its defined amount and before ore extraction. In addition, vertical pressure was stabled during extraction in each test.

The measurements of hang-ups for all experiments were carried out after 100 extraction cycles of LHD were achieved, which equalled approximately to five kilograms of extracted material. When a hang-up occurred, it was recorded and removed manually. For 2D tests, the extraction from both drawpoints was performed sequentially until a hang-up occurred. When a hang-up occurred at the drawpoint, drawing from the other drawpoint continued. If a hang-up did not clear naturally after drawing five cycles of LHD from the neighbouring drawpoint, the hang-up was removed manually.

**Table 4: Experiments carried out during this research**

Vertical pressure MPa	d <sub>50</sub> = 8.6 mm			d <sub>50</sub> = 10.8 mm			Bulk density t/m <sup>3</sup>	Void ratio #
	1D (Dry)	2D (Dry)	1D <sub>w</sub> (Wet)	1D (Dry)	2D (Dry)	1D <sub>w</sub> (Wet)		
0	2	2	2	2	2	2	1.38	0.91
0.8	1	-	-	1	-	-	-	-
1.5	2	1	1	2	1	1	1.48	0.77
3	2	1	1	2	1	1	1.56	0.69
5	1	1	-	1	-	-	1.69	0.56

## 4. Experimental results

Qualitative and quantitative analyses of hang-ups (frequency and height) were performed. For qualitative analysis, three classes for hang-up location were defined (see Figure 6). Type A illustrates a low hang-up characterised by a vertical slope of broken ore at the drawpoint brow. This type of hang-up was generally reported when coarse fragments collected at the top of the drawpoint. This type of hang up was mostly cleared by extracting material from the opposite drawpoint (2D tests). The type B hang-up in Figure 6 corresponds to the low hang-up over the drawpoint. The height of this hang-up was significantly affected by the magnitude of the applied vertical pressure. This type of hang-up must be removed manually. Type C represents a high hang-up located at the top of the drawbell. This type of hang-up often appears when high vertical pressure was applied. Due to the high compaction of material, it was not possible to clear type C hang-up manually. Therefore, the experiment finished when high hang up occurred.

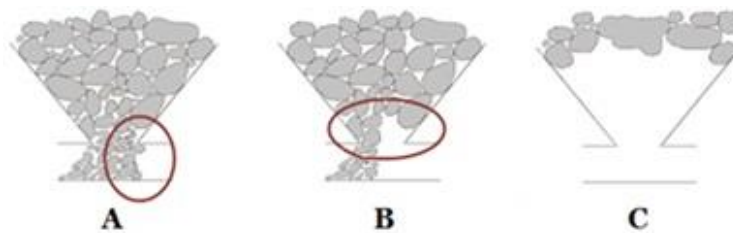


Figure 6: Types of hang-ups.

### 4.1 Hang-up frequency

The measurement unit of the hang-up frequency was the amount of material extracted between two consecutive hang-ups, *gram/hang-up*, which was subsequently scaled up to *ton/hang-up* to be comparable with the mine data.

Hang-up frequency of the samples (Curve 1 - C1 and Curve2 - C2) was measured when drawing from one drawpoint in dry (1D) and moisture (1D<sub>w</sub>) conditions as well as two drawpoints in dry condition (2D). The results from applying different vertical pressures are presented in Figure 7 **iError! No se encuentra el origen de la referencia.** and Figure 8 (for C1 and C2, respectively). Although three experiments were performed applying 5 MPa mean vertical pressures, these results have not been displayed in Figures 7 and 8 as, at the start of the test, a high hang-up occurred on the top of the drawbell due to the high stress conditions. As a result, a gap area was generated during the extraction of the material between the coarse arch and

the ore drawn. Therefore, the fragments under the gap area were drawn without confinement.

A downward trend for all drawing strategies (Figure 7 and 8) reveals that hang-up frequency decreases with increasing mean vertical pressures. The same results were obtained in previous studies with gravel material and circumferential extraction point [2]. Comparison of different tests shows that lower hang-up frequencies were obtained in wet condition. The hang up appeared in wet situation should probably be cohesive arches; since water creates an agglomerate of fine materials due to increasing the cohesion between fragments [3, 18]. Furthermore, when comparing two size distributions (Figure 6 and 7), it can be seen that hang-up frequency of finer material (C1) is significantly higher than the coarser fragmentation (C2). For instance, during the experiments without confinement, hang-up frequency increased 40% (C1) and 8% (C2) in 2D tests (compared with 1D tests). The increase in the hang-up frequency implies a better flow-ability of broken material. In addition, Curve 1 has less hang-up events due to a high ratio  $d_w/d_{50} = 6.2$  as compared to Curve 2 ( $d_w/d_{50} = 4.9$ ).

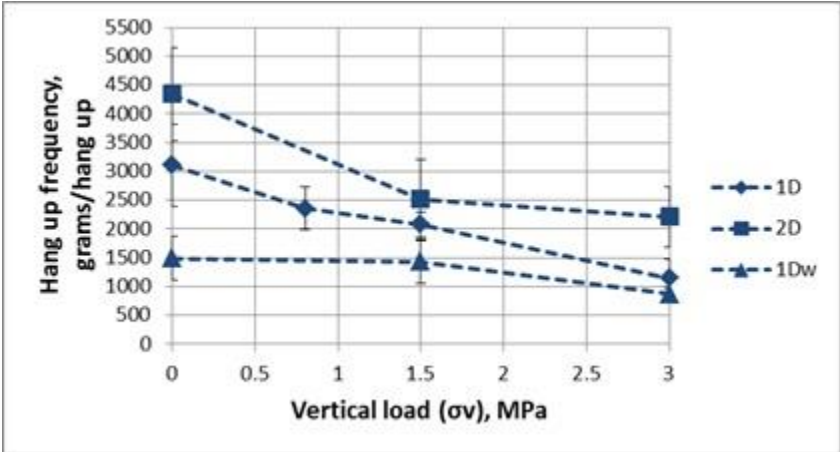


Figure 7: Curve 1 (d80 = 11.8 mm) hang-up frequency results.

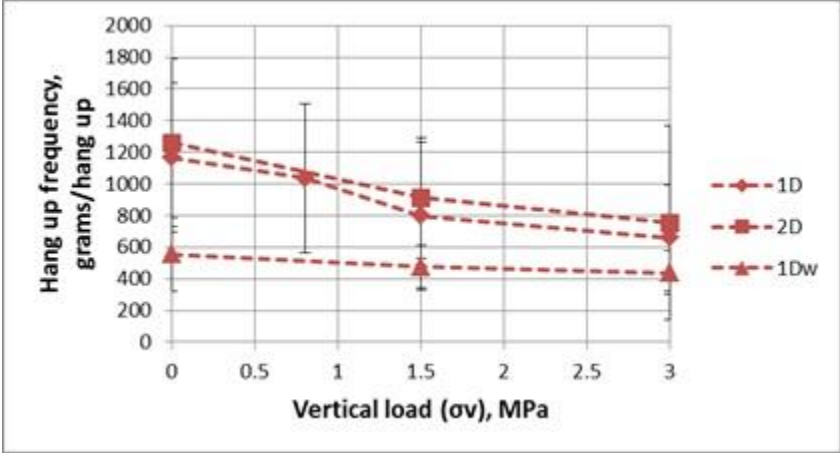


Figure 8: Curve 2 (d80 = 15.6 mm) hang-up frequency results.

A summary of results for the 2D test is shown in Table 5. The percentage of hang-ups, which were removed automatically, shows that the alternate extraction between drawpoints assists in the removal of hang-ups. Based on the experiments, approximately fifty percent of hang-ups were removed by alternating the extraction between the drawpoints, which shows that it is not always necessary to remove hang-ups manually when applying this draw strategy. Due to the interaction between drawpoints (at least for the ratios  $d_L/d_{50}$  tested between 8.9 and 11.2).

**Table 5: Hang-ups on Test 2D strategy**

Vertical pressure MPa	Total number of hang ups		Hang up remove Manu-ally		Hang up removal automatically, %	
	$d_{50} = 8.6$ mm	$d_{50} = 10.8$ mm	$d_{50} = 8.6$ mm	$d_{50} = 10.8$ mm	$d_{50} = 8.6$ mm	$d_{50} = 10.8$ mm
0	2	4	1	2	50%	50%
0	3	4	2	3	33%	25%
1.5	4	6	2	1	50%	83%
3	5	6	1	3	80%	50%
Mean					53%	52%

The size distribution has a significant impact on hang-up phenomena. During the experiments, fragmentation was measured before and after each test. Table 6 shows the degree of fragmentation as a function of the mean fragment size. Experimental results show that final fragmentation depends on the applied mean vertical pressure. Lower fragmentation is achieved with an increase in vertical pressure and, with a decrease in material size distribution under increased pressure, there is a reduction in hang-up frequency. In addition, the vertical stress influence on hang-up frequency is evident in Table 6. For example, in tests 1D, the same final ratio  $d_w/d_{50}$  of 6.2 was achieved for 0 MPa in Curve 1 ( $d_{50,initial} = 8.6$  mm) and 3 MPa in Curve 2 ( $d_{50,initial} = 10.8$  mm), yet the hang-up frequency in Curve 1 was almost five times that of Curve 2.

**Table 6: Summary of experimental results**

Test	Vertical pressure	d <sub>50</sub>	d <sub>50</sub>	d <sub>w</sub> / d <sub>50</sub>	Hang up frequency	Standard deviation
		initial	final	Final		
Condition	MPa	mm	mm	#	grams/hang up	grams/hang up
1D	0	8.6	8.6	6.2	3109.3	713.1
	0.8		7.8	6.8	2358.3	376.7
	1.5		7.6	7.0	2074.9	210.7
	3		7.1	7.4	1147.9	327.1
	0	10.8	10.8	4.9	1163.2	471.1
	0.8		9.1	5.8	1036.3	471.3
	1.5		8.7	6.1	797.6	465.8
	3		8.6	6.2	657.4	337.0
1Dw	0	8.6	8.6	6.2	1492.9	386.0
	1.5		8.4	6.3	1434.5	372.5
	3		8.2	6.5	866.4	104.0
	0	10.8	10.8	4.9	555.0	230.3
	1.5		10.5	5.1	476.5	134.6
	3		10.0	5.3	437.6	137.7
2D	0	8.6	8.6	6.2	4342.4	810.7
	1.5		7.6	7.0	2515.0	688.8
	3		7.1	7.4	2208.7	528.4
	0	10.8	10.8	4.9	1258.8	529.6
	1.5		8.7	6.1	910.7	382.5
	3		8.6	6.2	753.2	611.9

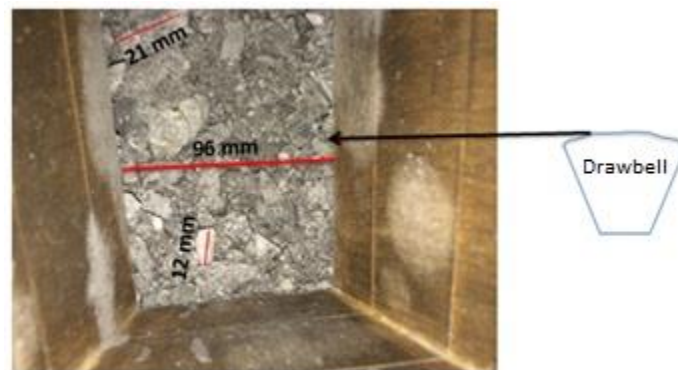
## 4.2 Hang-up height

The hang-up height is the distance between the top of the gallery and the arch of the hang-up. Table 7 shows the experimental results of the height of hang-ups with respect to the different mean vertical pressures. For all scenarios, increased vertical pressure resulted in the increase in the height of hang-ups. The results in Table 7 show that the height of hang-ups during 2D tests is less than during the 1D test. Therefore, the interaction between drawpoints (2D tests) not only decreases the number of hang-ups but also reduces hang-up height. Furthermore, in comparing the dry and wet conditions, the results show that the height of hang ups is more sensitive to vertical pressure under dry conditions in experiments carried out probably due to the frictional forces between fragment surfaces (of major sizes) decrease with the moisture content and the fragment sizes tested.

