International Journal of Mineral Processing 148 (2016) 59–64

Contents lists available at ScienceDirect

International Journal of Mineral Processing

journal homepage: www.elsevier.com/locate/ijminpro

The least energy and water cost condition for turbulent, homogeneous pipeline slurry transport

Christian F. Ihle

Department of Mining Engineering, Universidad de Chile. Tupper 2069, 8370451 Santiago, Chile
Advanced Mining Technology Center, Universidad de Chile, Tupper 2007, Santiago, Chile

ARTICLE INFO

Article history:
Received 2 January 2015
Received in revised form 5 December 2015
Accepted 20 January 2016
Available online 26 January 2016

Keywords:
Ore concentrate
Energy efficiency
Slurry pipelines
Long distance pipelines
Cost function

ABSTRACT

The efficient combined use of water and energy in long distance slurry pipelines is analyzed in light of the total cost function resulting from an energy and mass balance. Given a system throughput and a set of common slurry and flow properties associated with long distance cross country pipelines such as Krieger-type rheology and smooth wall turbulent flow with small yield-to-wall stress, the minimum cost condition is obtained at the minimum feasible transport mean velocity. This condition has been found irrespective of the particular value of the dissipation-to-pump station location difference, provided the required pumping power is positive.

Contents

1. Introduction

Long distance pipelines are often a cost-effective means of transporting a wide variety of ores and tailings between remote locations (Jacobs, 1991; Abulnaga, 2002). Considering that long distance slurry transport requires massive amounts of energy, the concept of energy efficiency in this kind of infrastructure (Wilson et al., 2006) is highly relevant. Motivated by the additional and recurrent constraint relating mining operations with water scarcity, the well-known problem of energy efficiency has been recast into a problem of minimum cost (Ihle, 2013; Ihle et al., 2013), which includes the cost of water. This new approach opens the question on whether the conditions for energy efficiency (Nguyen and Boger, 1998; Sofrá and Boger, 2002; Ihle and Tamburrino, 2012) are the same of those minimizing the sum of energy and water cost. The finding from the solution of a large-scale optimization problem

E-mail address: cihle@ing.uchile.cl.

http://dx.doi.org/10.1016/j.minpro.2016.01.011

that the minimum transport costs are apparently related to the minimum velocity (Ihle, 2013, Table 1), suggests to analyze in more detail the role of such condition. Although it has been previously suggested that the optimum concentration of solids increases with the throughput (Nguyen and Boger, 1998; Sofrá and Boger, 2002), and decreases with the velocity (Nguyen and Boger, 1998; Sofrá and Boger, 2002; Wu et al., 2010; Edelin et al., 2015), such experimental observations do not give a weight to the use of water. However, if water is considered, it has been shown that the optimal concentration is indeed an increasing function of the throughput (Ihle et al., 2014a). In this paper, a dimensionless formulation of the cost function is used to analyze the various possibilities of the cost and flow characteristics. Considering flow conditions and characteristics typical of fine-graded slurries such as bauxite, copper and iron concentrates, it is shown that it is precisely the minimum feasible velocity that which minimizes the combined water and energy cost. The result has been extended to cross country pipelines featuring an energy dissipation point or an elevated discharge.

2. Problem description

Consider an operating slurry transport system of length L with known internal diameter (D) where the throughput (G), defined as the dry solid rate, along with route and slurry properties are known. The total energy and water cost per unit time may be expressed as:

$$\Omega = c_e P + c_w Q_w, \tag{1}$$

where c_e and c_w represent the unit costs of energy and water, respectively. The variables P and Q_w are the pumping power and the water flow required to allow for the dispatch of the solids, respectively.

Assuming a final atmospheric discharge and neglecting minor singular pressure losses (except those intended to avoid column separation, consisting of the appearance of a gas phase in the slurry column, as explained below), the required pumping power is calculated using the energy conservation and the Darcy–Weisbach equation as:

$$P = \left(H_d - z_0 + \frac{8}{\pi^2}\frac{fLQ^2}{gD^5}\right)\frac{\rho g Q}{e}, \tag{2}$$

where f is the Darcy friction factor, $f = \pi^2 D^4 \tau_{\mathrm{wall}}/2\rho Q^2$, with Q and ρ the volume flow and density of the solid–liquid mixture. Here, D is the pipe internal diameter, g is the magnitude of the gravity acceleration vector, τ_{wall} the pipe wall shear stress and e is the overall efficiency of the pumping system. Here, z_0 is the altitude of the pump station (referred to a datum) and H_d is total singular energy consumption head required to ensure that at every point of the route the line pressure would exceed the vapor value to avoid column separation and the potentially harmful consequences of related flow transients (Bergant et al., 2006). The head loss H_d may be alternatively expressed in terms of a dimensionless coefficient (k_d) as $H_d = \frac{8k_d Q^2}{g\pi^2 D^4}$. A schematic of the importance of H_d is shown in Fig. 1, where the lines represent the hydraulic head, $E = p/\rho g + z_t + v^2/2g$. Here, z_t, p and v are the topographic altitude, line pressure and mean flow velocity, respectively. In long distance pipeline systems, commonly $v^2/2g$ is negligible in front of $p/\rho g$.

