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Herding Behavior and Default in Funded Pension Schemes: The Chilean Case

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ABSTRACT: In 1981, Chile replaced the former pay-as-you-go system with a new system based on individual capitalization, private administration of assets, free choice of fund managers, and state oversight of the normal functioning of the companies. The state imposes a minimum guaranteed return for investments and requires that companies hold assets as reserves to cover that guarantee. This requirement generates a herding behavior among companies. We simulate scenarios for pension fund administrators that deviate from the norm in their investment strategies. We find that the reserve requirement is overfunded under the actual conditions.

KEY WORDS: Chile, default, herding behavior, pension system

Introduction

Chile was the pioneer at the beginning of the 1980s in the change from a pay-as-you-go funding pension system to a private capitalization system. The traditional pension system was collapsing, and the state deficit was increasing quickly. The new pension system is based on individual accounts that are administered by private companies, called pension fund administrators (PFAs). These private companies manage assets valued at USD 160 billion as of December 2013, equivalent to 80 percent of the country's gross domestic product (GDP). Corbo and Schmidt-Hebbel (2003) quantify the effects of the Chilean pension fund system on the rate of growth, asset allocation, and development of the capital market. They conclude that the reform of pensions contributed 0.5 percentage points to the average of 4.6 percent in annual GDP growth over the period 1981–2001. The main contribution of the pension system is in the development of the capital market: specifically, increased depth in the equities market and the creation of company bonds, mortgages, and seed capital. Given the positive macroeconomic effects that the Chilean economy has experienced since the change, many other countries have replicated the Chilean experience.

PFAs are by far the largest institutional investor in the country. As fund managers, these companies must meet the requirements of strong regulations to guarantee workers' pension funds. In addition to controlling the investments, the state requires that PFAs satisfy a minimum guaranteed return (MGR). The MGR is constructed according to the average of the rate of return among companies, less 2 or 4 percentage points, depending on the type of fund managed. The MGR is, in practice, the main source of risk to PFAs. Consequently, PFAs have incentive to mimic other PFAs' portfolios to avoid deviating significantly from the investment policies of the norm and thus, in turn, to earn a close rate of return. In addition, the state does not have an incentive to break this herding behavior as it must cover the difference with public funds if a PFA cannot meet the MGR.

Given this discussion, this we examine the likelihood that a PFA that breaks from the herding behavior, which we term a *rebel*, may require public funds to cover the MGR. To distinguish differences in size, we analyze extreme scenarios using as the rebel the smallest and the largest PFA (there are six PFAs in total). We also analyze changes in capital requirements and different calculation

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periods for returns. We suggest that the pension system's kept reserves are excessive and could be rechanneled in a way to still protect the participants of the system.

Literature Review

Herding Behavior

Scharfstein and Stein (1990) develop one of the first models of herding behavior among managers. They show that managers could ignore their own information and make decisions following the behavior of other managers. Although this behavior may be inefficient from a social point of view, it may be perfectly rational from the perspective of managers who wish to avoid punishment in the labor market. By making the same decision as a group, managers avoid sole blame for bad results and receive the same benefits for good results. This herding mentality can cause managers to ignore their private information.

Roll (1992) shows that performance managers evaluated by a benchmark tend to minimize tracking error volatility on their asset allocation. However, this strategy also reduces the probability of receiving a superb performance. Graham (1999) extends Scharfstein and Stein's (1990) model adding reputation costs and finds that agents tend to herd to protect their current status and level of payments. Herding behavior has been studied in banks (Jain and Gupta 1987), mutual funds (Grinblatt, Titman, and Wermers 1995; Wermers 1999), pension funds (Fernandez 2013; Lakonishok, Shleifer, and Vishny 1992; Olivares 2008; Raddatz and Schmukler 2008), institutions (Sias 2004), and managers in general (Chang, Chen, and Jiang 2012; Chen, Wang, and Lin 2008; Villatoro 2009).

Default

The traditional models based on descriptive analysis of some accounting ratios (e.g., Beaver 1966), discriminant analysis (e.g., Altman 1968; Beaver 1968), scoring system (Altman 2005; Altman, Hartzell, and Peck 1997), or logit models (e.g., Ohlson 1980) cannot be used for PFA default because the failure is based on the relation between the return and the weighted return of the system.

Merton (1974) introduces market models and includes the use of option theory as a fundamental variable when considering the volatility of returns. Thus, in line with the ideas of Black-Scholes-Merton, the value of the company follows a geometric Brownian motion, and the implied probability of bankruptcy can be deduced as the probability of not satisfying the MGR rules.

The Chilean Private Pension System

By law D.L. 3.500, the Chilean government created pension fund administrators (PFAs) when it replaced the former pay-as-you-go system with a new system that is based on individual capitalization, private administration of the funds, free choice of fund managers, and state oversight of the normal functioning of the companies.

PFAs are private institutions whose sole objective is to manage five pension funds and to grant the payment of the benefits considered in law. To satisfy this objective, the system also has a reserve requirement of at least 1 percent of the total value of the fund. PFAs must satisfy this requirement to create a safeguard of a minimum yield on investments. In case a fund falls short of the minimum, the PFA must make up the difference by withdrawing funds from its reserves. If reserves are insufficient to bring the actual return to the minimum level, the institution is liquidated and the balances of the individual accounts transferred to another PFA. In this case, the government covers the difference with general revenues.

