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Microeconomic model of residential location incorporating life cycle and social expectations

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ABSTRACT

This paper is focused on the dynamics of residential location decisions based on the microeconomic theory of urban land use, in which we assume that each property is assigned to the agent with the highest bid. The agents' behavior includes expectations of their future based on the life cycle or social influence processes, which are anticipated or solved using a hypothesis of imitation of the behavior of other households currently living in those situations. Relocation decisions are then modeled, incorporating expected utilities by means of transition probabilities among households. An imitation multi-objective bid function is postulated for each alternative location depending on the expected income per unit of time, the current household value of amenities and the expected value obtained by the imitated agent in this location. A multinomial logit model is assumed to calculate the location equilibrium, where willingness to pay is determined by dwelling characteristics and spatial socioeconomic segregation (location externalities). Numerical examples and simulations are presented using linear bid functions to explain the proposed modeling approach and the impact of imitation on the dynamics of residential segregation.

1. Introduction

The study of land use dynamics in the context of an urban system involves the description and modeling of interactions between a variety of agents that change their socioeconomic characteristics and preferences over time and, therefore, make different decisions across both time and space during the residential relocation process. This dynamic feature appears as the result of various issues, such as the joint location decision of various households. These issues, in turn, affect the urban system's configuration, causing variations in real estate market behavior. In addition, there are some processes associated with the location decisions of households that are not explicitly considered in classical models of short- and long-term urban land use equilibrium, such as social learning, imitation, the formation of habits, the generation of expectations, uncertainty regarding availability of resources and fluctuation/disruption of the social and economic changes that establish a complex system.

In particular, households have internal dynamics such as changes in life cycles, changes in structure, and social interactions (new children, divorce, job changes, level of education, etc.), which affect consumption patterns and residential location. In addition, there are variations in urban land use due to the generation of new real estate projects in different areas of a city, motivating relocations. This type of expected dynamics associated with a household's life cycle induces transitions among possible states (or household types). Under the condition of uncertainty regarding the future, these states can be represented as a set of possibilities that can be anticipated and used as the basis for modeling the residential relocation process, not only in the short but also in the long term, such as residential relocation and intra-urban mobility (Li & Tu, 2011). Such relocation forces are denoted by Huff and Clark (1978) as cumulative inertia (resistance to movement) and residential stress (incentive for movement), given by possible dissatisfaction with certain attributes associated with the current household and its surroundings. This dissatisfaction can be generated by changes in the household life cycle and social network effects. For example, some empirical studies explain the residential relocation dynamics in urban areas through effects such as expected future salary or the importance that agents assign to the utility drawn by others through the consumption of various goods. One way to produce these decision changes is by assessing their anticipation through knowledge of the cluster, household type expectations or future change probabilities. For example, future expected revenues could be used as an estimate of the payment capacity of certain households (Kennan & Walker, 2011).

There are few studies that model residence choice using future expectations through stochastic dynamic programming. Among them, worth mentioning are the results of Bayer, McMillan, Murphy, and Timmins (2011) and Ortalo-Magne and Rady (2006). In Ortalo-Magne and Rady (2006), the authors design a model of the housing market

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life cycle with a focus on equilibrium outcomes, explicitly including credit constraints, particularly for young households. Bayer et al. (2011) develop a model of neighborhood choice in a dynamic environment that estimates housing preferences in dynamic evolution and neighborhood attributes. In addition, other influential factors in relocation decision making are possible changes in the long-term activities of any of the household members in terms of work, education, or other activities (Hooimeijer, 1996; Li & Tu, 2011) and changes in the household structure due to the departure or arrival of new members, etc. (Eluru, Sener, Bhat, Pendyala, & Axhausen, 2009).

An important concept in relation to residential relocation phenomena is transaction or moving costs, which depend on various factors, especially life cycle and various household characteristics (Bayer et al., 2011; Kennan & Walker, 2011). These relocation costs refer to not only monetary costs but also social, psychological and temporary losses due to the search for a new dwelling and the moving process. Moreover, some studies in the fields of psychology and sociology show that social and individual learning, along with socioeconomic changes in the life cycle, are important factors when a household makes decisions about intra-urban mobility (Rossi, 1955; Ritchey, 1976; Anderson & Milson, 1989; among others).

Another model type that analyzes the dynamics of urban location based on individuals' behaviors is the multi-agent model (see Benenson, 1998; Ettema, 2011; Filatova, 2014, etc.). In Benenson (1998) it is assumed that households can change their behavior according to their neighbors and to residential properties in their neighborhood as well as the entire city. In Ettema (2011), some economic concepts are integrated using multi-agent simulation models, which consider that household decisions are based on the perception of probabilities of the evolution of the real estate market. Filatova (2014) proposes a model that includes natural hazard risks and environmental amenities through hedonic econometric models.

In an interesting work, Parker and Filatova (2008) present a conceptual design for a residential market context for interaction between multiple buyers and sellers, adding the expectation formation prospect as a main subject in the decision-making process. In particular, the authors analyze and incorporate the effects associated with household life cycle such as age and size. In this way, agents can shape their own households, generating searches for independent residences or searching locations with attributes that are different from those of the current location. The interesting contribution of this model is the interaction between agent modeling and urban economic concepts.

Due to the low relevance that the literature gives to the life cycle effect of households in urban dynamics, in this paper, we propose a microeconomic model of residential location that incorporates some aspects of the agents' or decision makers' dynamics over time and their influence on the decision regarding a residential relocation. Here, we must highlight the importance of applying a microeconomic modeling approach to two key aspects related to imitation in the decision-making process. The first aspect is the possibility that some agents have (either households or firms) to evaluate expected life cycle dynamics, making urban location decisions at present dependent on the expected future, in terms of both future planned decisions (namely, education, workplace, expected number of children) and possible changes in the global economy. The second aspect is the social effects of decisions and valuations made by other individuals on the personal decisions of the decision maker. This latter feature makes the connection of the developed simulations (see Section 4) to life cycles a key aspect of social dynamics in the phenomenon of social segregation.

We use the aggregate model that assumes that households split into clusters according to a set of common features. Moreover, each agent of a specific cluster makes similar decisions regarding residential location under the assumption that these agents share a utility function with the same parameters. Idiosyncratic differences in decisions among members of the same cluster are captured by a Gumbel (Type I) stochastic distribution of willingness to pay. However, features defining such groups are dynamic over time for each agent (for example, age, number of household members, income, number of cars, and education level), leading households of the same cluster to evolve into different clusters in the subsequent time period. To trace this evolution, we develop a microeconomic model that has a formulation similar to stochastic dynamic programming models and is designed to capture some of these elements associated with the likelihood of changing clusters in the future. We also assume that the set of potential cluster changes and the likelihood of changing into each of them have an impact on the current assessment of the willingness to pay for the different dwelling types.

In building the willingness to pay, we hypothesize that households observe and imitate the behavior of other households. In other words, households build their expected or social network utility by observing the utilities of households in other clusters using a transition probability that reflects the chances of a change between clusters in the next time period or the social influence weight among socioeconomic groups. This imitation process allows us to simplify the forecasting problem, which under such conditions, becomes a static model of consumption at each period.

This process of imitation in decision-making is supported by other areas such as the economy, social science and engineering. For example, in the context of game theory, theoretical studies associate imitation and social learning dynamics with strategic decision making (Alós-Ferrer & Schlag, 2009). An interesting example in this area is the work developed by Berg (2010), in which firms make location decisions using information regarding the profit or utility obtained by other firms already in these locations.