Fig. 1a and b shows a schematic of the hydraulic head line that minimizes the energy and water cost for the cases of flat or very low topography. In both cases the energy line remains unchanged. However, if the topography is high enough the former energy line might cross it. As the absolute pressure must be greater than the vapor value, the pipeline extension that crosses the topography under this operational scenario must then be at the vapor pressure. This is denoted by the dashed line in Fig. 1c. To ensure that the whole energy line will not cross the topography in a fixed diameter pipeline, there are two possible mechanisms. One is to increase the volume flow, which effectively increases the

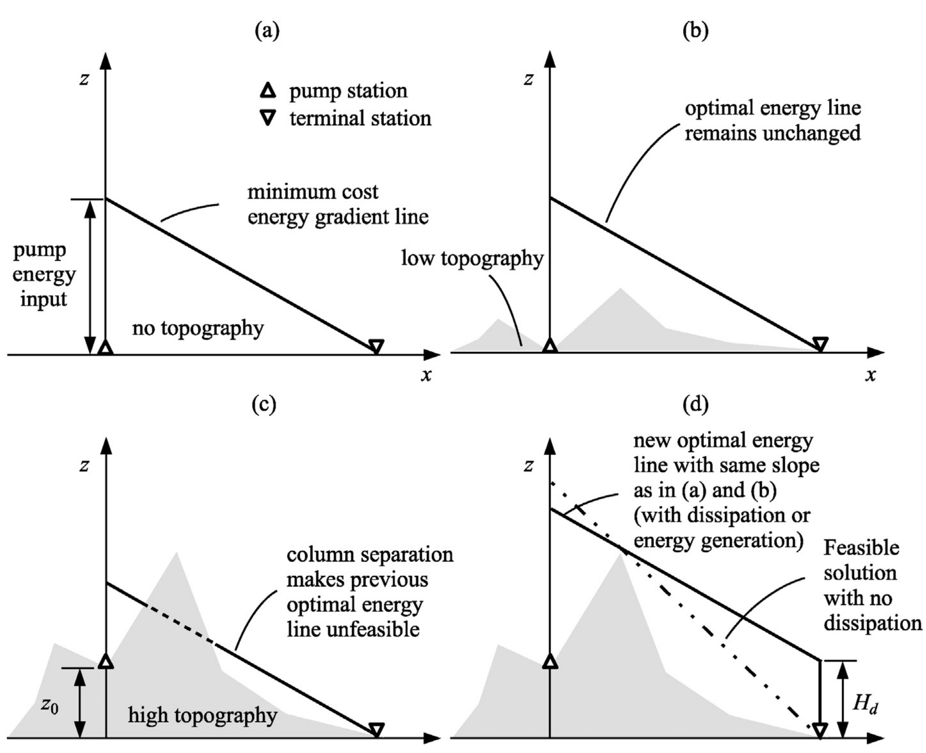

Fig. 1. Schematic of the hydraulic head, E, denoted by lines, in terms of the tubelength distance, x. The shaded area represents the topography. (a) Flat topography. (b) Topography with no influence on the optimal condition. (c) Topography with influence on the optimal energy line. (d) A correction to the optimal condition in (c), incorporating a singular head loss, H_d.

energy line slope, equal to $\frac{8fQ^2}{g\pi^2 D^5}$, given that models for the friction factor f in laminar and fully developed turbulent flow verify that $\partial(fQ^2)/\partial Q > 0$ (Darby, 2001; Abulnaga, 2002). A schematic of the resulting hydraulic head is shown in the dashed-dotted line of Fig. 1d. The second mechanism to raise the energy line is to keep the slope (*i.e.* the volume flow and the solid volume fraction) and introduce a singular pressure loss, either using energy dissipation devices or an inline turbine, to shift the energy line upwards. This situation, depicted in the solid line of Fig. 1d may be associated with a lower pump energy consumption, compared to the increased flow option, as seen from the crossing point of the $x = 0$ line. The graphical construction of Fig. 1 suggests that, under some conditions, the best energy gradient line may be determined independently of the topography or, equivalently, the particular value of $H_d - z_0$, as confirmed under some assumptions below. It is also noted that when energy dissipation is necessary, z_0 can be lesser or greater than H_d. Pumping, in lieu of a purely gravitational transport, is required only when $P \geq 0$.