PFAs are free to make their own investments but within a strict regulatory environment. The supervisory body of the state, the Superintendence of Pension Funds (*Superintendencia de Pensiones*), oversees the PFAs. This system is justified because workers, who are not, in general, knowledgeable about investment decisions, are compelled to contribute and are guaranteed a minimum pension by the state (Behrman et al. 2012; Mitchell and Ruiz 2011). In addition, the existence of strong regulation increases public trust in the new pension system. PFAs are compensated as a percentage of the active worker salaries. PFAs are not compensated for assets managed, and thus the system avoids the traditional moral hazard issues.

Article 45 of D.L. 3.500 explicitly states that investments must be realized to guarantee an adequate rate of return and safety. The main instruments applied by the regulator are the investment limits. Since August 2002, each PFA administers five funds differentiated according to the amount each is permitted to invest in equities, from fund type A, which can invested up to 80 percent in equities, to the fund type E, which can only invest 5 percent in equities.

Article 37 of D.L. 3.500 establishes that in each month, each PFA is responsible for ensuring that the past thirty-six months' annualized real rate of return of its funds does not fall below a limit; specifically, for fund types A and B (fund types C, D, and E), the past thirty-six months' annualized average return across funds of the same type minus 4 percentage points (2 percentage points) or the past thirty-six months' annualized average return across funds of the same type minus 50 percent of absolute value of this average return. To guarantee this condition, the law requires that PFAs hold 1 percent of the fund managed as reserve that must be invested in the same fund. When the rate of return is lower than the minimum required, a PFA can use these reserves until the MGR is achieved. If the reserves are not sufficient to guarantee this condition, the PFA has five working days to make up the difference and the required reserve. If the PFA cannot satisfy this condition, it is liquidated, and the state is responsible for the required difference. The affiliates are then assigned to other PFAs.

In practical terms, the MGR is one of the most important requirements of the pension system and the one that causes PFAs the most concern. Given the amount of funds administered, failing to meet this condition is extremely expensive to the owner of the PFA, and liquidation of the PFA is extremely likely. This requirement is well known in the market to create a herd behavior to minimize the failure probability.

Method and Data

Method

The aim of the methodology is to estimate the potential use of reserves and default probability of a PFA under different scenarios. First, we compare default probability between using the past thirty-six months required by law from the use of past twenty-four months for return calculations. Second, we introduce a rebel PFA that deviates from the other PFA investment decisions and consider scenarios in which the rebel has 1, 1.2, 1.5, and 2 times the volatility of historical PFA performance. These scenarios allow us to investigate the role of the herding effect on PFAs as they attempt to avoid default. Because the benchmark depends on the value of the fund, we analyze the PFA with the smallest and highest participation rate (USD 90 million and USD 19 billion, respectively). Third, we examine scenarios in which the PFA rebel deviates from the herd by three, six, nine, and twelve months. Finally, we change the capital required to MGR from 1 percent to 0.5 percent of the managed funds to investigate whether the margin requirements can be lowered without significantly affecting the system. By running these scenarios, we want to determine the importance of MGR of 1 percent of managed funds PFAs must maintain to avoid dropping below the required minimum return. We also estimate the conditional probability that given a PFA does not satisfy MGR, the reserve cannot cover the funds needed to ensure the required minimum return.

Given the complexity of the process involved in estimating the benchmark as the weighted return of the funds, estimating the probabilities using Monte Carlo methods as a theoretical estimation becomes impossible. To calculate the probabilities, we must make some assumptions about the PFA quote values and the probabilistic process following fund returns. We consider the six actively participating PFAs and assume that the quote values follow a diffusion process similar to Black–Scholes:

$$dS_t^i = rS_t^i dt + \sigma^i S_t^i dW_t^i, i = 1, \ldots, 6, \tag{1}$$

where $\left(W_t^1, \ldots, W_t^6\right)_{t \geq 0}$ are Brownian motions that have covariance between them and satisfy $d < W^i, W^j >_t = \rho_{ij} dt$. This covariance captures the correlation between quote funds.

The solution to this equation is a geometric Brownian motion:

$$S_t^i = S_0^i \exp^{\left(r - 0.5\sigma^{i^2}\right)t + \sigma^i W_t}, i = 1, \ldots, 6. \tag{2}$$

Thus, logarithmic returns are given as

$$R_t^{h,i} = \log\left(\frac{S_{t+h}^i}{S_t^i}\right) = \left(r - 0.5\sigma^{i^2}\right)h + \sigma^i (W_{t+h}^i - W_t^i), i = 1, \ldots, 6. \tag{3}$$

We can deduce that the logarithmic returns $R_t^h, \ldots, R_t^{nh}$, are independent, identically distributed (iid) multinormal variables with

$$R_t^{nh} \approx N((r - 0.5Diag(\Sigma))h, \Sigma), \tag{4}$$

where $\Sigma = (\Sigma)_{i,j} = \sigma^i \sigma^j \rho_{i,j} h$. This equation plays a fundamental role in performing simulations.

The goal of the Monte Carlo method is to perform a simulation of the trajectory of the quote funds. Once this path is established, we can determine whether a default event occurs given the return estimates and the monthly minimum return for the benchmark under each of the scenarios described. Finally, we can estimate the mean, standard deviation, and confidence intervals at 99 percent for all simulations.