**Table 7: Hang-up mean height results, mine scale.**

Vertical pressure MPa	1D		2D		1D <sub>w</sub>	
	Height C1 m	Height C2 m	Height C1 m	Height C2 m	Height C1 m	Height C2 m
0.0	0.0	0.5	0.7	0.2	1.1	2.5
0.8	3.2	1.1	-	-	-	-
1.5	0.1	4.0	0.0	0.4	0.3	3.5
3.0	5.1	5.5	0.0	3.1	2.3	4.4
5.0	9.8	5.6	-	-	-	-

The most frequent hang-ups observed during the experiments were type A (Figure 6) while high hang-up (type C) rarely appeared. Almost all high hang-ups appeared in conditions of high vertical pressure (greater than 3 MPa). Figure 9 displays a high hang-up under 5 MPa mean vertical pressure. The majority of the arch surface is covered by coarse fragments; however, the size of the largest fragment is at least five times less than the drawbell apex. According to the ore pass studies [8], these types of hang-ups wouldn't occur for the size distributions used in this research. However, it should be noted that the ore pass studies in [8] were performed in low stress condition. Therefore, the state of stresses and the geometry of the drawpoint must be considered in hang-up investigations.



**Figure 9: High hang-up on test.**

## **5. Hang-up analysis**

The experimental results were scaled up to enable a comparison with mine site data. A complete database of three caving mines was considered in this study. This database includes DOZ mine of Indonesia, Palabora mine of South Africa and four sectors of El Teniente mine in Chile. Due to the different drawbell geometry and fragment size distributions, the ratio of drawpoint apex ( $d_w$ ) and mean fragment size ( $d_{50}$ ) was selected as comparative parameters. Table **8iError! No se encuentra el origen de la referencia.** illustrates these ratios determined from experiments as well as mine site data. The  $d_w/d_{50}$  of mine site data ranges from 4.7 to 6.8 and the experimental ratios fall within this range.

**Table 8: Ratio between the drawpoint apex and the mean fragment size**

Data	Ratio ( $d_w/d_{50, initial}$ )
Curve 1 (Test)	6.2
Curve 2 (Test)	4.9
Palabora	4.7
Reno	6.8
Diablo Regimiento	6.8
Teniente 4 Sur	6.2
Esmeralda	ND
DOZ	5.1

ND: no data.

The experimental results illustrated the significant influence of mean vertical pressures over hang-ups. Since the mean vertical pressure of mine data is not available, it is necessary to calculate this parameter for each mine under study. According to the arching effect over the production level, it is not rigorous to calculate mean vertical pressures considering just weight and height of caved material ( $Y \cdot h$ ) [14]. Experimental study developed in [14] with fragmented gravel indicated mean vertical pressure was  $0.71 \cdot Y \cdot h$ . Numerical modelling also manifested stress arching on granular material [18]. Stress arching occurs when part of the material weight is transferred into the sidewalls, forces acting at the boundaries in an opposite direction to the material weight. Surrounding uncaved boundaries of a caved are generally rough, therefore, Janssen approach [19] was applied to estimate the mean vertical pressure at the bottom of the broken ore ( $\sigma_v$ ) in each mine, a general solution of Janssen's equation for mean vertical stresses is given by Nedderman [15]:

$$\bar{\sigma}_v = \frac{Rh \cdot \rho_b \cdot g}{\tan(\phi_w) \cdot k} \left( 1 - \exp\left(\frac{-k \cdot \tan(\phi_w) z}{Rh}\right) \right) \quad (1)$$

Where:

$Rh$ : Hydraulic radius of the drawn area (m); drawn area can be estimated through the ratio between production (tpd) and draw rate (tpd/m<sup>2</sup>),

$\rho_b$ : Bulk density (kg/m<sup>3</sup>),

$g$ : Gravitational constant (m/s<sup>2</sup>),

$k$ : Ratio of horizontal to vertical stress ( $k = 1 - \tan(\phi)$  [15],  $\phi$ : internal friction angle),

$\phi_w$ : Friction angle at the ore column boundary (rad) and

$z$ : Depth of broken ore (m), between production level to surface.

In order to compare the results obtained from the experiments with mine data, hang-up frequency was scaled up through geometric scale factor (1:75). Accordingly each ton per hang up equals (geometric scale factor)<sup>3</sup>/10<sup>6</sup> times of

one gram per hang up. For instance 1000 grams per hang up equivalent to 421.875 tons per hang up with the geometric scale factor of 75.

Subsequently, the mean vertical pressure was determined for the mines under analysis through equation (1), considering the parameters shown in Tables 9, 10 and 11. Hang-up frequency can be considered as a function of vertical pressure which is dependent on the height of draw column. In all mines, the hang-up frequency increased by advancing the extraction of the ore column. This was due to the reduction in the size of the fragments, which is known as secondary fragmentation [1].

**Table 9: Mine data of Diablo Regimiento, Reno, Teniente 4 Sur and DOZ mines.**

Mine		Draw column height	Hang up frequency	Depth	Drawn area	Mean Vertical pressure																																																												
Name	Sector	m	ton/hang up	M	m <sup>2</sup>	MPa																																																												
Diablo Regimiento [4]		0-45	605	590 [20]	16320 [4]	2.0																																																												
		0-93	784				Reno [4]	Fw	0-50	403	830 [20]	28026 [4]	2.5	0-100	531	0-200	522	0-300	434	0-400	782	Hw	0-50	458	0-100	596	0-200	830	0-300	914	0-400	914	Hw'	0-50	450	0-100	593	0-200	811	0-300	844	0-400	844	Teniente 4 Sur [4]		0-50	251	550-660 [20]	99360 [4]	4.4	0-100	327	0-200	461	0-300	514	DOZ [5]		0-60	348.2	1200 [21]	41550	3.2	60-120	294.2	120-180
Reno [4]	Fw	0-50	403	830 [20]	28026 [4]	2.5																																																												
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		0-300	514																																																															
DOZ [5]		0-60	348.2	1200 [21]	41550	3.2																																																												
		60-120	294.2																																																															
		120-180	321.4																																																															
		180-240	408.6																																																															
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**Table 10: Mine data of Esmeralda mine.**

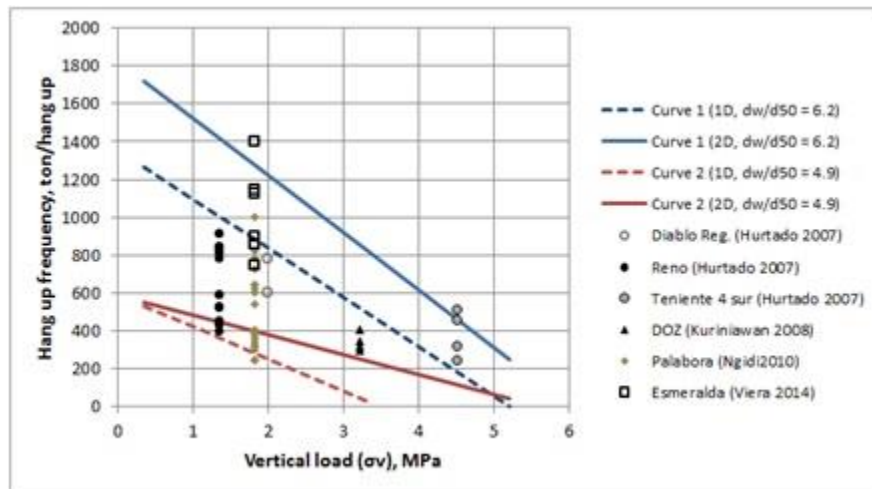
Mine		Extraction of primary column	Hang up frequency	Depth	Drawn area	Mean Vertical pressure
Name	Sector	%	ton/hang up	m	m <sup>2</sup>	MPa
Esmeralda [7]		10-20	750	650	30589 (B1) 13163 (B2) [7]	2.2
		10-20	860			
		10-20	900			
		20-30	1150			
		30-40	1130			
		40-50	1400			

**Table 11: Mine data of Palabora mine.**

Mine		Period	Hang up frequency	Depth	Drawn area	Mean Vertical pressure
Name	Sector	year	ton/hang up	M	m <sup>2</sup>	MPa
Palabora [6]	1	2004	240	500-1260 [20]	11071 [6]	2.0
		2005	300			
		2006	380			
		2007	390			
		2008	400			
		2009	380			
	2	2004	360		16850 [6]	
		2005	400			
		2006	620			
		2007	780			
		2008	640			
		2009	640			
	3	2004	400		27778 [6]	
		2005	540			
		2006	820			
		2007	1000			
		2008	600			
		2009	720			
	4	2004	240		11071 [6]	
		2005	240			
		2006	320			
		2007	320			
		2008	340			
		2009	320			

In Figure 10, hang-up frequency of different mines as well as experiments (1D and 2D tests for different size distribution C1 and C2) are plotted against the mean vertical pressure. Linear Regression was applied to the experimental results. The influence of the ore fragmentation on hang-up frequency is clearly

visible in Figure 10, which could be used as a practical approach to evaluate the hang-up frequency for different mean vertical pressures and fragmentation.



**Figure 10: Hang-up frequency graph. Ref.: Diablo Reg. [4], Reno [4], Teniente 4 sur [4], DOZ [5], Palabora [6], Esmeralda [7].**

## 6. Conclusions

The influence of fragmentation, mean vertical pressure, draw strategies and humidity on hang-up frequency and hang-up height was studied under confined conditions for a single drawbell. During the experiments conducted in the laboratory, hang-up frequency and hang-up height were recorded for different pressure, fragmentation, draw strategy and humidity scenarios.

The review of hang up frequency obtained in each vertical pressure shows the contrary relationship between these parameters. As can be seen from the graph in Figure 10, an increase in mean vertical pressure results a decrease in hang-up frequency. The Similar results were also obtained in previous studies [2]. Therefore, vertical pressure should be considered in the studies of hang-up frequency. Not even hang -up frequency, but also hang-up height was influenced by vertical pressure. According to Table 5, high hang-ups mostly appeared when applying high vertical pressure, due to the high compaction and increasing the friction forces between ore fragments.

Besides vertical pressure, this research shows that draw strategy also influence hang-up height and frequency. Higher hang-up frequency but lower hang-up heights are experienced when using two drawpoints with alternating extraction between the drawpoints due to interaction between the flow zones in

experiments. As a result, it would be possible to increase productivity by changing the draw strategies during ore extraction; provided that the drawpoints interaction takes place. In that case, the natural (due to draw) removal of hang-up is increased when alternated extraction between drawpoints is implemented.

