ICI Reduction in OFDM Systems Using a New Family of Nyquist-I Pulses

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Abstract—**In this paper we propose the improved parametric linear combination pulse (IPLCP); a new family of Nyquist pulses with two additional degrees of freedom to minimize the inter-carrier interference (ICI) power in orthogonal frequency-division multiplexing (OFDM) based systems due to frequency offset. Theoretical and numerical simulations are shown to verify that the performace of the IPLCP outperforms other existing pulses.**

Keywords—ICI, OFDM, Nyquist's first criterion, pulse-shaping.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has been widely used in wireless communications standards; such as LTE Advanced, WiFi, WPAN, and WiMAX [1]–[3]. Further, OFDM-based systems are being studied and proposed as one of the technologies to be implemented at the physical layer of 5G systems [4]–[6]. For example, OFDM-based systems combined with multiple-input multiple-output (MIMO) techniques are being suggested as the key technologies to be used at the physical layer of 5G cellular networks. This is due to the high data rate transmission capability, high bandwidth efficiency, the robustness to multi-path fading given by the ability to convert a frequency selective fading channel into several nearly flat fading channels, and also by the way OFDM-based systems deal with delay spread by using a guard interval [1], [2]. Despite all of the benefits offered by OFDM-based systems, there are some drawbacks. These systems are characterized by having high peak-to-average power ratio (PAPR) values [1], [2], [7]. Furthermore, OFDM-based systems are very sensitive to frequency offset errors caused by frequency differences between the local oscillators in the transmitter and the receiver, Doppler spread, and distortions within the channel, among others [1], [2], [8], [9]. Carrier frequency offset causes a number of impairments, such as attenuation or rotation of subcarriers and inter-carrier interference (ICI) between them, increasing the error probability rate as this offset becomes greater.

A number of methods have been developed to reduce the sensitivity to frequency offset, including windowing at the receiver side, pilot insertion, frequency domain equalization, and ICI self-cancellation schemes [1], [2], [10]. The use of Nyquist-I pulses to reduce the ICI power in OFDM-based systems has been studied, proposed, and implemented by several scholars [10]; such as the raised cosine (RC) pulse, "better than" raised cosine (BTRC) pulse [11], sinc power (SP) pulse

[12], improved sinc power (ISP) pulse [13], among others. The ISP pulse is characterized by having two additional design parameters. At the moment, the ISP is probably the pulse that has the best performance dealing with ICI due to frequency offset in OFDM-based systems. But as higher data rates and error-free communication links are going to be demanded in next generation networks, the design and implementation of new families of Nyquist-I pulses will become a fundamental research topic in the upcoming years.

In [14], the parametric linear combination pulse (PLCP) was proposed to minimize the PAPR in OFDM-based systems. The PLCP is characterized by having an additional design paramenter, giving an extra degree of freedom to minimize PAPR for a given roll-off factor. In this manuscript we propose an enhanced version of the PLCP to minimize the ICI power in OFDM-based systems. The proposed pulse is known as the improved parametric linear combination pulse (IPLCP). The IPLCP is characterized by having two new design parameters, adding extra degrees of freedom in its explicit time/frequency-domain expression for a certain roll-off factor. In this paper we will evaluate the IPLCP pulse in OFDM-based systems in terms of ICI power. The performance of the proposed pulse will be compared to those of other existing Nyquist-I pulses.

This paper has the following organization. The OFDM-based system model is introduced in Section II. Section III examines the PLCP family of pulses, and a derivation of the IPLCP family of pulses is done. Section IV analyses the ICI in an OFDM-based system, deriving the ICI power as a function of the pulse-shaping Fourier transform. Section V shows the performance of the proposed pulse by using theoretical expressions, and by implementing a real OFDM-based system via numerical simulations. Finally, conclusions are presented in Section VI.