Furthermore, in the context of the intertemporal modeling of goods consumption, several studies address the effect of social learning and imitation processes between agents (Allen & Carroll, 2001; Ballinger, Palumbo, & Wilcox, 2003; Páez, Scott, & Volz, 2008; Brown, Chua, & Camerer, 2009; Carbone & Duffy, 2014 among others). Note that in most of these studies, the imitation processes of other agents' choice are analyzed, not directly from the utility obtained but rather indirectly because the choice is effectively related to the achieved utility level. In contrast, we develop an imitation model that is combined with current socioeconomic characteristics, preferences and constraints to build a willingness to pay function for residential relocation. In this sense, the household willingness to pay reflects the annualized expected long-term value of the residence, which is built under the myopic assumption that consumers cannot forecast but can observe the dynamics of the population's socioeconomics and the different behaviors of the households of other clusters. In our model, households with such behavior enter the market in one period and submit bids, and an absentee auctioneer assigns locations in a static equilibrium market clearing process; equilibrium is attained by adjusting the households' utility levels such that all are allocated to available housing options in that period.

The paper is organized as follows. In Section 2, we include the theoretical background and initial considerations required to formulate the imitation model. In the third section, a discrete choice deterministic microeconomic model associated with residential location is developed, incorporating the household's change expectations by means of transition probabilities between household clusters during the life cycle. In the fourth section, numerical examples showing the proposed modeling and its effects in an urban configuration are developed. The paper concludes with a section that includes remarks and final discussions.

2. Mathematical and microeconomic fundamentals

This section describes the basic urban economy guidelines needed to understand the model developed in Section 3. In particular, the fundamentals of the Random Bidding and Supply Model (RB&SM) demand model are shown (Martínez & Henríquez, 2007). Urban economics offers two main approaches to explain location equilibrium. The first approach, called Bid-auction, assumes an auction-type market where agents bid for different locations, which are then assigned by the seller to the highest bidder (Alonso, 1964). The second approach, called Choice, assumes that agents choose locations that maximize their utility level (McFadden, 1978; Anas, 1982). In the present section, the two approaches are briefly explained, and some necessary assumptions are made to develop our proposed imitation model. Following Martinez's (1992) RB&SM, consider a household type $h \in H$ in period $t \in T$ choosing real estate $i \in D$ that maximizes its utility. Although the model can be specified as disaggregated at the level of each agent and location, in practice, aggregated versions such as that presented here are used. Consumers are classified into socioeconomically homogeneous categories (index h, and supply is described by location clusters (index i). The household values a real estate unit, indexed by i in D, using a set of distinctive attributes denoted by vector Z_i, which includes dwelling characteristics, accessibility and neighborhood quality (Louviere & Timmermans, 1990). Neighborhood quality includes the location of residents and commercial activities that affect the utility of other locators in the neighborhood, called location externalities Martínez & Henríquez, 2007).

The following static and deterministic microeconomic problem of residential location, based on the discrete choice theory, represents the household's location problem:

$$\max_i \max_x U_h^t(x, Z_i^t) \quad \text{subject to } p^t x + r_i^t \leq I_h^t \tag{1}$$

where r_i^t is the rent for property (i), I_h^t is the exogenous income of household h, and p^t is the price vector associated with the set of market goods x. As it is described by the microeconomic theory of discrete choices (Jara-Díaz, 2007), by solving only the problem in x conditional in i, it is possible to obtain the Marshallians demands

$$x^*(p; I_h^t - r_i^t; Z_i^t).$$

By replacing that result in the direct utility function, we obtain

$$U_h^t(x^*(p; I_h^t - r_i^t; Z_i^t), Z_i^t) \equiv V_h^t(I_h^t - r_i^t, Z_i^t, p^t)$$

where, $V_h^t(I_h^t - r_i^t, Z_i^t, p^t)$ is the indirect utility function associated with the solution of the problem (1) for real estate (i).

For a given utility levelU_h, if the inverse function of V_h^t exists with respect to the rent variable, then we obtain

$$r_i^t = I_h^t - V_h^{-1}(U_h, Z_i^t, p^t)$$

which represents the value that the consumer is willing to pay for location (i) to attain a utility level U_h (Ellickson, 1981), denoted by:

$$B_{hi}^t = I_h^t - V_h^{-1}(U_h, Z_i^t, p^t). \tag{2}$$

Based on that, it is possible to prove that if the direct utility function is quasilinear, then the indirect utility function becomes (see Appendix A),:

$$V_{hi}^t(Z_i^t, I_h^t - r_i^t) = \lambda_h^t * (I_h^t - r_i^t) + \lambda_h^t\, b_{hi}^t\,(Z_i^t) \tag{3}$$

where λ_h^t is the marginal utility of income and $b_{hi}^t(Z_i^t)$ is a function that measures the household h's valuation of the property attributes. In expression (3), by equalizing $V_{hi}^t(Z_i^t, I_h^t - r_i^t)$ to a given utility level U_h^t and clearing the rent variable (r_i^t) we obtain

$$r_i^t = I_h^t + b_{hi}^t\,(Z_i^t) - \frac{\bar{U}_h^t}{\lambda_h^t}.$$

Therefore, under a competitive economic behavior considering a market of bids and assuming a quasi-linear direct utility, we obtain that the willingness to pay of an agent of type h for location/dwelling (i) is

$$B_{hi}^t = I_h^t + b_{hi}^t\,(Z_i^t) - \frac{U_h^t}{\lambda_h^t}. \tag{4}$$

Following (Martínez & Henríquez, 2007), we denote B_{hi}^t as:

$$B_{hi}^t = a_h^t + b_{hi}^t\,(Z_i^t) \tag{5}$$

where $a_h^t = I_h^t - \frac{U_h^t}{\lambda_h^t}$, and the utility level is obtained from the market equilibrium defined by the condition that every agent should be located in the city. Additionally, assuming idiosyncratic variability among households in the cluster represented by stochastic bids, $\hat{B}_{hi}^t = B_{hi}^t + \varepsilon_{hi}^t$, with ε_{hi}^t being identically and independently Gumbel distributed with dispersion parameter μ, and B_{hi}^t being the deterministic bid. Under these assumptions, the probability that household h be the highest bidder for property i in period t is:

$$Q_{h|i}^t = \frac{\exp(\mu B_{hi}^t)}{\sum_{g \in C} \exp\left(\mu B_{gi}^t\right)}. \tag{6}$$

For household clusters of different sizes given by H_h^t, McFadden (1978) proposes a size correction that yields:

$$Q_{h|i}^t = \frac{H_h^t \exp(\mu B_{hi}^t)}{\sum_{g \in C} H_g^t \exp\left(\mu B_{gi}^t\right)}. \tag{7}$$

In this way, the number of type h households allocated to type i locations in period t is $H_{hi}^t = S_i^t Q_{h|i}^t$, where S_i^t is the exogenous supply, i.e., the number of real estate units of type i in period t. Given that the model is intertemporal, H_h^t in Eq. (7) can change over time depending on in and out migration and on the existence of households that move from one cluster to another during that period.

This resulting bid-auction probability was first proposed by Ellickson (1981). Martinez (1992) demonstrates that it is equivalent to the utility maximization approach of McFadden (1978) and Anas (1982).

In any static equilibrium model (e.g., RB&SM, Martínez & Henríquez, 2007), location externalities in each zone mean that the utilities, and therefore bids, of one household depend on the land use of the neighborhood; hence, they depend on instantaneous location equilibrium. Therefore, $B_{hi}^t\,(H_{gj}^t, \forall g, j)$, such that $Q_{h|i}^t$ depends on $Q_{g|i}^t$, generating a fixed point system of equations. In a dynamic model (e.g., Martínez & Hurtubia, 2006), the information is assumed to be delayed, e.g., in one period, such that location externalities affect bids as $B_{hi}^t\,(H_{gj}^{t-1}, \forall g, j)$, which avoids the fixed point problem of the static model.

