

# Is the baryon acoustic oscillation peak a cosmological standard ruler?

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## ABSTRACT

In the standard model of cosmology, the Universe is static in comoving coordinates; expansion occurs homogeneously and is represented by a global scale factor. The baryon acoustic oscillation (BAO) peak location is a statistical tracer that represents, in the standard model, a fixed comoving-length standard ruler. Recent gravitational collapse should modify the metric, rendering the effective scale factor, and thus the BAO standard ruler, spatially inhomogeneous. Using the Sloan Digital Sky Survey, we show to high significance ( $P < 0.001$ ) that the spatial compression of the BAO peak location increases as the spatial paths' overlap with superclusters increases. Detailed observational and theoretical calibration of this BAO peak location environment dependence will be needed when interpreting the next decade's cosmological surveys.

**Key words:** cosmological parameters – cosmology: observations – dark energy – distance scale – large-scale structure of Universe.

## 1 INTRODUCTION

The choice of a space–time coordinate system for the Universe that enables the expansion to be represented via a global scale factor  $a$  as a function of just one coordinate (cosmological time  $t$ ; Lemaître e.g. 1927) is extremely convenient. On large enough comoving-length scales, statistical spatial patterns are fixed in this coordinate system. Thus, the theory of primordial density fluctuations leads to that of baryon acoustic oscillations (BAOs; Eisenstein & Hu 1998). The BAO peak in the two-point spatial auto-correlation function  $\xi$  was clearly detected in the Sloan Digital Sky Survey (SDSS; Eisenstein et al. 2005) and the Two-Degree Field Galaxy Redshift Survey (Cole et al. 2005; earlier surveys may have detected this too; Einasto et al. 1997). BAOs now constitute one of the most important tools for making cosmological geometry measurements, especially for upcoming observational projects such as the space mission *Euclid* (Refregier et al. 2010) and the ground-based projects extended Baryon Oscillation Spectroscopic Survey (eBOSS; Zhao et al. 2015), Dark Energy Spectroscopic Instrument (DESI; Levi et al. 2013), 4-metre Multi-Object Spectroscopic Telescope (4MOST) (de Jong et al. 2012), and the Large Synoptic Survey Telescope (LSST) (Tyson

et al. 2003). The BAO peak location of about  $105 h^{-1}$  Mpc (where  $h$  is the Hubble constant  $H_0$  expressed in units of  $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ) is commonly expected to be a large enough comoving-length scale for it to provide a fixed comoving ruler in the real Universe.

However, the validity of the BAO peak location as a standard ruler depends on the validity of the assumed cosmological metric (differential rule for measuring lengths) in the context of the real Universe, which is lumpy (cf. the ‘fitting problem’; Ellis & Stoeger 1987). Scalar averaging is a general-relativistic formalism that extends beyond the standard cosmological model, by allowing a spatial section of the Universe at a given time to have an inhomogeneous metric and calculating background-free volume-weighted averages of scalar variables (Buchert 2001, 2008). The univariate scale factor  $a(t)$  is replaced by an effective, environment-dependent volume-based scale factor  $a_{\mathcal{D}}(t) \propto V_{\mathcal{D}}^{1/3}$ , dependent on both the choice of compact spatial domain  $\mathcal{D}$  of volume  $V_{\mathcal{D}}$  and on time. Without this extension, the cosmological, comoving metric is forced (by definition) to be rigid in comoving coordinates, i.e. inhomogeneities in the matter distribution are not allowed to ‘tell comoving space how to curve’ (e.g. Buchert & Carfora 2008). Applying scalar averaging to an observationally standard power spectrum that statistically represents density fluctuations at an early epoch implies that even for spatial domains as large as the BAO peak length scale, the environment dependence of the scale factor should be observationally detectable, i.e.  $a_{\mathcal{M}} < a_{\mathcal{E}}$  is expected, where  $\mathcal{M}$  (‘Massive’) and  $\mathcal{E}$  (‘Empty’) represent overdense and

