Similitudes for the structural response of flexural plates

ARTICLE *in* ARCHIVE PROCEEDINGS OF THE INSTITUTION OF MECHANICAL ENGINEERS PART C JOURNAL OF MECHANICAL ENGINEERING SCIENCE 1989-1996 (VOLS 203-210) · FEBRUARY 2015

Impact Factor: 0.56 · DOI: 10.1177/0954406215572436

3 AUTHORS:

Sergio De Rosa
University of Naples Federico II

Francesco Franco
University of Naples Federico II

V. Meruane
University of Chile

Similitudes for the structural response of flexural plates

S. De Rosa*(⊕), F. Franco (⊕), V. Meruane (⊗)

(⊕) pasta-Lab, Department of Industrial Engineering, Aerospace Section
Università degli Studi di Napoli "Federico II"
Via Claudio 21, 80125 Napoli, Italy

(⊗) DIMEC, Department of Mechanical Engineering
Universidad de Chile
Av. Beauchef 850, Santiago 8370448, Chile

Abstract

This article presents an investigation into exact and distorted similitudes and the related scaling laws for the analysis of the dynamic response of rectangular flexural plates. The response of a given model in similitude is determined from a generalization of the modal approach, which allows the use the mode shapes and natural frequencies in order to establish scaling laws. Analytical models of simply supported rectangular plates are used to produce both the original and distorted model responses. Some highlights about the distribution of the natural frequencies, the forced response and the energy response are given. The results show that with the proposed methodology it is possible to reproduce with good confidence the response of the reference plate, even if distorted models are used.

Keywords: `similitude, plate, modal approach`

*corresponding author: `sergio.derosa@unina.it`

Contents

1 Introduction **4**

2 Modal approach and similitudes **6**

3 Application to flexural plates **8**

4 Results **9**

4.1 Modal response 10

4.1.1 Exact similitudes 10

4.1.2 Avatars 11

4.2 Modal Density 12

4.3 Energy response 13

5 Concluding remarks **13**

List of Figures

1 Scheme of the plates: (a) Reference, (b) Replica, (c) Proportional Sides, (d) Avatar 1, (e) Avatar 2, (e) Avatar 3. 17

2 Plates M and R, natural frequencies. 18

3 Plates M and R, dimensionless responses. 19

4 Plates M and R, remodulated responses. 20

5 Plates M and E, natural frequencies. 21

6 Plates M and E, Dimensionless Responses. 22

7 Plates M and E, remodulated responses. 23

8 Plates M and A1, natural frequencies. 24

9 Plates M and A1, dimensionless responses. 25

10 Plates M and A1, remodulated responses. 26

11 Plates M and A2, natural frequencies. 27

12 Plates M and A2, dimensionless responses. 28

13 Plates M and A2, remodulated Responses. 29

14 Plates M and A3, natural frequencies. 30

15 Plates M and A3, dimensionless responses. 31

16 Plates M and A3, remodulated Responses. 32

17 Plates M and A3, SEA Original and remodulated responses. 33

1 Introduction

In many engineering applications, it is often desired to have assemblies to test some design choices and/or to verify possible enhancements. Nevertheless, for certain objects sizes these verifications can become very difficult. Thus, it is frequently required to change, if it is possible, the scales of the models being tested.

This topic has emerged with strong evidence in most engineering fields (naval, aerospace, civil and railway ones) where some investigations on new prototypes are highly necessary, often mandatory. These prototypes should be able to move to sizes, easy to manage in a laboratory, by either increasing or reducing the original ones. In these cases, it is very important to define similarity conditions or scaling laws. Similarity conditions specify relations between a full-scale prototype and its scaled models, whereas with analogies a given problem is solved by looking for similar equations. A general view of the problems of similitude and analogies are presented in [1, 2].

Several studies have investigated the problem of predicting the dynamic characteristics of a prototype from those of its scale models. Rezaeepazhand and Simitses [3] investigate the feasibility of scaled models to predict the vibration response of laminated cylindrical shells. The authors apply similitude theory to the governing equations of the structural system to develop similarity conditions. According to the researchers, similitude theory, based on governing equations, is more direct than the conventional method based on dimensional analysis. A similar investigation is presented by Simitses [4] for a simply supported laminated plate subjected to transverse, buckling and free vibration. Wu et al. [5] derive scaling laws using the principles of dimensional analysis and similitude theory applied to the equations of motion in modal coordinates. The methodology is validated using both free and forced vibrations of a uniform beam and a gantry crane. The same approach is followed by Wu [6, 7, 8] to predict the response of flat plates under different boundary conditions when they are subjected to dynamic moving loads. Singhatanadgid and Songkhla [9] performe an experimental investigation on the predicted natural frequencies of a rectangular plate using the results of its scaled models. Torkamani et al. [10] derive scaling laws based on the similarity theory to predict the natural frequencies of orthogonally stiffened cylindrical shells. Numerical and experimental examples are used to validate the derived scaling laws.

De Rosa and Franco [11, 12] develope a scaling method that uses the Energy Distribution Approach (EDA) to predict the scaled responses of linear dynamic systems. This method, called ASMA, represents an incomplete similitude, which focuses on the possibility to scale a given model in order to generate artificially high modal overlap factor conditions while reducing the computational cost. The results are very attractive when comparing them to those obtainable with large finite element models. A similar approach is followed by De Rosa et al. [13, 14, 15], where the similitude is defined by invoking the EDA while scaling laws are identified by looking for equalities in the structural responses. Particularly, in [13], some preliminary analysis, aimed at the complete similitude for simple plate and assemblies of plates, are presented. In [14] recent and successful steps involving simple vibro-acoustic systems: infinite flexural cylinder/finite flexural coupled with the internal acoustics are presented. Lastly, in [15] the scaling between structural components with different modal density is introduced (a coupled beam-plate system).

The present work focuses on tuning exact and distorted similitudes and the related scaling laws for the analysis of the dynamic response of simple flat flexural plates. Thus, the attention is concentrated on the possible developments concerning thin plates, [16, 17]. It has to be underlined that chapter 19 in [16] is fully devoted to the similitudes of plates and shells, as well as the work in [18].

Here, the response of a given model in similitude is determined from a generalization of the modal approach, which allows the use of the mode shapes and natural frequencies in order to establish scaling laws. Some highlights about the distribution of the natural frequencies and the forced response are given for flat plates. Furthermore, some possibilities to assemble distorted models are discussed, which are very interesting from an engineering point of view. Analytical models are used throughput the whole work to produce both the original and distorted model responses.

The results are very encouraging, especially considering that the modal approach, invoked here for the theoretical investigations, can be extended with relative simplicity to commercial codes working with the Finite Element Methods. The complete procedure is named SAMSARA, *Similitude and Asymptotic Models for Structural-Acoustic Researches and Applications.*

This work is the first in which the distorted similitudes are carefully analysed for engineering purposes and in parallel some experimental investigations are already ongoing for validating the these initial findings. SAMSARA is far to be considered a validated tool but the the results are encouraging.

After these initial remarks, the work is continued in Section 2, which contains the general frame of the modal response approach. The flat and unstiffened plate models are introduced in Section 3, where also the type of similitudes are defined. Section 4 presents the results in terms of the variation of the main scaling parameters, additionally some considerations about the similitudes for the energy response are also given. A concluding summary closes the work in Section 5.

2 Modal approach and similitudes

The response of a generic linear system can always be expressed in terms of its N_M eigensolutions through a summation. For a force located at a point S of the given spatial domain, the velocity response at a point R is given as follows:

$$V(R,S;\omega) = (\mathrm{j}\omega F(\omega)) \sum_{i=1}^{N_M} \frac{\phi_i(R)\phi_i(S)}{\mu_i \left[\left(\omega_i^2 - \omega^2 \right) + \mathrm{j}\eta_i \omega_i^2 \right]} \tag{1}$$

being ϕ_i, ω_i, η_i and μ_i the i-th eigenvector, radian natural frequency, structural modal damping and generalised mass, respectively. $F(\omega)$ represents the force amplitude, j the imaginary unit and i the mode index. The force, $f(S;t)$ is considered acting at a point S and is expressed as follows:

$$f(S;t) = F(\omega)\delta(S_0 - S)e^{\mathrm{j}\omega t} \tag{2}$$

being δ the Kronecker delta function and t the time coordinate. The velocity has been assumed in the same form:

$$v(R,S;t) = V(R,S;\omega)e^{\mathrm{j}\omega t} \tag{3}$$

For the aims of the present work a useful simplification of the Eq. (1) can be used by assuming that the modal damping is constant and also the force can be considered with a given constant amplitude.

$$V(R,S;\omega) = \mathrm{j}\omega F \sum_{i=1}^{N_M} \frac{\phi_i(R)\phi_i(S)}{\mu_i\left[\left(\omega_i^2 - \omega^2\right) + \mathrm{j}\eta\omega_i^2\right]} \tag{4}$$

The velocity response for a generic system in similitude can be written as:

$$\hat{V}(R,S;\omega) = \mathrm{j}\omega \hat{F} \sum_{i=1}^{N_M} \frac{\hat{\phi}_i(R)\hat{\phi}_i(S)}{\hat{\mu}_i\left[\left(\hat{\omega}_i^2 - \omega^2\right) + \mathrm{j}\hat{\eta}\hat{\omega}_i^2\right]} \tag{5}$$

The symbol $\hat{()}$ will always denote the parameter in similitude.

Let define a set of scaling parameters, r, as the ratios of the similitude parameter to the original one, thus: $r_F = \frac{\hat{F}}{F}$; $r_\eta = \frac{\hat{\eta}}{\eta}$; $r_\omega = \frac{\hat{\omega}_i}{\omega_i}$; $r_{mass} = \frac{\hat{\mu}_i}{\mu_i}$. The spatial dependence, $\phi_i(R)\phi_i(S)$, presents the dimensionless positions of the source and receiver points and thus they do not need to be posed in similitude. Some of the scaling parameters represent a choice, while others are derived from this one.

The generic similitude can be rewritten as:

$$\hat{V}(R,S;\omega) = \mathrm{j}\omega\ r_F F \sum_{i=1}^{N_M} \frac{\phi_i(R)\phi_i(S)}{r_{mass}\mu_i\ \left[\left(r_\omega^2\omega_i^2 - \omega^2\right) + \mathrm{j}r_\eta\ \eta r_\omega^2\omega_i^2\right]} \tag{6}$$

rearranging Eq. (6):

$$\hat{V}(R,S;\omega) = \left(\frac{r_F}{r_{mass}r_\omega}\right)\left[\mathrm{j}\left(\frac{\omega}{r_\omega}\right)\ F\right] \sum_{i=1}^{N_M} \frac{\phi_i(R)\phi_i(S)}{\mu_i\ \left[\left(\omega_i^2 - \left(\frac{\omega}{r_\omega}\right)^2\right) + \mathrm{j}r_\eta\ \eta\omega_i^2\right]} \tag{7}$$

If $r_\eta = 1$, the response of the original system can be recovered from the response of its similitude:

$$V(R,S;\omega) = \frac{r_{mass}r_\omega}{r_F}\hat{V}(R,S;r_\omega\omega) \tag{8}$$

A final consideration is that any variation of the geometry and/or the material will alter the distribution of the natural frequencies, it is further assumed that the boundary conditions and the mode shapes are left unaltered.

Rather than trying to give further theoretical considerations, it is useful to pass to the investigation on simple plate models.