II. OFDM SYSTEM MODEL

An OFDM symbol is formed by the sum of N data symbols (M-PSK, M-QAM, or other type of digital modulation) each transmitted on a different orthogonal subcarrier. The complex envelope of the OFDM symbol is expressed as

$$s(t) = \text{Re}\left\{ e^{j2\pi f_c t} \sum_{k=0}^{N-1} p(t) d_k e^{j2\pi f_k t} \right\}, \qquad (1)$$

where j is the imaginary unit ($\sqrt{-1}$), N is the number of subcarriers, d_k represents the k-th data symbol (which should
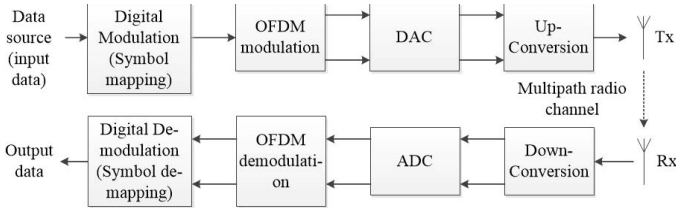
Fig. 1. Scheme of an OFDM system.

Fig. 3. Simplified graph of an OFDM signal spectrum.

not be confused with the OFDM symbol), f_k is the k-th subcarrier frequency, f_c is the central carrier frequency, and $p(t)$ is the pulse-shaping function that limits or narrows each data symbol in a certain interval of time. A simplified scheme of an OFDM-based system is shown in Fig. 1. For the system we assume that the data symbols are uncorrelated, where $\{d_k\}$ is a zero-mean independent sequence of unit variance and normalized symbol energy. This is

$$E[d_k d_m^*] = \begin{cases} 1, & \text{if } k = m \\ 0, & \text{if } k \neq m. \end{cases} \quad (2)$$

As it was mentioned before, one very important characteristic of an OFDM-based systems is the fact that the subcarriers must be orthogonal with one another. To ensure orthogonality the following relationship must be fulfilled

$$f_k - f_m = \frac{k - m}{T}, \quad (k, m) \in \{0, 1, \dots N - 1\}, \quad (3)$$

where $1/T$ is the minimum required subcarrier frequency spacing to satisfy orthogonality. Therefore, individual subcarrier frequencies could be defined as

$$f_k = \frac{k}{T} \quad \forall k \in \{0, 1 \dots N - 1\}. \quad (4)$$

III. NYQUIST PULSE-SHAPING FUNCTION

Based on the fact that each data symbol is transmitted on a different subcarrier, the signal analysis can not be made in the time domain, because the envelope given by (1) is the result
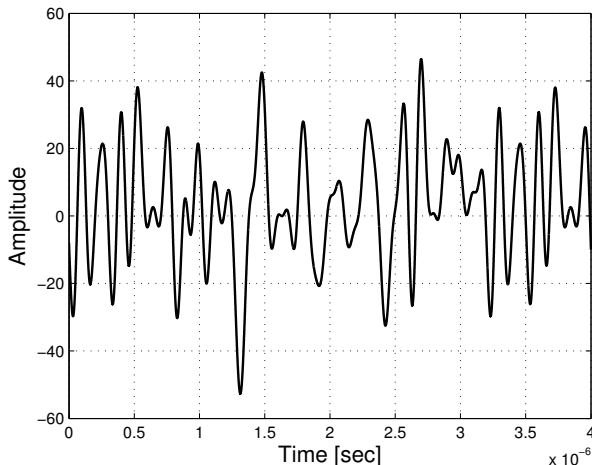

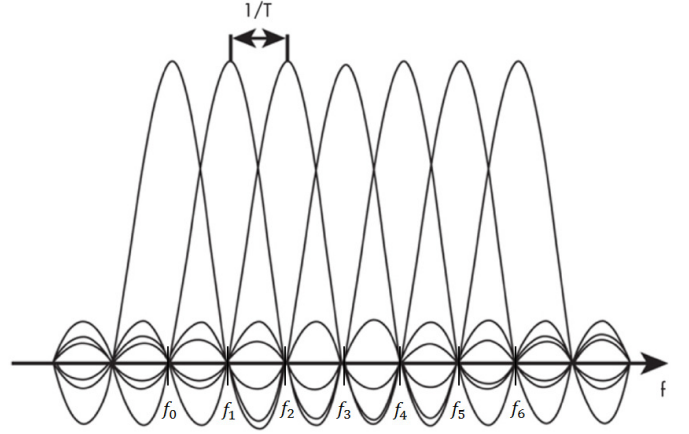Fig. 2. OFDM signal based on IEEE 802.11a specifications.

of the sum of N orthogonal signals which do not give much information about the behavior of the digital data, as seen in Fig. 2. In Fig. 2 we can not ensure that the subcarriers really satisfy (3), fact that could produce data interference resulting in probables information losses.