The presence of water mostly change geomechanical and geotechnical concepts of rock. Consequently, the influence of water on hang-up characterization were analysed in this study, considering one active drawpoint. The results revealed that Hang-up frequency is lower in wet conditions ( $1D_w$ ) as compared to dry rock ( $1D$ ) due to higher cohesion in humid conditions. Moreover, hang-up height recorded in wet conditions is lower than in dry situation for the same mean vertical pressure due to lower frictional force between fragments causing instability in coarse arches, which decreases the hang-up height.

Comparison of the experimental results of the samples with two different size distributions showed that coarser fragmentation results in fewer number of hang-up frequency. With the increase in the number of large fragments, the probability of blockage at drawpoints increases.

In addition to laboratory experiments, field data were also used to compare with experimental results. The scaled up hang-up frequency fell within the range of the mine site data. It is thus concluded that experimental results could be used as a practical guide for the analysis and prediction of the hang-up events.

## **Acknowledgements**

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# **Paper 2: Experimental and comminution model framework for secondary fragmentation quantification for block caving**

## **Abstract**

Block Caving is the most economic underground methods for excavating massive ore bodies, due to the low production costs. The success of block caving operation depends on the fragmentation and draws control as large blocks decrease the equipment efficiency, inhibit the material flow, and may lead to extensive secondary blasting. Even though ore handling system design as well as overall economic results critically rely on secondary fragmentation, this parameter continues to be one of the least understood in the caving operations.

The work presented in this paper focuses on the experimental study of secondary fragmentation, considering the impact of vertical loads and flow. Experiments using a LHD extraction system where conducted under lab conditions to quantify the influence of load and particle size. Results demonstrated that as the vertical load (simulating deeper conditions) increase the larger the fragmentation that occurs and fines generation during draw. From the experimental results a mathematical model was developed using the Kinetic Matrix Model used in comminution processes to adjust the fragmentation observed in the experiments. The results showed good correlation to experimental data showing the usefulness of the method to predict secondary fragmentation. Finally a framework was developed to calculate the fragmentation for a block caving applications which was compared to mine data for secondary fragmentation estimates.

Keywords: Secondary fragmentation, Kinetic matrix model, autogenous comminution, hang-ups.

## **1. Introduction**

Block caving fragmentation can be classified as: in-situ, primary and secondary fragmentation. The in-situ fragmentation is represented by the blocks naturally present within the rock mass before any mining activity commences. The primary fragmentation consists of the blocks in the vicinity, but separate from the cave back; this fragmentation occurs when the undercut is mined and caving phenomenon is initiated. The subsequent fragmentation occurring as

the blocks move down through the draw column to the drawpoints is known as secondary fragmentation (Brown, 2004). The nature and degree of secondary fragmentation is expected to vary with the stress regime within the caved mass, the composition and mechanical properties of the ore body, the rate of draw, the height through which the material is drawn and the residence time in the draw column.

Many attempts have been made to model secondary fragmentation in block caving (Chitombo, 2010). One of the most widely used methods of assessing in-situ, primary and secondary fragmentations in block caving operation is Block Cave Fragmentation (BCF, Esterhuizen, 1999). BCF was specifically developed to generate rock size distribution using, as input data, the cave face orientation (dip and dip direction), stress data, MRMR, joint set orientations and joint set spacing. Using actual data from Premier and Codelco caves in Chile, Ngidi and Pretorius concluded that BCF predictions are generally conservative (Ngidi and Pretorius, 2011). Laubscher (2001) established a practical graph, which shows how the decrease in particle size is influenced by different draw heights (or travel of fragmented rock) and coarse (less jointed) to fine (well jointed) rock mass. The influence of fragments' travel has been observed in ore passes, where the size of the rock representing the 50% of mass after passing through 257 m rise has a reduction of 78% (Verden and Henderson, 2004). The same results were obtained at the Brunswick and Kidd Creek mines at which the maximum particle size was reduced by around 50% per 300 m vertical distance of fragments' travel (Yu TR, 1989).

Empirical models lack of a theoretical background to explain the influence of compression (load) and shear (flow) in the fragmentation process of block caving. There is evidence that compression and shear have influence on fragmentation (Pierce, Waetherley and Kojovic, 2010). Flow models such as Rebob have included the empirical breakage model of Bridgwater to estimate the shearing-induced secondary fragmentation within a caving context (Pierce, 2004). Some authors have used methods that describe breakage mechanisms based on comminution process to describe the secondary fragmentation process (Merino, 1986; Hustrulid, 2000). The Matrix Model is a method that could be used to study the process of comminution (Broadbent and Callcot, 1956; National Materials Advisory Board, 1981; Lynch 1997). This method has been successfully applied to comminution in grinding machines, however, because of its generality and simplicity, it is suitable for the study of any breakage process including secondary fragmentation in caving. The Kinetic Model is, in concept, very similar to the Matrix Model except for the selection function being replaced by the rate of breakage (Lynch, 1997). The rate of breakage can vary with fragment size and the time the fragments stay inside

the draw column. Despite being postulated to caving environments there is not experimental evidence of the efficiency of the Kinetic Breakage Model based on experimental data.

Given the difficulties to estimate and measure shear and compression within the ore column, in this paper, the authors investigated the efficiency of Kinetic Breakage Model to model secondary fragmentation. For this purpose gravity flow experiments under compression were conducted and based on two particle size distributions of crushed ore material and compared to the predictions of a proposed mathematical model.

## 2. Principals of Matrix Model

The Matrix Model can be used to analyze the breakage of heterogeneous particles for any process that results in changes to the distribution of a characteristic in an assembly (Broadbent and Callcot, 1956). To use this method for predicting the secondary fragmentation in caving operations, the draw column is represented as a comminution machine. This model was developed to describe the breakage mechanism for solids. Breakage event consisted of a single step in the degradation process (Epstein, 1948). If the breakage process is considered as a succession of discrete events, two basic functions can be used to determine the progress of a breakage process:

- Selection function, which determines the proportion of particles that undergo breakage while the remaining fragments are left unbroken;
- Breakage function, which provides the probability of the mill output containing a broken particle of size  $y$ ,  $P(y)$ . The probability that particle of size  $y$  is broken to size  $x$  in the  $n$ th step of the breakage process is defined by function  $F(x,y)$ .

Any comminution device grinds a set of particles having a specific size distribution with the larger particles having a greater probability of being broken than the smaller particles. Thus, smaller particles require more events of breakage. The breakage behavior of particles in a comminution device depends on their size, shape and homogeneity and on other variables, which are related to the equipment (National Materials Advisory Broad, 1981). If the feed of the breakage machine is represented by a cumulative continuous distribution function,  $F(y)$ , and this function is differentiable, the frequency function,  $f(y)$ , can be calculated as:

$$f(y) = \frac{dF(y)}{dy} \quad (1)$$



In which  $f(y)$  is the proportion of particles of size  $y$ . If the material undergoes a breakage process, represented functionally by  $b(x,y)$ , the output frequency function  $f(x)$ , for each size  $y$ , can be obtained by (Merino 1986):

$$P_1(x) = \int b(x,y)dFy = \int b(x,y)f(y)dy \quad (2)$$

In Equation 2,  $b(x,y)$  is defined as the breakage matrix, with elements  $b_{ij}$ , where  $i, j = 1, \dots, n$ ; correspond to the proportion of a typical particle with size between  $a_{j-1}$  and  $a_j$  before breakage, which falls between a size class  $a_{i-1}$  and  $a_i$  after breakage.

There is no recirculation of fragments in caving operation. Therefore, if a proportion  $S_i$  is selected for breakage from  $f_i$ , the mass of fragments selected for breakage in size range  $i$  is represented by  $S_i f_i$  and  $(1-S_i)f_i$  represents the fragment that remains unbroken.  $S_i$  is defined as the selection function (Reid, 1965).

## 2.1 Modelling of secondary fragmentation

The secondary fragmentation model presented in this paper is able to predict the particle size distribution based on the ore characteristics and operating conditions. The model structure has its basis within a population mass balance model, which has been widely accepted and is in common usage for many comminution systems (King, 2001) including systems using the pressure as the main fragmentation mechanism (Torres and Casali, 2009).

The linear size-discretised model for breakage kinetics, in its general form (King, 2001), is as follows,

$$p_i = \int_0^{\infty} f_i(t) E(t) dt \quad (3)$$

Where:

$p_i$  is the ore mass fraction in the  $i^{th}$  size interval, at the draw point.

$E(t)$  is the residence time distribution in the fragmentation environment.

$f_i(t)$  is the ore mass fraction in the  $i^{th}$  size interval of the material being fragmented at time  $t$ . It is known as the batch grinding kinetic equation (King, 2001), expressed as follow,

$$\frac{d[H f_i(t)]}{dt} = -S_i H f_i(t) + \sum_{j=1}^{i-1} b_{ij} S_j H f_j(t) \quad (4)$$

Where:

$H$  (holdup) is the total mass of material being fragmented.

$S_i$  is the selection function and denotes the fractional rate at which material is broken out of the  $i^{th}$  size interval.

$b_{ij}$  is the breakage function and represents the fraction of the primary breakage product of material in the  $j^{th}$  size interval which appears in the  $i^{th}$  size interval.

Considering that during the experiments pressure is applied to the material confined in a steel cylinder, a plug flow has been assumed, neglecting any vertical mixing in the system. For this case, the  $E(t)$  term has a value of 1 for a fixed residence time  $\tau$  and 0 for all other times. Then, Eq. 3 can be re-written as follows,

$$p_i = f_i(\tau) \quad (5)$$

To find  $p_i$ , the solution of the batch grinding kinetic equation (Eq. 2) is needed. Considering  $H$  large enough to be assumed constant, Eq. 4 has been solved analytically (Reid, 1965). The system solution for the  $N$  size classes and  $t = \tau$  is written as follows,

$$p_i = \sum_{j=1}^i A_{ij} e^{-S_j \tau} \quad (6)$$

Where:

$$A_{ij} = \begin{cases} 0 & \text{if } i < j \\ f_{i0} - \sum_{k=1}^{i-1} A_{ik} & \text{if } i = j \\ \sum_{k=j}^{i-1} \frac{b_{ik} S_k}{S_i - S_j} A_{kj} & \text{if } i > j \end{cases}$$

$f_{i0}$  is the initial particle size distribution.

For the breakage functions,  $b_{ij}$ , the functional expression (Austin and Luckie 1972) presented in Eq. 7 was used,

$$B_{i1} = \alpha_1 \left(\frac{x_i}{x_2}\right)^{\alpha_2} + (1 - \alpha_1) \left(\frac{x_i}{x_2}\right)^{\alpha_3} \quad (7)$$

Where:

$a_1$ ,  $a_2$  and  $a_3$  are model parameters ( $a_3 > a_2$ ) representing the shape of the fragment size distribution and need to be adjusted with the experimental data.

$B_{i1}$  is the cumulative form of the  $b_{ij}$  functions.

$x_i$  is a mesh size for a given size interval.