3 Application to flexural plates

The application of the modal approach is here reported to a homogeneous material rectangular plate. The boundary conditions are such that the edges are all simply supported. The thickness is h and the side lengths are L_x and L_y; the plate belongs to a generic Oxy reference plane. This leads to the following expression:

$$V(R,S;\omega) = \frac{4\mathrm{j}\omega F}{\rho L_x L_y h} \sum_{i=1}^{N_M} \frac{\sin(\frac{m_i \pi x_R}{Lx}) \sin(\frac{n_i \pi y_R}{L_y}) \sin(\frac{m_i \pi x_S}{Lx}) \sin(\frac{n_i \pi y_S}{L_y})}{\left(\omega_i^2 - \omega^2\right) + \mathrm{j}\eta\omega_i^2} \tag{9}$$

where the integers m_i and n_i are the number of half waves of the i-th mode.

Introducing the dimensionless coordinates, $\xi = \frac{x}{Lx}$ and $\zeta = \frac{y}{L_y}$:

$$V(\xi_R, \zeta_R, \xi_S, \zeta_S, ; \omega) = \frac{4\mathrm{j}\omega F}{\rho L_x L_y h} \sum_{i=1}^{N_M} \frac{\sin(m_i \pi \xi_R) \sin(n_i \pi \zeta_R) \sin(m_i \pi \xi_S) \sin(n_i \pi \zeta_S)}{\left(\omega_i^2 - \omega^2\right) + \mathrm{j}\eta\omega_i^2} \tag{10}$$

with the natural frequencies:

$$\omega_i = \sqrt{\frac{Eh^2}{12\rho(1-\upsilon^2)}} \left[\left(\frac{m_i \pi}{L_x}\right)^2 + \left(\frac{n_i \pi}{L_y}\right)^2 \right] \tag{11}$$

where E, υ and ρ denote the material Young modulus. Poisson modulus and mass density, respectively.

In order to explore the achievable similitudes, a further choice has been made to keep the material and damping values, $r_\eta = r_E = r_\rho = 1$. This is easily justified thinking to the extreme engineering complexity of introducing also variation of the materials and the condition of the damping.

Thus, only variation of the force and the sizes of the plates are investigated so that the generic parent plate response can be written as:

$$\hat{V}(\xi_R, \zeta_R, \xi_S, \zeta_S, ; \omega) = \frac{4\mathrm{j}\omega r_F F}{\rho r_x r_y r_h Lx L_y h} \sum_{i=1}^{N_M} \frac{\sin(m_i \pi \xi_R) \sin(n_i \pi \zeta_R) \sin(m_i \pi \xi_S) \sin(n_i \pi \zeta_S)}{\left(r_\omega^2 \omega_i^2 - \omega^2\right) + \mathrm{j}\eta r_\omega^2 \omega_i^2} \tag{12}$$

with,

$$\hat{\omega}_i = r_h h \sqrt{\frac{E}{12\rho(1-\upsilon^2)}} \left[\left(\frac{m_i \pi}{r_x Lx}\right)^2 + \left(\frac{n_i \pi}{r_y L_y}\right)^2 \right] \tag{13}$$

Rearranging Eq. 12:

$$\hat{V}(\xi_R, \zeta_R, \xi_S, \zeta_S, ; \omega) = \frac{r_F}{r_x r_y r_h r_\omega} \frac{4\mathrm{j}F}{\rho Lx L_y h} \left(\frac{\omega}{r_\omega}\right) \sum_{i=1}^{N_M} \frac{\sin(m_i \pi \xi_R) \sin(n_i \pi \zeta_R) \sin(m_i \pi \xi_S) \sin(n_i \pi \zeta_S)}{\left(\omega_i^2 - \left(\frac{\omega}{r_\omega}\right)^2\right) + \mathrm{j}\eta\omega_i^2} \tag{14}$$

thus,

$$V\left(\xi_R, \zeta_R, \xi_S, \zeta_S, ; \omega\right) = \frac{r_x r_y r_h r_\omega}{r_F} \hat{V}\left(\xi_R, \zeta_R, \xi_S, \zeta_S, ; r_\omega \omega\right) = \frac{r_{mass} r_\omega}{r_F} \hat{V}\left(\xi_R, \zeta_R, \xi_S, \zeta_S, ; r_\omega \omega\right) \tag{15}$$

The parameter r_F works as a simple correction factor. Thus, by looking at the last expression and having in mind to keep the material and the damping, a generic parent plate can be investigated in order to recover the response of the original one. The only condition to obtain an exact similitude is that the natural frequencies of the parent plate can be written proportional to those of the original plate. This can be accomplished if $r_x = r_y$, in this case:

$$\hat{\omega}_i = \frac{r_h}{r_x^2} \sqrt{\frac{Eh^2}{12\rho(1-\upsilon^2)}} \left[\left(\frac{m_i \pi}{Lx}\right)^2 + \left(\frac{n_i \pi}{L_y}\right)^2\right] = \frac{r_h}{r_x^2} \omega_i \tag{16}$$

The parent plate is obtained by varying both the lengths of each sides and the thickness.

If $r_x = r_y = r_h$ a *replica* is obtained. If $r_x = r_y$ for any r_h a *proportional side* type of similitude is obtained. Both are exact similitudes.

An *avatar*, that is a distorted similitude, is obtained for $r_x \neq r_y$

In the next section, the results will be reported for *replicas*, *proportional side* and *avatar* plates in order to understand the several possibilities.

4 Results

The tests concern several plates summarised in Table I. Figure 1 presents a scheme of the plates. The *replica* and the *proportional sides* types are exact similitudes, whereas the *avatar* types are distorted ones.

Table I: Plates

Plate Id	Type of similitude	Dimensions $[m]$			Coefficients		
		Lx	L_y	h	r_x	r_y	r_h
M	Reference Model	0.70	0.30	0.001	1	1	1
R	Replica	2.10	0.90	0.003	3.0	3.0	3.0
E	Proportional Sides	0.35	0.15	0.002	0.5	0.5	2.0
A1	Avatar	0.35	0.90	0.002	0.5	3.0	2.0
A2	Avatar	0.35	0.30	0.002	0.5	1.0	2.0
A3	Avatar	0.35	0.21	0.001	0.5	0.7	1.0

All the results, for a given couple of plates, are represented by three graphs: (i) the first relates the natural frequencies of the reference plate and that in similitude, before and after remodulation; (ii) the second reports the dimensionless responses of each plate; (iii) the last chart reports the frequency response of each plate remodulated by using the scaling laws.

Specifically the label *original* on the natural frequencies charts means that the values of the eigenvalues of both the reference plate and the related parent are reported. The scaling factor r_ω is used to remodulated those frequencies associated to the parent plate. For exact similitudes, the natural frequencies of the reference plate and those of the remodulated parent have to be equal.

The dimensionless responses serve to understand how different is the behaviour of each plate. The frequency axis reports $\omega h/c_L$, being c_L the longitudinal wave speed of the selected material. The response axis presents the dimensionless group: $V(\xi_R, \zeta_R, \xi_S, \zeta_S, ; \omega)\frac{\rho Lx L_y c_L}{F}$.

4.1 Modal response

4.1.1 Exact similitudes

Figures (2-7) present the expected results. In both the *replica* and the *proportional sides* cases, the similitude models are able to recover exactly the response of the reference plate. It has to be again underlined that the frequency axis has been scaled by r_ω and the response axis by $r_\omega\, r_{mass}$.

The natural frequencies, after being remodulated, are perfectly located along the bisecting axis of the natural frequencies charts, as presented in Figures (2) and (5). Furthermore, the responses are perfectly collapsed after having remodulated the response values, too.

These are not surprising findings but represent a good check in view of the *avatar* responses.

4.1.2 Avatars

The first set of results concerns the relation between the reference plate and the first avatar. As shown in Figure (8), this avatar is drastically different from the reference plate and thus the distribution of the natural frequencies even after the remodulation does not present a good agreement. The dimensionless responses, presented in Figure (9), confirm that the differences are huge. Nevertheless, in the response of both plates presented in Figure (10), it is possible to see how the remodulation allows recovering the response by using the reference plate for getting the avatar response and viceversa. The results are quite acceptable from an engineering point of view up to 1000 Hz. For higher frequencies, the differences are increasingly great.

The second set, referring to the second avatar, presents only an altered value of r_y while r_x and r_h are the same as those used in the first avatar. This second avatar is less distorted than the first one, and this can be read on each of the related Figures (11-13). Specifically, the approximation is acceptable in all the investigated frequency ranges. Figure (13) shows that the second avatar reproduces with a good confidence the response of the reference plate with the only exception of the first peaks.

From an engineering point of view, the most interesting results are those referred to the third avatar in which the thickness has been kept and both sides have been reduced. The results presented in Figure (14) show that, even being different plates as demonstrated in Figure (15), the distribution of the natural frequencies after remodulation is very close to the exact ones. Furthermore, the recovered responses presented in Figure (16) are the best of the avatar group.

4.2 Modal Density

Another useful parameter to analyse the results is the modal density, the statistical number of modes resonating in a given frequency band. In case of flexural plates, it is a constant value and it is given as simply as follows:

$$n \propto \frac{L_x\, L_y}{h} \tag{17}$$

and thus for any model in similitude

$$\hat{n} \propto \frac{r_x\, r_y}{r_h} \frac{L_x\, L_y}{h} \tag{18}$$

This is the situation for the plates reported in Table I:

Table II: Modal Density of Plates

Plate Id	Type of similitude	$\frac{r_h}{r_x\, r_y}$
R	Replica	0.33
E	Proportional Sides	8.0
A1	Avatar	1.33
A2	Avatar	4.0
A3	Avatar	2.86

The models in complete similitudes, R and E, cannot be interpreted with the variations of the modal densities since they replicated exactly the natural frequencies over wider or smaller frequency ranges.

For the *avatars*, it is still under investigations the possibility to define an index of correlation in order to compare the original and similitude responses, but the modal density can represent a first useful support. In fact, the A2 is the worst of the group since it is associated to both large variations of the modal density and the thickness. The A1 should be the best one since it is associated to the minor variations of the modal density. Nevertheless, the A3 even having a value of the modal density larger than the A1, keeps the original thickness and thus is associated to more acceptable variation of the natural frequencies.

The initial lesson learnt from this simplified schemes is to consider avatars in which the natural frequencies are distorted by modifying only the in-plane lengths and keeping the original thickness.

4.3 Energy response

A consideration is finally needed for the response of the plates for increasing excitation frequency. In this case, the local behaviour disappears and the response becomes global. The Statistical Energy Analysis, SEA [19], can be applied to estimate the flexural response of a plate:

$$V_{SEA}^2(\omega) \approx \frac{F^2}{\rho^2 \omega L_x L_y h^3 \omega c_L \eta} \tag{19}$$

and thus by introducing the scaling parameters

$$\hat{V}_{SEA}^2(\omega) \approx \left(\frac{r_f^2}{r_x r_y r_h^3} \right) \frac{F^2}{\rho^2 \omega L_x L_y h^3 \omega c_L \eta} \tag{20}$$

it is thus always possible to get a complete similitude between any couple of flexural plates. This is here certified by including only a comparison between plates M and A3 in Figure (17).

5 Concluding remarks

This article presents an investigation into exact and distorted similitudes and the related scaling laws for the analysis of the dynamic response of rectangular flexural plates. The response of a given model in similitude is determined from a generalization of the modal approach, which allows the use the mode shapes and natural frequencies in order to establish scaling laws. The complete procedure is named SAMSARA, *Similitude and Asymptotic Models for Structural-Acoustic Researches and Applications*. Analytical models of simply supported rectangular plates are used to produce both the original and distorted model responses.

It is demonstrated that, if the original damping values and the distribution of natural frequencies in the parent models are kept, then it is always possible to switch from the original model to the parent model and viceversa, being the similitude complete. These conditions are fulfilled for the replica and proportional side cases.