The rent at each location is obtained endogenously at equilibrium as the expected value of the maximum bid, which under independent and identically Gumbel distributed bids is given by the log sum expression (Jara-Díaz, 2007; Martinez, 1992; Martínez & Henríquez, 2007):

$$r_i^t = \frac{1}{\mu}\left\{ \ln\left(\sum_{h \in C} H_h^t \exp(\mu\, B_{hi}^t) + \gamma \right) \right\}, \quad \forall i \tag{8}$$

where γ is Euler's constant. Finally, the residential market equilibrium condition is that every household is located in the available housing, which is attained imposing $\sum_i Q_{h|i}^t S_i^t = H_h^t$. For this condition to hold, households must adjust their utilities to comply with the following equation (Martínez & Henríquez, 2007):

$$a_h^t = -\frac{1}{\mu} \ln\left(\sum_{i \in I} S_i^t \exp\left(\mu \left(b_{hi}^t - r_i^t\right)\right)\right), \quad \forall \ h \tag{9}$$

which represents a fixed point system of equations whose solution yields the maximum utility levels attainable at equilibrium by each cluster of households.

Finally, it is worth mentioning that extensions of the RB&SM model have been successfully applied in different social and regional contexts. In particular, we can mention the Mussa model (Martínez & Donoso, 2010) from Santiago, Chile that bases its theoretical foundations on the guidelines described in this section. In that model, the relevant variables incorporated for the bid function are average income per zone (location externalities), accessibility and attractiveness attributes for each area, number of rooms, etc. In addition, in that model, the important features for generating the socioeconomic groups are household income, education, age and household structure (number of members). A commercial extension of the RB&SM model applied in different cities is known as CUBE-LAND within the CUBE modeling platform that was internationally commercialized by CITILABS.

3. Expectations and social effects in residential location

This section develops an expectation or social dynamic model of residential land use based on the bid-auction trade process and inspired by the RB&SM model. The novel condition is that households foresee that their socioeconomic status or life cycle social behavior will change over the time that they reside in the new location; hence, they anticipate changes in their own valuation of housing attributes over time such that they can assess their bids while accounting for both their current and future values. Households anticipate their future or social perceptions by observing and imitating the behavior of households currently in different clusters. Here, we study the effects on residential location under the condition of this imitation behavior. The literature reports several factors that explain why the decisions of one economic agent (*firm*, *school*, *household*, *individual*) are similar to the decisions or the valuations made by other agents. For example, the households that belong to a socioeconomic cluster incorporate some preferences for locations observed from other socially related households (Páez et al., 2008); other agents make decisions based on future income, employment, education, and other expectations (Kennan & Walker, 2011). In the context of housing choice or residential relocation, all of these behaviors are called imitation, similar to the process studied in research on game theory and strategic behavior (Alós-Ferrer & Schlag, 2009). To understand the effect of imitation on residential location choices and on the emerging urban equilibrium in both the short and long term, we propose incorporating the expectations associated with life cycle dynamics into location choices, which can be obtained based on an endogenous decision (changing jobs, having children, etc.) or an exogenous shock (layoffs, economic shocks, etc.).

In our model, we consider a generic agent representing the behavior to be a locator, which may be a household, an individual or a firm. For simplicity, hereafter, we refer to a household. In a dynamic context, we assume that the agent at period t makes a decision about location and goods consumption. The utility of goods consumption is based on current perceptions or values, that is, the perception associated with the socioeconomic cluster h in period t, while the utility of residential location incorporates future possible changes in life cycle, i.e., changes to another cluster in the next period. We define the following residential location problems:

$$\max_i \ \max_x \ \theta_h^t U_h^t(x, Z_i^t) + (1-\theta_h^t) \sum_{f \in C} P\left(f^{t+1} | h^t\right) V_{fi}^{t+1} \qquad \textit{subject to} \quad p^t x + r_i^t \le I_h^t \tag{10}$$

where θ_h^t defines the social valuation of utility or, in a temporal context, represents a discount rate.

Problem (10) has two interesting interpretations. In an intertemporal context, we can assume that an agent belonging to cluster h in period t faces a potential change in life cycle during period $t + 1$, moving from cluster h to cluster f with transition probability $P(f^{t+1}|h^t)$, which represents changes in the household's daily life (job, income, education, car ownership, etc.). However, in a social context, we can interpret $(1-\theta_h^t)$ $P(f^{t+1}|h^t)$ as the assessment or influence of the utility of household type f over type h agents. $(1-\theta_h^t)$ $P(f|h)$ can also be interpreted as a weight that measures the social relationship among these types of households. Note that for simplicity, in a dynamic or intertemporal context, it is assumed that the individual anticipates transition possibilities only between consecutive periods, but this assumption can be easily generalized to a long-term transition process. Additionally, household f expects an (indirect) utility associated with real estate (i) given by $V_{fi}^{t+1} = V_f(E(Z_i^{\{t+1\}}), E(I_f^{t+1} - r_i^{t+1}))$, where V_f is household type f's indirect utility conditional on the location, assuming that the expected attributes of the residential location i in period $t+$, is denoted as $E(Z_i^{t+1})$, and the expected disposable income after paying residential rent, is denoted as $E(I_f^{t+1} - r_i^{t+1})$. That is, the agent solves the expected value of the future utilities associated with location i, similar to the classical formulation of dynamic intertemporal stochastic programming; in this case, the stochastic state is associated with changes in the agent's own socioeconomic status.

To estimate the expected values in $t + 1$, we assume a myopic agent that assesses the future based only on current and past information. Under this condition, the rational agent observes how other agents value attributes and assesses the disposable income of each agent at each location i during current period t. Then,

$$V_{fi}^{t+1} = V_f\left(E\left(Z_i^{t+1}\right), E\left(I_f^{t+1} - r_i^{t+1}\right)\right) \approx V_f\left(Z_i^t, I_f^t - r_i^t\right). \tag{11}$$

This model assumes that all agents share the same information regarding the behavior of other agents at any time period and that they are all rational and, thus, forecast their behavior in the future in the same way; what makes agents different is the transition matrix. Additionally, myopic consumers estimate transitions by observing current changes in demography, which implies the assumption of $P(f^{t+1}|h^t)$ as a homogeneous transition matrix, i.e., $P(f^{t+1}|h^t) = P(f^t|h^{t-1}) = P(f|h)$. Therefore, the consumer's problem can be formulated as follows:

$$\max_i \ \max_x \ \theta_h^t U_h^t(x, Z_i^t) + (1-\theta_h^t) \sum_{f \in C} P(f|h) V_f\left(Z_i^t, I_f^t - r_i^t\right) \quad \text{subject to} \quad p^t x + r_i^t \le I_h^t. \tag{12}$$

The consumer's problem (12) is static in t because it includes dynamic effects in either the utility function or the income constraint. It assumes that agents belonging to cluster h know the utility parameters (tastes) of the other potentially imitable agents. We note that the myopic assumption simplifies the consumer's long-term dynamic problem associated with households' residential location decisions. It replaces the intertemporal dependency of the current and future states of the land use system and the need to assume that consumers are able to forecast the future (called perfect foresight assumption) with the myopic assumption that all information for consumers is currently available and the same for all agents (later, we will relax the assumption of equal information). Note that this approach is not only plausible but also highly convenient for the modeler because it simplifies calculations, which in some dynamic models, become very difficult to solve analytically. Furthermore, the forecasting assumptions are difficult to defend given high levels of uncertainty in both the economy and future behavior. In addition, formulation (12) allows us to study and incorporate other social dynamics such as influence among socioeconomic groups, social learning and bi-level processes in decision making among leaders and followers.