The density ρ depends on the concentration as $\rho(\phi) = \rho_w \sigma$, with $\sigma = \phi(S-1)+1$. From the mass balance in the flow,

$$G = S\rho_w \phi Q, \tag{3}$$

where G is the (dry) solid throughput, S is the specific gravity of solids, ρ_w the liquid phase density and ϕ the solid volume fraction in the flow. On the other hand, the process objective of the concentrate line is to deliver a fixed amount of solids per unit time. That is, the throughput G is a process parameter that effectively acts as a restriction, *via* Eq. (3), for both the flow and the solid concentration. The mass balance also implies that

$$Q_w = (1-\phi)Q. \tag{4}$$

Besides the turbulent flow regime requirement, concentrate pipelines are commonly designed to operate above the deposit velocity, corresponding to the lower threshold below which there is sediment formation in the pipe section (Durand and Condolios, 1952; Abulnaga, 2002). This requisite configures a common restriction in long distance ore pipelines, in terms of the minimum attainable slurry volume flow, as:

$$Q > Q_{\mathrm{mf}}, \tag{5}$$

with $Q_{\mathrm{mf}} = \max\{Q_{\mathrm{dv}}, Q_{\mathrm{lt}}\}$, where Q_{dv} and Q_{lt} are the volume flows corresponding to the deposit formation threshold and the minimum value to ensure turbulent transport, respectively. The inequality corresponding to expression (5) is an operational restriction of the optimization problem given by Eq. (1). The deposit velocity may be expressed as a function of the solid volume fraction *via* the bulk density ρ and the mixture viscosity (Gillies et al., 2000; Poloski et al., 2010), and is commonly a decreasing function of the volume fraction (Ihle and Tamburrino, 2012). Conversely, the laminar-turbulent transition depends on a critical Reynolds number. At constant critical Reynolds number, an increase in concentration causes a greater increase in viscosity than in density—this will be evident in the next section—,and thus will tend to increase the mean flow velocity corresponding to the laminar-turbulent transition. If the flow has not significant cross-sectional variations of the solid fraction, for small enough concentrations the deposit mechanism tends to dominate over the laminar-turbulent one, whereas for mid- to high concentrations, associated with a high water cost regime, the laminar-turbulent transition controls the minimum flow (Ihle and Tamburrino, 2012; Ihle et al., 2014b).

3. Hypotheses

A smooth wall, fully developed turbulent flow regime with a mean flow velocity deposit limit is considered. This assumption gives a good description of a number of pipeline concentrate flows and, in particular, those copper and iron concentrates that exhibit low yield stresses, given their typical Reynolds number ranges, between 10^4 and 10^5 (see Abulnaga, 2002, for a comprehensive list of related slurries). In these cases, the very high wall shear stresses occurring during turbulent concentrate pipeline flows (τ_w) imply that $\xi = \tau_y/\tau_{\mathrm{wall}} << 1$ (Ihle and Tamburrino, 2012; Ihle et al., 2014b), and therefore the pressure losses are weakly dependent on the yield stress. This observation is not to be confused with what happens in strongly heterogeneous or bed forming high concentration coarse particle flows (Doron and Barnea, 1993; Matoušek, 2002; Pullum et al., 2006), where the manifest segregation in the vertical makes the present hypothesis inapplicable.

According to the smooth wall turbulent friction model by Chilton and Stainsby (1998), $f = f_N/(1-\xi) \approx f_N$, where f_N is the corresponding Darcy friction factor for a viscous Newtonian fluid, in this case the Blasius empirical model. It is possible to obtain a similar conclusion out of the model proposed by Thomas and Wilson (1987) (also, Wilson and Thomas, 1985), based on the logarithmic velocity profile.

In the present problem, the solid–liquid mixtures are assumed as non-cohesive, and thus the viscosity of such suspensions (μ) may be reasonably described following a Krieger-type model $\mu/\mu_w = \eta$, with μ_w the viscosity of the liquid phase, and the dimensionless function η given by:

$$\eta = \left(1 - \frac{\phi}{\phi_m}\right)^{-\beta_\eta}, \tag{6}$$

with $\beta_\eta > 0$ and ϕ_m the maximum attainable concentration of the flowing mixture (Krieger and Dougherty, 1959). This expression predicts a solid-like behavior in the limit $\phi \to \phi_m$, *i.e.*, $\eta \to \infty$ in this case. This type of viscosity model thus yields a growth rate higher than that of an exponential function of η. The exponent $\beta_\eta = 2$ is valid for either spheres or elongated particles (Maron and Pierce, 1956; Quemada, 1977; Ovarlez et al., 2006; Mueller et al., 2010). The existence of particle size distributions is accounted for *via* the maximum packing fraction ϕ_m (Stickel and Powell, 2005). This has been confirmed with viscosity measurements of fine comminution slurries at moderate volume fractions (a data compilation is given in the Appendix of Ihle, 2013). The latter parameter has also been related by Mueller et al. (2010) to the particle shape with the empirical relation for the particle size ratio (r_p), $\phi_m = 2/(0.321 r_p + 3.02)$. Comprehensive reviews for the viscosity determination in the infinite Péclét number limit, where hydrodynamic forces are dominant over Brownian ones as in the present case, are given in Quemada (1977), Stickel and Powell (2005) and, more recently, Mueller et al. (2010).