Given that returns are independent variables, we estimate Σ as the variance–covariance matrix of actual daily returns of the funds. Then, we simulate a series of six independent, identically distributed standard normal variables in Matlab $X_1, \ldots, X_n$ and estimate Cholesky decomposition of $\Sigma = L^T L$, so the variable $\left(r - 0.5\sigma^{i^2}\right)h + LX$ is distributed as daily returns.

For the simulation of a trajectory, we first simulate the standard normal variables $X_1, \ldots, X_n$ and we get a daily return. We then estimate the value of the fund for the day according to the values of good funds from April 1, 2008 to February 29, 2012 and reestimate the value of Σ. The process is iterated and stops after thirty-six months simulated or 36*20 days. We increase the volatility of the rebel PFA by changing the corresponding coordinates on the diagonal of Σ(squared because it is the covariance matrix) for the duration of the corresponding stages (three, six, nine, and twelve months).

For these simulations, to estimate odds using a Monte Carlo method, we must have independent, identically distributed binary variables; that is, we need Bernoulli variables with mean p and variance p (1–p). The length of confidence intervals at 99 percent is given as

$$2 * 2.756 * \frac{\sigma}{\sqrt{n}} = 2 * 2.756 * \frac{\sqrt{p(1-p)}}{\sqrt{n}} \leq 2 * 2.756 * \frac{\sqrt{0.5 * 0.5}}{\sqrt{n}} = \frac{2.756}{\sqrt{n}}. \tag{5}$$

The inequality comes from the fact that $p \mapsto p(1-p)$ is a quadratic function with a maximum of 0.5. Thus, to obtain x percent accuracy, we must run at least $\left(\frac{2.756}{x\%}\right)^2$ simulations. As a result, our accuracy is 1 percent using 64,400 simulations.

Data

We use daily quote values collected on the funds from the Superintendence of Pension Funds (*Superintendencia de Pensiones*) for fund type C, which is the most representative type of the pension funds, managed for each PFA for the period from April 1, 2008 to February 29, 2012 in Chilean pesos. To obtain real values, we use the *unidad de fomento* (UF).[1] The MGR is defined according to the weighted average of the rate of return of the pension funds according to the size of the PFA, which is set by law.

Results

Table 1 provides the estimates of probability of using reserves for the smallest and largest PFA rebels under different periods of calculation for minimum return guaranteed. We find that, for the smallest rebel, the default probability increases according to the months of being a rebel and decreases as the period of calculation increases. In addition, the default probability for rebels increases with more volatility. With the actual level of volatility, the smallest PFA has a 6 percent probability to use reserves being a rebel. We find the same pattern for the largest rebel PFA. However, the probability of using reserves for the largest rebel PFA is about 0 percent with the actual volatility. If the largest PFA becomes rebel, the probability of using reserves goes up faster not only for this PFA, but also for the rest of the PFAs. Finally, the results show that the group without a rebel increases (decreases) the default probability when the rebel PFA is the largest (smallest). The latter result implies that the smallest does not affect significantly the other probability default. The more months the largest PFA deviates, the higher is the default probability for the rest of the PFAs.

Table 1. Probability of using reserves according to different months for calculations and deviations for rebel and nonrebel PFA

	Smallest PFA rebel						Group without rebel					
Rule	24 months			36 months			24 months			36 months		
Time rebel	1σ	1.5σ	2σ	1σ	1.5σ	2σ	1σ	1.5σ	2σ	1σ	1.5σ	2σ
3 months	17.14	34.34	43.67	6.20	18.36	27.58	0.039	0.030	0.018	0.003	0.000	0.000
6 months	16.99	44.62	56.76	6.13	26.02	37.10	0.039	0.024	0.024	0.002	0.002	0.000
9 months	17.02	52.18	65.24	6.19	31.14	43.18	0.018	0.021	0.018	0.000	0.002	0.000
12 months	17.15	58.41	71.22	6.23	35.51	47.73	0.029	0.024	0.018	0.002	0.002	0.003

	Largest PFA rebel						Group without rebel					
Rule	24 months			36 months			24 months			36 months		
Time rebel	1σ	1.5σ	2σ	1σ	1.5σ	2σ	1σ	1.5σ	2σ	1σ	1.5σ	2σ
3 months	0.00	2.71	11.95	0.00	0.18	3.09	17.37	19.08	21.84	6.15	7.22	9.11
6 months	0.00	10.18	26.32	0.00	2.16	11.10	17.45	21.06	25.90	6.20	8.31	11.74
9 months	0.00	17.88	36.65	0.00	5.34	18.23	17.50	22.97	29.18	6.16	9.52	14.21
12 months	0.00	24.59	43.39	0.00	8.88	24.20	17.47	24.86	31.45	6.19	10.49	16.18

Notes: PFA = pension fund administrator (*administradora de fondos de pensiones*). Results are in percentage points. $k\sigma$ means times the volatility assumed for the PFA.
Source: Calculations based on the authors' own simulations.