Alternatively, the OFDM analysis is done in the frequency domain where the measure of interference between data is given by the ICI power instead of the inter symbol interference (ISI). Graphically, and in its most simplified way, the spectrum of an OFDM signal is mainly the Fourier transform of $p(t)$, which we will write as $P(f)$, placed on each subcarrier frequency, as seen in Fig. 3. Hence, to achieve zero interference between subcarriers, $P(f)$ has to behave as a Nyquist-I pulse. This fact could be mathematically derived by using equations (1) and (3), leading to the following expression [11]

$$\int_{-\infty}^{\infty} p(t) e^{j2\pi(f_k - f_m)t} dt = \begin{cases} 1, & \text{if } k = m \\ 0, & \text{if } k \neq m, \end{cases} \quad (5)$$

which indicates that $P(f)$ should have spectral nulls at the frequencies $\pm 1/T, \pm 2/T, \dots$ to ensure subcarrier orthogonality. Formally this could be expressed as

$$P(f) = \begin{cases} 1, & \text{if } f = 0 \\ 0, & \text{if } f = \pm 1/T, \pm 2/T, \dots, \end{cases} \quad (6)$$

which is basically Nyquist's first criterion, but written in the frequency domain.

In (6), the peak power of $P(f)$ is associated with the main lobe, whereas the ICI power is associated with the side lobes, as will be seen later. Therefore, the main goal of the pulse-shaping function is to reduce the amplitude of the sidelobes.

A. Improved Parametric Linear Combination Pulse

The PLCP derived in [14] could also be used for minimizing ICI in OFDM-based systems. The frequency domain expression of the PLCP is given as follows

$$P_{PLCP}(f) = \mu P(f)_{PLP_{n=1}} + (1 - \mu) P(f)_{PLP_{n=2}}, \quad (7)$$

where n is a parameter that defines different parametric linear pulses (PLP) [15] with different decay rates. On the other hand, μ corresponds to the linear combination constant and it is defined for all real number. This constant adds an additional degree of freedom for a given roll-off factor α, which is
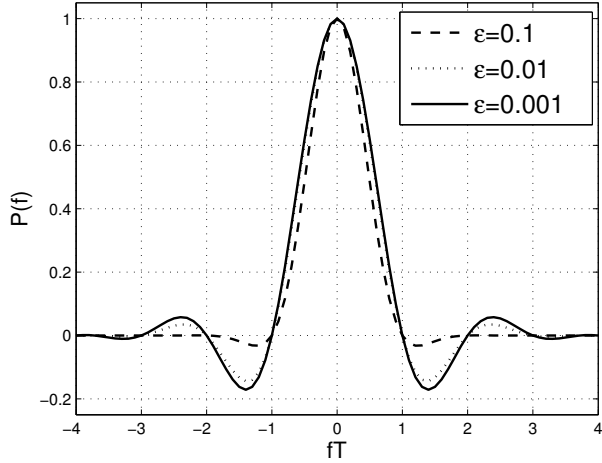
Fig. 4. IPLCP frequency function as ε varies for $\alpha = 0.22$, $\mu = 1.6$ and $\gamma = 1$.

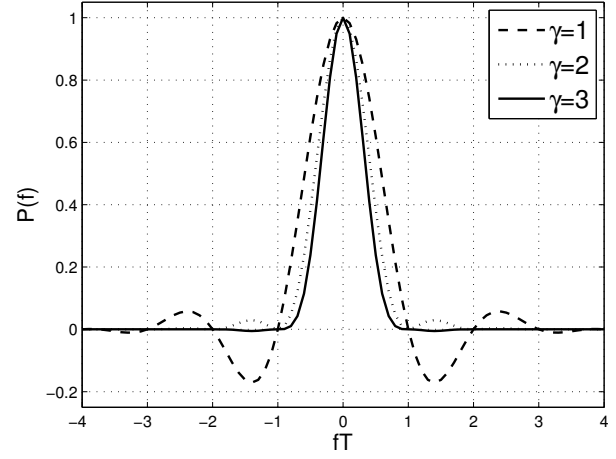Fig. 6. IPLCP frequency function as γ varies for $\alpha = 0.22$, $\mu = 1.6$ and $\varepsilon = 0.001$.