In the case of a distorted parent, the distribution of natural frequencies is altered when compared to those of the original plate, and thus only a partial similitude is achieved. Nevertheless, even in this case, it is possible to reproduce with good confidence the response of the reference plate. These results are very encouraging, especially considering that the modal approach invoked here can be extended with relative simplicity to commercial codes working with the Finite Element Methods.

Finally, it is shown that when the energy response of a plate is computed using statistical energy analysis, it is always possible to get a complete similitude between any couple of flexural plates.

Further work is necessary to move to experimental validations tests, where the boundary and damping conditions between the original and parent models are not necessarily the same.

Acknowledgements

Part of this work has been developed during a short stay of the first author in the Department of Mechanical Engineering at Universidad de Chile, which was financed by the Chilean National Fund for Scientific and Technological Development (Fondecyt) under Grant No. 11110046.

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

References

[1] E. Szucs, Similitude and Modelling, Elsevier Science Ltd., ISBN: 0444997806, 1980.

[2] S. J. Kline, Similitude and Approximation Theory, Springer-Verlag, ISBN 0387165185, 1986.

[3] J. Rezaeepazhand, G. Simitses, Design of scaled down models for predicting shell vibration response, Journal of Sound and Vibration 195 (2) (1996) 301–311.

[4] G. J. Simitses, Structural similitude for flat laminated surfaces, Composite structures 51 (2) (2001) 191–194.

[5] J. Wu, M. Cartmell, A. Whittaker, Prediction of the vibration characteristics of a full-size structure from those of a scale model, Computers & Structures 80 (18) (2002) 1461–1472.

[6] J. Wu, The complete-similitude scale models for predicting the vibration characteristics of the elastically restrained flat plates subjected to dynamic loads, Journal of Sound and Vibration 268 (5) (2003) 1041–1053.

[7] J. Wu, Dynamic analysis of a rectangular plate under a moving line load using scale beams and scaling laws, Computers & Structures 83 (19) (2005) 1646–1658.

[8] J. Wu, Prediction of the dynamic characteristics of an elastically supported full-size flat plate from those of its complete-similitude scale model, Computers & Structures 84 (3) (2006) 102–114.

[9] P. Singhatanadgid, A. Na Songkhla, An experimental investigation into the use of scaling laws for predicting vibration responses of rectangular thin plates, Journal of Sound and Vibration 311 (1) (2008) 314–327.

[10] S. Torkamani, H. Navazi, A. Jafari, M. Bagheri, Structural similitude in free vibration of orthogonally stiffened cylindrical shells, Thin-Walled Structures 47 (11) (2009) 1316–1330.

[11] S. De Rosa, F. Franco, A scaling procedure for the response of an isolated system with high modal overlap factor, Mechanical Systems and Signal Processing 22 (7) (2008) 1549–1565.

[12] S. De Rosa, F. Franco, On the use of the asymptotic scaled modal analysis for time-harmonic structural analysis and for the prediction of coupling loss factors for similar systems, Mechanical Systems and Signal Processing 24 (2) (2010) 455–480.

[13] S. De Rosa, F. Franco, T. Polito, Structural similitudes for the dynamic response of plates and assemblies of plates, Mechanical Systems and Signal Processing 25 (3) (2011) 969–980.

[14] S. De Rosa, F. Franco, X. Li, T. Polito, A similitude for structural acoustic enclosures, Mechanical Systems and Signal Processing 30 (2012) 330–342.

[15] S. De Rosa, F. Franco, T. Polito, Partial scaling of finite element models for the analysis of the coupling between short and long structural wavelengths, Mechanical Systems and Signal Processing,doi:10.1016/j.ymssp.2014.06.007.

[16] W. Soedel, Vibrations of shells and plates, CRC Press, 2004.

[17] A. W. Leissa, Vibration of plates, Acoustical Society of America, ISBN: 9781563962943, 1969.

[18] G. Simitses, J. Starnes Jr, J. Rezaeepazhand, Structural similitude and scaling laws for plates and shells: A review, in: Advances in the Mechanics of Plates and Shells, Springer, 2002, pp. 295–310.

[19] R. Lyon, Statistical energy analysis of dynamical systems: theory and applications, MIT press Cambridge, ISBN: 97802626217551975, 1975.

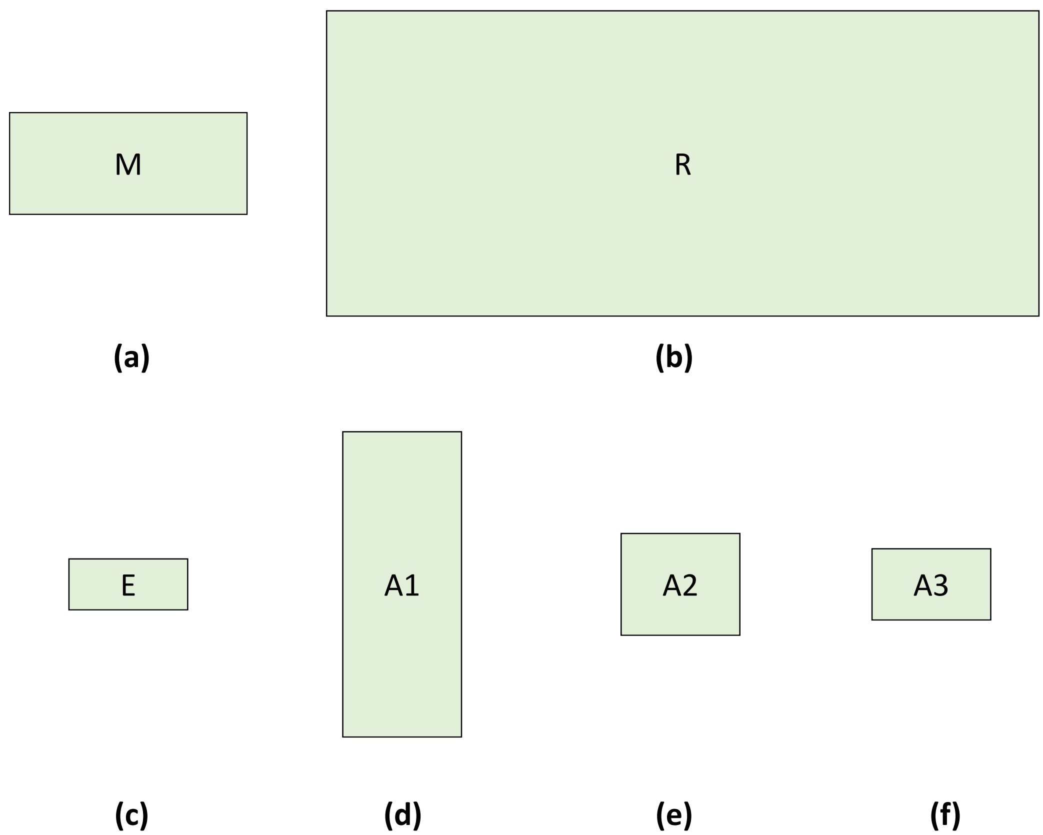

Figure 1: Scheme of the plates: (a) Reference, (b) Replica, (c) Proportional Sides, (d) Avatar 1, (e) Avatar 2, (e) Avatar 3.

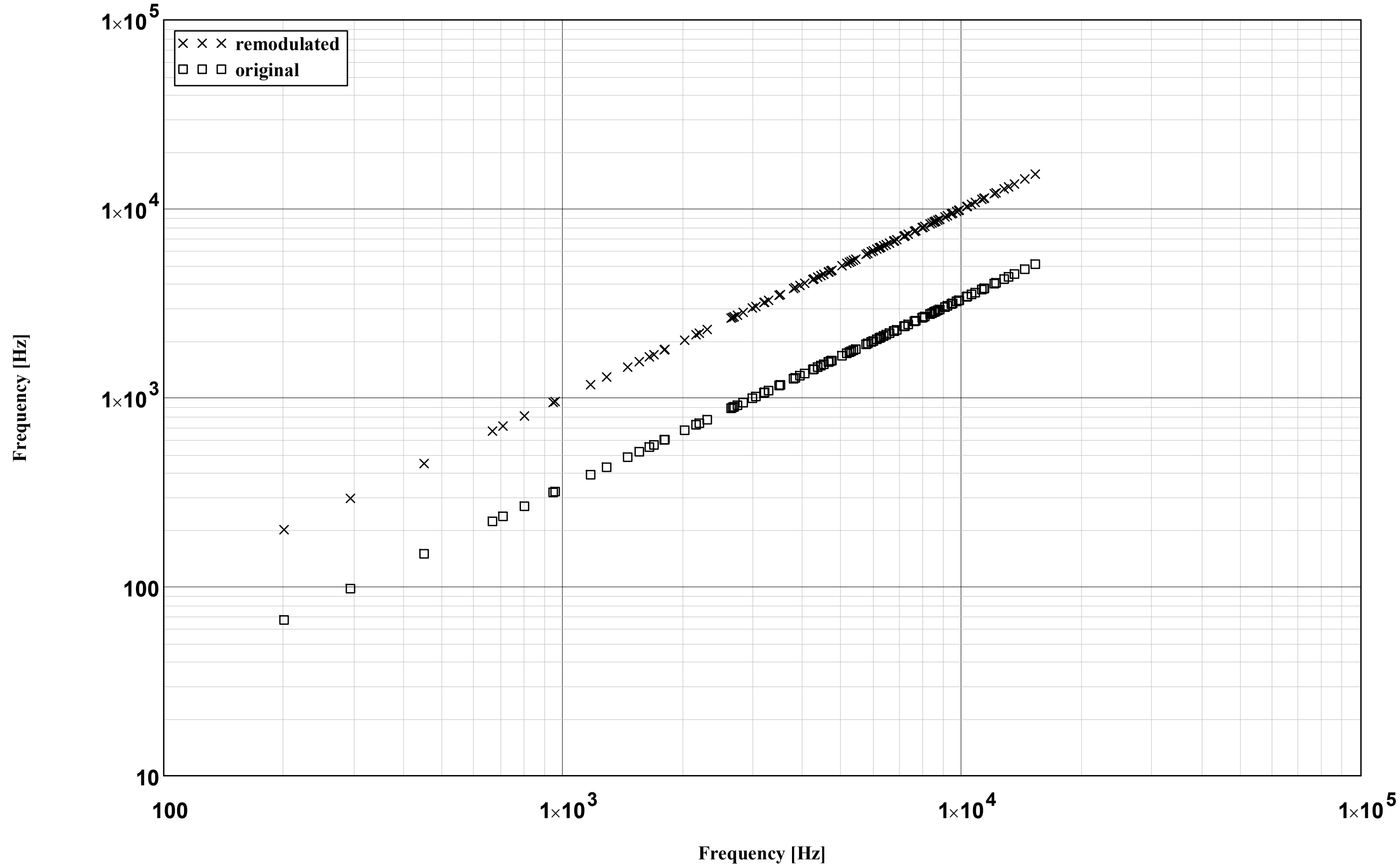

Figure 2: Plates M and R, natural frequencies.