Table 2. Probability of a PFA not meeting the required guarantee

Smallest PFA	0.5% guarantee as reserve						1% guarantee as reserve					
Rule	24 months			36 months			24 months			36 months		
Time rebel	1σ	1.5σ	2σ	1σ	1.5σ	2σ	1σ	1.5σ	2σ	1σ	1.5σ	2σ
3 months	1.27	8.83	17.80	0.25	2.17	6.63	0.17	2.76	8.56	0.02	0.37	2.16
6 months	1.29	14.66	28.30	0.23	4.27	12.10	0.17	4.81	14.32	0.02	0.69	4.10
9 months	1.28	19.25	34.68	0.24	6.08	15.66	0.18	6.46	17.78	0.02	0.96	5.32
12 months	1.24	22.71	39.34	0.23	7.56	18.30	0.17	7.74	20.48	0.02	1.20	6.34
Largest PFA	**0.5% guarantee as reserve**						**1% guarantee as reserve**					
Rule	**24 months**			**36 months**			**24 months**			**36 months**		
Time rebel	1σ	1.5σ	2σ	1σ	1.5σ	2σ	1σ	1.5σ	2σ	1σ	1.5σ	2σ
3 months	1.27	1.94	5.47	0.20	0.25	0.60	0.19	0.23	1.16	0.02	0.03	0.06
6 months	1.35	3.53	11.97	0.21	0.36	2.29	0.19	0.38	2.96	0.02	0.03	0.25
9 months	1.31	5.53	17.14	0.20	0.62	4.17	0.20	0.60	4.33	0.02	0.04	0.55
12 months	1.32	7.39	20.86	0.20	0.88	5.79	0.20	0.80	5.49	0.02	0.06	0.80

Notes: PFA = pension fund administrator (*administradora de fondos de pensiones*). Results are in percentage points. $k\sigma$ means times the volatility assumed for the PFA.
Source: Calculations based on the authors' own simulations.

Table 2 provides the results for the probability of not meeting the MGR. Under the current rules, to keep 1 percent of reserves with a calculation period of thirty-six months, the default probability for a smallest PFA is from 0.02 percent (3 month rebel (3 m), 1 times the volatility (σ)) to 6.34 percent (12 m, 2σ). If we reduce the capital requirement to 0.5 percent of the managed funds, the default probability increases from 0.23 percent (12 m, σ) to 18.30 percent (12 m, 2σ). Given the same period of thirty-six months, the default probability for the largest PFA goes from 0.02 percent (any rebel period, σ) to 0.80 percent (12 m, 2σ) at the 1 percent minimum reserve level and from 0.20 percent to 5.79 percent (12 m, 2σ) at the 0.5 percent minimum reserve level. When we reduce the calculation period to twenty-four months for MGR, the default probability for the smallest PFA ranges from 0.17 percent to 20.48 percent (12 m, 2σ) at the 1 percent minimum capital requirement and from 1.24 percent to 39.34 percent (12 m, 2σ) at the 0.5 percent minimum capital requirement. For the same period, the probability of default for the largest PFA is from 0.19 percent to 5.49 percent at the 1 percent minimum capital requirement and from 1.27 percent to 0.86 percent at the 0.5 percent minimum capital requirement.

Table 3 reports the default probability conditional on the use of reserves for the PFA. Under the current rules, to keep 1 percent of reserves with a calculation period of thirty-six months, the default probability for a smallest PFA is from 0.36 percent (3 m, σ) to 13.28 percent (12 m, 2σ). If we reduce the capital requirement to 0.5 percent of the managed funds, the default probability increases to 3.75 percent (12 m, 2σ) to 38.34 percent (12 m, 2σ). Given the same time period of thirty-six months, the default probability for the largest PFA goes from 0.34 percent (anytime rebel, σ) to 1.97 percent (12 m, 2σ) at the 1 percent minimum reserve level and from 3.21 percent (anytime rebel, σ) to 14.34 percent (12 m, 2σ) at the 0.5 percent minimum reserve level. When we reduce the calculation period to twenty-four months for MGR, the default probability for the smallest PFA ranges from 1.02 percent (3 m or 12 m, σ) to 28.75 percent (12 m, 2σ) at the 1 percent minimum capital requirement and from 7.21 percent (12 m, σ) to 55.23 percent (12 m, 2σ) at the 0.5 percent

Table 3. Conditional probability of default according to percentage of guarantee required

Smaller AFP rebel						Group without rebel					
24 months			**36 months**			**24 months**			**36 months**		
1	1.5	2	1	1.5	2	1	1.5	2	1	1.5	2
3.18	15.64	25.38	1.21	7.47	14.87	0.65	0.12	0.07	0.30	0.05	0.03
16.07	72.66	81.79	5.71	52.98	65.91	1.11	0.50	0.40	0.45	0.25	0.21
Larger AFP rebel						Group without rebel					
24 months			36 months			24 months			36 months		
1	1.5	2	1	1.5	2	1	1.5	2	1	1.5	2
0.00	1.34	7.71	0.00	0.08	1.74	3.83	4.94	7.03	1.51	1.90	2.53
7.71	35.64	45.41	1.74	23.09	35.86	17.18	43.20	48.44	6.16	22.70	36.28
0.5% guarantee as reserve						1% guarantee as reserve					
24 months			36 months			24 months			36 months		
1	1.5	2	1	1.5	2	1	1.5	2	1	1.5	2
7.48	43.06	58.76	3.25	25.23	42.35	1.28	16.02	31.14	0.24	4.06	14.53
21.54	45.98	60.55	12.73	26.99	43.08	5.35	18.59	34.02	0.99	4.52	15.25
0.5% guarantee as reserve						1% guarantee as reserve					
24 months			36 months			24 months			36 months		
1	1.5	2	1	1.5	2	1	1.5	2	1	1.5	2
7.48	18.03	30.21	3.25	7.57	14.05	1.28	2.74	7.63	0.24	0.31	1.46
21.54	20.93	32.56	12.73	12.74	17.15	5.35	4.34	9.84	0.99	1.39	2.53
0.5% guarantee as reserve						1% guarantee as reserve					
24 months			36 months			24 months			36 months		
1σ	1.5	2	1	1.5	2	1	1.5	2	1	1.5	2
0.238	6.735	14.91	0.039	1.885	6.297	0.008	0.019	0.022	0.001	0.002	0.004
3.461	33.41	49.52	0.727	14.3	28.39	0.059	0.093	0.136	0.004	0.011	0.032
0.5% guarantee as reserve						1% guarantee as reserve					
24 months			36 months			24 months			36 months		
1	1.5	2	1	1.5	2	1	1.5	2	1	1.5	2
0	0.242	2.329	0	0.006	0.244	0.049	0.135	0.536	0.004	0.006	0.037
1.661	7.459	14.79	0.222	2.942	6.15	0.919	1.875	4.766	0.061	0.316	0.918
0.5% guarantee as reserve						1% guarantee as reserve					
24 months			36 months			24 months			36 months		
1σ	1.5σ	2σ	1σ	1.5σ	2σ	1σ	1.5σ	2σ	1σ	1.5σ	2σ
7.40	25.69	40.74	3.98	11.83	24.03	1.02	8.03	19.59	0.36	2.03	7.82
7.57	32.84	49.85	3.81	16.40	32.63	1.01	10.76	25.22	0.37	2.65	11.06
7.52	36.87	53.14	3.87	19.51	36.26	1.05	12.37	27.24	0.37	3.08	12.31
7.21	38.87	55.23	3.75	21.28	38.34	1.02	13.25	28.75	0.36	3.39	13.28