An initial analysis of Eq. (12) shows that for a household in cluster h, the value of $(1-\theta_h^t)\sum_{f\in C} P(f|h)V_f\ (Z_i^t, I_f^t - r_i^t)$ is constant in the continuous optimization problem in x. Then, the optimal allocation of goods consumption is conditional only on the current status of agent h:

$$x_h^t(p^t, I_h^t - r_i^t, Z_i^t, \theta_h^t)$$

and the agent's indirect utility function conditional on real estate i is:

$$V_{hi}^t \equiv \theta_h^t V_h^t(I_h^t - r_i^t, Z_i^t) + (1-\theta_h^t)\sum_{f\in C} P(f|h)V_f\left(Z_i^t, I_f^t - r_i^t\right). \tag{13}$$

It is possible to prove that the willingness to pay for the social behavioral problem with expectation, denoted $\tilde{B}_{hi}^t$, has the following form:

$$\tilde{B}_{\mathrm{hi}}^t = \theta_h^t \frac{\lambda_h^t}{\bar{\lambda}_h^t} B_{\mathrm{hi}}^t + (1-\theta_h^t)\sum\nolimits_{f\in C} P(f|h)\frac{\lambda_f^t}{\bar{\lambda}_h^t} B_{\mathrm{fi}}^t \tag{14}$$

where $\bar{\lambda}_h^t = \theta_h^t \lambda_h^t + (1-\theta_h^t)\sum_{f\in C} P(f|h)\lambda_f^t$ represents the expected marginal utility of income between periods t and $t+1$. In the case that $P(f|h)$ represents long-term probabilities, $\bar{\lambda}_h^t$ would represent the expected marginal utility per period in the long term. Note also that if $P(f|h)$ represents the effect of social behavior, then $\bar{\lambda}_h^t$ would represent this social influence over individual income.

By using the following change in notation:

$$\psi_{hh} = \theta_h^t \lambda_h^t + (1-\theta_h^t)\ P(h|h)\lambda_h^t, \qquad \psi_{hf} = (1-\theta_h^t)\ P(f|h)\lambda_f^t, \qquad \text{with } f \neq h.$$

Note that $\bar{\lambda}_h^t = \sum_{f\in H}\psi_{\mathrm{hf}}$, and the bid function can be written as:

$$\begin{aligned}\tilde{B}_{\mathrm{hi}}^t &= \sum\nolimits_{f\in H}\frac{\psi_{\mathrm{hf}}}{\bar{\lambda}_h^t}B_{\mathrm{fi}}^t = \sum\nolimits_{f\in H}\frac{\psi_{\mathrm{hf}}}{\bar{\lambda}_h^t}\left(a_h^t + b_{\mathrm{hi}}^t\ (Z_i^t)\right) = \sum\nolimits_{f\in H}\frac{\psi_{\mathrm{hf}}}{\bar{\lambda}_h^t}\tilde{a}_h^t \\ &+ \sum\nolimits_{f\in H}\frac{\psi_{\mathrm{hf}}}{\bar{\lambda}_h^t}\tilde{b}_{\mathrm{hi}}^t \\ &= \tilde{a}_h^t + \tilde{b}_{\mathrm{hi}}^t\end{aligned} \tag{15}$$

where $\frac{\psi_{\mathrm{hf}}}{\bar{\lambda}_h^t}$ is the valuation percent attached by agent h to the willingness to pay of the imitated agent f for location i. Note that $\sum_{f\in H}\frac{\psi_{\mathrm{hf}}}{\bar{\lambda}_h^t} = 1$. Thus, in building her own willingness to pay, agent h weights the willingness to pay of other clusters that she/he imitates; the weights reflect the marginal utilities of income and the transition probabilities for future socioeconomic conditions. The bid with imitation (15) has two interesting terms, $\tilde{a}_h^t$ and $\tilde{b}_{hi}^t$, with

$$\tilde{a}_h^t = \frac{1}{\bar{\lambda}_h^t}\left(\sum_f \psi_{\mathrm{hf}} a_f^t\right) = \frac{1}{\bar{\lambda}_h^t}\left(\sum\nolimits_f \psi_{\mathrm{hf}} * \left\{I_f^t - \frac{\bar{U}_f^t}{\lambda_f^t}\right\}\right) \tag{16}$$

The right side of Eq. (16) is obtained by replacing a_f^t with $I_f^t - \frac{U_f^t}{\lambda_f^t}$, where I_f^t is income in period t, and $\frac{U_f^t}{\lambda_f^t}$ is the ratio between the utility level obtained in equilibrium and the marginal utility of income (see Eq. (5)). Thus, in terms of the willingness to pay function of an agent h, both the level of income and the income associated with each property good bid are directly influenced by the utility level and the income of the socially imitable agents.

As previously mentioned, formulations (12) to (16) can be extended or analyzed in other contexts to study life cycles or social dynamics. For example, if we interpret $(1-\theta_h^t)\ P(f|h)$ by a weight that measures a social relationship between these types of households (Páez et al., 2008), then $\frac{\psi_{\mathrm{hf}}}{\bar{\lambda}_h^t}$ would indicate agent h's valuation in terms of willingness to pay based on the social influence of agent f. Thus, the formulation of utility (13) and willingness to pay (15) can represent an implementation of classic assumptions in social network studies and represent a model of the impact of such assumptions on location decision making. These social interactions represent the dynamic version of location externalities mentioned above. In general, these imitation phenomena are understood as a collective learning form (Alós-Ferrer & Schlag, 2009), which uses information from other agents for decision making. For example, in terms of firms' location, the profit of other firms already at the location can be used for decision making (e.g., imitation location).

The effect of imitation—or social networks—on location choices differentiates the utility of agents that belong to the same cluster but have different imitation or social tie behavior. Furthermore, the previously proposed modeling strategy can be used when it is unclear to which specific cluster a household belongs (called fuzzy clustering; see, for example, Hwang & Thill, 2009), and thus, the model can generate more representative willingness to pay for such agents. In that sense, in micro simulation processes, where it is assumed that each agent has a value belonging to each distinctive cluster, a more specific utility and bid can be obtained for each household and for each property good.

In the stochastic version of the imitation model, we assume that the willingness to pay $\tilde{B}_{hi}^t$ is a random variable, where the stochastic term is assumed to be i.i. Gumbel distributed with scale parameter μ. It is important to note that the bid with expectations includes an expected income within the formulation, as noted in (16). However, the bid at time t should be constrained by income during this period, regardless of the expectation levels. Therefore, we impose:

$$\tilde{B}_{hi}^t \le I_h^t \tag{17}$$

Regardless of social expectation levels, if static constraint (17) is not satisfied with the current income at location i for agent h at equilibrium, then the agent should not participate in the auction, i.e., the agent is eliminated from the set of potential bidders for that location. The income constraint (17) includes wages, loans, savings, etc. Similar to other works related to research on land use, we can model (17) using the constrained logit model (Martínez, Aguila, & Hurtubia, 2009) with "cutoff" functions on location probabilities denoted as ϕ_{hi}^t. In addition, the population distribution H_{hi}^t at period t is:

$$H_{hi}^t = S_i^t Q_{h|i}^t = S_i^t \frac{H_h^t \phi_{hi}^t\, exp\left(\mu \tilde{B}_{hi}^t\right)}{\sum_{g\in C} H_g^t \phi_{gi}^t\, exp\left(\mu \tilde{B}_{gi}^t\right)} \tag{18}$$

where S_i^t is the exogenous supply during period t. Although the constrained logit model was already used and previously suggested in land use models in different contexts, it is worth mentioning its importance to the bid posture or willingness to pay for each property in our formulation with social expectations or imitation effects because in addition, the level of expectation that includes the endogenous bid should not exceed the current income (period t). That is, two individuals with the same level of future expectations but with different incomes may not necessarily have the same willingness to pay for each type of housing. In that sense, it is important to note that although the function $\tilde{B}_{hi}^t$ depends on the expected incomes contained within $\tilde{a}_h^t$ (see Eqs. (5) and (16)), the income constraint ϕ_{hi}^t permits the elimination of infeasible alternatives given households' current socioeconomic status.