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underdense spatial regions, respectively. For example, adopting  $1\sigma$  initially overdense ( $\mathcal{M}$ ) and underdense ( $\mathcal{E}$ ) fluctuations in a spherical domain of diameter  $\approx 105 h^{-1}$  Mpc and using equations (2), (13), (32), (50), and (54) of Buchert, Nayet & Wiegand (2013) to integrate the Raychaudhuri equation (9) of the same paper gives a relativistic Zel'dovich approximation estimate of

$$a_{\mathcal{M}}/a_{\mathcal{E}} \approx 0.91. \quad (1)$$

In other words, in the scalar-averaging approach, one way in which matter inhomogeneities are expected to affect the large-scale geometry and dynamics is to shrink the curved-space volume of overdensities on the BAO scale by about  $1 - (a_{\mathcal{M}}/a_{\mathcal{E}})^3 \approx 24$  per cent, leading to an expected shift in the BAO peak location to a lower scale by somewhat below or above 9 per cent for galaxy pairs that weakly or strongly, respectively, overlap with typical BAO-scale overdensities.

An environment-dependent effect has recently been detected as a six percent compression of the BAO peak location for spatial paths that touch or overlap superclusters of luminous red galaxies (LRGs) in the SDSS (Roukema et al. 2015; environment dependence of  $\xi$  at smaller scales has also been detected in the SDSS, Chiang et al. 2015).

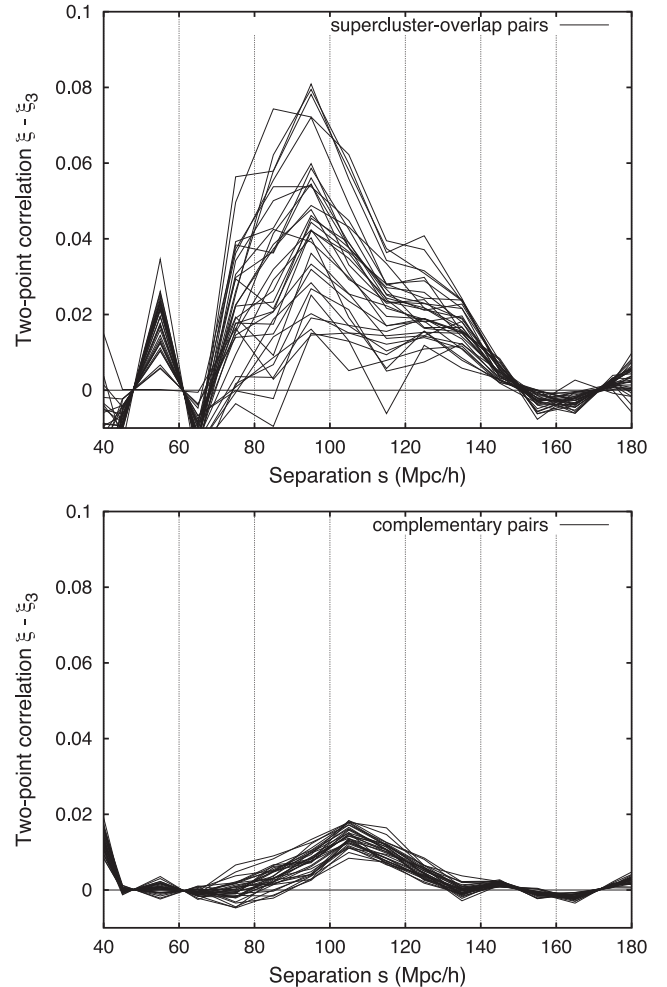
In this Letter, we check whether the compression is dependent on the minimum overlap between spatial paths and superclusters, as it should be if the effect is induced by the statistically overdense nature of the superclusters.