Fig. 5. IPLCP time function as ε varies for $\alpha = 0.22$, $\mu = 1.6$ and $\gamma = 1$.

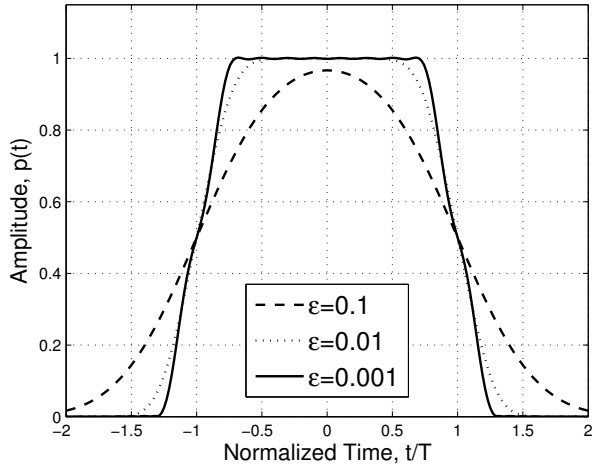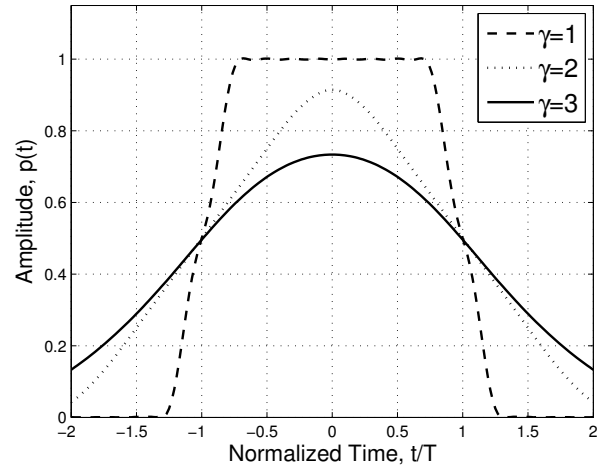Fig. 7. IPLCP time function as γ varies for $\alpha = 0.22$, $\mu = 1.6$ and $\varepsilon = 0.001$.

the only independent variable (degree of freedom) in pulses like the RC or the BTRC. The roll-off factor α is defined as $0 \leq \alpha \leq 1$. After some algebraic manipulations, the ICI-free PLCP is given as follows

$$P_{PLCP}(f) = \frac{\sin(\pi fT)}{\pi fT} \times \left[\frac{4(1-\mu)\sin^2(\pi\alpha fT/2)}{\pi^2\alpha^2(fT)^2} \right. $$
$$\left. + \frac{\pi\alpha\mu fT \sin(\pi\alpha fT)}{\pi^2\alpha^2(fT)^2} \right]. \quad (8)$$

The PLCP correspond to the base of the IPLCP; therefore, a completely new family of Nyquist-I pulses is defined as follows

$$P(f)_{IPLCP} = \exp(-\varepsilon\pi^2(fT)^2) \times P_{PLCP}(f)^\gamma, \quad (9)$$

where we can notice that the IPLCP is equivalent to the γ-th power of a PLCP multiplied by the exponential factor $\exp(-\varepsilon\pi^2(fT)^2)$. These last two operations add two extra degrees of freedom to the pulse-shaping function: γ and ε,

both defined for all real numbers. To prove that the IPLCP still meets (6), lets notice that (9) evaluated for $f = 0$ and for any value of μ, ε or γ, is always equal to one. Additionally, the proposed pulse, evaluated for $f = \pm 1/T, \pm 2/T, \ldots$, is always equal to zero. Therefore, the IPLCP fulfills Nyquist's first criterion.