For the selection function,  $S_i$ , the functional expression (Herbst and Fuerstenau, 1980) presented in Eq. 8 was used,

$$S_i^E = S_1^E \exp \left\{ Z_1 \ln \left( \frac{d_i}{d_1} \right) + Z_2 \left[ \ln \left( \frac{d_i}{d_1} \right) \right]^2 \right\} \quad (8)$$

Where:

$Z_1$ ,  $Z_2$  and  $S_1^E$  are the model parameters.  $S_1^E$  considered invariant for each size, and representing the specific rate of breakage for particles of the maximum size *fraction in the sample* and  $Z_1$  and  $Z_2$  representing the shape of the variation of the specific rate of breakage with particle size. These parameters need to be adjusted with the experimental data.

$d_i$  is the geometric mean of the  $i^{th}$  size class,  $d_i = \sqrt{x_i \cdot x_{i+1}}$

To obtain the selection function,  $S_i$ , a scale-up relationship (Herbst and Fuerstenau, 1980) among the specific rate of breakage and the quotient between Power,  $P$  and Holdup,  $H$ , was used, as it is shown in Eq. 9.

$$S_i = S_i^E \left( \frac{P}{H} \right) \quad (9)$$

As was mentioned above, in the case of block caving  $H$  was assumed constant and could be demonstrated that the relation of  $Power/H$  could be equated to Pressure/constant. Therefore, as  $S_i^E$  is invariant for each size, the scale-up relationship (Eq. 9) can be transformed to Eq. 10 as follows,

$$S_i = S_i^E (\sigma_V) \quad (10)$$

Where  $\sigma_V$  is the vertical load in MPa.

In summary, the secondary fragmentation model in block/panel caving mines consists of Eqs. 6, 7, 8 and 9. This model considers 7 parameters  $a_1$ ,  $a_2$ ,  $a_3$ ,  $T$ ,  $Z_1$ ,  $Z_2$  and  $S_1^E$ . All of these parameters can be directly determined by experiment.

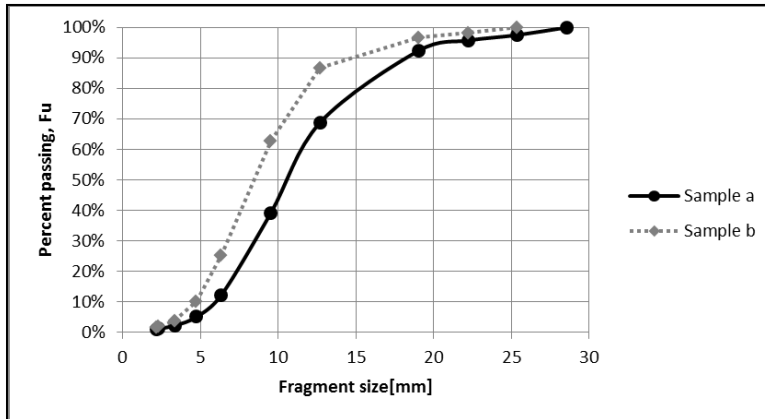
### 3. Laboratory equipment, material and experimental conditions

Confined flow experiments were conducted in order to study the secondary fragmentation. Experimental set up has been presented in (Castro, Fuenzalida and Lund, 2014). Equipment was selected that could deal with 60-70 kg of crushed ore per test. To avoid the concentration of stresses at corners, a cylindrical shape was selected for the model. A 1,800 KN hydraulic press can apply high pressure to material confined in the cylinder (Figure 1). A steel cylinder with a 340 mm inner diameter was selected to apply maximum pressure of 14 MPa (Figure 1). The height of the cylinder was 700 mm to hold the desired volume of ore and to suit the stroke length of the hydraulic press. In the base of the cylinder there is a drawbell (1:75 scale) located in the center of the model to ensure that the flow zones would not intersect the model's walls. The outlet with a rectangular opening of 53mm x 96mm was set to allow the broken material to flow. The extraction system represents a Load Haul Dump (LHD) of 14 yd<sup>3</sup>.



**Figure 1: Cylindrical model in a press machine to apply different vertical loads,  $\sigma_v$ .**

The material used in the experimental tests was crushed ore with a sphericity of 0.58 and a roundness of 0.25, both values were measured on 30 random samples. Two different particle size distributions of sulfide ore were prepared and tested. The basic sample with a wide distribution with the  $d_{80}$  of 15.6 mm (shown in Figure 2) was scaled from the size distribution predicted for Chuquicamata Underground Project, Chile (Codelco VP, 2009). The other initial size distribution (also shown in Figure 2), were constructed with the characteristic sizes ( $d_{80}$ ) of 11.8 mm. Both samples have a coefficient of uniformity of 2 ( $d_{60}/d_{10}$ ), density of 2.63 t/m<sup>3</sup>, point load index of 6.2 MPa and friction angle of 39°.



**Figure 2: Particle size distributions of samples: (a)  $d_{80}=15.6$  mm, (b)  $d_{80}=11.8$  mm.**

The experimental conditions under which the experiment were conducted are listed in Table 1. Vertical loads were chosen regarding different heights of the broken column to represent what is expected in caving operations. Janssen formula (Nedderman, 1992) was applied to calculate vertical loads. Moreover, experiments carried out under 0, 1.5, and 3 MPa were duplicated and tested for wet condition. In the case of wet condition 1.6% of water content was applied, this percentage is enough to saturate the fragments surface (4.3% degree of saturation).

**Table 1: Summary of the experimental conditions**

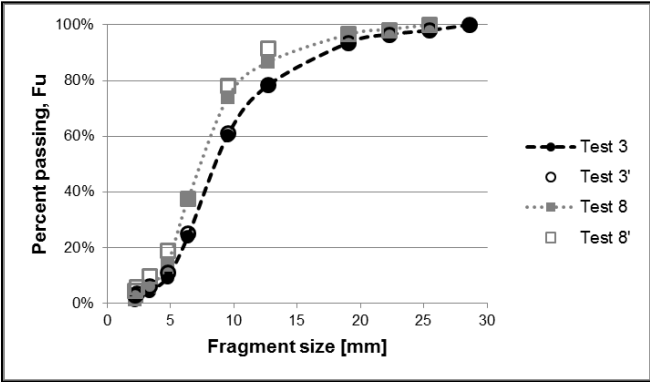
Test	Vertical load $\sigma_v$ , MPa	Initial size distribution		
		Sample	$d_{80}$ , mm	$d_{50}$ , mm
1	0			
2	0.8			
3	1.5	a	15.6	10.8
4	3			
5	5			
6	0			
7	0.8			
8	1.5	b	11.8	8.6
9	3			
10	5			

## 4. Results

### 4.1 Reproducibility evaluation

Since the physical model of confinement flow implemented for the first time to study secondary fragmentation, it is necessary to test reproducibility of the results. Thus, the results of each duplicated test were compared to investigate the differences between the results of a specific test in completely similar condition. Figure 3 shows the results of tests 3 and 8 (Table 1) and their

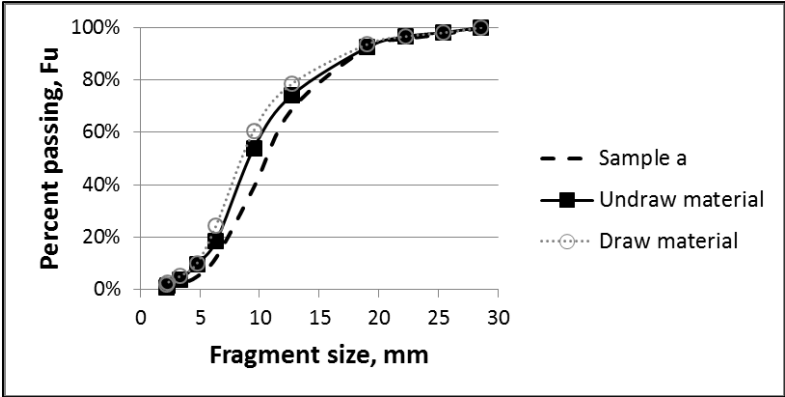
duplicate tests (3' and 8'). The size distributions of both tests are similar to their replicate tests. Therefore, the reproducibility of both is quite acceptable. The same results were obtained from the other tests.



**Figure 3: Experimental tests reproducibility. Tests (3-3') Sample "a"  $d_{80}=15.6$  mm,  $\sigma_v= 1.5$  MPa; Tests (8-8') Sample "b"  $d_{80}=11.6$  mm,  $\sigma_v= 1.5$  MPa.**

**4.2 Extraction effect**

In block caving mining, the abrasion due to the ore extraction is a key parameter that influence on secondary fragmentation (Laubscher, 2001), then, must be consider for experimental studies. In this investigation, the experiments were conducted under draw conditions keeping vertical load constant, where almost the 10 percent of the total mass was extracted. Tests in draw are undrawn conditions were carried out at 3 MPa of vertical load to demonstrated if the ore extraction influences on fragmentation. Figure 4 shown results of both test conditions, fragmentation of characteristics sizes increased around 10 on tests (related to undrawn condition).



**Figure 4: Fragmentation due to draw, 3 MPa of vertical load conditions.**

**4.3 Fragmentation results**

During the experiments the influence of vertical loads, water content, initial size distribution and extraction on secondary fragmentation was analysed.

Figures 4 and 5 illustrate that size distribution always decreased by increasing the vertical load; even though, the applied load is significantly lower than the fragment strength. Increasing the compression and abrasion by vertical load, is the main mechanism of breaking fragments during the experiment.

Comparing Figures 4 and 5 shows that coarser material show higher percentage of fragmentation. This is due to the fact that coarser material has higher probability to push over each other. Moreover, the abrasion of coarse fragments is more because of the more surface area.

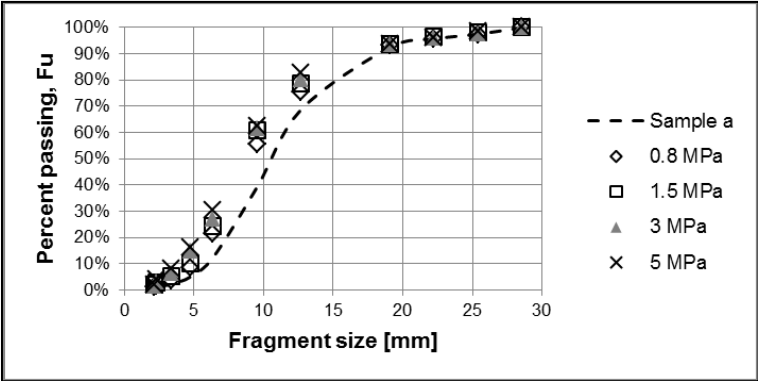


Figure 5: Fragmentation results for sample "a" (d<sub>80</sub>=15.6 mm).