## 2 METHOD

We modify the previous method (Roukema et al. 2015, section 2) in order to allow stronger overlaps. As in the original method, we calculate the correlation function  $\xi$  of the ‘bright’ sample of LRGs in the SDSS Data Release 7 (DR7) for pairs of LRGs selected for overlap (Roukema et al. 2015, section 2.3, fig. 1) (or non-overlap) of superclusters in the survey (Nadathur & Hotchkiss 2014), using the Landy & Szalay estimator (Landy & Szalay 1993) on real and ‘random’ (artificial) catalogues (Kazin et al. 2010). Comoving separations  $s$  are calculated assuming the standard  $\Lambda$  cold dark matter ( $\Lambda$ CDM) model (Spergel et al. 2003; Planck Collaboration XVI 2014) with matter-density parameter  $\Omega_{m0} = 0.32$  and dark-energy parameter  $\Omega_{\Lambda 0} = 0.68$ . A best-fitting cubic  $\xi_3(s)$  over separations  $s \leq 70 h^{-1}$  Mpc and  $s \geq 140 h^{-1}$  Mpc (i.e. excluding the peak), is found for the tangential signal (pairs  $\leq 45^\circ$  from the sky plane). The procedure is repeated, bootstrap resampling the supercluster catalogue (Nadathur & Hotchkiss 2014) and the ‘random’ LRG catalogue, several times. The BAO peak location is estimated from the medians and median absolute deviations of the centres of the best-fitting Gaussians to  $\xi(s) - \xi_3(s)$ . In this work, minimum overlaps  $\omega_{\min}$  in the range  $10 h^{-1}$  Mpc  $\leq \omega_{\min} \leq 100 h^{-1}$  Mpc rather than  $\omega_{\min} = 1 h^{-1}$  Mpc are considered. In order that  $\xi$  be defined for  $s \leq \omega_{\min}$  for these high values of  $\omega_{\min}$ , we consider a pair of LRGs joined by a comoving spatial path entirely contained within a supercluster to satisfy the overlap criterion.

## 3 RESULTS

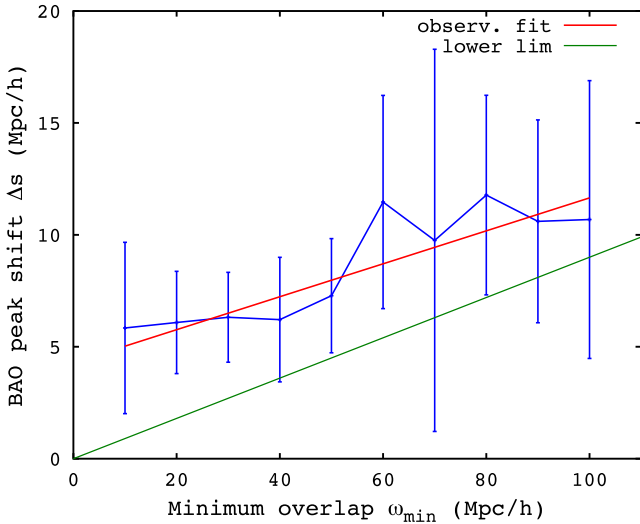
Fig. 1 shows that for a minimum overlap  $\omega_{\min} = 60 h^{-1}$  Mpc, the BAO peak is shifted to lower separations  $s$  (panel a) than for the complementary set of LRG pairs (panel b). The shift is clearer than for the earlier analysis, which had  $\omega_{\min} = 1 h^{-1}$  Mpc (Roukema et al. 2015, fig. 8). Requiring a stronger overlap yields a stronger shift.



**Figure 1.** Compression of the BAO peak. Upper panel: BAO peak (cubic-subtracted correlation function) for pairs of LRGs whose paths either overlap with superclusters by  $\omega \geq \omega_{\min} = 60 h^{-1}$  Mpc or are entirely contained within the superclusters. The overlap  $\omega$  is the chord length defined (Roukema et al. 2015, section 2.3, fig. 1) by the intersection of the path joining two LRGs and the supercluster modelled as a sphere. Individual curves represent 32 bootstrap resamplings of the supercluster catalogue (235 objects) and the ‘random’ galaxies (484 352 selected from 1521 736). The real galaxies (30 272) are not resampled. Most of the curves peak sharply at  $95 h^{-1}$  Mpc; a few peak at  $85 h^{-1}$  Mpc. The high amplitude (in comparison with the lower panel) is consistent with biasing that modifies the amplitude of  $\xi$ . Lower panel: BAO peak for the complementary subset of galaxy pairs. The peak occurs at the standard value of about  $105 h^{-1}$  Mpc.