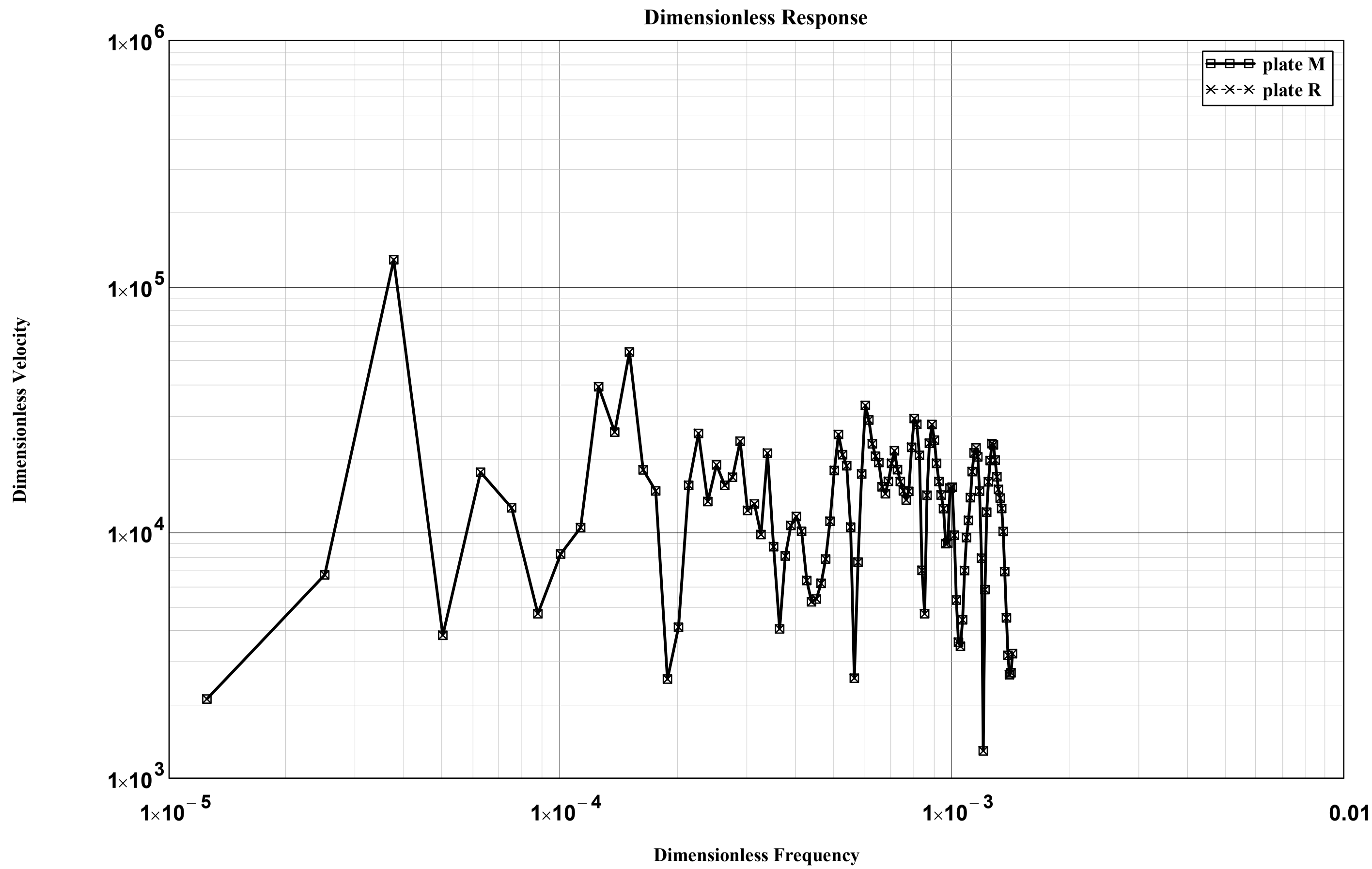

Figure 3: Plates M and R, dimensionless responses.

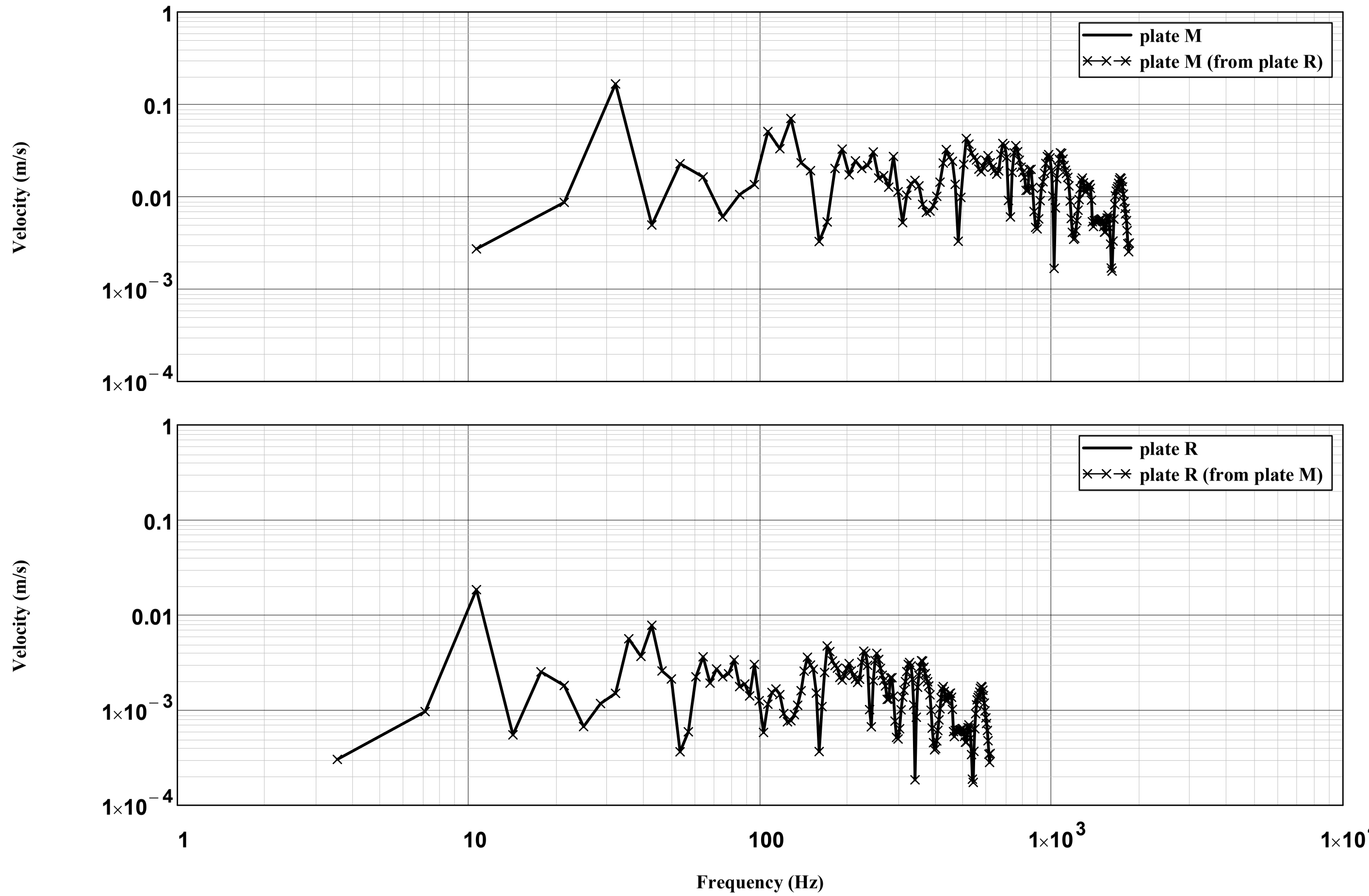

Figure 4: Plates M and R, remodulated responses.

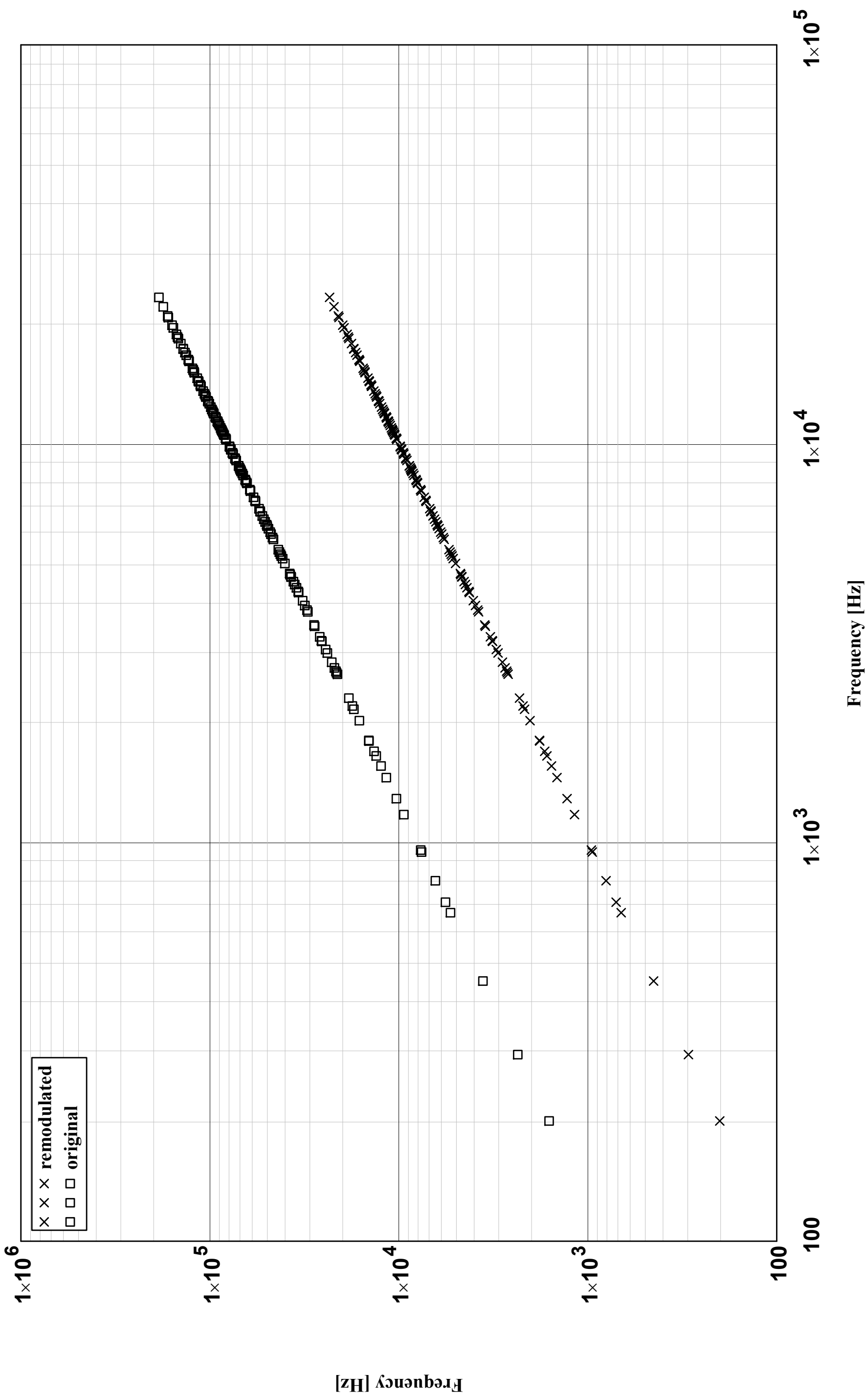

Figure 5: Plates M and E, natural frequencies.