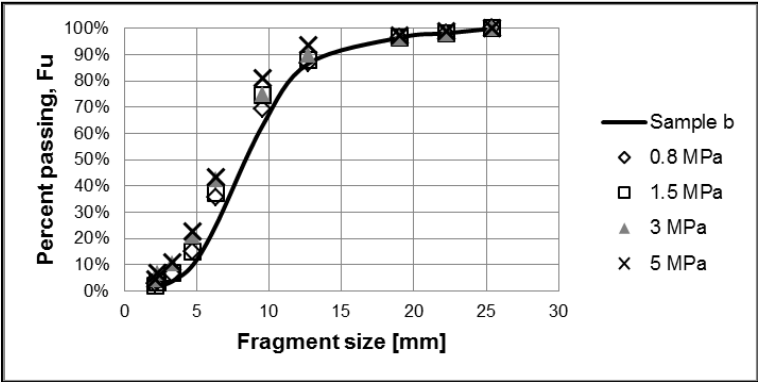
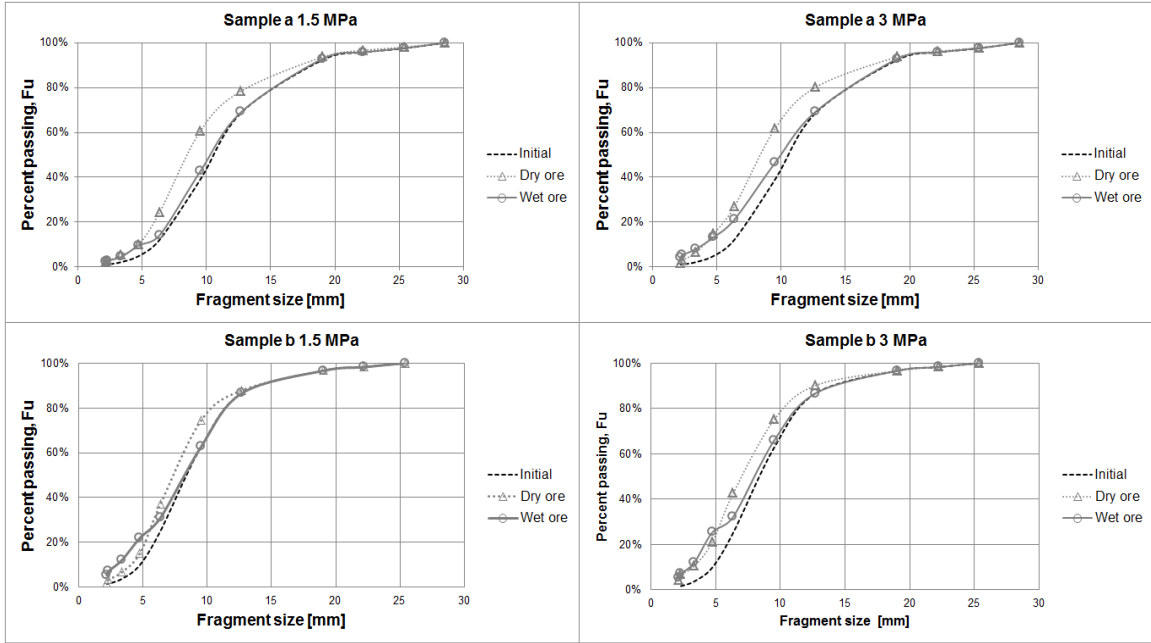


Figure 6: Fragmentation results for sample "b" (d<sub>80</sub>=11.8 mm).

The secondary fragmentation results in wet condition reveal that fragmentation occurs only in the fine section of size distribution curve. Figure 6 shows the results obtained on dry and wet conditions for samples "a" and "b". The humidity avoided comminution of coarser fragments on both samples. This phenomenon is due to lubricating of fragments' surfaces, which decrease the friction between particles. In addition, comminution by compression also decreases regarding to greasing the surface of fragments. Therefore, the main fragmentation mechanisms, compression and abrasiveness, are decreased in wet condition and inhibit the coarser fragmentation effect.



**Figure 7: Secondary fragmentation of dry and wet material: Sample "a" (d<sub>80</sub> = 15.8 mm) and Sample "b" (d<sub>80</sub> = 11.6 mm).**

#### 4.4 Modelling

In order to define the appropriate model of secondary fragmentation prediction, the constant values of breakage and selection functions (Eq. 7 and 8) were adjusted based on experimental results. The adjustment was performed using the minimum square error and the fitting parameter results are presented in Table 2 (for all tests) and graphically for 4 tests in Figures 7 and 8. When applying adjusted parameters in breakage function, the second part of the equation approached to zero. Therefore, only the first part of the breakage function (Eq. 7) was considered in this research with  $\alpha_1$  and  $\alpha_2$  as:

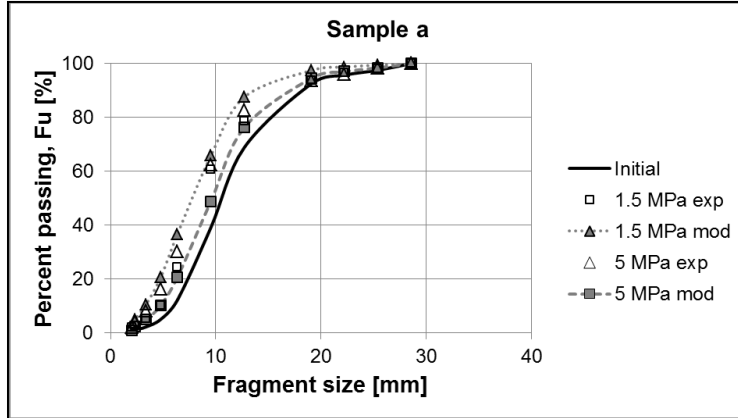
$$B_{i1} = \alpha_1 \left( \frac{x_i}{x_2} \right)^{\alpha_2} \quad (11)$$

**Table 2: Adjusted parameters of the comminution model**

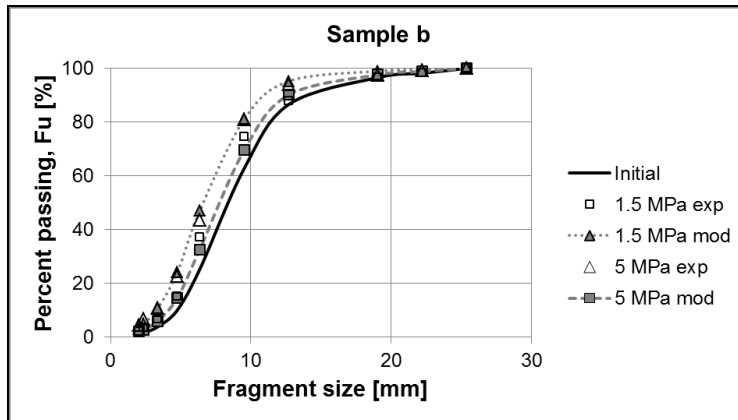
$S_1^E$	$Z_1$	$Z_2$	$\alpha_1$	$\alpha_2$	$\tau$
0.53	0.01	-1.03	0.95	3.72	0.52

Figures 7 and 8 display the comparison of the predicted model and experimental results. It is obvious that the developed model capable to predict secondary fragmentation for different ranges of vertical loads in both initial sizes distribution.





**Figure 8: Experimental and modelled values for Initial particle size distribution:  $d_{80} = 15.6$  mm. Vertical load ( $\sigma_v$ ) = 1.5 and 5 MPa.**



**Figure 9: Experimental and modelled values for initial particle size distribution:  $d_{80} = 11.8$  mm. Vertical load ( $\sigma_v$ ) = 1.5 and 5 MPa.**

The Goodness of Fit of the model was evaluated using the correlation coefficient,  $R^2$ , as well as the Mean Squares Error,

$$MSE = \frac{1}{N} \sum_{i=1}^N (y_{i,exp} - y_{i,mod})^2 \quad (12)$$

Where:

$y_{i,exp}$  and  $y_{i,mod}$  are the cumulative size fractions for size  $i$ , measured and modelled, respectively.

The fitness values illustrated in Table 3 show that the comminution model could be effectively applied to represent the secondary fragmentation under confined flow.

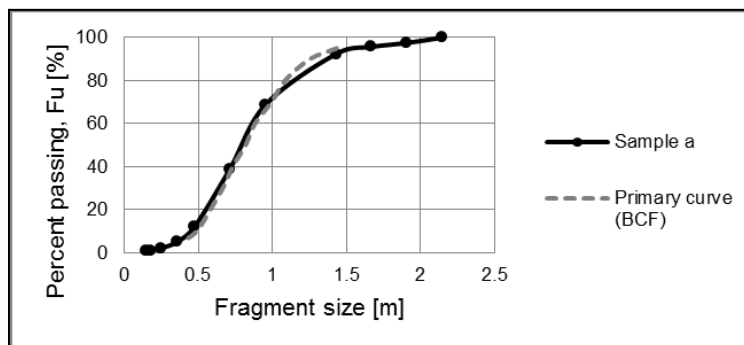
**Table 3: Model Goodness of Fit**

Test	Sample	Vertical load MPa	Correlation coefficient $R^2, \%$	MSE $\%^2$
1	a	0.8	99.70	13.66
2		1.5	99.64	15.16
3		3	99.90	3.44
4		5	99.91	9.95
5	b	0.8	99.89	6.30
6		1.5	99.88	4.86
7		3	99.97	4.09
8		5	99.92	3.19

## 5. Mine application

### 5.1 Model calibration based on BCF

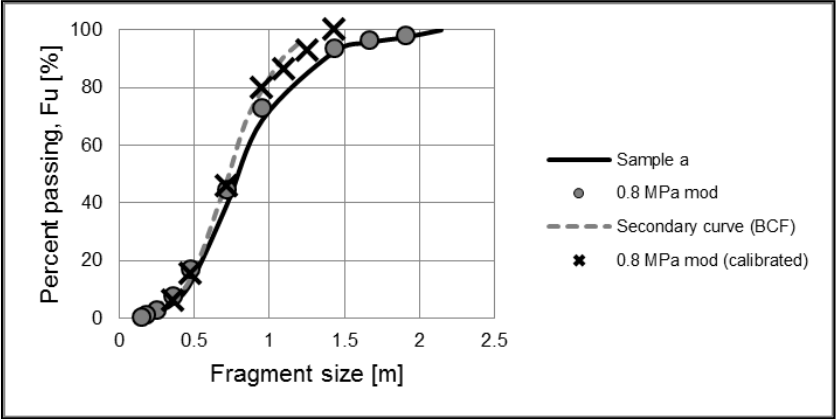
Since the comminution model developed based on experimental data, it is required to be evaluated considering mine-scale information. Therefore, secondary fragmentation module of BCF software was used to scale the presented comminution model. As representing the finer fragmentation is more difficult in BCF, in this paper only considered Sample "a" to scale up the model. Thus, the size distribution of Sample "a" was scaled up based on similitude analysis (Castro, Orellana and Pineda, 2012) with the geometric factor (1:75). Sample "a" was also simulated in BCF (Figure 9).



**Figure 10: Primary curve made in BCF and Sample "a", mine scale.**

BCF software used geometric and draw parameters to predict secondary fragmentation, while the developed model consider vertical loads and rock characteristics besides geometric and draw parameters. In order to consider vertical load in BCF, the draw zone geometry used in the software was correlated with vertical load through Janssen approach (Nedderman, 1992). According to the scaled up geometrical parameters of physical model, the

vertical load of 0.8 MPa was obtained by Janssen equation in mine scale. Hence, the secondary fragmentation result obtained by BCF was compared with the scaled up size distribution obtained through the experiment of Test 2 (Sample "a",  $\sigma_v = 0.8$  MPa). Figure 10 shows that predicted fragmentation of the model and BCF results are different in coarse fragments. In general, BCF tends to predict fragmentation finer than reality. This difference is related to the scaled factor. In order to eliminate the difference between two curves, the constant parameters of the developed model were also scaled up (Table 4). The calibrated model is also presented in Figure 10.