4. A minimum cost condition

4.1. Dimensionless cost formulation

The cost function, given by Eq. (1), may be rendered dimensionless using the scale $\Omega = (\pi/4)D\mu_w c_w/\rho_w$. On the other hand, the expression for the energy consumption (Eq. (2)), which depends on the slurry volume flow (Q), may be expressed in terms of the (fixed) system throughput, G, the solid volume fraction, ϕ, and the solid density ($S\rho_w$) using Eq. (3) as $Q = G/S\rho_w\phi$. Additionally, the water volume flow (Q_w) may be expressed in terms of ϕ and Q—and hence $G/S\rho_w\phi$—using the water mass balance relation (Eq. (4)). As a result, the following dimensionless expression is obtained:

$$\frac{\omega}{\omega_0 R_0{}^3} = \frac{\sigma}{\phi^3} f + \frac{1-\phi+\sigma\omega_0\mathscr{H}}{\phi\omega_0 R_0{}^2}, \tag{7}$$

where:

$$\omega_0 = \frac{1}{2}\frac{\mu_w{}^2 L c_e}{\rho_w D^3 e c_w}; \quad R_0 = \frac{4}{\pi}\frac{G}{SD\mu_w}, \text{and} \quad \mathscr{H} = \frac{2g(H_d - z_0)\rho_w{}^2 D^3}{\mu_w{}^2 L}. \tag{8}$$

In hydraulically smooth pipes the friction factor f is typically expressed as a sole function of the slurry Reynolds number, which may be expressed as $Re = (4/\pi)(\sigma/\phi\eta)(\rho_w Q/\mu_w D)$ or, equivalently, $Re = (\sigma/\phi\eta)R_0$. The latter relation also holds as an equation for ϕ when $Re = Re_c$, with Re_c the laminar-turbulent transitional Reynolds number (see Appendix A for an analytical derivation of a particular solution corresponding to the case when $\beta_\eta = 2$ in Eq. (6). So far, it is noted that the cost formulation remains independent of the particular form of η, except for the term bearing the friction factor.

From Eq. (3), the slurry flow is inversely proportional to the solid volume fraction and, in particular, it becomes singular in the no-solid condition, with $\omega \to \infty$ as $\phi \to 0$. On the opposite side, noting that if $f \approx \alpha_f / Re^{\beta_f}$ in hydraulically smooth turbulent flow, with $\alpha_f > 0$ and $0 < \beta_f < 1$ (Chilton and Stainsby, 1998; Darby, 2001; Ihle et al., 2014b), then $\omega/\omega_0 > \alpha_f(\eta/\sigma)^{\beta_f} R_0{}^{3-\beta_f} + \mathscr{H}R_0$. Using Eq. (6), recalling that η grows unboundedly and $\sigma \to \phi_m(S-1)+1$ as $\phi \to \phi_m$, then $\omega \to \infty$ as $\phi \to \phi_m$. The cost function is continuous everywhere except at $\phi = 0$ and ϕ_m and thus, if f and η admit the definitions given here, there is a non-trivial optimal concentration that minimize the cost, for fixed values of the fluid and solid properties, the energy and water unit costs. The same conclusion also holds if η is bounded for $0 \le \phi \le 1$ and, in general, for any frictional law such as $\partial f/\partial \phi > 0$.

For a relatively broad set of system parameters associated with the transport of concentrates obtained from comminution processes, the minimum flow condition described in Eq. (5) also sets a minimum cost condition. The mass conservation statement corresponding to Eq. (3) implies that given the throughput, valid transport concentrations must verify $\phi < \phi_{mf}$, where $\phi_{mf} = \phi(Q = Q_{mf})$, *i.e.*, the volume fraction corresponding to the minimum flow condition. Thus, if $d\omega/d\phi \le 0$ for $0 < \phi \le \phi_{mf}$ then $\min_\phi \Omega = \Omega(\phi_{mf})$ or, equivalently, $\min_Q \Omega = \Omega(Q_{mf})$. From Eq. (7) it is seen that this result depends on the problem input parameters ω_0, R_0 and $\mathscr{H}$, where the three of them are independent of the volume fraction.

The optimal cost depends also on the friction factor, a function of R_0, σ and η, and on the particular choice of the relation to obtain the friction factor and η.

4.2. Base case: $z_0 = H_d = 0$

Before analyzing the role of $\mathscr{H}$, the simpler case of a flat topography with no energy dissipation is considered. Differentiating Eq. (7) with respect to ϕ reads:

$$\frac{1}{\omega_0 R_0{}^3}\frac{d\omega}{d\phi} = \frac{f}{\phi^4}\left(-1 - 2\sigma + \frac{\phi\sigma}{f}\frac{df}{d\phi}\right) - \frac{1}{\phi^2 \omega_0 R_0{}^2}. \tag{9}$$

The dimensionless cost function becomes a decreasing function of the solid volume fraction and the concavity if its derivative with respect to ϕ is negative or zero. This is equivalent to the following condition for the friction factor f:

$$\frac{1}{f}\frac{df}{d\phi} \le \frac{1+2\sigma}{\phi\sigma}. \tag{10}$$

If so, it is straightforward to obtain that $d^2\omega/d\phi^2 \ge 0$ and to conclude that, in this case, $\phi = \phi_{mf}$ minimizes the cost function (1) if R_0 and ω_0 are positive.