0.5% guarantee as reserve						1% guarantee as reserve					
24 months			36 months			24 months			36 months		
1σ	1.5σ	2σ	1σ	1.5σ	2σ	1σ	1.5σ	2σ	1σ	1.5σ	2σ
7.29	8.89	16.19	3.28	3.32	4.93	1.10	1.07	3.42	0.34	0.47	0.49
7.74	11.31	22.92	3.35	3.40	10.01	1.10	1.23	5.68	0.34	0.33	1.11
7.47	13.54	26.05	3.25	4.18	12.85	1.14	1.47	6.58	0.34	0.28	1.69
7.55	14.94	27.87	3.21	4.56	14.34	1.13	1.63	7.34	0.34	0.31	1.97

Notes: PFA = pension fund administrator (*administradora de fondos de pensiones*). Results are in percentage points. $k\sigma$ means times the volatility assumed for the PFA.

minimum capital requirement. For the same period, the probability of default for the largest PFA is from 1.10 percent (3 m or 6 m, σ) to 7.34 percent (12 m, 2σ) at the 1 percent minimum capital requirement and from 7.29 percent (3 m, σ) to 27.87 percent (12 m, 2σ) at the 0.5 percent minimum capital requirement.

These findings show that the risk of a PFA using its reserves to cover the minimum return is low under the current rules of using thirty-six months of historical returns for the MGR estimation and the low dispersion of investments by PFAs. In addition, the state could reduce the reserve requirement from the current 1 percent to 0.5 percent of the fund management without greatly varying the probability of use of those reserves.

We also find that the largest PFA may have more incentive to behave as a rebel as its probability of falling below the MGR is much smaller than that of other PFAs. This result is justified, given that larger PFAs have a greater weight in the calculation of MGR and their movements thus have greater risk coverage because they can move the system on its way.

Robustness Checks

We perform several robustness checks using a multivariate stochastic model with jumps. The structure presented in the Method section induces high correlation in returns. If we use a jump diffusion process the correlations are allowed to be much lower.

Merton's Model

Merton (1976) introduces an asset diffusion process as

$$S_t = S_0 Exp\left(\left(\mu - 0.5\sigma^2\right)t + \sigma W_t\right) Exp\left(\sum_{i=1}^{N_t} J_i\right),$$

where J_i are normal variables iid with media 0 and variance σ_s^2; W_t are Brownian motions; and N_t is the jump numbers until time t. Each jump comes with an additional normal variable added to the normal asset evolution. We have assumed a Poisson process:

$$N_\delta \approx N_{2\delta} - N_\delta \approx N_{3\delta} - N_{2\delta} \approx \ldots \approx N_{(k+1)\delta} - N_{k\delta} \approx Poisson(\lambda\delta)\ N_0 = 0.$$

Thus, the logarithmic returns are

$$\begin{aligned} R_t^{t+h} &= Ln\left(\frac{S_{t+h}}{S_t}\right) = \left(\mu - 0.5\sigma^2\right)h + \sigma(W_{t+h} - W_t) + \sum_{i \geq N_t}^{N_{t+h}} J_i \\ &= \left(\mu - 0.5\sigma^2\right)h + \sigma(W_{t+h} - W_t) + Z_{t,h} \approx N\left(\left(\mu - 0.5\sigma^2\right)h, \sigma^2 h + (\mathrm{N_{t+h}} - \mathrm{N_t})\sigma_\mathrm{s}^2\right). \end{aligned}$$

Comparison Between Models

We consider volatility increases k times of its volatility in the Method section, while the model with jumps does so but only if a jump occurs in a period of rebellion. Therefore, the jump model increases the volatility for a lower period. For the first model,

$$\frac{\rho_{R_1,R_{b(nonrebel)}}}{\rho_{R_1 y R_{b(rebel)}}} = \frac{\frac{Cov(R_1,R_b)}{\sqrt{var(R_1)var(R_b)}}}{\frac{Cov(R_1,R_b)}{\sqrt{var(R_1)k^2 var(R_b)}}} = k^2.$$

Then, correlation decreases in $1/k^2$ for rebel periods. We use multiples $k = 1$, 1.5 and 2, then correlations are 100 percent, 44 percent, and 25 percent, respectively.