However, similar to RB&SM (Martínez & Henríquez, 2007), we can consider in our modeling that existing externalities associated with the population distribution in each zone affect households' willingness to pay.

Furthermore, the rent at each location is obtained endogenously by the expected maximum bid, given by the log sum expression:

$$r_i^t = \frac{1}{\mu}\left\{ \ln\left(\sum_{g\in C} H_g^t \phi_{gi}^t \exp\left(\mu \tilde{B}_{gi}^t\right) + \gamma\right)\right\}, \quad \forall i. \tag{19}$$

Finally, the following equilibrium condition ensures that every household is allocated to a location in the city, i.e., $\sum_i S_i^t Q_{h|i}^t = H_h^t$:

$$a_h^t = -\frac{\sum_g \psi_{hg}}{\mu\psi_{hh}} \ln\left(\sum_i S_i^t \phi_{hi}^t \exp\left(\mu\left\{\frac{\psi_{hh}}{\sum_g \psi_{hg}} b_{hi}^t - r_i^t + \sum_{f\neq h} \frac{\psi_{hf}}{\sum_g \psi_{hg}} B_{fi}^t\right\}\right)\right), \quad \forall h.$$

Similarly,

$$a_h^t = -\frac{\lambda_h^t}{\mu\psi_{hh}} \ln\left(\sum_i S_i^t \phi_{hi}^t \exp\left(\mu\left\{\frac{\psi_{hh}}{\bar{\lambda}_h^t} b_{hi}^t - r_i^t + \sum_{f\neq h} \frac{\psi_{hf}}{\bar{\lambda}_h^t} B_{fi}^t\right\}\right)\right), \quad \forall h. \tag{20}$$

Note that rents r_i^t depend on each a_g^t, each a_g^t depends on ϕ_{hi}^t, and each B_{fi}^t depends on both a_f^t and on the utility levels of the imitable individuals. Then, (20) constitutes a nonlinear fixed-point system of equations for a_h^t, defining the maximum levels of utility feasible at equilibrium for agent h given by $\tilde{a}_h^t$ (Eq. (16)). This value is conditioned by the other agents' utility levels (both by the equilibrium condition and by the imitation effect) and by the expected incomes

$$\tilde{I}_h^t = \frac{1^t}{\bar{\lambda}_h}\left(\sum_f \psi_{hf} I_f^t\right)$$

Thus, current rents and household utility levels in period t depend on future incomes. This result would explain one of the causes of speculation and uncertainty in the property market, for example, the observed *overvaluation* of some real estate rents associated with household expectations and the state of the economy.

To briefly analyze the imitation effect on urban distribution, consider the case of two clusters f and h with an expectation or imitation normalized factor denoted as $1>\psi_{hf}>0$.

If for a given location i occurs such that $B_{fi}\geq B_{hi}$ and under the assumption that $\phi_{hi}^t = \phi_{fi}^t = 1$ (income constraint is fulfilled), then, we have $H_{hi}\leq H_{hi}^{imi}$, where H_{hi} is the urban distribution with no imitation ($\psi_{hf}=0$) and $H_{\{hi\}}^{\{imi\}}$ is the urban distribution with imitation or expectations in i ($\psi_{hf}>0$).

Note that if the income constraint is not met, then $H_{hi}\leq H_{hi}^{imi}$ will not necessarily be true. Therefore, results are not particularly intuitive and cannot be concluded directly regarding the effects of imitative behavior on urban distribution. Thus, a more detailed analysis of such restrictions and their effect on both imitation dynamics and household life cycles is relevant.

However, the dynamic modeling of residential relocation allows the inclusion of bids (or utilities), generating other strategies to integrate the interaction between different temporal and spatial information sources. For example, Habib and Miller (2009) and Martínez and Hurtubia (2006) use information from former periods to build the valuations of each real estate type during the current period. Similarly, a way to formulate bids with location externalities is to assume that every bid function B_{hi}^t depends on the urban distribution observed in the previous period t − 1, i.e., $B_{hi}^t\,(H_{gj}^{t-1}, \forall g,j)$. These one period lagged interactions avoid the calculation of location externalities as a fixed point. This simplification, however, does not avoid the fixed point computation of the utility $\tilde{a}_h^t$ at equilibrium, which must be obtained in each period using Eq. (19). It is important to note that under an intertemporal context, the change in expectations is renovated and updated. Hence, it is necessary to perform the analysis period by period for the number of agents that actually change household type and for the sizes of the new clusters because they affect the urban distribution. Note that even in a city experiencing slow relocation processes, if households change their socioeconomic characteristics over time, then the city will have different population distributions due to the agents' endogenous dynamics.

It is important to note that the imitation process described in this section differs somewhat from the classic imitation processes of the dynamic economy because in those cases, the process imitates the choice but does not directly imitate the utility. Thus, we extend this concept based on the following hypothesis and concepts: In the theory of deterministic discrete choice, imitating choice implies somehow imitating utility provided that the level of utility achieved commands the optimal choice. Thus, intuitively, we could think of a direct relationship between choice and achieved utility. Provided that our problem is similar to a dynamic programming process with variable utility over time (intertemporal utilities), if we wish to anticipate future events, it is plausible to associate future utilities with current knowledge. In other words, our imitation process allows us to solve the dynamic problem in the long term. Moreover, by considering only imitation of the choice of other agents, we would not be able to solve the microeconomic formulation either empirically or econometrically. Furthermore, in the theory of stochastic discrete choice (logit protocol), imitating the choice would imply imitating the choice probability of each good, which depends on the utility level of each alternative. Therefore, it is reasonable to think that imitating choices implies a process of valuing the utility of other agents. Finally, the process of imitation allows us to also analyze other types of key factors for urban modeling, such as fuzzy clustering, in which an agent could have a certain degree of membership degree in each cluster. With our formulation, it is feasible to include such modeling aspects in both the utility functions and the willingness to pay for the goods.

4. Numerical examples: analysis of residential segregation models

In this section, we present numerical examples based on dynamic urban segregation models of the proposed formulation. The imitation model proposed in the experiments is sensitive to the social and life-cycle construction of the dynamic valuation effects and influenced by different socioeconomic groups with respect to others.

In particular, the purpose of this section is to analyze the urban system dynamics (long-term configuration) with the inclusion of social imitation processes described by Eqs. (15) to (20) using the stochastic modeling (logit protocol or structure) of the bid functions. We address the following question: If agents expect to change their clusters over time (life cycle evolution) or expect their behavior to generate a process of social imitation for other agents' valuations, does this process reduce segregation levels? To develop a numerical example to explain the proposed modeling of imitation and its effects on urban configuration, we use the utility and bid functions previously used in the literature of dynamic models of residential segregation (see Zhang, 2011; Grauwin, Goffette-Nagot, & Jensen, 2012). Such works analyze the dynamics of residential segregation in an urban system through simple numerical examples in which agents' behaviors are defined by a utility function associated with their set of neighbors in a given period based on relocation rules. In O'Sullivan (2009), triangular bid functions (asymmetric preferences for integration) are used to analyze the dynamic models of segregation from a deterministic and a disaggregated perspective.

However, Gravel and Oddou (2014) analyze the properties of endogenous segregation dynamic processes in the presence of a competitive land market using a particular economic model. In this paper, the authors show that segregation by income is stable. In a particular applied work, Bayer and McMillan (2012) use simulations based on real parameters to conclude that reductions in commuting costs generate an increase in the racial segregation and a moderate increase in income segregation. While decreasing the preferences associated with housing characteristics increases racial segregation, it reduces income segregation.