Fig. 2 shows the dependence of the shift  $\Delta s$  on  $\omega_{\min}$ . The BAO peak shift for supercluster-overlapping LRG pairs appears to increase from  $\Delta s \approx 6\text{--}7 h^{-1}$  Mpc for  $\omega_{\min} \lesssim 50 h^{-1}$  Mpc to  $\Delta s \approx 11 h^{-1}$  Mpc for greater overlaps. The Pearson product-moment correlation coefficient of  $\Delta s$  and  $\omega_{\min}$  is 0.87, with a probability of  $P \approx 0.0008$ , i.e. a positive correlation is detected to high significance. Thus, the environment dependence of the BAO peak shift is confirmed. Unfortunately, the uncertainties shown in Fig. 2 are too high to infer the details of this correlation from the present data and analysis. The slope and zero-point of the linear best fit are  $0.073 \pm 0.040$  and  $4.3 \pm 2.0 h^{-1}$  Mpc, respectively.

As a rough guide to what is expected from scalar averaging, we can use the 9 per cent shift estimate from equation (1), which would



**Figure 2.** Overlap dependence of the BAO peak shift  $\Delta s := s_{\text{non-sc}} - s_{\text{sc}}$ , where  $s_{\text{sc}}$  and  $s_{\text{non-sc}}$  are the median estimates of the centres of the best-fitting Gaussians to  $\xi - \xi_3$  for LRG pairs that overlap superclusters (sc) and those that do not (non-sc), respectively. The error bars show a robust estimate of the standard deviation,  $\sigma(\Delta s)$ , defined here as 1.4826 times the median absolute deviation of  $\Delta s$ . At each  $\omega_{\text{min}}$ , these statistics are calculated over 32 bootstrap resamplings of the observational data. Four out of the 320 Gaussian fits in the supercluster-overlap case failed and were ignored in calculating these statistics; no failures occurred for the 320 non-supercluster-overlap cases. A linear least-squares best-fitting relation  $\Delta s = 4.3 h^{-1} \text{ Mpc} + 0.07 \omega_{\text{min}}$  is shown with a red line. The most significant individual rejection of a zero shift is  $\Delta s = (6.3 \pm 2.0) h^{-1} \text{ Mpc}$  for  $\omega_{\text{min}} = 30 h^{-1} \text{ Mpc}$ , a  $3.1\sigma$  (Gaussian) rejection. Since bootstraps are used, this estimate is conservative:  $\sigma(\Delta s)$  is expected to be an overestimate of the true uncertainty (Fisher et al. 1994, section 2.2). A 9 per cent shift [equation (1)] would give  $\Delta s = 0.09\omega$ . Since  $\omega \geq \omega_{\text{min}}$ , a scalar-averaging lower expected limit  $\Delta s > 0.09\omega_{\text{min}}$  is shown as a green line.

give  $\Delta s = 0.09\omega$ , where the overlap path lengths are approximated as corresponding to  $1\sigma$  overdense regions on the BAO scale, even though in reality, the overdense regions are superclusters, some smaller and some larger than this scale. Since  $\omega \geq \omega_{\text{min}}$ , this implies a rough scalar-averaging lower expected limit of  $\Delta s > 0.09\omega_{\text{min}}$ , shown as a green line in Fig. 2.

## 4 DISCUSSION

Since the BAO peak location serves as a major tool for cosmological geometry measurements, it is clear that its environment dependence will need to be observationally calibrated and correctly modelled theoretically. It is possible that the effect could also be interpreted within the standard  $\Lambda$ CDM model, as is the case for many large-scale phenomena. For example, observed supervoids on the 200–300  $h^{-1} \text{ Mpc}$  scale (Nadathur & Hotchkiss 2014; Szapudi et al. 2015) can be interpreted within the  $\Lambda$ CDM model (Hotchkiss et al. 2015), although their occurrence is expected to be rare (Szapudi et al. 2015). In contrast, the study of SDSS DR7 ‘dim’ (or ‘bright’) LRGs via Minkowski functionals on scales ranging up to the BAO peak scale, within a 500  $h^{-1} \text{ Mpc}$  (or 700  $h^{-1} \text{ Mpc}$ , respectively) diameter region, shows  $3\sigma$ – $5.5\sigma$  (or  $0.5\sigma$ – $2.5\sigma$ ) inconsistencies with  $\Lambda$ CDM simulations (Wiegand, Buchert & Ostermann 2014, table 1). Minkowski functionals have more statistical power than lower order statistics that are commonly used in analysis of large-scale structure,

such as the two- and three-point correlation functions, the correlation dimension or percolation (‘friends-of-friends’) analyses. This is because all the  $n$ -point correlation functions would be needed in order to represent the statistical geometrical information that the Minkowski functionals contain.