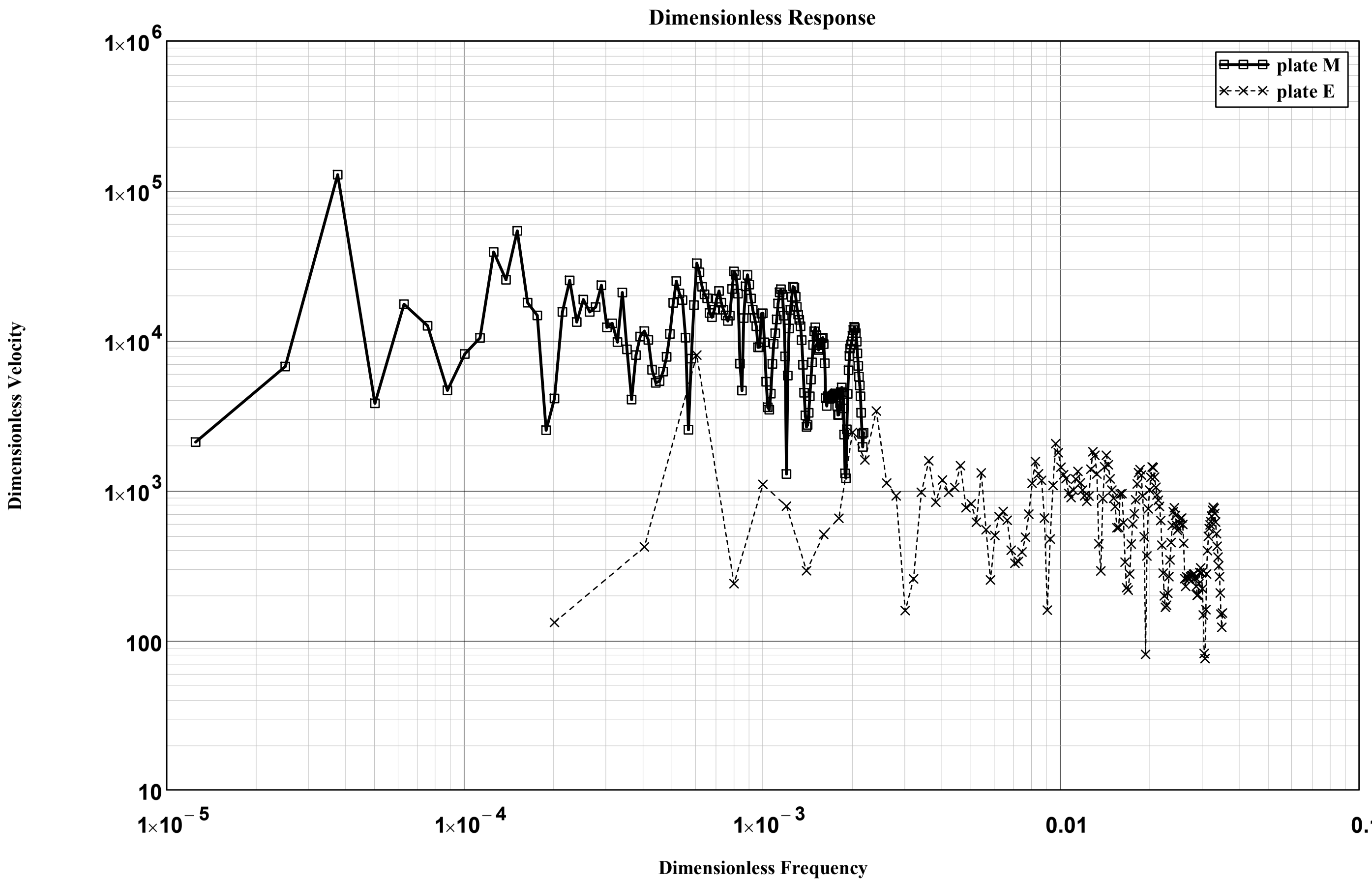

Figure 6: Plates M and E, Dimensionless Responses.

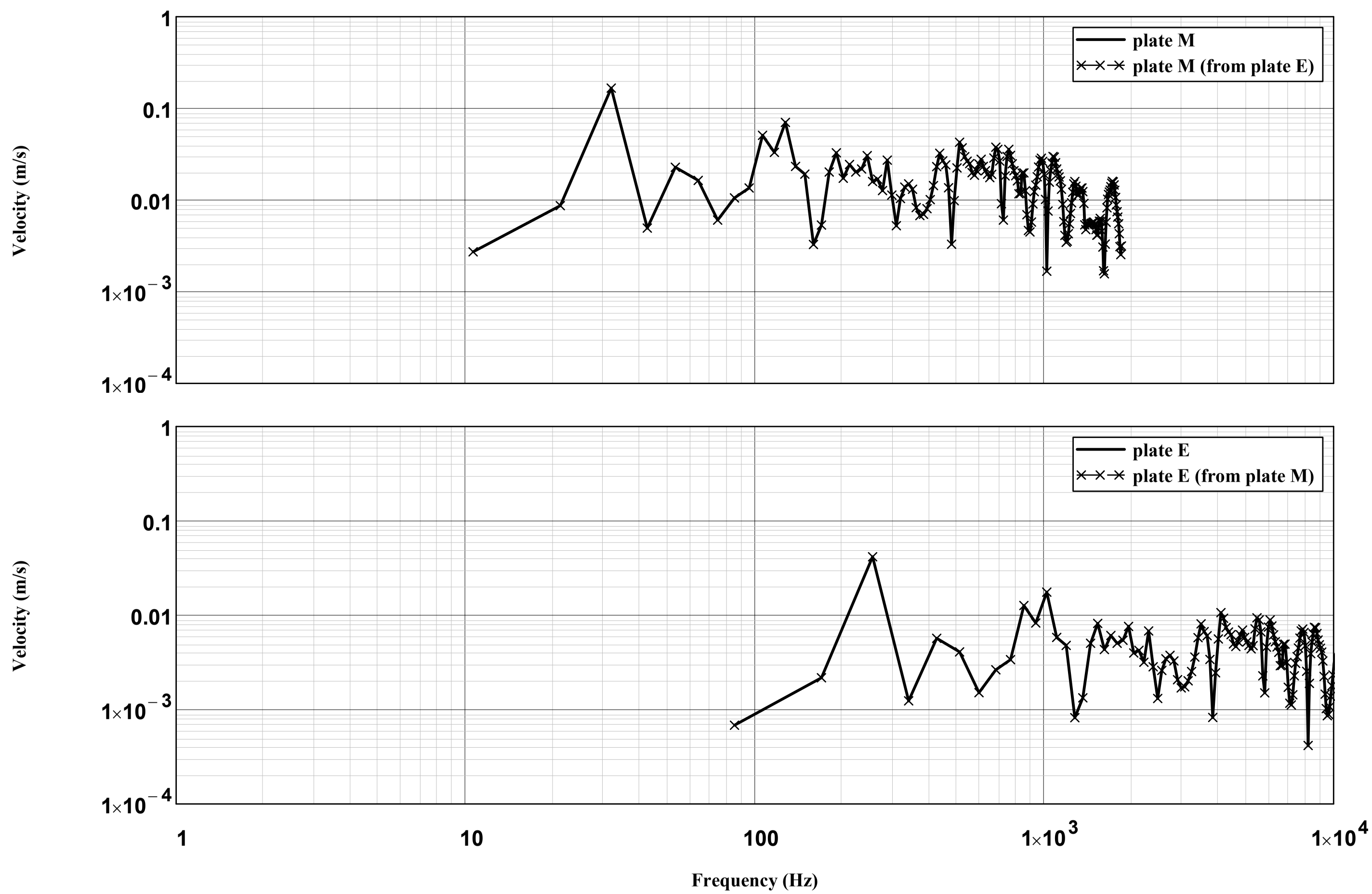

Figure 7: Plates M and E, remodulated responses.

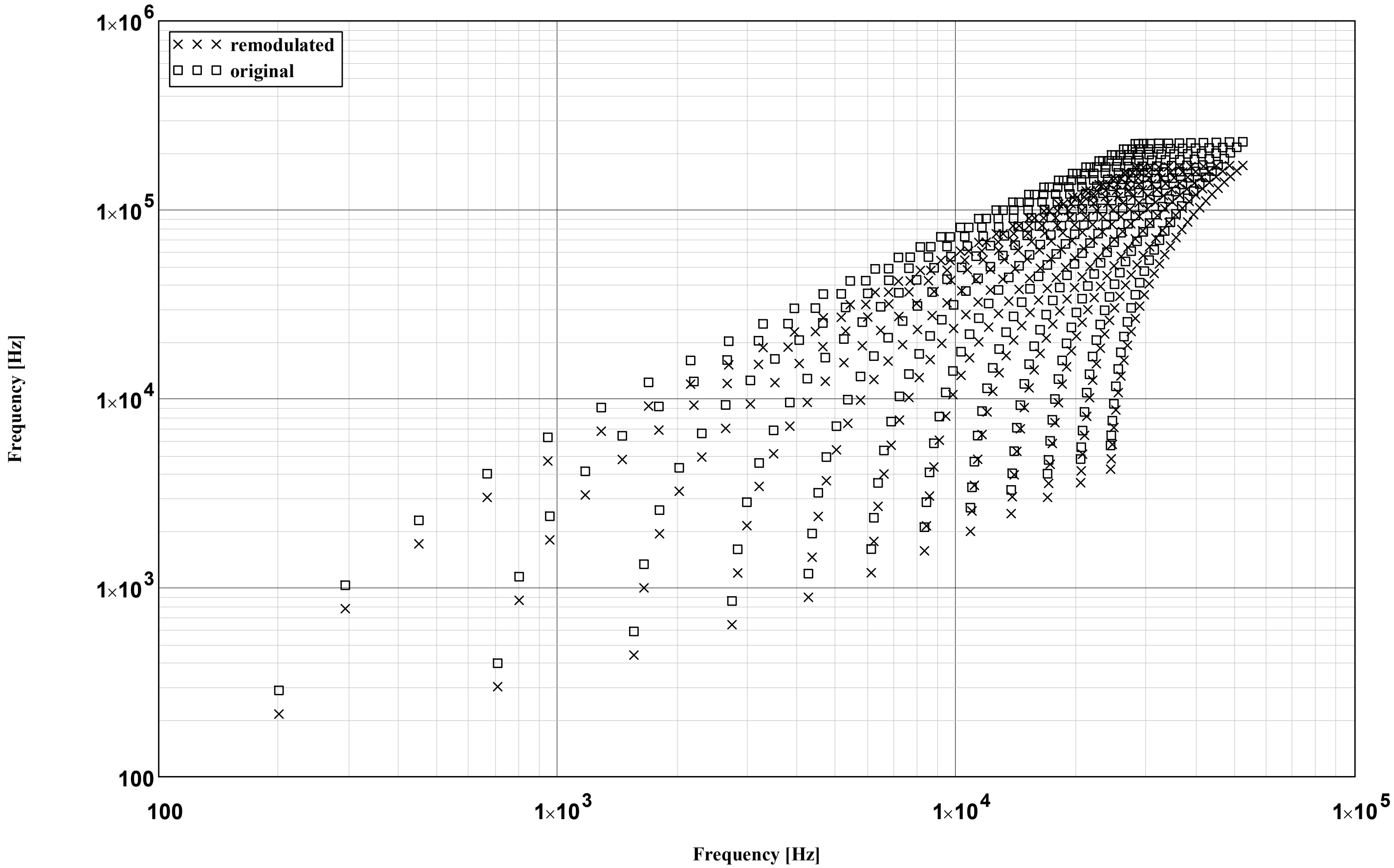

Figure 8: Plates M and A1, natural frequencies.