**Figure 11: Comminution model curves on mine scale.**

As it was mentioned, the comminution model included breakage and selection function. The breakage function presents how the size distribution function changes after breakage. This function is invariant to the scale for the same material and mechanisms of fragmentation. However, selection function is scaled dependent. Therefore, the constant parameters of selection function as well as residence time ratio ( $\tau$ ) were considered to adjust the secondary fragmentation curve obtained by BCF (Table 4).

Even though, there are uncertainties in secondary fragmentation prediction by BCF, this software was the most common tool to validate the developed model. It should be mentioned that the comminution model enables to calibrate easily regarding to the variables mentioned in table 4. It is suggested that this model calibrated based on the mean fragmentation obtained applying image analyses of the muck piles in drawpoints.

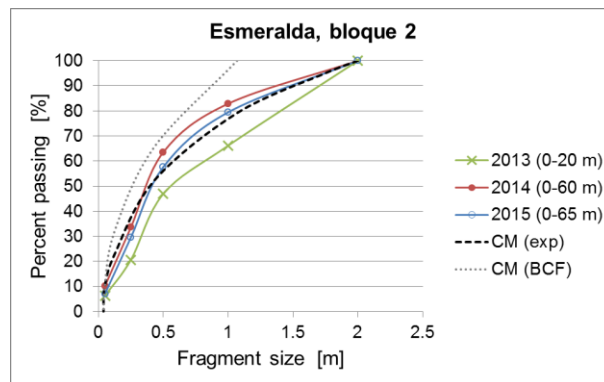
**Table 4: Selection function parameter on mine scale**

Parameter	Experimental value	Calibrated value
$S_1^E$	0.53	39.74
Z1	0.01	0.01
Z2	-1.03	-21.67
$\tau$	0.52	4.52

## 5.2 Comminution model based on field data

While BCF is the most widely used model for predicting fragmentation, in this research BCF outcomes were considered to scale up the constant of comminution model; although, the accuracy of BCF software is being discussed. Thence, the fragmentation of three sector of El Teniente mine (Esmeralda, Reno-Fw and Reno-Hw) were considered to evaluate the developed model.

Figures 11 illustrates the mean fragmentation of Esmeralda sector which measured during two recent years (2013 and 2014). The size distribution curve of 2013 representing the fragmentation of 0 and 20 m extraction column. This distribution was considered as input and comminution model was used to predict mean fragmentation of next years. The result of predicted size distribution curves reveals that the Comminution Model (CM) without scaling up the constant parameters always predicted the secondary fragmentation while the calibrated model has imprecise representation of fragmentation.



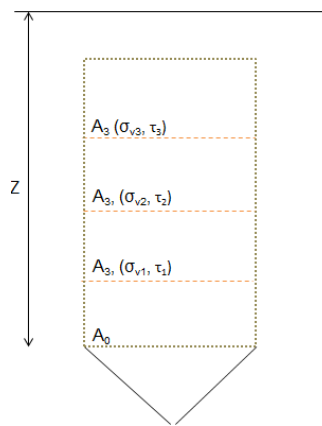
**Figure 12: Fragmentation curves of Esmeralda, El Teniente mine (Codelco, 2015), and the comminution model. CM (exp): constant calibrated through test; CM (BCF): constant calibrated through BCF.**

The adequate prediction of the main comminution model reveals that the constant parameters obtained in laboratory experiment are competent to predict fragmentation and it is not necessary to scale them up. Moreover, it is not necessary to use the exact material to define the constant parameters, only material with the same strength could be sufficient.

### 5.3 Height consideration

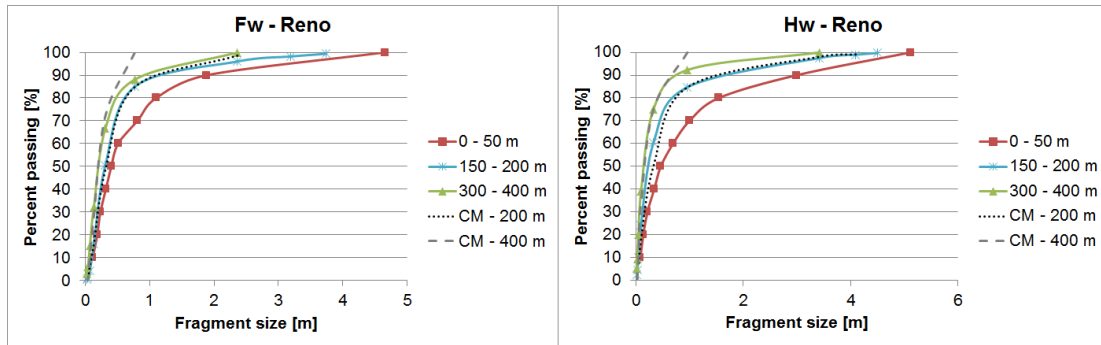
In order to evaluate the comminution model in various heights (for different extraction period), the caved column was divided into several horizontal zones and the comminution model was applied to each zone. Figure 12 shows a scheme of an ore column divided into different areas or zones ( $A_i$ ,  $i= 1, 2, 3... n$ ) as a function of height.

For each area, the comminution model changes based on vertical load  $\sigma_v$  and the constant parameter of  $\tau$ . Vertical load of each zone was calculated regarding to its height and obtained based on rock properties and the geometry of the caved column (Janssen approach). While  $\tau$  is related to the residence time that fragment are in the draw column. In practical, the residence time is not conceivable in caving operation; as a result, this parameter was calibrated with mine data as a function of the height of the draw column.



**Figure 13: Schematic view of fragmentation in various heights of draw column.**

In order to evaluate the comminution model in various heights, this model was performed to estimate the secondary fragmentation of each column heights of Reno, El Teniente (Hurtado and Pereira, 2009). Figure 13 illustrates the size distribution of two heights of draw column for two sectors of Reno (Footwall (Fw) and Hanging wall (Hw)). The comminution model was also applied for different values of  $\sigma_v$  and  $\tau$ , based on the mine situation. Figure 13 indicates that the comminution model enables to estimate secondary fragmentation in both heights. The validation of model for the draw heights more than 400 meters was not achievable due to the lack of mine data in high draw columns.



**Figure 14: Fragmentation back-analysis from Reno (Hurtado and Pereira, 2009) and comminution model (CM) estimations.**

Despite vertical load, calculation of residence time ratio ( $\tau$ ) is somehow ambiguous. Secondary fragmentation occurs while material flow through draw column. In high caved columns, long travel distance, results more residence time and hence finer fragments will be obtained during the extraction. Even though measuring the residence time of material is not possible, this parameter could be presented by geometrical properties. In this research, mine data was utilized to develop an empirical relationship between  $\tau$  and distance of ore transportation as follow:

$$\tau = 0.52 \cdot \left( 355.57 \cdot \frac{dh}{A_d} + 0.51 \right) \quad (13)$$

Where:

$dh$ : is the height of the draw column where the secondary fragmentation want to be predicted [m],

$A_d$ : is the area under draw [ $m^2$ ].

Equation 13 models the behaviour between the height of the draw column and the area under draw of this column, for higher columns the material inside the column travelled large distance, then, the parameter  $\tau$  increase to represented more time in the breakage process. Mine data used to calibrate  $\tau$  show an inverse correlation between this parameter and the area under draw.

Finally, secondary fragmentation can be estimated for various heights applying the comminution model. The parameter  $\tau$  can be approached with Eq. 13, a simplification form of parameter  $\tau$  of 0.52 (obtained from experimental result) can be use getting an acceptable fitting.

## 6. Conclusions

Despite the high influence of secondary fragmentation on block caving mine design, equipment selection and production planning, few studies have been conducted to predict the final size distribution. This paper used the concept of the comminution process to predict rock fragmentation in draw columns of block caving mines with vertical loads (through  $\sigma_v$ ). Therefore, several experiments were undertaken using a confined flow test with various vertical loads and size distributions to calibrate the comminution model. Comparison of the size distribution before and after each test provides the percentage of breakage during the test. The comminution model was modified to consider vertical load in fragmentation prediction and then experimental results were applied to determine the coefficients of the model.

The height of caved column was also considered in this model. The result of the comparison of model outcomes with mine data shows that this model enables to estimate secondary fragmentation in different heights. The methodology presented in this paper was developed of comminution analysis for secondary fragmentation estimation and evaluate the fragmentation. However, there are limitations regarding to the representativeness of caving operations and a limited number of conditions such as rock type. This would require more experiments and full scale tests which would give more insight into the meaning and range of fitted parameters for secondary fragmentation prediction.

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## **Paper 3: Use of experiments to quantify the flow-ability of caved rock for block caving**

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### **Abstract**

*Block/panel caving mining is a massive underground method, in which above the production level an ore column of broken rock is generated as the cave propagates upwards through the ore body. As reserves deplete from near surface, the next generation of block caves will be carried out in deeper condition than currently known, with large column heights and therefore higher vertical stresses. There are unknowns related to the flow characteristics that deeper caves would face. The aim of this study is to quantify the impact of large vertical pressure on the flow-ability of fragmented rock. For this reason, experiments representing the stress and geometry conditions of deep caves were conducted under a range of vertical pressures, materials and humidity conditions. The results indicate that the flow-ability of caved rock depends on the vertical stresses, fines content and humidity conditions.*

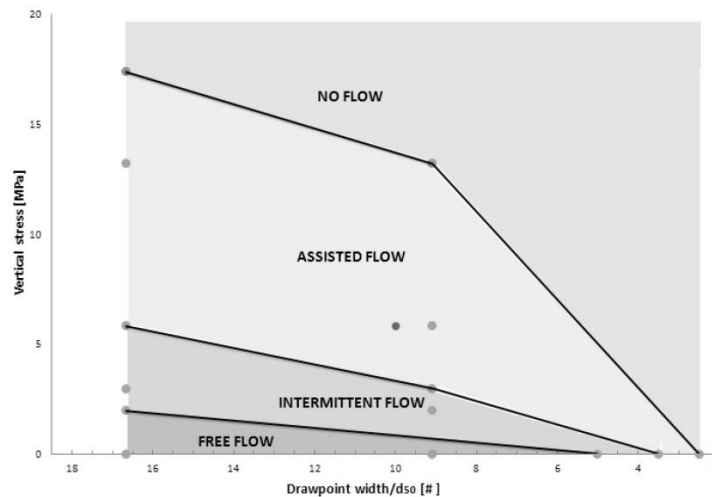
*Keyword: caving mining, flow-ability, hang up, vertical stresses, fines and humidity content.*

### **1. Introduction**

In block/panel caving, ore production is affected by interferences associated with the caving process, especially those related to the gravity flow, such as hang ups and over size rocks. The mine design capability to provide a given production rate is affected, among other factors, by the ore flow.

Flow-ability is defined as the flow condition or ability of a granular material to flow under a given set of material properties, infrastructure geometry, and stress conditions. The flow-ability can be classified into free flow, intermittent flow, assisted flow, and no-flow (Castro, 2014). Kvapil (2008) indicates that flow-ability depends on many parameters including particle size, extraction rate, particles shape, surface roughness between particles, friction between particles, moisture content, compressibility, compaction, particle resistance, and magnitude, distribution and direction of external loads, and forces. However despite being listed the flow-ability under all those sets of parameters has not been quantified.