The fitness of the condition given by the inequality (10) to real long distance concentrate pipelines is yet to be verified. In particular, assuming here the Blasius-like friction law referred to above, and η as given in Eq. (6), then the upper bound condition given by the inequality (10) becomes:

$$\beta_\eta \le F(\phi | S, \phi_m, \beta_f), \tag{11}$$

with

$$F(\phi | S, \phi_m, \beta_f) = \frac{3 - \beta_f + 2\phi(S-1)}{\beta_f[\phi(S-1)+1]}\left(\frac{\phi_m}{\phi} - 1\right). \tag{12}$$

The function F is a monotonically decreasing function of the solid fraction and is undefined in $\phi = 0$. On the other hand, its lowest possible value is given by the least upper bound, $\sup_\phi \beta_\eta = F(\phi_{mf})$. Some numeric examples are given in Fig. 2, where each group of curves corresponds to a specific value of ϕ_m. It is seen that for a wide variety of values of S and

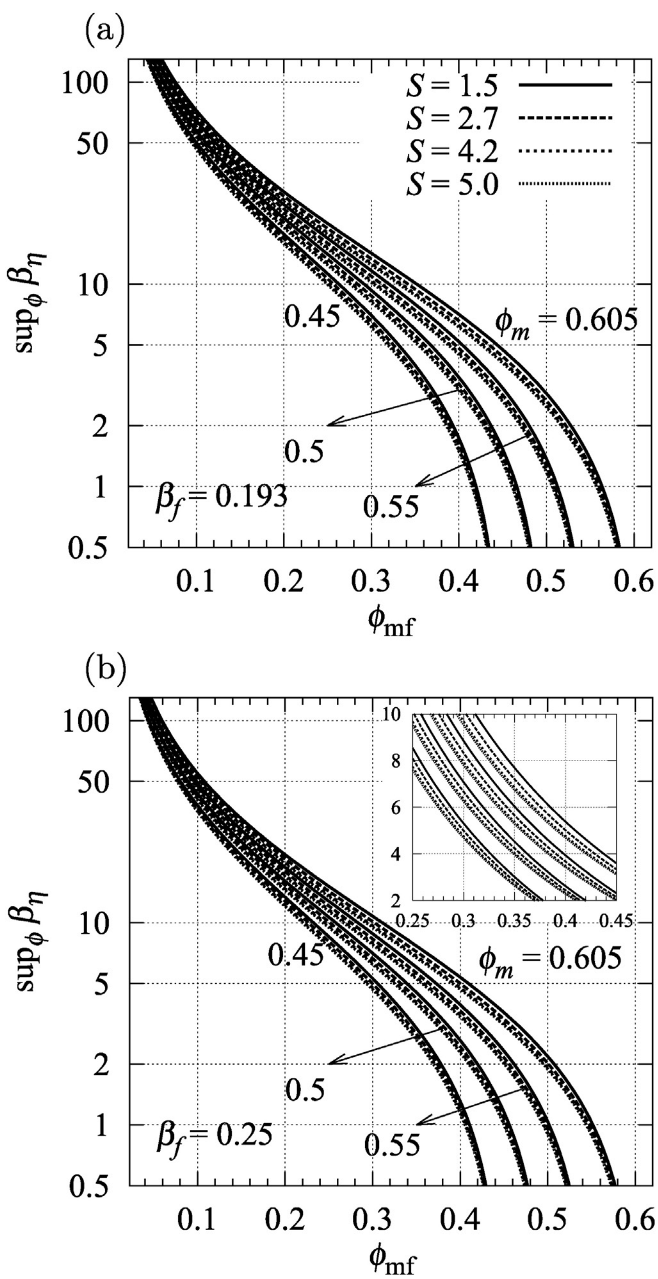

Fig. 2. Numerical values of $F(\phi_{mf}) = \sup_\phi \beta_\eta$, with $F(\phi)$ given by Eq. (12). (a) $\beta_f = 0.193$ (Darby, 2001) (additional comments in Ihle et al. (2014b)). (b) $\beta_f = 0.25$ (Chilton and Stainsby, 1998). In (b), the inset shows a detail of F for ϕ_{mf} between 0.25 and 0.45.

ϕ_m, the exponent $\beta_\eta = 2$ (Ovarlez et al., 2006; Ihle, 2013) is reached with very high solid fractions. This means that for these cases a reasonable viscosity model is compatible with optimal energy and water utilization rates.

A similar analysis may be done assuming a friction law based on the logarithmic velocity profile which, in smooth pipelines reads (Wilson and Thomas, 1985; Thomas and Wilson, 1987; Mansour and Rajie, 1988): $f^{-1/2} = -\alpha_1 \log_{10}(\alpha_2/Ref^{1/2})$. This equation is implicit on the friction factor but for Newtonian fluids several expressions have been proposed to avoid the iterative process (Clamond, 2009, and references therein). Following a similar procedure to that leading to the inequality (11), a similar upper bound condition for β_η is found as $\beta_\eta \le (1/\phi - 1)(1 - 1/2\sigma)(1 + k/f^{1/2})(\phi_m - \phi)$, with $k = \ln 10/\alpha_1$, evaluated at $\phi = \phi_{\mathrm{mf}}$. Although the results are similar to those depicted in Fig. 2, to proceed it is necessary to identify the flow Reynolds number to assess the value of f in the upper bound.