To analyze the effect that the parameters ε and γ have on the behavior of the pulse-shaping function, lets fix the value of one of them and vary the other one. Throughout the manuscript we will normally use a roll-off factor, α, equal to 0.22. This is because the 3rd Generation Partnership Project (3GPP) has suggested the implementation of $\alpha = 0.22$ for the pulse-shaping filter at the transmitter and receiver sides [16], [17]. We have also fixed the value of the parameter μ equal to 1.6 because it was the value used in [14]. As seen in Fig. 4, if the value of ε increases, the sidelobes of the IPLCP frequency function are considerably reduced without significantly affecting the central lobe width. In Fig. 5 we can notice that in the time domain, as ε increases, the rectangular behavior is almost maintained; therefore, by increasing the
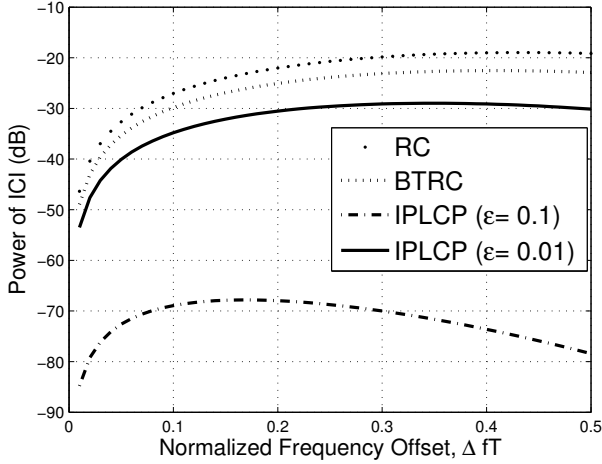
Fig. 8. Theoretical ICI Power for 64 subcarriers, $\alpha = 0.22$, $\mu = 1.6$, $\gamma = 1$ and different values of ε.

TABLE I.   SIMULATION PARAMETERS

Parameter	Value
Modulation	BPSK
Number of Subcarriers	64
Input Data Block Size	52
Transmission Bandwidth	20 MHz
Block Oversampling	10
Bit-to-noise Ratio	30 dB
Roll-off Factor	$\alpha = 0.22, 1.0$

value of ε, an important ICI reduction would be achieved without considerably affecting the time domain performance.

On the other hand, if one fixes the value of ε and increases γ, the sidelobes are rapidly reduced, but the central lobe width narrows as seen in Fig. 6. The fact of having a pulse with a narrow central lobe and almost no sidelobes (similar to a Dirac delta function) highly reduces the ICI power, but on the other hand increases the bit errror rate (BER) of the system because the synchronization between the oscillator of transmitter and the receiver needs to be almost perfect to recover the desired signal [13]. It can be seen in Fig. 7 that the time domain behavior is not preserved as γ increases, and the time domain function of the IPLCP ends up looking more like a triangle. Therefore, the pulse is not able to effectively limit the symbol duration period. Further, its amplitude is considerably reduced, attenuating the data symbols and increasing the BER.

IV.   ICI EVALUATION METRIC

Frequency offset, Δf, and phase error, θ, are introduced due to channel distorsion and desynchronization between the crystal oscillator of the transmitter and receiver in OFDM-based systems. The received signal is given by [11]

$$r(t) = e^{2\pi(\Delta f)t + \theta} \sum_{k=0}^{N-1} d_k p(t) e^{j2\pi f_k t}. \tag{10}$$

For the transmitted symbol d_m, the decision variable is given as [12]

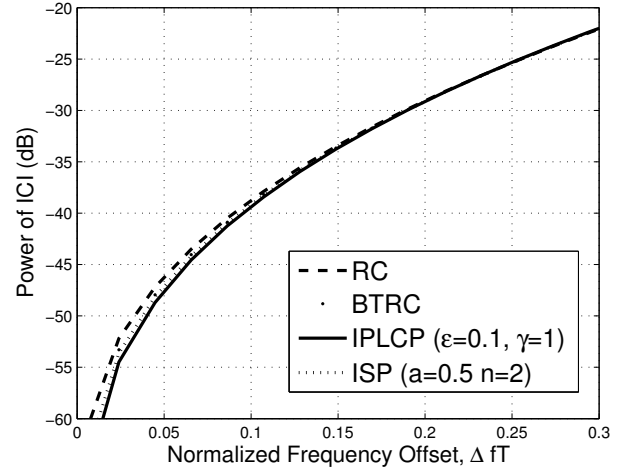$$\hat{d}_m = \int_{-\infty}^{+\infty} r(t) e^{-j2\pi f_m t} dt, \tag{11}$$