Feitosa, Le, and Vlek (2011) propose a multi-agent model of urban segregation for exploring the impacts and consequences of some mechanisms on the emergence of segregation patterns.

In our illustrative example, we assume 3 equally sized zones (number of dwellings are $S_1 = S_2 = S_3 = 100$), with the only differentiating factor being the initial inhabitants in each zone, where the zone defines the neighbor. Every agent evaluates each zone $i = (1,2,3)$ according to the proportion of each household type in the zone. There are three agent types with the same sizes ($H_1 = H_2 = H_3 = 100$) although showing different preferences and income level. In addition, it is assumed the following order of marginal utilities of income ($\lambda_1 > \lambda_2 > \lambda_3$); that is to say, the agents in group 3 belong to the highest socioeconomic income group as they have the highest marginal utility of income. We impose that their willingness to pay satisfies the budget constraint (it is not necessary to use the cutoff probabilities).

The bid function with externalities is as follows:

$$B_{\text{hi}}^t = \sum_f \frac{\psi_{\text{hf}}}{\bar{\lambda}_h^t} * \left(a_f^t + b_{\text{fi}}^t\right) \quad \text{with} \quad b_{\text{fi}}^t = \sum_g \beta_{\text{fg}} \frac{H_{\text{gi}}^t}{\sum h' H_{h'i}^t} \tag{21}$$

$$\text{and} \quad \beta = \begin{pmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{pmatrix}$$

That is, H_{hi}^* is the result of solving the system of equations described by (18) to (20) (location probabilities, expected rents and utility). Moreover, we must note that B_{hi}^t depends on b_{fi}^t, and this value is a function of the urban distribution (H_{gi}^t) at time t; therefore, to find H_{gi}^t through (18), we must solve a fixed point system of equations that indicates the urban externality affecting the location decision.

In addition, H_{hi}^* represents a fully segregated benchmark solution in our stochastic context. For our analysis, we propose the following simple index of segregation

$$dis^{imi} = \frac{\sum_{h,i} \left| H_{hi}^* - H_{hi}^{imi} \right|}{\sum_h H_h}$$

with $dis \in [0,1]$, which indicates an increasingly segregated system as it approaches 0 and a fully integrated system at 1. For further information about other spatial segregation indices, refer to Song, Merlin, and Rodriguez (2013); Rey and Folch (2011), and Meng, Hall, and Roberts (2006), among others.

Under this setup, several scenarios of imitation are analyzed below:

Scenario 1: Agents 1 and 2 imitate agent 3 with a valuation of their bid function of form:

$$\text{Imitation normalized matrix}: \left\{ \frac{\psi_{\text{hf}}}{\bar{\lambda}_h^t} \right\}_{h,f \in H} = \begin{pmatrix} n & 0 & 1-n \\ 0 & n & 1-n \\ 0 & 0 & 1 \end{pmatrix} \tag{22}$$

where $0 \le n \le 1$. In this expression, n is the percentage of valuation of bid function $\frac{\psi_{\text{hh}}}{\bar{\lambda}_h^t}$ when $h = 1$ and $h = 2$. That is, in this case, $1 - n = \frac{\psi_{h3}}{\bar{\lambda}_h^t}$ corresponds to the valuation that agents 1 and 2 give to the bid of agent 3. However, we also have $\frac{\psi_{3f}}{\bar{\lambda}_3^t} = 1$ if $f = 3$ and 0 if $f = 1$ or 2, that is, agent 3 does not imitate the behavior of other groups. In a first analysis, we consider n values between a high (0.6) and a low 0.9 imitation effect.

In a context of life-cycle, this matrix (22) represents an economy experiencing successful progress (associated with socioeconomic groups); in addition, the groups with lowest income intend to imitate the behavior of the groups with higher income. This is accomplished by means of debts (mortgages, credits, etc.) in order to improve their education, health, car ownership as well as other durables, with the objective of reaching their long-term expectations (a raise in their income) under socially accepted behaviors. Note that the likelihood of reducing the income of these agents is not null; the issue is that such agents do not value such an option in their bid functions.

The results of dis^{imi} are shown in Fig. 1.

According to the results, a global social integration index of 14% is achieved when $n = 0.6$. Additionally, we observe an interesting phenomenon: at zone 3, where agent 3 (the imitable agent) is most often located, the lowest integration is achieved. For example, when $n = 0.6$, the following population distribution is obtained:

$H_{hi}^{imi}(n = 0.6) =$

	Zone 1	Zone 2	Zone 3
Household 1	92	5.4	2.6
Household 2	5.4	92	2.6
Household 3	2.6	2.6	94.8

This result is persistent even though Households 1 and 2 strongly imitate Household 3, showing that the self-preference of households in cluster 3 (the peer attraction effect) overrules the integration impulse of clusters 1 and 2. Additionally, this result also shows a reduction in segregation between Households 1 and 2 (see Zones 1 and 2), which is a less expected result because there is no imitation behavior between these two clusters. The rent results for this case are shown in Fig. 2.

The results for rents in Fig. 3 show that as the imitation of clusters 1 and 2 increases (e.g., see $n = 0.6$), rents diverge between zone 3 and the other two zones; i.e., the willingness to pay of agents 1 and 2 decreases in the zones where they are mostly located, while cluster 3 values the segregated environment in zone 3. On the right extreme of Fig. 3, imitation effects disappear and all prices converge, but to a higher value, revealing the valuation of the peer attraction effect in this case. Moreover, such rent dynamics are obtained through a combined effect because real estate prices fall due to the emergence of more integrated neighborhoods; in turn, the difference between the rents of Zone 3 with respect to Zones 2 and 1 increases due to agent 3's higher willingness to pay. This can also be the result of natural segregation due to agents' desire to live among peers.

Scenario 2 Agents 1 and 2 imitate each other and imitate agent 3, with the following imitation factors:

$$\text{Imitation normalized matrix}: \left\{ \frac{\psi_{\text{hf}}}{\bar{\lambda}_h^t} \right\}_{h,f \in H} = \begin{pmatrix} n & \frac{1}{2}(1-n) & \frac{1}{2}(1-n) \\ \frac{1}{2}1-n & n & \frac{1}{2}(1-n) \\ 0 & 0 & 1 \end{pmatrix}$$

where $0 \le n \le 1$. That is, n is the percentage of valuation of bid function $\frac{\psi_{\text{hh}}}{\bar{\lambda}_h^t}$ when $h = 1$ and $h = 2$. Furthermore, $\frac{1}{2}(1-n) = \frac{\psi_{\text{hf}}}{\bar{\lambda}_h^t}$ if $f \ne h$ and $h \ne 3$, corresponding to the valuation that agents 1 and 2 give to the bids of the agents of the other two groups.

Fig. 3 shows the segregation results of this scenario. In this case, the social integration factor increases with respect to scenario 1; for example, a social integration factor of 18% is achieved when $n = 0.6$. In Zone 3, where type 3 agents are concentrated, we observe lower integration levels compared with scenario 1 because the imitation preferences of agents 1 and 2 over agents 3 are less valued

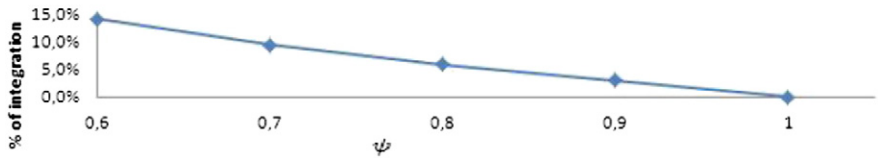

Fig. 1. Level of long-term integration (Scenario 1) by varying *n*.

than in scenario 1. For $n = 0.6$, we obtain the following urban distribution:

$$H_{hi}^{imi}(\boldsymbol{n}=0.6) =$$

	Zone 1	Zone 2	Zone 3
Household 1	87.6	10.9	1.5
Household 2	10.9	87.6	1.5
Household 3	1.5	1.5	97.1

Results with regard to rents are similar to those found in Scenario 1.