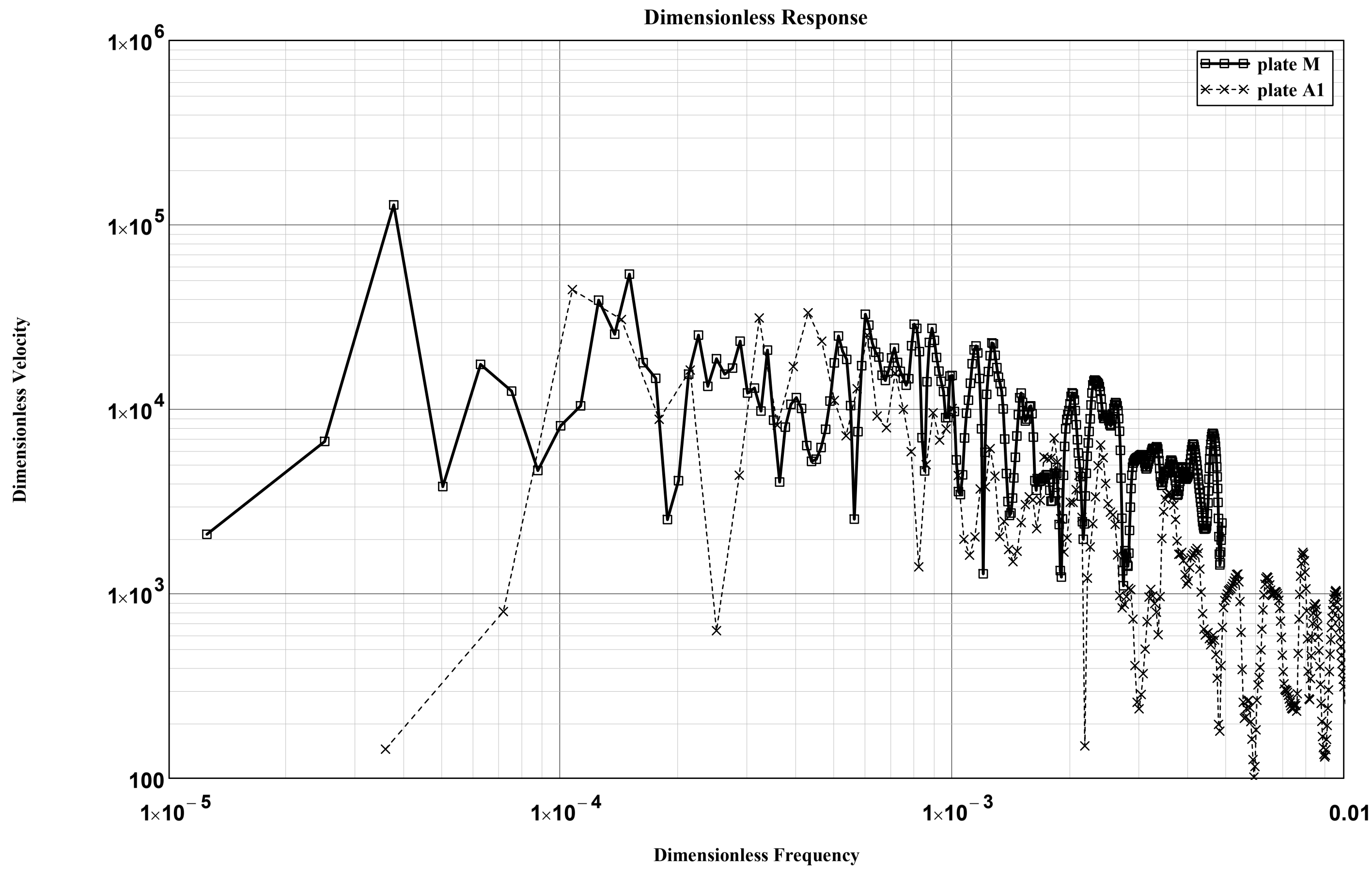

Figure 9: Plates M and A1, dimensionless responses.

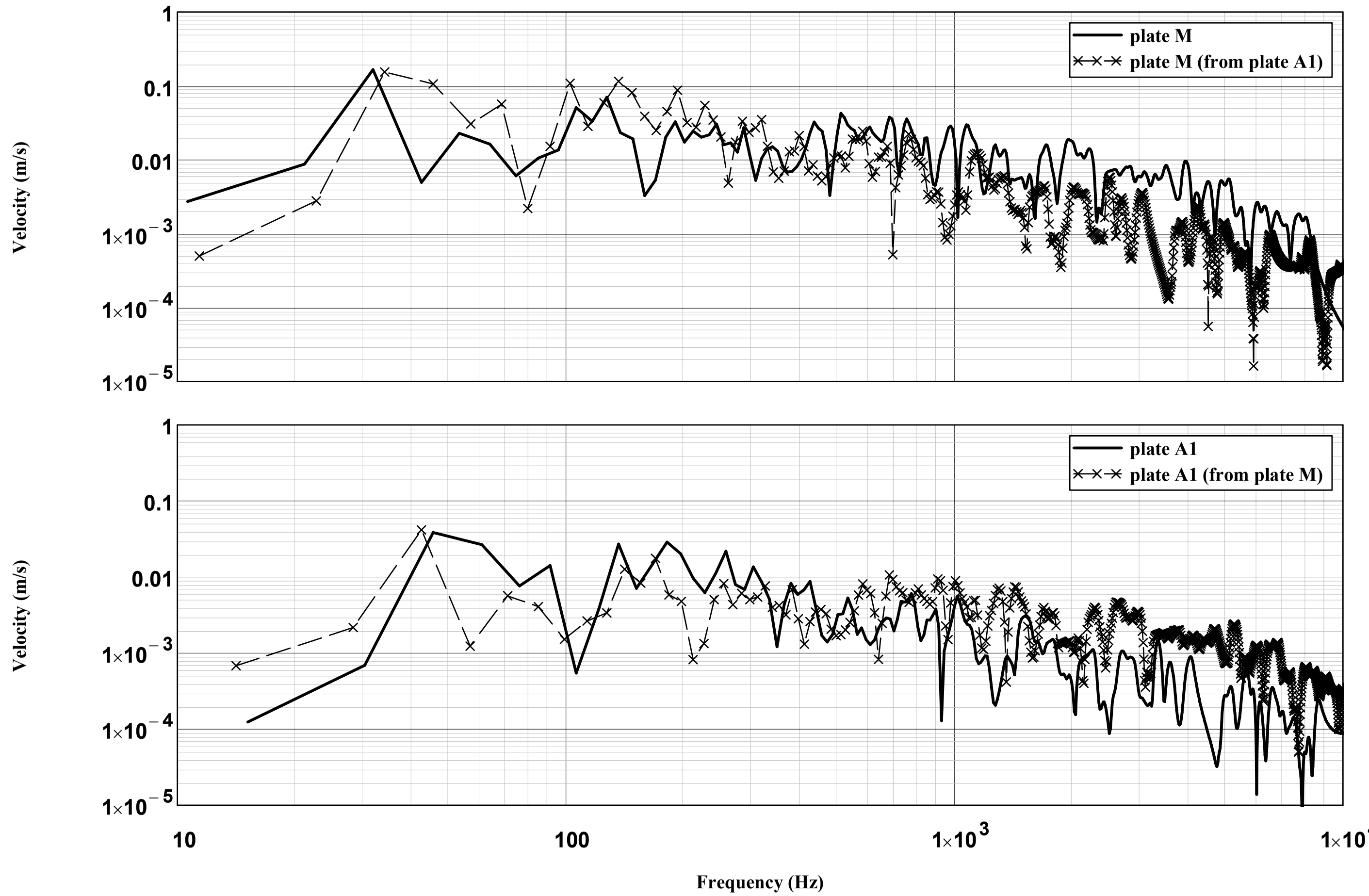

Figure 10: Plates M and A1, remodulated responses.

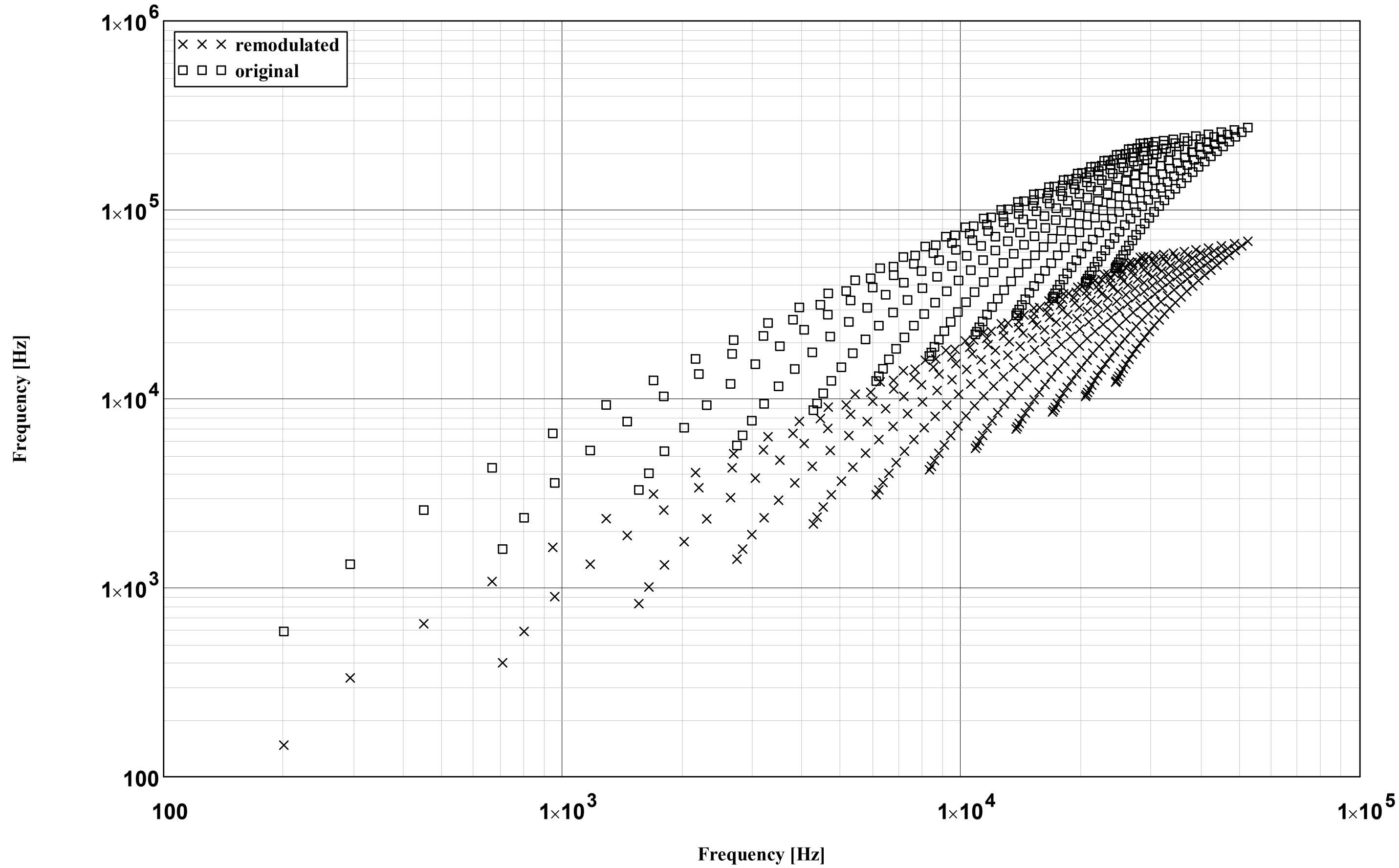

Figure 11: Plates M and A2, natural frequencies.