The inequality (10) may be understood as a base condition for this analysis. Given that it is the minimum velocity that which minimizes this base problem, it is left to introduce the parameter $\mathscr{H}$ and see whether the optimal condition changes with it.

4.3. Case 1: $\omega_0\mathscr{H} + 1 \ge 0$

The parameter $\mathscr{H}$ is assumed to take a constant value such as when the pressure exceeds the vapor value everywhere. Graphically, this parameter does not modify the slope of the energy line, but only offsets it vertically, as depicted in Fig. 1d (solid line). Differentiating Eq. (7) with respect to ϕ, the following expression is obtained:

$$\frac{1}{\omega_0 {R_0}^3}\frac{\mathrm{d}\omega}{\mathrm{d}\phi} = \frac{f}{\phi^4}\left(-1 - 2\sigma + \frac{\phi\sigma}{f}\frac{\mathrm{d}f}{\mathrm{d}\phi}\right) - \frac{1}{\phi^2 {R_0}^2}\left(\frac{1}{\omega_0} + \mathscr{H}\right). \tag{13}$$

Given that $\omega_0\mathscr{H} + 1 \ge 0$ the second term of the right hand side of Eq. (13) is negative, then again Eq. (13) is negative if the inequality (10) holds, regardless of the value of R_0.

In real long distance pipeline systems it is common that $z_0 > H_d$, which means that the geographic altitude of the pump station is above the total point energy dissipation head. From Eq. (13), it is noted that if $-1/\omega_0 \le \mathscr{H} \le 0$, then $\mathrm{d}\omega/\mathrm{d}\phi \ge 0$. The case $\mathscr{H} < -1/\omega_0 < 0$ is treated on Section 4.4.

4.4. Case 2: $\omega_0\mathscr{H} + 1 < 0$

Although $\mathscr{H}$ may take negative values (*i.e.* the dissipation energy level is below the altitude of the pump station), $\mathscr{H}$ must be such that the pumping power is positive. If otherwise, the available potential energy would suffice to satisfy the throughput requirement with pure gravitational transport. Any excess favorable value of z_0 may be compensated with additional energy consumption, either through energy dissipation or energy generation. In other words, from the expression (2), it is required that $P \ge 0$ or, in terms of the present dimensionless variables,

$$\mathscr{H} \ge -\frac{f{R_0}^2}{\phi^2}. \tag{14}$$

If $P = 0$, the cost function (1) takes the simpler form $\omega = R_0(1-\phi)/\phi$, whose minimum corresponds to the maximum attainable concentration, *i.e.*, the minimum flow condition.

Replacing the condition (14) in Eq. (13) yields

$$\frac{1}{\omega_0 {R_0}^3}\frac{\mathrm{d}\omega}{\mathrm{d}\phi} \le \frac{f}{\phi^4}\left(-2\sigma + \frac{\phi\sigma}{f}\frac{\mathrm{d}f}{\mathrm{d}\phi}\right) - \frac{1}{\phi^2 \omega_0 {R_0}^2}. \tag{15}$$

Combining the condition $\mathscr{H} < -1/\omega_0$ with the inequality (14) yields:

$$f > \frac{\phi^2}{\omega_0 {R_0}^2}. \tag{16}$$

On the other hand, from the inequality (15), $\mathrm{d}\omega/\mathrm{d}\phi \le 0$ if $-2\sigma + (\phi\sigma/f)\mathrm{d}f/\mathrm{d}\phi \le \phi^2/f\omega_0 {R_0}^2$ or, equivalently, $\mathrm{d}\omega/\mathrm{d}\phi \le 0$ if

$$\frac{1}{f}\frac{\mathrm{d}f}{\mathrm{d}\phi} \le \frac{1+2\sigma}{\phi\sigma} + \frac{1}{\phi\sigma}\left(\frac{\phi^2}{f\omega_0 {R_0}^2} - 1\right) < \frac{1+2\sigma}{\phi\sigma}, \tag{17}$$

where the rightmost bound is obtained noting that the term between brackets is negative due to Eq. (16). This condition corresponds to the inequality (10), which leads to expressions (11) and (12). Therefore, in this case, under the present hypotheses, again the minimum feasible velocity minimizes the transport cost.

5. Conclusions

Cases 1 and 2 show that if the minimum feasible velocity minimizes the cost, then any arbitrary vertical offset of it—either through modifying the pump altitude or introducing a dissipation—will yield the minimum cost with the minimum velocity as well.