Fig. 9. Comparison of ICI power for different pulse-shaping functions in a 64-subcarrier OFDM system and $\alpha = 1$.

which could be decomposed into [13]

$$\hat{d}_m = d_m e^{j\theta} P(-\Delta f) + e^{j\theta} \sum_{\substack{k=0 \\ k \neq m}}^{N-1} d_k P\left(\frac{m-k}{T} - \Delta f\right). \tag{12}$$

The power of the desired signal can be calculated as

$$\sigma_m = |d_m|^2 |P(\Delta f)|^2, \tag{13}$$

and the ICI power is given as follows [18]

$$\sigma_{ICI}^m = \sum_{\substack{k=0 \\ k \neq m}}^{N-1} \sum_{\substack{n=0 \\ n \neq m}}^{N-1} d_k d_n^* P\left(\frac{k-m}{T} + \Delta f\right) P\left(\frac{n-m}{T} + \Delta f\right). \tag{14}$$

Considering (2) and (14), the average ICI power is finally given by

$$\overline{\sigma_{ICI}^m} = \sum_{\substack{k=0 \\ k \neq m}}^{N-1} \left| P\left(\frac{k-m}{T} + \Delta f\right) \right|^2, \tag{15}$$

where one can notice that the ICI power mainly depends on the value of the frequency offset Δf and the pulse-shaping function $P(f)$, which is the main reason of why is so important to choose the right pulse-shaping function for the OFDM-based system.

V.   PERFORMANCE EVALUATION

To evaluate the theoretical ICI performance of the IPLCP, we can choose a certain number of subcarriers and plot (15) as the frequency offset varies. In Fig. 8 we plot the ICI power of different pulse-shaping functions for 64 subcarriers. It is easy to notice that the IPLCP outperforms the RC and BTRC pulses in terms of ICI power, because it exhibits a considerably lower ICI for any frequency offset. The previous figure also demonstrates that increasing the value of ε improves even further the ICI performance of the IPLCP.
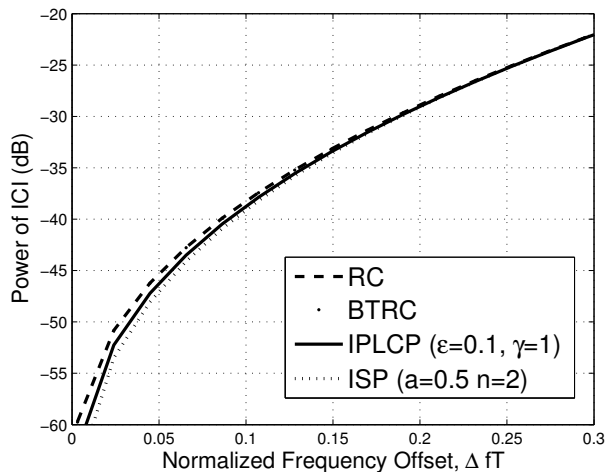
Fig. 10. Comparison of ICI power for different pulse-shaping functions in a 64-subcarrier OFDM system and $\alpha = 0.22$.
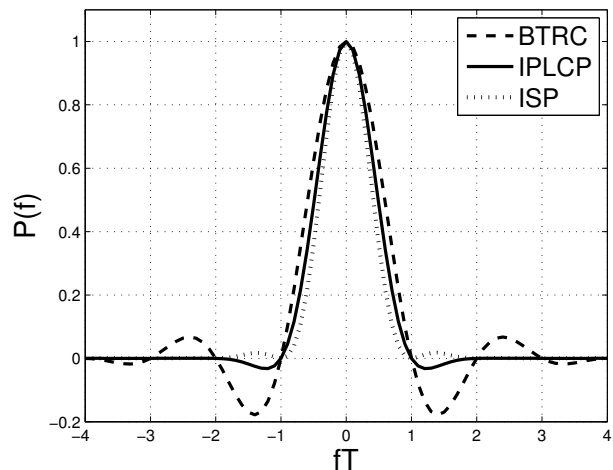

Fig. 11. Frequency behavior of BTRC for $\alpha = 0.22$, IPLCP with $\mu = 1.6$, $\varepsilon = 0.1$, $\gamma = 1$ and ISP with parameters $a = 0.5$, $n = 2$.