The model with jumps assumes logarithmic return follows $\approx N\left((\mu - 0.5\sigma^2), \sigma^2 + (\mathrm{N}_1)\sigma_s^2\right)$. Then,

$$Var(R_b) = E\left(R_b^2\right) - E(R_b)^2 = E\left(E\left(R_b^2|N_1\right)\right) - E(R_b)^2$$
$$= E\left(E\left(\sigma^2 + (\mathrm{N}_1)\sigma_s^2|N_1\right) + E(R_b)^2\right) - E(R_b)^2 = \sigma^2 + \lambda\sigma_s^2$$

$$Cov(R_1, R_b) = E(R_1R_b) - E(R_1)E(R_b) = E(E(R_1R_b|N_1)) - E(R_1)E(R_b)$$
$$= E(E(R_1)E(R_b|N_1)) - E(R_1)E(R_b) = E(E(R_1)E(R_b)) - E(R_1)E(R_b) = 0.$$

Thus, covariances are 0 between rebel PFA and others PFAs. Also, we can get the same variances if $k = \sqrt{(1 + \lambda\theta)}$. Finally, the θ values for the combination of λ and σ parameters are

λ	σ	1.5σ	2σ
1	0	1.25	3
1/3	0	3.75	9
1/6	0	7.5	18

Results

Table 4 provides the estimates for the smallest and largest PFA rebels under different periods of calculation for minimum return guaranteed and jumps. All robustness results are consistent with previous findings. The probability of using reserves increases slightly under this robustness analysis.

Table 5 provides the results for the probability of not meeting the MGR using a diffusion process with jumps while estimations reported in Table 6 show the probability of extending beyond the capital kept in companies as reserves. Results are comparable with results in Tables 2 and 3, respectively. The default probability increases for this analysis since correlations in returns are much lower now. It is more probable to get worse returns when the volatility increases. However, when the smallest (largest) PFA does not behave as a rebel, the probability is in the range 0.02–0.03 percent (0.02–0.03 percent) with a rule of thirty-six past months' returns to 0.19–0.23 percent (0.19–0.21 percent) with a rule of twenty-four past months' returns at a 1 percent of capital requirement.

Conclusion

We discuss the effect of PFAs that deviate in their investments policy on the probability that they will use the reserves kept in the pension funds to cover the MGR. The results of the simulations show that the probability of using reserves is low for the actual rules and volatiles (6 percent for the smallest PFA and 0 percent for the largest PFA). We also investigate changes in the rate of the capital required; specifically, from a 1 percent to a 0.5 percent minimum reserve. The probability of using reserves increases to 17 percent for the smallest PFA and continues being 0 percent for the largest PFA. We can conclude that with the actual rules, smaller PFAs have more incentive to herd.

Table 4. Probability of using reserves according to different months for calculations and deviations for rebel and nonrebel PFAs (using jumps)

	Smallest PFA rebel						Group without rebel					
Rule	24 months			36 months			24 months			36 months		
Time rebel	1σ	1.5σ	2σ	1σ	1.5σ	2σ	1σ	1.5σ	2σ	1σ	1.5σ	2σ
Jump = 1 month												
3 months	17.31	29.23	41.47	6.26	15.42	29.35	0.03	0.02	0.02	0.003	0.000	0.002
6 months	17.31	41.34	60.45	6.26	24.58	44.21	0.03	0.02	0.02	0.003	0.000	0.002
9 months	17.31	50.05	71.04	6.26	30.63	52.36	0.03	0.02	0.02	0.003	0.000	0.002
12 months	17.31	57.04	77.39	6.26	35.37	57.63	0.03	0.02	0.02	0.003	0.000	0.002
Jump = 3 months												
3 months	17.26	29.87	33.82	6.16	18.70	24.30	0.03	0.03	0.04	0.000	0.002	0.000
6 months	17.26	44.09	51.77	6.16	30.89	41.40	0.03	0.03	0.03	0.000	0.002	0.000
9 months	17.26	54.13	64.00	6.16	38.93	51.50	0.03	0.02	0.03	0.000	0.000	0.000
12 months	17.26	61.93	72.26	6.16	44.92	58.09	0.03	0.02	0.03	0.000	0.000	0.000
Jump = 6 months												
3 months	17.24	26.07	27.35	6.19	15.79	17.79	0.02	0.04	0.02	0.003	0.000	0.000
6 months	17.24	37.70	40.34	6.19	27.13	31.24	0.02	0.03	0.02	0.003	0.000	0.003
9 months	17.24	47.16	50.75	6.19	35.84	41.25	0.02	0.03	0.03	0.003	0.000	0.005
12 months	17.24	54.77	59.31	6.19	42.33	49.05	0.02	0.03	0.03	0.003	0.000	0.005