Flow-ability could be characterised both qualitatively and quantitatively. In terms of qualitative characterization, the flow could be qualified as free flow, intermittent flow and no flow depending on the ratio between particle size and opening (Laubscher, 2006). Studies on gravel have shown also that the flow-ability of granular material is influenced also by the vertical load (Fuenzalida, 2012). Castro et. al (2014) have proposed a flow-ability chart for coarse and dry rock which is presented in terms of vertical stress and drawpoint width/ $d_{50}$ .



**Figure 15: Influence of vertical stress in flow-ability.**

In quantitative terms flow-ability can be characterised in terms of the number of hang ups every 1000 tons or broken rock drawn. Hang ups are one of the flow interferences that affect productivity. Moreover, hang ups can be used to measure the flow-ability of material because when a hang up occurs it means the flow of material has been interrupted and the broken rock cannot be extracted (Troncoso, 2006). Two kinds of hang ups can be formed in coarse material: cohesive and mechanical (Kvapil, 2008. Beus et. al, 2001. Hadjigeorgiou & Lessard, 2007). The formation of arches on a rough wall is generated by the rotation of the principal stresses on the wall and by induced wall pressures (Handy, 1985). The dimensions of the arch depends on the friction angle of the material, depth or vertical stress, inclination of the walls in a drawpoint, draw rate, shape and strength of the particles, and humidity (Kvapil, 2008). At the mine it has been observed that as more material is extracted from a drawpoint the frequency of the hang ups decreases (Maass, 2013). This phenomenon is probably related to the decrease of the particle size during the extraction of an ore column (Montecino, 2001).

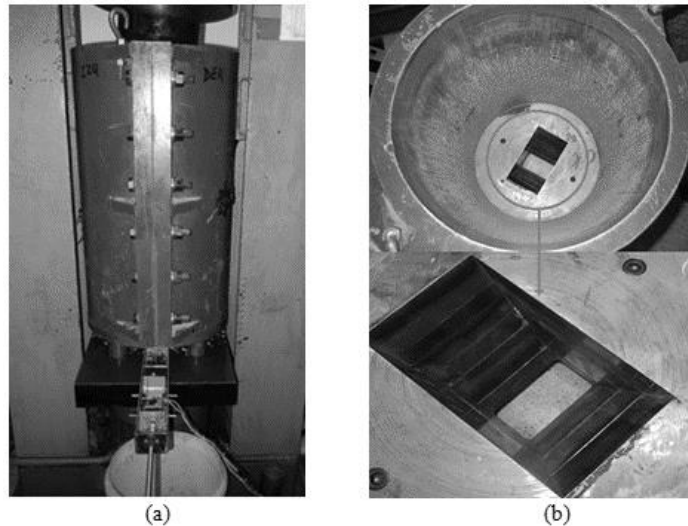
There are many unknowns related to the flow of materials especially under confined conditions. For example what is the role of the fines, water and stresses on the flow-ability of the broken rock. In this article we present the experiments conducted to evaluate the flow-ability of caved rock under high

vertical load for different fines and humidity conditions. Extraction is carried out by a scaled LHD system to represent current caving characteristics.

## 2. Laboratory scaled model and material characterization

### 2.1 Experimental set up and materials

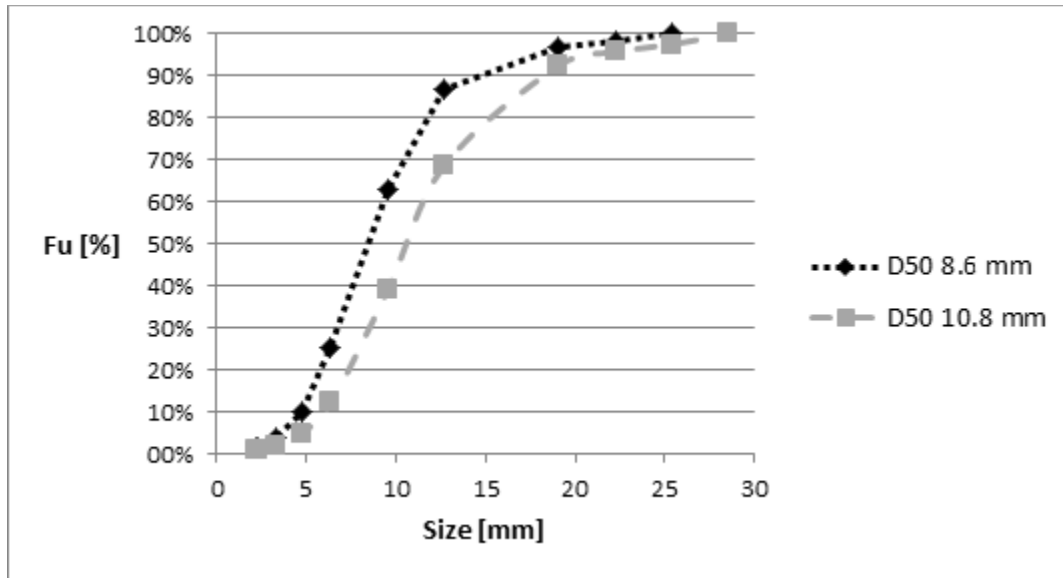
The experiments were conducted in a set up to study confined flow. This consist of a steel cylinder which is filled with broken rock (70-80 kg of crushed ore) under a hydraulic press machine with a capacity of 1,800 kN. The steel cylinder has diameter of 340 mm, as shown in Figure 2. A cylindrical shape was chosen to avoid the concentration of stresses at singularities. The height of the designed cylinder is 700 mm in order to hold the desired volume of gravel and to suit the emulated Andina mine drawbell with a scale of 1:75 (see Figure 2-b), with a rectangular opening of 53 x 96 mm<sup>2</sup>. Since the drawbell is located in the center of the cylinder, flow zones will not intersect the walls of the model. A steel extraction system was built to replicate the extraction the same as LHD does from an extraction point (see Figure 2-a).



**Figure 16: Experimental equipment: (a) cylindrical model in a press machine which changes the vertical load,  $\sigma_v$ , and (b) extraction system, located in the bottom, center of the model.**

The material used in the experimental tests was crushed sulphide ore with a high aspect ratio to represent the geometry of caved rock (sphericity of 0.58 and a roundness of 0.25). Two different particle size distributions of this material were prepared and tested: one with a passing size  $d_{80}$  of 11.8 mm and the other one with a  $d_{80}$  of 15.6 mm. Both samples have the same uniformity coefficient ( $C_u = d_{60}/d_{10}$ ) of 2. Those particle size distributions were

scaled from the size distribution of the primary fragmentation curve of the underground's Chuquicamata project (Figure 3). Table 1 summarizes the characteristics of the samples.



**Figure 17: Particle size distribution of samples used in the experiments**

**Table 1: Samples characteristics.**

Media	Average size $d_{50}$ [mm]	$d_{80}$ [mm]	Uniformity coefficient Cu	Drawpoint width*/ $d_{50}$	Point load index $I_{s50}$	Initial humidity [%] (solid by weight)	Density [kg/m <sup>3</sup> ]
9 mm	8.6	11.8	2.0	7.9	6.2	0.8	2,600
11 mm	10.8	15.6	2.0	6.3			

\*Drawpoint width for non-square geometry can be represented by hydraulic diameter (Jennings & Parslow, 1988).

## 2.2 Experiments

A total of 18 experiments have been carried out to date described in Table 2. Ten experiments were performed without fines or humidity in order to define a base case considering different vertical loads. Then humidity and fines were added to the samples to measure their impact on the flow-ability. Fine material used in this study has a uniform distribution with  $d_{100}$  equals 1 mm. Humidity used in this study is 1.5 liters per 10kg of fine material that is 15% of water.

**Table 2: Summary of experimental conditions.**

Test	Material	Vertical load $\sigma_v$ [MPa]	Humidity [%]	Fines [%]	Size $d_{80}$ [mm]
1	A <sub>1</sub>	0	0	0	11.8
2		1.5	0	0	11.8
3		3	0	0	11.8
4		6	0	0	11.8
5		10	0	0	11.8
6	A <sub>2</sub>	0	0	0	15.6
7		1.5	0	0	15.6
8		3	0	0	15.6
9		6	0	0	15.6
10		10	0	0	15.6
11	B	0	0	20	15.6
12		6	0	20	15.6
13	C	0	0	40	15.6
14		6	0	40	15.6
15	D	0	15	20	15.6
16		6	15	20	15.6
17	E	0	15	40	15.6
18		6	15	40	15.6

### 3. Results

The experiments were performed twice for coarse ore and once for fine material (the latter only for one particle size distribution). During the tests, the flow-ability, the hang up frequency and the hang up height were recorded.

#### 3.1 Flow-ability

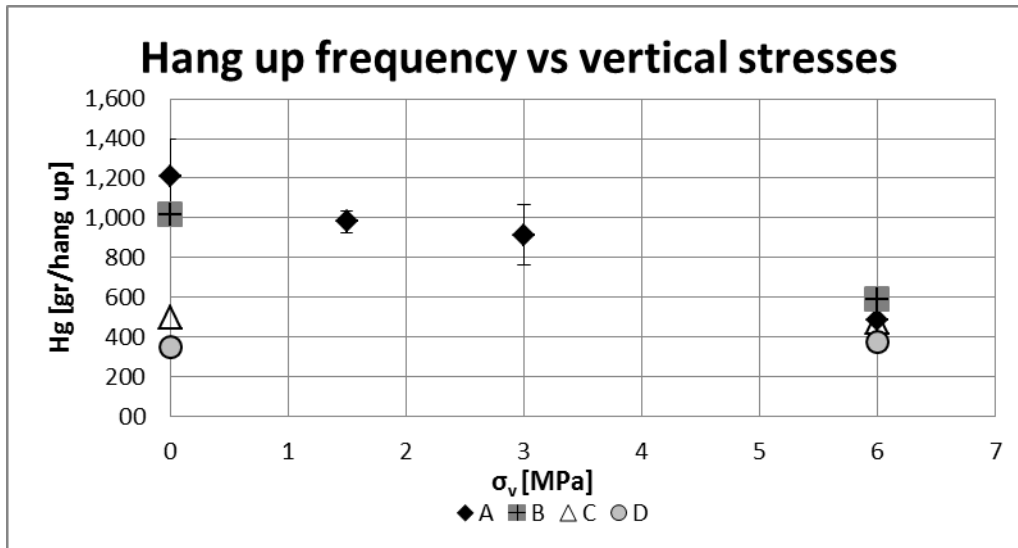
Flow-ability is classified into free flow, intermittent flow, assisted flow, and no-flow (Figure 1). In terms of flow-ability the results (Table 3) indicate that for materials A<sub>1</sub> and A<sub>2</sub> flow-ability decreased from free flow to no flow when vertical load increased from 0 to 10 MPa. When fines were added without humidity, the flow condition was intermittent or assisted. When water was added the flow was assisted flow and, when there were a 40% of fines, the flow condition was no flow at all.