Next, we evaluate the performance of the IPCLP in a real OFDM-based system simulation scenario. Table I illustrates the parameters implemented in the simulation. To further validate our results, we simulated the system using two different roll-off factors, $\alpha = 0.22, 1.0$, to comply with the parameters used in [11]–[13]. Fig. 9 and 10 compare the ICI power for different pulse-shaping functions in a 64-subcarrier OFDM system plotted as functions of the normalized frequency offset, ΔfT, for $\alpha = 1.0$ and $\alpha = 0.22$, respectively. It can be seen that for $\alpha = 1.0$ the IPLCP outperforms the other pulses; whereas for $\alpha = 0.22$, the optimized ISP with parameters $a = 0.5$ and $n = 2$ [13] exhibits the best ICI performance, but it is closely followed by the IPLCP.

Even though the performance of the ISP is better than the performance of the IPLCP in terms of ICI power for $\alpha = 0.22$, the ISP pulse will cause some other problems to the OFDM-based system. Fig. 11 shows that both the ISP and IPLCP have almost no sidelobes, but the ISP has a narrower central lobe, which will probably cause a higher BER [13]. It can also be seen in Fig. 12 that the time-domain behavior of the ISP with parameters $a = 0.5$ and $n = 2$ is not able to limit the symbol duration period. Further, the function $p(t)_{ISP}$ will clearly attenuate each digital symbol, making it more complex to send information to the receiver, and increasing the BER.

The preliminary results that this work exhibits are quite promising, but the fact that the optimal values of ε and γ have not been calculated yet for a given μ and α, does not allow the full potential of the IPLCP to be shown. A complete BER analysis, and also the optimization of the novel IPCLP for ICI power reduction in OFDM-based systems will be considered as future work. Optimization techniques, as well as extensive computer simulations will be implemented to fully optimize the novel IPCLP pulse and verify that it outperforms other existing pulses not only in terms of the ICI power, but also in terms of the BER.

VI. CONCLUSION

In this paper, a novel family of Nyquist-I pulses has been derived by enhancing the PLCP. The new family of pulses is
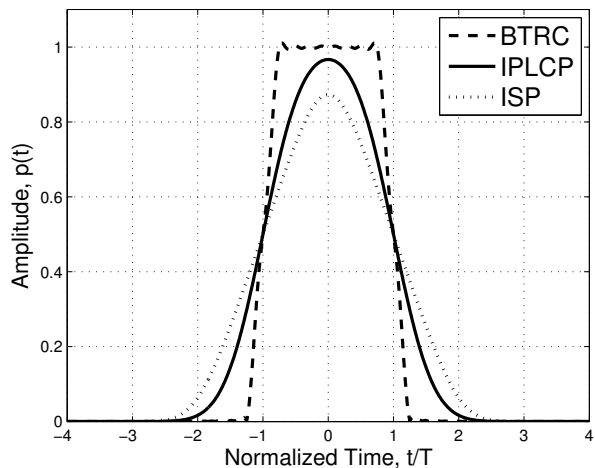

Fig. 12. Time behavior of BTRC for $\alpha = 0.22$, IPLCP with $\mu = 1.6$, $\varepsilon = 0.1$, $\gamma = 1$ and ISP with parameters $a = 0.5$, $n = 2$.

characterized by two new design parameters, ε and γ, adding two extra degrees of freedom to minimize the ICI power, for a certain μ and a given roll-off factor, α, in an OFDM-based system. Overall, it was found that the parameter ε is the one that has the biggest effect in reducing the ICI power. As the constant ε increases, the sidelobes of the IPLCP frequency function are considerably reduced without signicantly affecting the central lobe width. Theoretical and system simulations showed that there is a great potential in the IPLCP pulse for ICI power reduction in OFDM-based systems. As future work, an in-depth analysis will be done to determine the optimum pulses.

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