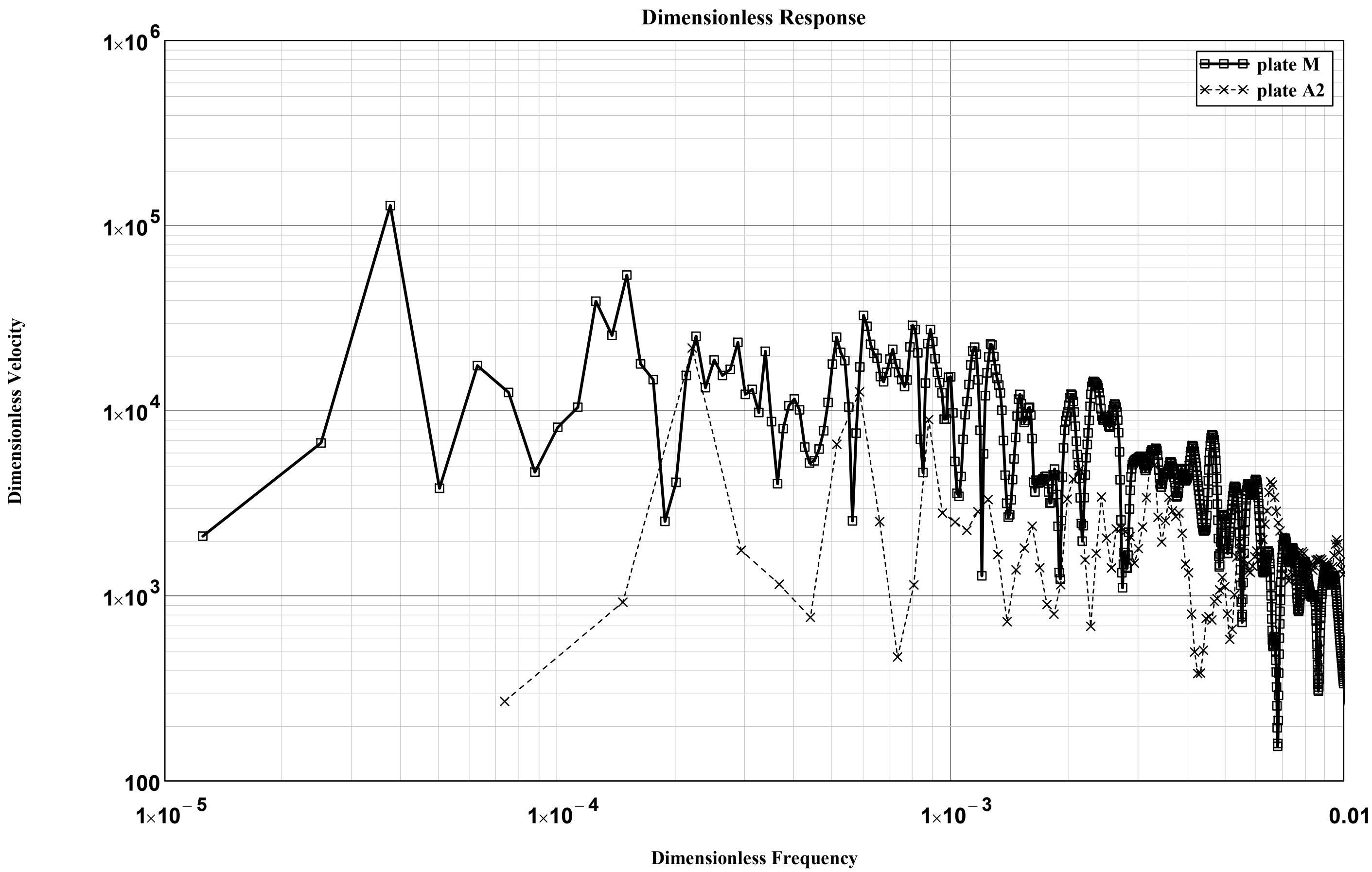

Figure 12: Plates M and A2, dimensionless responses.

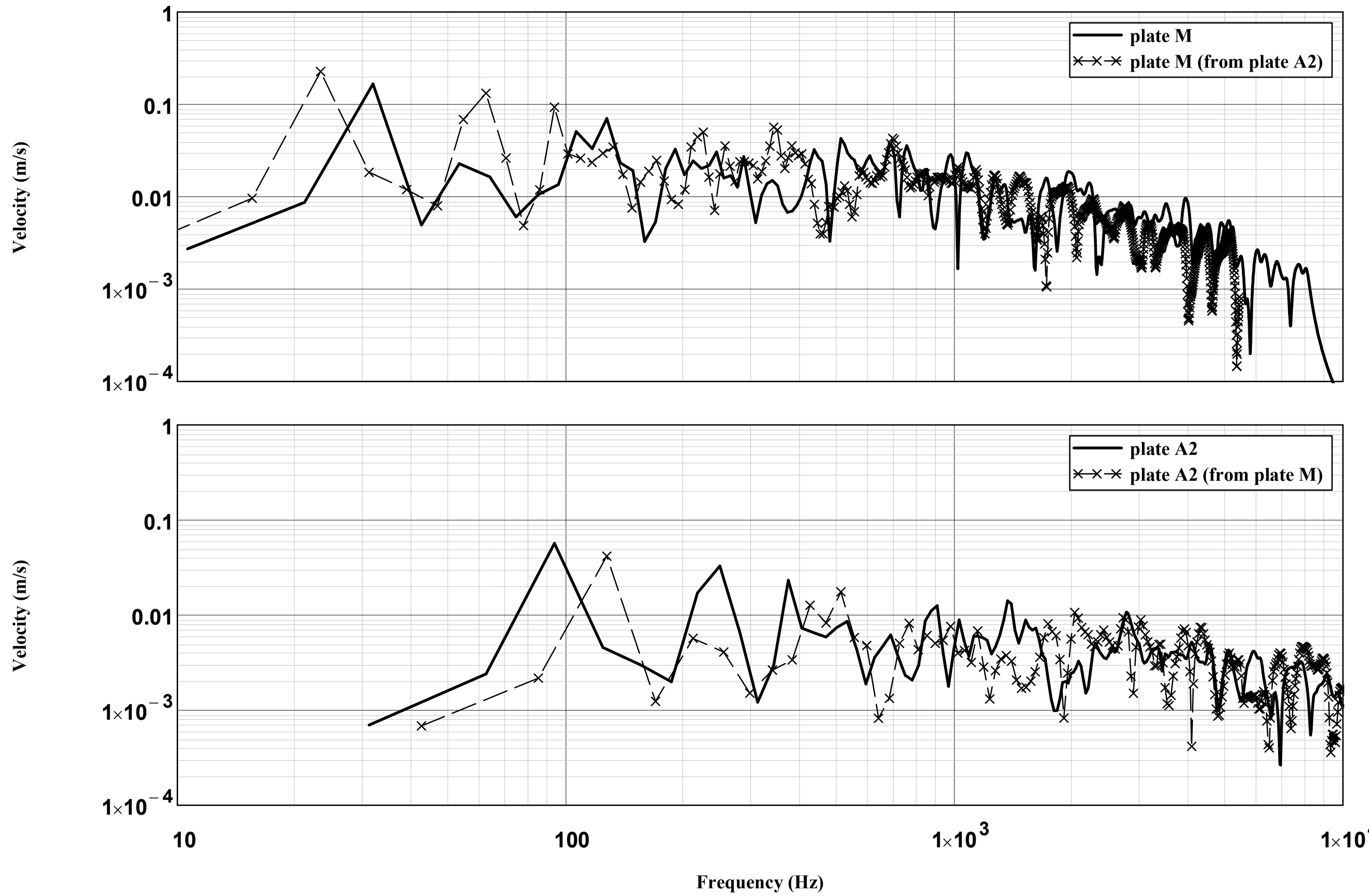

Figure 13: Plates M and A2, remodulated Responses.

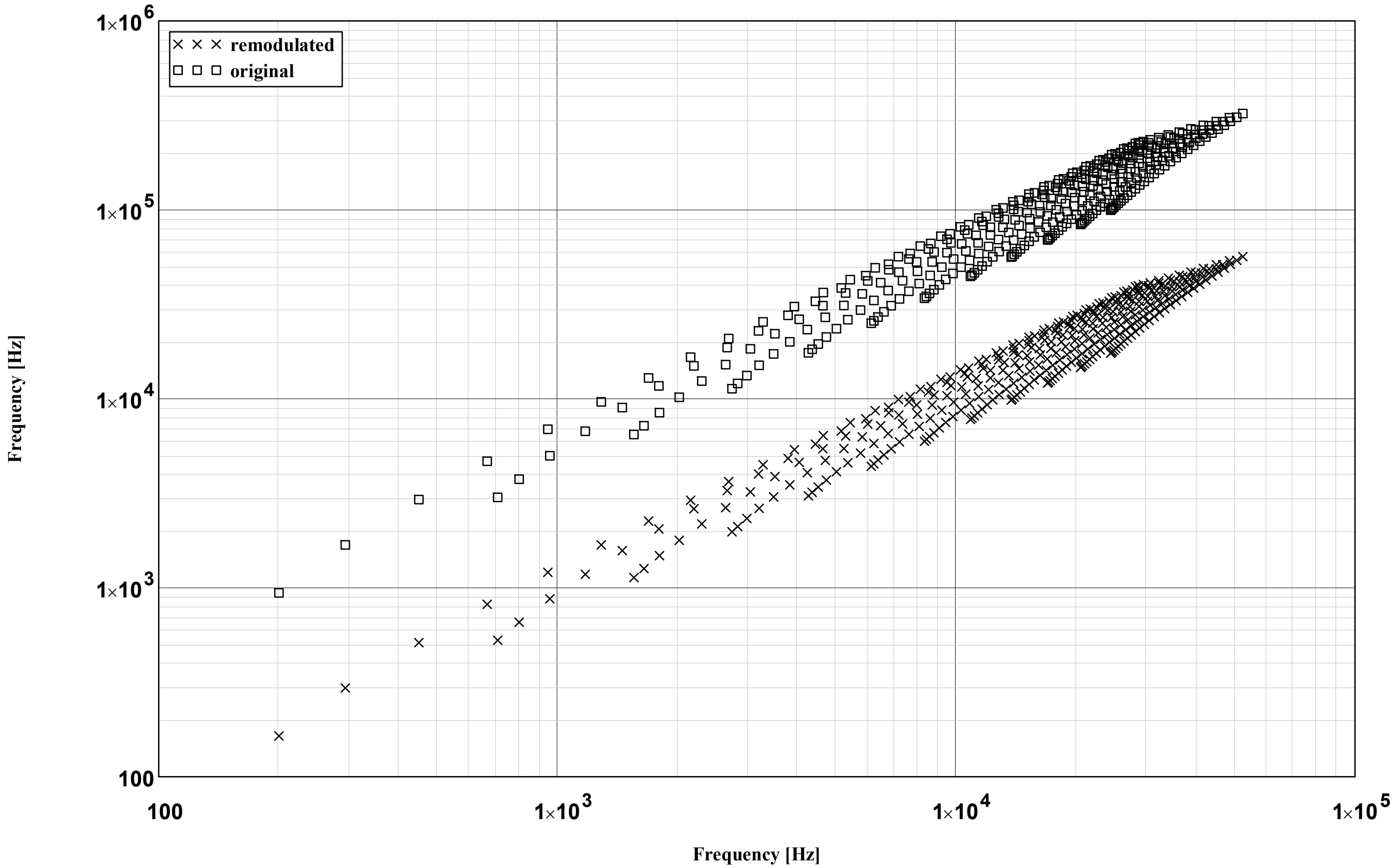

Figure 14: Plates M and A3, natural frequencies.