Scenario 3: In this last case, the socioeconomic hierarchy increases with the cluster number, and the following social imitation factor matrix is assumed:

$$\text{Imitation normalized matrix}: \left\{\frac{\psi_{hf}}{\bar{\lambda}_h^t}\right\}_{h,f\in H} = \begin{pmatrix} n & \frac{1}{2}(1-n) & \frac{1}{2}(1-n) \\ 0 & n & 1-n \\ 0 & 0 & 1 \end{pmatrix}$$

with $0 \le n \le 1$.Here, agent 1 imitates agents 2 and 3 with the same valuation ($\frac{1}{2} * (1-n)$), and agent 2 imitates agent 3 with valuation $1-n$. In this case, the agents of lowest income have expectation of improving in the future, analyzing the probability of changing to any socioeconomic groups of higher income.

The simulation results show that the social integration index increases with respect to Scenario 1 but decreases with respect to Scenario 2. In addition, a higher segregation level is obtained in Zones 2 and 3 than in Zone 1, where cluster 1 is primarily located. Moreover, the rents in each zone display the behavior shown in Fig. 4.

Rent results indicate that, similar to previous scenarios, the stronger the agents' imitation, the more differentiated the rents between zones and the lower the rents in all zones compared with the no imitation case $n = 1$ because of the strong peer attraction or segregation effect in all clusters. However, compared to Scenario 1, rents are lower in this scenario under high imitation (e.g., see $n = 0.6$) because the more disperse imitation matrix reduces to half of agent 1's value of other clusters in the same zone. Moreover, the results associated with the rent process shown in our simulations are analogous to the results of Bayer and McMillan (2012), who empirically and theoretically show that the immigration of residents with higher incomes (for example, agent 3 in our simulations) to the surrounding areas of a city generate an increase in income and willingness to pay, causing poorer original residents to migrate. This process is known as endogenous gentrification. This result is explained partly by the preference to live among high-income neighbors (positive externality), as also found in our imitation simulations.

The results of Scenarios 1, 2 and 3 show that in a city with agents having high imitation values and changing socioeconomic expectations, segregation levels will tend to reduce over time independent of the number of residents with low imitation values, i.e., the existence of imitators is a sufficient condition to lower segregation. Even in the presence of agents that pursue segregation, social integration occurs because this effect is reduced due to the action of imitators.

Another case: an indifferent agent and externalities.

In the following case, we assume one agent that is indifferent to the neighborhood (agent 1), another imitator agent (agent 2) and finally one imitated agent (agent 3). The bids without imitation are:

$$B_{hi}^t = \sum_f \frac{\psi_{hf}}{\bar{\lambda}_h^t} * \left(a_f^t + b_{fi}^t\right) \quad \text{with} \quad b_{fi}^t = \sum_g \beta_{fg} \frac{H_{gi}^t}{\sum h' H_{h'i}^t} \quad \text{and}$$

$$\beta = \begin{pmatrix} 20/3 & 20/3 & 20/3 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{pmatrix}.$$

Additionally, the imitation matrix is as follows:

$$\text{Imitation normalized matrix}: \left\{\frac{\psi_{hf}}{\bar{\lambda}_h^t}\right\}_{h,f\in H} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & n & 1-n \\ 0 & 0 & 1 \end{pmatrix}$$

with $0 \le n \le 1$. In this case, $\frac{\psi_{22}}{\bar{\lambda}_2^t} = n$ and $\frac{\psi_{23}}{\bar{\lambda}_2^t} = 1-n$.

That is to say, in case of agents 1 and 3, even though life-cycle changes were observed, they would not incorporate such processes in their valuation of goods and zones. Unlike agents 1 and 3, agents 2 do consider life-cycles changes, thinking always in the possibility of improving their income.

Fig. 5 shows the population size in each zone, varying the imitation factor *n*. For example, the first graph in Fig. 5 shows the distribution of agent type 1 in each zone for different n values.

In the case of $n = 1$, an integration level of 15.2% is obtained, which is higher than in previous cases. Thus result is caused by the indifference of agent 1 to urban distribution, which increases in the case of $n = 0.7$ to 42%. However, in Zone 3, a concentration effect that is similar to the above scenarios is observed because only 13% of type 3 households are located outside of Zone 3. Additionally, the number of indifferent type 1 households located in Zone 3 decreases (from 5 to 1 agents) with the increase of imitation effects, while the imitator type 2 households located in zone 3 increase (from 1 to 12). That is, Zone 3 has a social integration of 13% compared to 6% in the segregated solution in the same zone. In addition, segregation decreases faster in Zone 2 with the reduction of *n*,

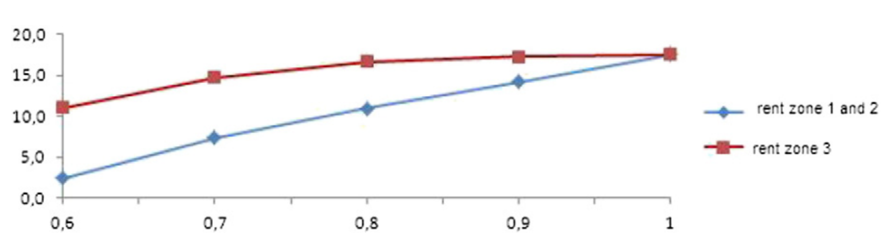

Fig. 2. Rents in each zone (Scenario 1) by varying *n*.

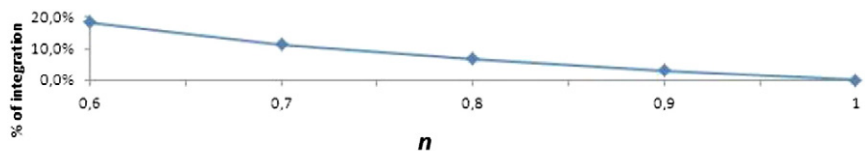

Fig. 3. Level of long-term integration (Scenario 2) by varying ***n***.

mainly because those agents that are mostly located in this region (agents 2) imitate agent 3. This reduces their peer attraction effect and creates the possibility of integration with type 2 and 3 households, which is the goal of agent 2's process of imitation. Indifferent agents (type 1) have an important impact because they reduce segregation in all cases and allow agent 2 to achieve a greater integration with agent 3.

In general, for all of the analyzed scenarios, the aggregated segregation level has a minimum reduction independent of including the imitation effect with the different types of valuations. Thus, high levels of segregation are maintained due to the imitated agents' bid function (agent 3) associated with always living among their peers. This result confirms and extends the work of Zhang (2011), who demonstrates that segregation is stable using tools of evolutionary game theory under conditions of the different valuations (asymmetric) of two types of agents (2 colors). What is interesting about an imitation model based on a maximum bid stochastic model under a multinomial logit protocol is that we clearly show the effect on the rents that must be paid by the non-segregator (or imitator households) that want to live close to segregator agents (agent 3). In all cases, these rents are higher than the rents in areas where such agents are not primarily located. Thus, if the social objective is to seek higher levels of integration, incentives must be found such that agent 3 decreases its claims to remain segregated (taxes, coordination, etc.). Some of these tools are proposed and discussed in Grauwin et al. (2012) in the context of utility functions and game theory.