	Largest PFA rebel						Group without rebel					
Rule	24 months			36 months			24 months			36 months		
Time rebel	1σ	1.5σ	2σ	1σ	1.5σ	2σ	1σ	1.5σ	2σ	1σ	1.5σ	2σ
Jump = 1 Month												
3 months	0.00	2.40	17.61	0.00	0.42	9.41	17.22	18.95	23.80	6.04	7.25	11.22
6 months	0.00	9.73	36.39	0.00	2.73	23.59	17.22	21.28	30.40	6.04	8.62	17.18
9 months	0.00	17.47	46.96	0.00	6.19	33.31	17.22	23.64	34.13	6.04	9.89	21.54
12 months	0.00	24.49	52.48	0.00	9.95	39.92	17.22	25.59	36.34	6.04	11.17	24.60
Jump = 3 Months												
3 months	0.00	9.37	17.61	0.00	4.77	14.71	17.15	21.04	25.77	6.08	8.98	14.58
6 months	0.00	20.94	34.41	0.00	12.20	29.70	17.15	25.58	33.60	6.08	12.53	22.88
9 months	0.00	30.38	43.89	0.00	18.92	39.10	17.15	29.17	37.90	6.08	15.55	28.06
12 months	0.00	37.55	49.45	0.00	24.68	45.30	17.15	31.95	40.25	6.08	17.94	31.24
Jump = 6 Months												
3 months	0.00	9.10	12.10	0.00	7.00	11.23	17.18	21.72	24.25	6.21	10.25	14.02
6 months	0.00	19.81	25.04	0.00	15.67	23.66	17.18	26.67	31.69	6.21	14.88	22.56
9 months	0.00	28.29	34.05	0.00	22.79	32.60	17.18	30.81	36.70	6.21	18.72	28.39
12 months	0.00	34.78	40.16	0.00	28.61	39.09	17.18	33.81	40.08	6.21	21.73	32.40

Notes: PFA = pension fund administrator (*administradora de fondos de pensiones*). Results are in percentage points. $k\sigma$ means times the volatility assumed for the PFA.
Source: Calculations based on the authors' own simulations.

The real effect in the use of reserves comes with the decision to be a rebel and deviate from the investment decisions of the PFAs. The probability of using reserves and the default probability go up faster according the number of rebel months and the increased volatility of the PFA.

Table 5. Probability of a PFA not meeting the required guarantee (using jumps)

Smallest PFA	**0.5% guarantee as reserve**						**1% guarantee as reserve**					
Rule	**24 months**			**36 months**			**24 months**			**36 months**		
Time rebel	1σ	1.5σ	2σ	1σ	1.5σ	2σ	1σ	1.5σ	2σ	1σ	1.5σ	2σ
Jump = 1 month												
3 months	1.32	7.40	22.17	0.23	2.24	12.94	0.19	3.08	16.05	0.03	0.68	8.42
6 months	1.32	15.22	40.53	0.23	5.52	25.12	0.19	6.84	30.07	0.03	1.70	16.67
9 months	1.32	21.06	50.81	0.23	8.16	32.28	0.19	9.76	38.19	0.03	2.58	21.60
12 months	1.32	25.70	56.67	0.23	10.35	36.65	0.19	12.06	42.88	0.03	3.35	24.68
Jump = 3 months												
3 months	1.25	12.27	19.85	0.19	7.15	16.07	0.17	8.84	17.34	0.02	4.75	14.04
6 months	1.25	25.02	39.41	0.19	15.23	31.83	0.17	18.67	35.26	0.02	10.35	27.98
9 months	1.25	34.11	52.14	0.19	21.36	41.61	0.17	25.71	46.94	0.02	14.54	36.69
12 months	1.25	40.98	60.66	0.19	25.96	48.00	0.17	31.12	54.83	0.02	17.68	42.43
Jump = 6 months												
3 months	1.28	10.79	13.69	0.19	7.90	11.75	0.16	8.75	12.14	0.02	6.51	10.93
6 months	1.28	22.70	28.57	0.19	17.32	25.13	0.16	19.37	26.52	0.02	14.55	23.59
9 months	1.28	32.39	40.19	0.19	24.83	34.97	0.16	28.02	37.67	0.02	21.00	32.88
12 months	1.28	39.97	49.45	0.19	30.55	42.68	0.16	34.86	46.62	0.02	25.86	40.22

Largest PFA	**0.5% guarantee as reserve**						**1% guarantee as reserve**					
Rule	**24 months**			**36 months**			**24 months**			**36 months**		
Time rebel	1σ	1.5σ	2σ	1σ	1.5σ	2σ	1σ	1.5σ	2σ	1σ	1.5σ	2σ
Jump = 1 month												
3 months	1.28	2.16	13.64	0.21	0.29	4.68	0.16	0.37	7.08	0.03	0.03	2.01
6 months	1.28	4.69	29.56	0.21	0.68	13.11	0.16	1.00	16.08	0.03	0.09	6.02
9 months	1.28	7.73	39.27	0.21	1.35	19.82	0.16	1.81	21.83	0.03	0.19	9.31
12 months	1.28	10.58	44.74	0.21	2.20	24.50	0.16	2.58	24.98	0.03	0.34	11.53
Jump = 3 months												
3 months	1.28	7.70	21.89	0.19	2.41	13.91	0.17	3.80	16.53	0.02	1.02	10.07
6 months	1.28	16.62	42.69	0.19	6.39	29.35	0.17	8.91	32.85	0.02	2.89	21.28
9 months	1.28	24.39	54.78	0.19	10.56	39.65	0.17	13.42	42.44	0.02	4.91	28.79
12 months	1.28	30.51	61.98	0.19	14.20	46.69	0.17	16.94	48.15	0.02	6.69	33.94
Jump = 6 months												
3 months	1.27	11.02	19.08	0.19	5.91	14.26	0.16	7.49	15.81	0.02	3.90	11.87
6 months	1.27	22.76	38.32	0.19	13.33	30.44	0.16	16.17	32.64	0.02	8.94	25.33
9 months	1.27	32.24	51.72	0.19	19.86	42.46	0.16	23.15	44.41	0.02	13.42	35.31
12 months	1.27	39.46	60.84	0.19	25.31	51.22	0.16	28.66	52.49	0.02	17.12	42.64

Notes: PFA = pension fund administrator (*administradora de fondos de pensiones*). Results are in percentage points. $k\sigma$ means times the volatility assumed for the PFA.
Source: Calculations based on the authors' own simulations.