**Table 3: Summary of experimental results.**

Test	Material	Vertical load $\sigma_v$ [MPa]	Flow condition	Interferences [g/hang up]	Standard dev. [g/hang up]
1	A <sub>1</sub>	0	Free Flow	1246	640
2		1.5	Assisted Flow	928	371
3		3	Intermittent flow	1068	256
4		6	Assisted Flow	368	246
5		10	No Flow	0	-
6	A <sub>2</sub>	0	Free Flow	1177	471
7		1.5	Intermittent flow	1036	471
8		3	Intermittent flow	761	356
9		6	Assisted Flow	599	276
10		10	No Flow	0	-
11	B	0	Intermittent flow	1014	248
12		6	Assisted Flow	586	312
13	C	0	Assisted Flow	501	153
14		6	Assisted Flow	475	180
15	D	0	Assisted Flow	352	97
16		6	Assisted Flow	378	112
17	E	0	No flow	0	-
18		6	No flow	0	-

### 3.2 Hang up frequency

During the experiments it was possible to detect flow interruptions or hang ups which were recorded in terms of mass and height. The hang up frequency ( $Hg$ ) is defined as the amount of material that can be drawn before an interruption happens. The experimental results of  $Hg$  as a function of the vertical stress for each laboratory test are presented in Figure 4.



**Figure 18: Hang up result in laboratory test. A: coarse material test (duplicates included), B: 20% of fine material, C: 40% of fine material and D: 20% of fine material with humidity**

The experimental results in coarse material (A) shows that increasing the vertical stress decreases the flow-ability of material. For the media B, that is the one with 20% of dried fines, the hang up frequency number decreased. For materials C and D, the vertical load had no significant influence in the frequency of hang ups as they were not able to flow.

Field measured hang ups are quantified by their hang ups index (number of hang ups per 1000 ton of ore). The measured hang ups index in the experiments is similar to the observed index of primary sulphides in mines. The index varies from 1.6 to 3.6 in mines and, as can be seen in Table 4, the scaled experimental index varies from 0.75 to 3.95.

**Table 4: Scaled hang ups index**

Vertical load [MPa]	Hang ups index [# hang up/ 1000 ton]	
	A <sub>1</sub>	A <sub>2</sub>
0	1.05	2.05
1.5	1.26	2.29
3	1.09	3.11
6	3.17	3.95
10	0.75	3.32

For the experiments with vertical load 10 MPa the material got strongly compacted over the drawbell generating a great hang up above the drawbell and almost no hang ups of lower height.

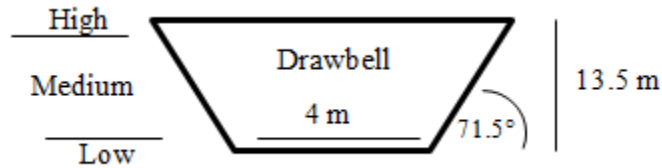


### 3.3 Height of hang ups

A classification was made according to the height of the hang up:

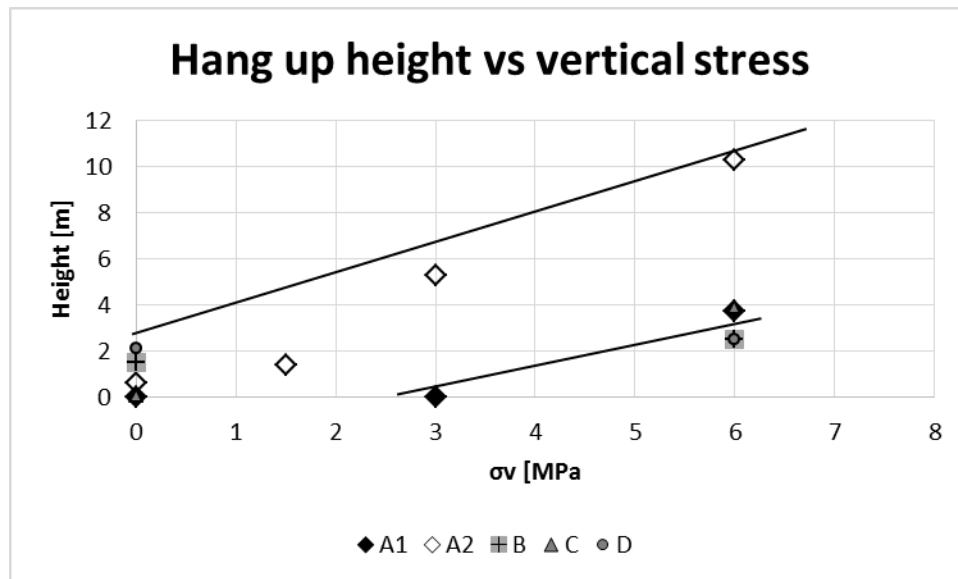
- Low: in extraction point.
- Medium: in drawbell (0-13.5m from the roof of the production level)
- High: Above drawbell (over 13.5 m).

The geometry of the drawbell from which the dimensions were scaled is represented in Figure 5.



**Figure 19: Drawbell geometry in**

Average hang up height of each experiment based on the vertical stress is shown in Figure 6. It can be seen that as the vertical stress increases, the height of the hang ups increases simultaneously. The dimensions of each observed hang up were scaled in order to quantify their height in the mine.



**Figure 6: Hang up heights in laboratory test. On coarse material A<sub>1</sub>: d<sub>80</sub>=15.6 [mm], A<sub>2</sub>: d<sub>80</sub>=11.8 [mm], B: 20% of fine material, C: 40% of fine material and D: 20% of fine material with humidity**

In general, the height of the hang ups increases with the vertical pressure for the coarse material (A1 and A2). In the case of the addition of fine (materials B and C) the results indicate that vertical load has a small impact in the increase of the height of the hang ups. When fines and water were added (material D) there is no effect of the vertical load on the height of the hang ups.

#### **4. Conclusions and discussion**

Based on experimental tests, this paper shows that particle size, as well as the moisture content, and vertical stress have a noticeable impact on the flow-ability of caved rock. The number and height of hang ups increases as the vertical load increases. Fines and humidity increase the number of hang ups. Also, high hang ups occurs when a vertical pressure was applied or when humidity and fines were acting together.

The scaled model was successful in understanding how confinement, particle size distribution, humidity and fines presence affect the flow-ability of material. It is expected that as block caves get deeper the number of hang ups would increase if fragmentation keeps constant. The results of the above experiments shows that the hang up's number for mine design applications could be obtained from this kind of experiments. This would require further research and analysis. It is expected that this kind of experiments would be in the future standards to the caving industry, especially for future and unknown conditions.

#### **Acknowledgement**

We want to thank Patricio Ávila and Alonso Vives for their help in these experiments, the staff of Block Caving Laboratory for their support and encouragement and Asieh Hekmat for her advices. This project was conducted under the partial funding of the basal project for Basal Excellences, which is an initiative of the Chilean government.

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## **General Conclusion**

The research presented in this thesis is an experimental study which main objective is to understand the influence of key parameters such as stresses, fragments size, ore extraction and humidity on hang up phenomena and secondary fragmentation in caving mining. To achieve this purpose, a set up was developed to represent the extraction in a drawbell of Block Caving mines.

This chapter summarizes the main findings regarding the hang up events and fragmentation results obtained in the confined flow model as well as a comparison of the model performance to real mine data from several Block Caving mines.

### **1. Hang Up occurrence**

The process of ore fragments flowing through drawpoints implies a discrete sequence of events, each one having a probability to create interference such as a coarse arch. Typically this probability depends on certain variables like the ratio between drawpoint apex and fragment size, geometry of drawpoints, stress condition, presence of fine material, moisture content, draw rate and interaction between neighbor drawpoints.

Previous research using the confined flow model with a circular opening indicate that the stress condition has an extraordinary effect on hang ups thus implying that hang up studies should also consider stresses along with the ratio between drawpoint apex and fragment size. Other studies in the field have frequently missed or overlooked testing and simulating the mine conditions, especially those related with the geometry of drawpoints and extraction system, both considerations introduced in the present work. The impact of these variables on the hang up frequency and the height of the hang up were quantified on tests.

Hang up frequency was analyzed for different load, fragmentation, draw strategy and humidity scenarios. A raise in vertical load increases the hang up occurrences in drawpoints due to ore compaction creating stable arches of fragments. Fragmentation also influences the hang up interferences, with larger fragment size inducing more hang up events, associated with a reduction of the ratio between drawpoint apex and fragment size. When extraction occurs in both drawpoints of the drawbell a better flow conditions took place, because the distance among the drawpoints let interaction and helps destabilizing arches formed. An increase in ore moisture content also has

and impact on hang ups, increasing the quantity of hang up. This effect is linked to the enhanced cohesion in wet ore.

On the other hand, the height of hang ups increases with higher vertical load, due to the intensification of friction forces between ore fragments. The opposite is true under wet conditions, the height of hang up decreases due to lower frictional force between fragments.

Field data was used to back up experimental results. The scaled up hang up frequency fell within the range of the mine site data. It is thus concluded that experimental results could be used as a practical guide for the analysis and prediction of hang-up events.

## **2. Secondary fragmentation**

The comminution model was developed using the Kinetic Matrix Model originating at mill comminution processes. In this study the model was implemented to adjust the fragmentation observed in the experiments due to the similitude between the breakage mechanisms. Additionally, the mathematical model also incorporates the stress as parameter to estimate fragmentation. The model was calibrated with experimental results and compared with fragmentation of mine data.

Experimental results showed major fragmentation for higher vertical load, although the high differences between the rocks strength of the ore tested (~140 MPa) and vertical loads applied on test (0-5 MPa). Abrasion and compression were the main breakage mechanisms during the experiments. Moreover, these mechanisms were individually identified when tests with and without draw were carried out. Secondary fragmentation is affected by ore draw, thus, the extraction must be considered in experiments of secondary fragmentation. These results allowed adjusting the comminution model parameters for mine implementation.

Even though the comminution model can be used for any fragment size (because it is an input of the model), the scale factor from experimental result to mine was considered. In the first instance, Block Caving Fragmentation (BCF) software was used to adjust the comminution model's parameters in mine scale. Then two set of parameters was available: parameters calibrated just with experimental results and parameters calibrated with BCF. However, when the comminution model was compared with fragmentation from mine data, parameters adjusted with experimental results showed grater fitness due to the comminution model can be represented by any distribution curve shape, while BCF only displaces the primary fragmentation curve.

### **3. Future work**

#### **3.1 Hang up events**

Few experimental studies are found in the literature on the subject of hang ups and even fewer representing the mine condition of caving operations in a reliable manner. This work presents new findings that shed light on the behaviour of hang up events but a number of open questions still remain unsolved in this field, therefore it would be recommendable to continue with this line of research.

The ore internal friction angle is known to be a relevant parameter in the hang up analysis; hence, material with different friction angles should be tested to determine its real influence. Cohesion is also an important characteristic of the hang up process, experiments adding fines and humidity could contribute to our understanding on hang ups.

Drawpoint geometry is another interesting parameter, it would be adequate to design various experiments testing drawbells with different angles and shapes, based on typical operational designs. Furthermore, not only drawpoint interaction should be studied but also the interaction between drawbells in search for a draw strategy based on the experimental results.

#### **3.2 Fragmentation**

To strengthen the application of the comminution model the following recommendations should be considered in future research:

Ore fragmentation due to extraction should be quantified under experimental and controlled conditions. Additionally, it would be interesting to measure the ore fragmentation that occurs due to interaction between drawpoints and/or drawbells.

Ore with different rock properties should be tested to improve the prediction accuracy of the model and defining the parameters based ore properties. A long term goal for this line of research is to be able to represent the parameters of the model only based on rock properties as strength, friction angle, shape factor, density.

Finally, coding the comminution model in a user friendly interface or software would be highly recommended to facilitate mine application. It should be programmed considering the equations presented in the "Experimental and

comminution model framework for secondary fragmentation quantification for block caving” paper.