A possible avenue to studying the environment-dependent BAO shift within the standard approach would be to carry out a Fourier analysis rather than using the two-point correlation function. This would require the development of a supercluster-overlap-dependent Fourier analysis method. Another alternative, to avoid having to determine the position of the peak itself, would be comparison of radial to tangential correlation functions directly. This would require correcting for peculiar velocity effects, which are highly anisotropic with respect to the observer.

Interpreting the environment dependence of the BAO peak location reported in this Letter within the standard  $\Lambda$ CDM model would require the comoving-length scale at which the Universe is rigid in comoving coordinates to be pushed up to a scale greater than 105  $h^{-1} \text{ Mpc}$ . The environment dependence (e.g. the  $\Delta s(\omega_{\text{min}})$  relation) would have to be modelled within a rigid comoving background that can only exist at larger scales, at which no sharp statistical feature that can function as a standard ruler is presently known. This leads to a Mach’s principle type of concern that at recent epochs, it is difficult to have confidence that the standard comoving coordinate system is correctly attached to an observational extragalactic catalogue (peculiar velocity flow analyses indicate similar concerns; Wiltshire et al. 2013). Interpretation within the scalar-averaging approach should be easier because its description of fluctuation properties and the cosmological expansion rate is environment dependent.

Nevertheless, within the rigid comoving background framework (i.e. the standard model), small shifts in the BAO peak location have been predicted analytically and from  $N$ -body simulations (Desjacques et al. 2010; Sherwin & Zaldarriaga 2012), while BAO reconstruction techniques (Padmanabhan & White 2009; Padmanabhan et al. 2012; Schmittfull et al. 2015) have been developed to attempt to evolve galaxies’ positions backwards in cosmological time, using a blend of theoretical calculations and  $N$ -body models. The expected mean shift in the BAO peak location is less than one percent, i.e. an order of magnitude less than what we find for the shift conditioned on  $\omega_{\text{min}} \geq 60 h^{-1} \text{ Mpc}$ . The amplitude of the shifts found in these calculations is constrained by the assumption that curvature averages out on the assumed background, i.e. that a conservation law for intrinsic curvature holds globally (Buchert & Carfora 2008).

Theoretical work is underway in the scalar-averaging approach, which general relativistically extends the standard model, allowing the restrictive assumption of a conservation law for intrinsic curvature to be dropped (Buchert & Carfora 2008). The physical origin of curvature deviations from the background on scales as large as the BAO scale can then be thought of as following from the non-existence of a conservation law for intrinsic curvature. To reconstruct the primordial comoving galaxy positions more accurately than in the standard model, i.e. to allow flexible comoving curvature that varies with the matter density and the extrinsic curvature tensor across a spatial slice, relativistic Lagrangian perturbation theory (Buchert & Ostermann 2012; Buchert et al. 2013; Alles et al. 2015) is available for analytically guided calculations.  $N$ -body simulations in which the growth of inhomogeneities is matched by inhomogeneous metric evolution will most likely also be needed to develop numerical confidence in what could be called ‘relativistic BAO reconstruction’.

## 5 CONCLUSION

No matter which approach is chosen, analytical, numerical, and observational work will be required if the BAO peak location is to correctly function as a standard ruler for cosmological geometrical measurements, since the evidence is strong ( $P < 0.001$ ) that it is strongly affected by structure formation. Moreover, the formation of superclusters – in reality, filamentary, and spider-like distributions of galaxies (Einasto et al. 2014) rather than the spherically symmetric objects assumed here for calculational speed – can now be tied directly to a sharp statistical feature of the primordial pattern of density perturbations.

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