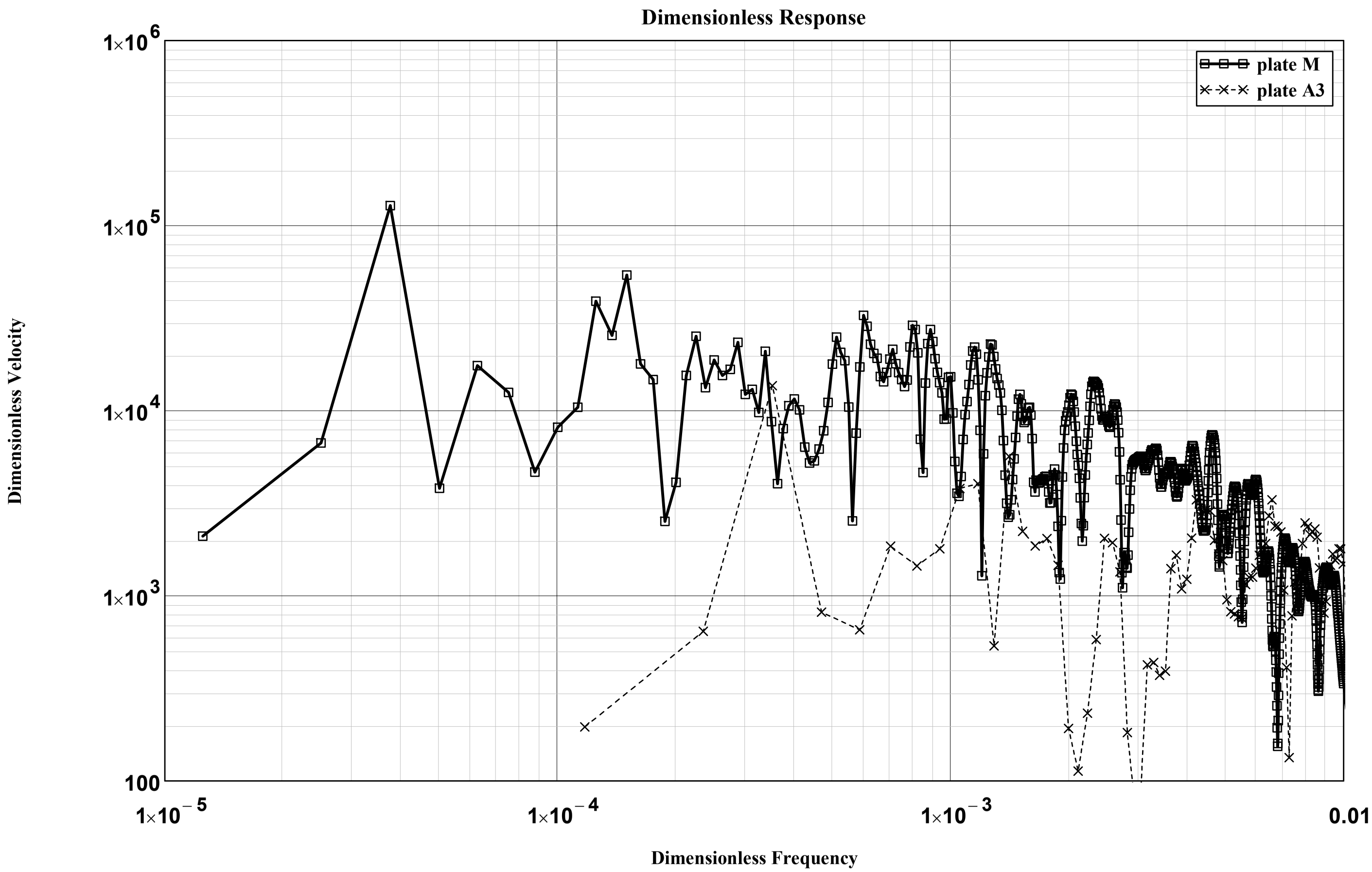

Figure 15: Plates M and A3, dimensionless responses.

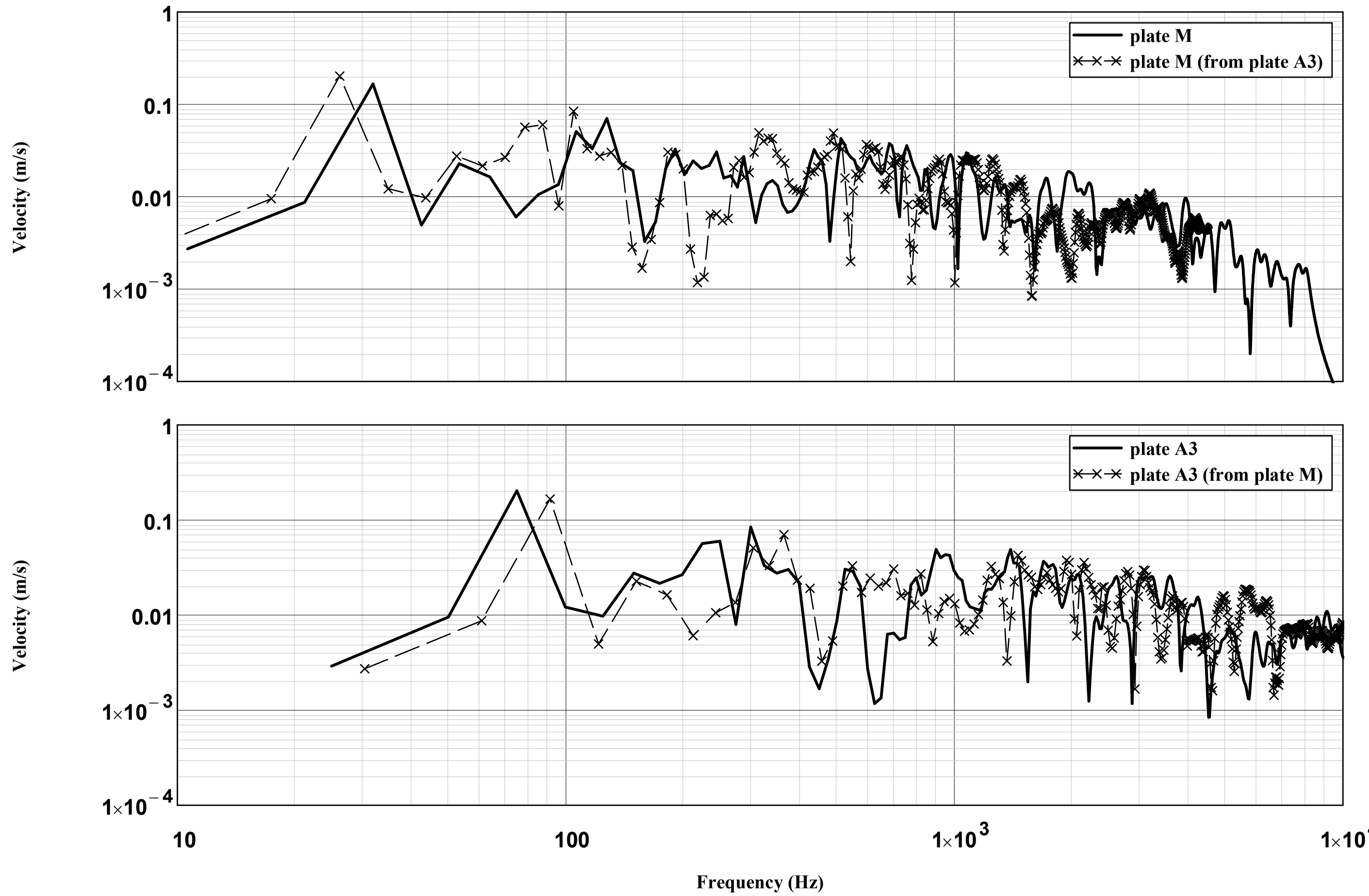

Figure 16: Plates M and A3, remodulated Responses.

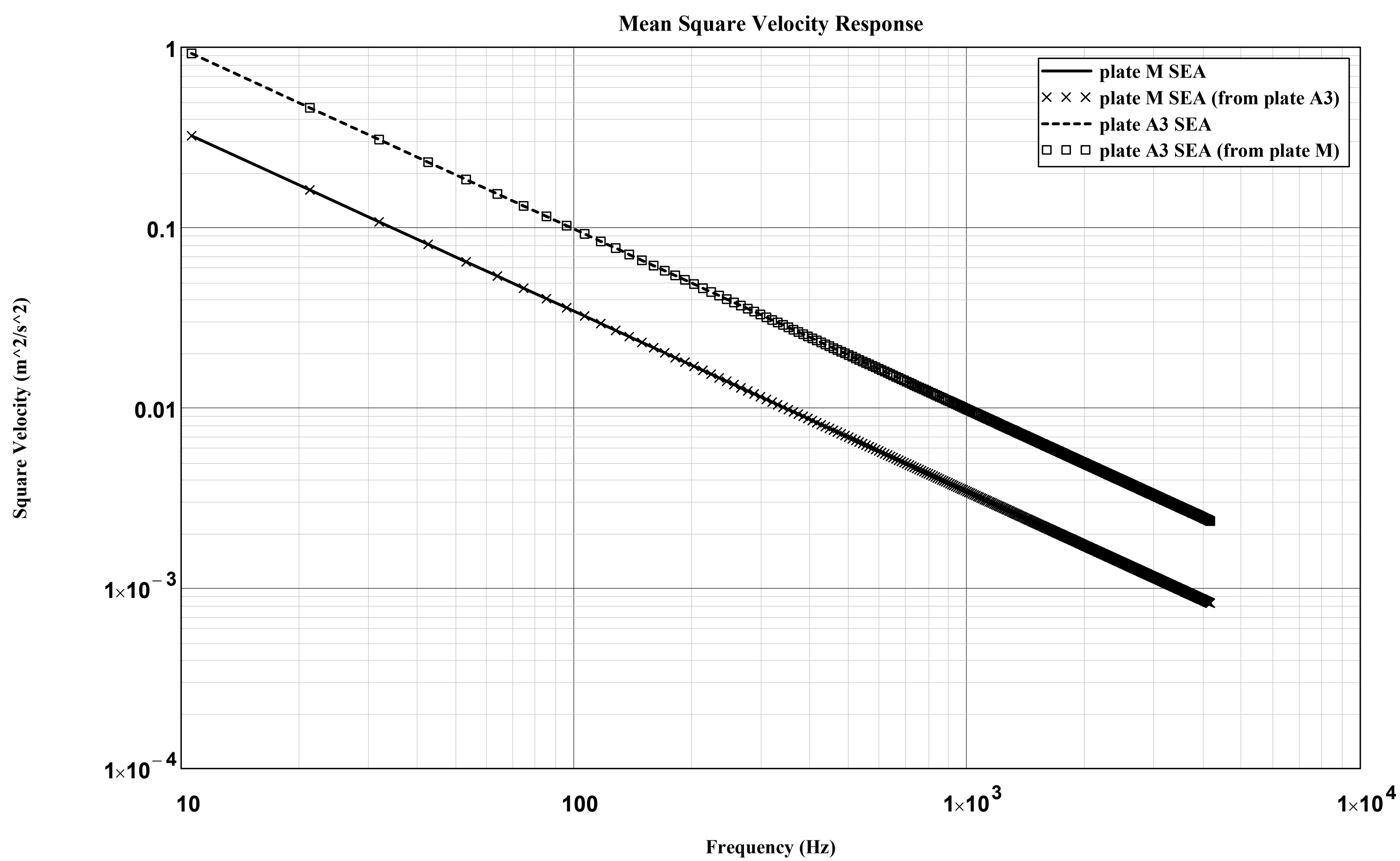

Figure 17: Plates M and A3, SEA Original and remodulated responses.