5. General remarks

The use of household life cycle or social expectations in residential location decisions is incorporated in a discrete choice microeconomic formulation by means of transition probabilities or weights among household clusters considering life cycle and social dynamics as well as the hypothesis of the imitation behavior of such agents under the consideration that they behave rationally and myopically in the temporal dimension. This premise considers that the tastes of imitable agents are known. Based on a microeconomic consumer problem formulation, a multi-objective bid function is obtained, which includes an expected income per unit of time and a utility consistent with the behavior of those agents that are potentially imitable. For this model, it is necessary to include an income constraint per period because such valuations can generate infeasible willingness to pay, as they do not match the net income of each household in that period. In the simulations conducted in this work, we show that adding these dynamic effects of expectations in the agents' behavior as an endogenous valuation decreases the level of segregation when there are possibilities for interaction with other agents. In other words, in the case that one agent partly imitates another agent's behavior, with the feature that she or he also strongly values coexisting among her or his peers, it is very difficult to drastically increase social integration.

Complementarily, simulations were conducted considering a group that is indifferent to the neighborhood together with two other groups of agents interacting such that one is a follower (or imitator) of the other's behavior. These results found that the indifferent agent's role is to provide some integration possibilities between the imitator agent (its goal is integration as it imitates) and the imitated agent (its goal is segregation).

However, the simulation results showed that the segregation effect is quite stable even if there are differences in either the individuals' valuation (asymmetric effects) of the neighborhood or in the configuration of the area or zone, independent of the existence of indifferent agents (or agents that value living in an integrated manner, called imitators). This result confirms what was mathematically proved by Zhang (2011), who concludes that when there is less asymmetry in the valuation, segregation will occur endogenously, supporting work on evolutionary game theory with two types of agents (two colors). However, a key aspect in the formulation proposed in the present paper is the analysis of the effect on the rents of such valuations, which shows that higher rents are associated with groups that prefer to live among their peers. As a further development, we plan to conceive a more disaggregated method of modeling, the groups or clusters of agents with different types of preferences under the context of imitative behavior or life cycle, as postulated in Clark and Fossett (2008), where higher levels of integration are obtained using models based on agents.

Finally, the microeconomic formulation and the willingness to pay of the model with imitation could be extended to other contexts, such as social networks or fuzzy clustering, due to the interactions among different agents and the effect an agent's ability to incorporate others' perceptions (valuations) in its own valuation.

Acknowledgments

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Appendix A

Suppose the following microeconomic consumers' problem associated with discrete choices under de consideration of a quasi-linear di-

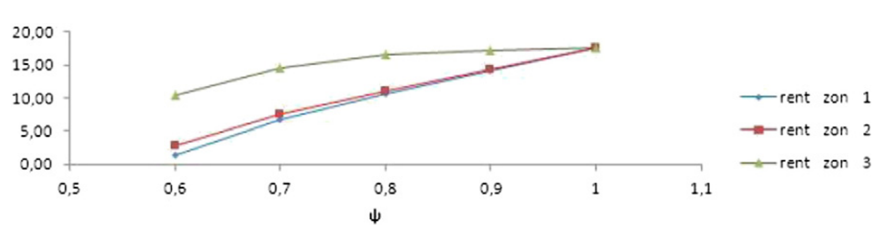

Fig. 4. Rents in each zone (Scenario 3) by varying n.

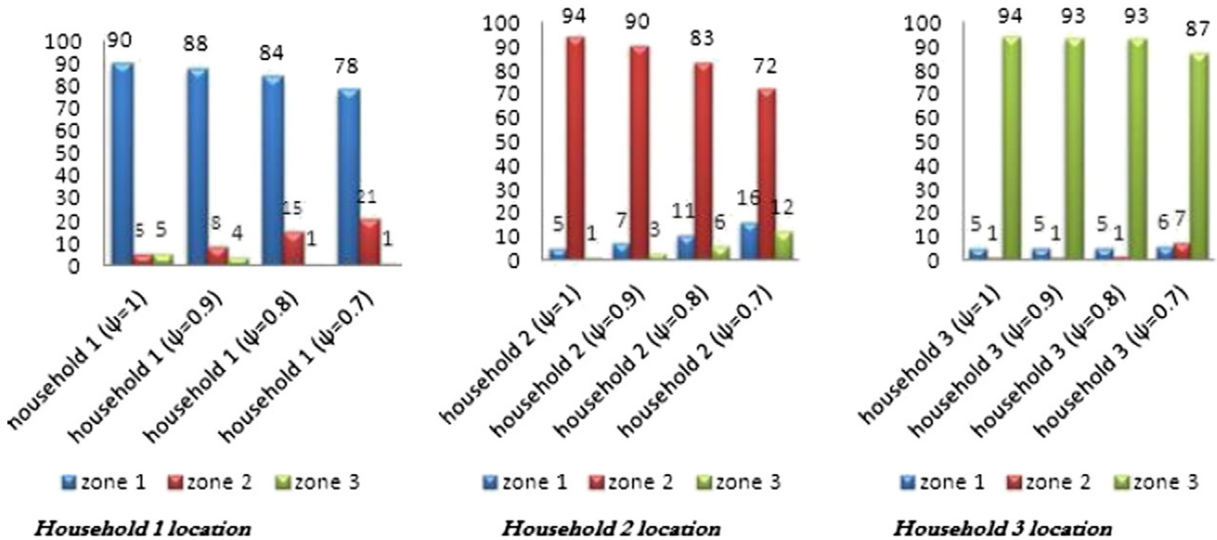

Fig. 5. Long-term population distribution by varying *n*.

rect utility function

$$\max_i \max_x U(x, Z_i) = \alpha x_o + f(x_1, x_2, \ldots, x_n, Z_i) \tag{A.1}$$

Subject to $x_o + \sum_{j=1}^{n} p_j x_j + r_i \leq I$.

Without losing generality, we suppose that the price per unit of market good x_o is 1. Given that f is a concave and increasing function, then the income constraint (A.1) will saturate (constraint becomes active), by which

$$x_o = I - r_i - \sum_{j=1}^{n} p_j x_j$$

Thus, replacing x_o in the utility function of (A.1) we obtain the following unconstrained problem:

$$\max_i \max_x U(x, Z_i) = \alpha * \left(I - r_i - \sum_{j=1}^{n} p_j x_j \right) + f(x_1, x_2, \ldots, x_n, Z_i) \tag{A.2}$$

Solving the consumer problem (A.2) with respect to variable x conditional on discrete good i, it is possible to get that

$$\alpha p_j = \frac{\partial f}{\partial x_j}$$

or, equivalently

$$\alpha = \frac{\frac{\partial f}{\partial x_j}}{p_j}.$$

Therefore α is the marginal utility of income

On the other hand, by solving (A.2) we obtain the optimal consumptions that depend on prices, income and each discrete good features, that means $x_j^*(p, Z_i)$. By replacing x_j^* in the direct utility function $U(x^*, Z_i)$, we obtain

$$U(x^*, Z_i) = \alpha * \left(I - r_i - \sum_{j=1}^{n} p_j x_j^*(p, Z_i) \right) + f(x^*((p, Z_i)), Z_i)$$

Given that, the indirect utility function conditional to discrete good i is defined as $V \equiv U(x^*, Z_i)$. Then

$$V_i = a^*(I - r_i) + a^* \left(\frac{f(x^*(p, Z), Z_i)}{\alpha} - \sum_{j=1}^{n} p_j x_j^*(p, Z_i) \right). \tag{A.3}$$

In addition, we can assume that market prices are exogenous to discrete good i; then

$$b_i(Z_i) = \frac{f(x^*((p, Z_i), Z_i)}{\alpha} - \sum_{j=1}^{n} p_j x_j^*(p, Z_i)$$

For which, the formulation of the indirect utility function becomes

$$V_i = \alpha * (I - r_i) + \alpha * b_i(Z_i) \tag{A.4}$$

The previous result proves Eq. (3).

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