The minimum flow condition thus represents the key to minimize the cost associated with the energy and water required to hydraulically transport solids as a pseudo-homogeneous mixture, especially over long distances, where the energy cost becomes relevant. It is noteworthy that the present result, stating that the most economic mode of transport corresponding to the minimum safe transport velocity, has not been linked to any particular model for deposit velocity or laminar turbulent transition. Supporting the present results, previous experimental measurements have concluded that higher concentrations and lower velocities are bonded to lower transport costs per unit mass of solids. Thus, the present study somewhat extends the previous knowledge on the optimality of the minimum velocity in energy efficiency to the concept of cost, associated simultaneously with water and energy. In particular, given a fixed throughput, the higher the concentration and the lesser the velocity, the lower the transport cost is. The present analysis does not take into account the computation of the actual minimum, which has been discussed elsewhere for a variety of examples (Ihle, 2013; Ihle et al., 2013; Ihle et al., 2014a) and depends, in particular, on the design criteria to set the minimum velocity condition. However, this result opens the door to narrowing down future optimization schemes, especially to identify economic infrastructure, by analogy with the concept of economic pipeline diameter in water networks.

List of symbols

a	constant
c_e	energy unit cost
c_w	water unit cost
D	pipeline internal diameter
e	overall pump efficiency
E	hydraulic head
F	Eq. (12)
f	Darcy friction factor
f_N	Darcy friction factor for a Newtonian fluid
g	magnitude of gravity acceleration vector
H_d	dissipation head
$\mathscr{H}$	dimensionless number (Eq. (8))
L	pipeline length
G	dry solid flow (throughput)
p	pressure
Q	flow rate

R_0	dimensionless number (Eq. (8))
Re	Reynolds number
Re_c	critical Reynolds number
p	pressure
P	pumping power (Eq. (2))
Q	slurry volume flow
Q_{dv}	slurry volume flow (deposit velocity)
Q_{lt}	slurry volume flow (laminar-turbulent transition)
Q_{mf}	minimum slurry volume flow
Q_w	water volume flow (Eq. (4))
R_0	dimensionless number (Eq. (8))
S	specific gravity of solids
z_0	altitude of the pump station
z_t	altitude

Greek letters

α_f	parameter for the relation $f=\alpha_f/Re^{\beta_f}$
β_f	parameter for the relation $f=\alpha_f/Re^{\beta_f}$
β_η	parameter for Eq. (6)
μ_w	water dynamic viscosity
η	viscosity concentration function (Eq. (6))
ϕ	solid volume fraction
ϕ_m	maximum solid volume fraction
ρ	density
ρ_w	water density
σ	$\phi(S-1)+1$
τ_{wall}	shear stress at the pipe wall
τ_y	yield stress
ξ	yield to wall stress ratio
Ω	dimensional cost function
Ω	cost scale
ω	dimensionless cost function
ω_0	dimensionless number (Eq. (8))

Acknowledgments

The author gratefully acknowledges support from the Department of Mining Engineering of University of Chile and Project VID Enlace 2015 (Univ. Chile).

Appendix A. An analytical solution of the solid volume fraction, corresponding to a specific critical Reynolds number

If $\beta_\eta=2$ in expression (6), then the relation $Re_c=(\sigma/\phi\eta)R_0$, where Re_c is the critical Reynolds number that marks the laminar-turbulent flow transition, is a 3rd degree polynomial equation in ϕ:

$$\phi^3+a_2\phi^2+a_1\phi+a_0=0, \tag{A.1}$$

where:

$$a_2=\frac{1-2\phi_m(S-1)}{S-1} \tag{A.2}$$

$$a_1=\frac{{\phi_m}^2(S-1)-2\phi_m-\theta}{S-1} \tag{A.3}$$

$$a_0=\frac{{\phi_m}^2}{S-1}, \tag{A.4}$$

with $\theta={\phi_m}^2 Re_c/R_0$. This equation has three real roots (Abramowitz and Stegun, 1965), whose only physically meaningful one is:

$$\phi=q^{1/2}\left[\cos\left(\frac{\psi}{3}\right)-\sqrt{3}\sin\left(\frac{\psi}{3}\right)\right]-\frac{\phi_m}{3}\left(\frac{1}{\sigma_m-1}-2\right), \tag{A.5}$$

where

$$q=\frac{\phi_m\left[\phi_m\sigma_m^2+3\theta(\sigma_m-1)\right]}{9(\sigma_m-1)^2} \tag{A.6}$$

$$\psi=\arctan\sqrt{\frac{q^3}{r^2}-1} \tag{A.7}$$

$$r=\frac{{\phi_m}^2\left[9\theta(2\sigma_m-3)(\sigma_m-1)-2\sigma_m^3\phi_m\right]}{54(\sigma_m-1)^3}, \tag{A.8}$$

with $\sigma_m=\phi_m(S-1)+1$.

References

Abramowitz, M., Stegun, I.A., 1965. Handbook of Mathematical Functions: With Formulas, Graphs, and Mathematical Tables. Dover Publications.

Abulnaga, B.E., 2002. Slurry Systems Handbook. McGraw-Hill, New York.

Bergant, A., Simpson, A.R., Tijsseling, A.S., 2006. Water hammer with column separation: a historical review. J. Fluids Struct. 22 (2), 135–171.

Chilton, R.A., Stainsby, R., 1998. Pressure loss equations for laminar and turbulent non-Newtonian pipe flow. J. Hydraul. Eng. 124 (1929), 522–529.

Clamond, D., 2009. Efficient resolution of the Colebrook equation. Ind. Eng. Chem. Res. 48 (7), 3665–3671.