The Chilean pension system has accumulated USD 160 billion, and 1 percent of capital that PFAs must hold to cover the MGR represents USD 1.6 billion. Thus, we find that the reserves may be overfunded under the actual rules. An opportunity exists to release significant resources into the capital market that could be placed in better investments than pension funds without affecting in a substantial way the default probability.

Table 6. Conditional probability of default according to percentage of guarantee required (using jumps)

Smallest PFA	0.5% guarantee as reserve						1% guarantee as reserve					
Rule	24 months			36 months			24 months			36 months		
Time rebel	1σ	1.5σ	2σ	1σ	1.5σ	2σ	1σ	1.5σ	2σ	1σ	1.5σ	2σ
Jump = 1 month												
3 months	7.60	25.28	53.43	3.63	14.51	44.09	1.08	10.53	38.67	0.43	4.39	28.69
6 months	7.60	36.79	67.02	3.63	22.44	56.83	1.08	16.54	49.72	0.43	6.91	37.72
9 months	7.60	42.05	71.51	3.63	26.63	61.65	1.08	19.49	53.74	0.43	8.41	41.25
12 months	7.60	45.04	73.21	3.63	29.25	63.60	1.08	21.15	55.39	0.43	9.47	42.82
Jump = 3 months												
3 months	7.22	41.06	58.64	3.06	38.23	66.11	1.00	29.56	51.21	0.32	25.40	57.78
6 months	7.22	56.70	76.08	3.06	49.29	76.89	1.00	42.33	68.07	0.32	33.50	67.58
9 months	7.22	62.98	81.44	3.06	54.87	80.79	1.00	47.48	73.32	0.32	37.36	71.24
12 months	7.22	66.15	83.92	3.06	57.78	82.64	1.00	50.23	75.85	0.32	39.36	73.04
Jump = 6 months												
3 months	7.39	41.33	50.03	3.12	50.01	66.07	0.93	33.52	44.35	0.24	41.24	61.46
6 months	7.39	60.15	70.80	3.12	63.83	80.43	0.93	51.35	65.72	0.24	53.65	75.50
9 months	7.39	68.65	79.14	3.12	69.28	84.77	0.93	59.38	74.18	0.24	58.60	79.70
12 months	7.39	72.94	83.33	3.12	72.18	87.01	0.93	63.61	78.56	0.24	61.08	82.00

Largest PFA	0.5% guarantee as reserve						1% guarantee as reserve					
Rule	24 months			36 months			24 months			36 months		
Time rebel	1σ	1.5σ	2σ	1σ	1.5σ	2σ	1σ	1.5σ	2σ	1σ	1.5σ	2σ
Jump = 1 month												
3 months	7.46	10.12	32.93	3.49	3.79	22.69	0.94	1.71	17.09	0.55	0.45	9.73
6 months	7.46	15.11	44.25	3.49	5.97	32.17	0.94	3.23	24.07	0.55	0.76	14.75
9 months	7.46	18.81	48.42	3.49	8.42	36.14	0.94	4.40	26.92	0.55	1.19	16.97
12 months	7.46	21.12	50.37	3.49	10.43	37.98	0.94	5.15	28.12	0.55	1.60	17.87
Jump = 3 months												
3 months	7.46	25.34	50.44	3.05	17.55	47.49	1.00	12.49	38.10	0.40	7.44	34.37
6 months	7.46	35.73	62.77	3.05	25.84	55.82	1.00	19.16	48.29	0.40	11.70	40.46
9 months	7.46	40.96	66.97	3.05	30.64	59.04	1.00	22.53	51.89	0.40	14.24	42.86
12 months	7.46	43.90	69.10	3.05	33.31	60.99	1.00	24.37	53.68	0.40	15.68	44.34
Jump = 6 months												
3 months	7.41	35.77	52.48	2.98	34.24	56.49	0.94	24.30	43.49	0.27	22.59	47.02
6 months	7.41	48.96	67.55	2.98	43.61	65.87	0.94	34.78	57.52	0.27	29.27	54.81
9 months	7.41	54.55	73.11	2.98	47.83	69.62	0.94	39.17	62.77	0.27	32.33	57.89
12 months	7.41	57.52	75.83	2.98	50.28	71.64	0.94	41.78	65.42	0.27	34.01	59.63

Notes: PFA = pension fund administrator (*administradora de fondos de pensiones*). $k\sigma$ means times the volatility assumed for the PFA.
Source: Calculations based on the authors' own simulations.

Note

1. This unit of accounting takes into consideration variation in the consumer index prices. One UF is equivalent to 23.310 Chilean pesos at the end of December 2013, or around USD 40.

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