Darby, R., 2001. Chemical Engineering Fluid Mechanics. CRC.

Doron, P., Barnea, D., 1993. A three-layer model for solid–liquid flow in horizontal pipes. Int. J. Multiphase Flow 19 (6), 1029–1043.

Durand, R., Condolios, E., 1952. The Hydraulic Transport of Coal and Solid Materials in Pipes. In: Proc. a Colloq. Hydraul. Transp. coal. Natl. Coal Board, London, UK, pp. 39–52.

Edelin, D., Czujko, P.-C., Castelain, C., Josset, C., Fayolle, F., 2015. Experimental determination of the energy optimum for the transport of floating particles in pipes. Exp. Thermal Fluid Sci. 68, 634–643 (Nov.).

Gillies, R.G., Schaan, J., Sumner, R.J., McKibben, M.J., Shook, C.A., 2000. Deposition velocities for Newtonian slurries in turbulent flow. Can. J. Chem. Eng. 78 (4), 704–708.

Ihle, C.F., 2013. A cost perspective for long distance ore pipeline water and energy utilization. Part I: optimal base values. Int. J. Miner. Process. 122, 1–12.

Ihle, C.F., Montserrat, S., Tamburrino, A., 2013. A cost perspective for long distance ore pipeline water and energy utilization. Part II: effect of input parameter variability. Int. J. Miner. Process. 122, 54–58.

Ihle, C.F., Tamburrino, A., 2012. Variables affecting energy efficiency in turbulent ore concentrate pipeline transport. Miner. Eng. 39, 62–70.

Ihle, C.F., Tamburrino, A., Montserrat, S., 2014a. Computational modeling for efficient long distance ore transport using pipelines. Miner. Eng. 63, 73–80.

Ihle, C.F., Tamburrino, A., Montserrat, S., 2014b. Identifying the relative importance of energy and water costs in hydraulic transport systems through a combined physics- and cost-based indicator. J. Clean. Prod. 84, 589–596.

Jacobs, B.E.A., 1991. Design of Slurry Transport Systems. Routledge.

Krieger, I.M., Dougherty, T.J., 1959. A mechanism for non-Newtonian flow in suspensions of rigid particles. Trans. Soc. Rheol. 3, 137.

Mansour, A.R., Rajie, O.T., 1988. Generalized explicit equation for the friction factor for Newtonian and non-Newtonian fluids in both circular and non-circular ducts. Chem. Ing. Tech. 60 (4), 304–306.

Maron, S.H., Pierce, P.E., 1956. Application of Ree-Eyring generalized flow theory to suspensions of spherical particles. J. Colloid Sci. 11 (1), 80–95.

Matoušek, V., 2002. Pressure drops and flow patterns in sand-mixture pipes. Exp. Thermal Fluid Sci. 26 (6–7), 693–702.

Mueller, S., Llewellin, E.W., Mader, H.M., 2010. The rheology of suspensions of solid particles. Proc. R. Soc. London Ser. A 466 (2116), 1201–1228.

Nguyen, Q.D., Boger, D.V., 1998. Application of rheology to solving tailings disposal problems. Int. J. Miner. Process. 54 (3–4), 217–233.

Ovarlez, G., Bertrand, F., Rodts, S., 2006. Local determination of the constitutive law of a dense suspension of noncolloidal particles through magnetic resonance imaging. J. Rheol. 50 (3), 259–292.

Poloski, A.P., Etchells, A.W., Chun, J., Adkins, H.E., Casella, A.M., Minette, M.J., Yokuda, S.T., 2010. A pipeline transport correlation for slurries with small but dense particles. Can. J. Chem. Eng. 88 (April), 182–189.

Pullum, L., Graham, L., Rudman, M., Hamilton, R., 2006. High concentration suspension pumping. Miner. Eng. 19 (5), 471–477.

Quemada, D., 1977. Rheology of concentrated disperse systems and minimum energy dissipation principle. Rheol. Acta 16 (1), 82–94.

Sofrá, F., Boger, D.V., 2002. Environmental rheology for waste minimisation in the minerals industry. Chem. Eng. J. Biochem. Eng. J. 86 (3), 319–330.

Stickel, J.J., Powell, R.L., 2005. Fluid mechanics and rheology of dense suspensions. Annu. Rev. Fluid Mech. 37 (1), 129–149.

Thomas, A.D., Wilson, K.C., 1987. New analysis of non-Newtonian turbulent flow and yield-power-law fluids. Can. J. Chem. Eng. 65 (2), 335–338.

Wilson, K.C., Addie, G.R., Sellgren, A., Clift, R., 2006. Slurry Transport Using Centrifugal Pumps. Chapman and Hall, London.

Wilson, K.C., Thomas, A.D., 1985. A new analysis of the turbulent flow of non-Newtonian fluids. Can. J. Chem. Eng. 63 (4), 539–546.

Wu, J., Graham, L., Wang, S., Parthasarathy, R., 2010. Energy efficient slurry holding and transport. Miner. Eng. 23, 705–712.