# Contents


**1 General Introduction** **1**
1.1 Thesis description and motivations . . . 1
1.2 Some important notions in bioprocess engineering . . . 3
1.2.1 Bioreactors: Design, kinetics and operation mode . . . 5
1.2.2 Microalgae cultivation: light influence and photobioreactors . . . 14
1.2.3 Multiple bioreactors connected: The general gradostat . . . 21

**2 Productivity optimization of microalgae cultivation in a batch photobioreactor process** **26**
2.1 Introduction . . . 26
2.2 Optimization problem formulation . . . 28
2.2.1 Model construction . . . 28
2.2.2 Problem statement . . . 30
2.3 Main results . . . 31
2.3.1 Constant light . . . 31
2.3.2 Dark/light cycles . . . 34
2.4 Numerical approach . . . 40
2.4.1 Chlamydomonas reinhardtii study case . . . 40
2.5 Discussion . . . 44
Appendices . . . 46
A Considering light incidence . . . 46
B Simplification and reduction of the system . . . 47
C Parameter estimation . . . 49

**3 Modelling and stability analysis of a microalgal pond with nitrification** **52**
3.1 Introduction . . . 52
3.2 Problem statement . . . 53
3.2.1 Stoichiometric Equations . . . 54
3.2.2 Kinetic Equations . . . 54
3.2.3 Mass Balance Equations . . . 55
3.3 Model analysis . . . 55
3.3.1 Equilibrium existence and local stability . . . 57
3.3.2 Model reduction . . . 64
3.4 About the global behaviour . . . 67
3.5 Application . . . 72

3.6 Discussion . . . 74
Appendices . . . 76
A Jacobian Matrix . . . 76

**4 Equivalence of finite dimensional input-output models of solute transport and diffusion 77**
4.1 Introduction . . . 77
4.2 Notations and preliminary results . . . 80
4.2.1 About controllability and observability . . . 82
4.3 The Multi-Rate Mass Transfer and Multiple INteracting Continua configurations . . . 83
4.4 Equivalence with MRMT structure . . . 86
4.5 Equivalence with MINC structure . . . 90
4.6 Examples: Reduction and the importance of minimal representation . . . 95
4.7 Discussion . . . 99
Appendices . . . 100
A Explicit equivalences for two models of three compartments . . . 100
B Invariance of total volumes by MINC and MRMT transformations . . . 104
C A direct method to obtain an MINC structure . . . 108

**5 Conclusions and future perspectives 110**

**6 Annexes 113**
A Linear Algebra results . . . 113
B Realization Theory Fundamentals . . . 114
C The unsymmetric Lanczos procedure and tridiagonalization . . . 119
D Compartmental Systems . . . 125

**Bibliography 131**

# List of Tables


2.1 Parameters and functions used in the general model. . . . . 28
2.2 optimal values in different light environments for *C. reinhardtii* considering constant light. . . . . 43
2.3 Parameter estimations for different incidental light settings for *C. reinhardtii*. . . . . 44
2.4 Optimal values in different incidental light settings for *C. reinhardtii* considering constant light. . . . . 45
2.5 Parameter values for *C. reinhardtii* obtained from [57]. . . . . 49
2.6 Linear regression $f(x) = axL$ . . . . . 50
2.7 Parameters estimated for *C. reinhardtii* in this model. . . . . 51

3.1 Variables used in the algal pond model . . . . . 54
3.2 Equilibrium existence conditions for break-even concentrations . . . . . 61
3.3 Local stability conditions of non-coexistence equilibrium points for break-even concentrations . . . . . 63
3.4 Equilibrium existence and local stability conditions for break-even concentrations in system (3.18). . . . . 67
3.5 Parameter values for the system (3.6). . . . . 72

# List of Figures


1.1 Effect of nutrient concentration on the specific growth rate of E. coli (source [74]). . . 8
1.2 Batch bioreactor in a laboratory experiment (source Laboratoire de Biotechnologie de l'Environnement, INRA, Narbonne, France ). . . . . 10
1.3 Growth of phytoplankton in a continuous reactor (source Laboratoire d'Ecologie du plancton, Marin, CNRS, Villefranche sur mer, France). . . . . 11
1.4 Batch growth profile (source [17]). . . . . 15
1.5 Light-response curve of photosynthesis (P-curve). The intercept on the vertical axis is the measure of O2 uptake due to dark respiration. $I_c$, light compensation point; $I_s$, light saturation intensity; $I_h$, light intensity value at which photoinhibition occurs (source [84]). . . . . 18
1.6 An open raceway pond is a photobioreactor with natural light (source Laboratoire de Biotechnologie de l'Environnement, INRA, Narbonne, France ). . . . . 19
1.7 Multiple chemostats connected in a laboratory experiment (source Laboratoire de Biotechnologie de l'Environnement, INRA, Narbonne, France ). . . . . 21
1.8 The standard $n$-vessel gradostat. The left vessel labeled $R$ is a reservoir containing nutrient at concentration $S_0$, $C$ is an overflow vessel, and $D$ denotes the dilution rate. All vessels have the same volume (source [87]). . . . . 22
1.9 Irreducible "dead-end" gradostat. Note that the inflow to each vessel balances the outflow (source [87]). . . . . 25

2.1 Monod growth function. . . . . 29
2.2 Phase plane of the differential equation (2.7) for microalgae *C. reinhardtii* with parameter values $\overline{\mu} = 2.34\ [d^{-1}]$, $\rho = 0.34\ [d^{-1}]$, $c = 0.253\ [g.L^{-1}]$. and different initial conditions. . . . . 33
2.3 Solutions of the differential equation (2.9) for microalgae *C. reinhardtii* with parameter values $\overline{\mu} = 2.34\ [d^{-1}]$, $\rho = 0.34\ [d^{-1}]$, $c = 0.253\ [g.L^{-1}]$. considering regular intervals of 12 hours (0.5 $[d]$) and different initial conditions. A stable set exist in the interior of the interval $[2.35, 2.8]$ over biomass concentration axis. . . . . 38
2.4 Surface and level curves of the net rate of production (mean biomass volumetric productivity) for the optimization problem (2.6) for *C. reinhardtii* at the parameter values $\overline{\mu} = 2.34\ [d^{-1}]$, $\rho = 0.34\ [d^{-1}]$, $c = 0.253\ [g.L^{-1}]$. . . . . 41
2.5 Surface and level curves of the net rate of production for the optimization problem (2.6) for *C. reinhardtii* with the same parameter values in summer period. . . . . 42
2.6 Surface and level curves of the net rate of production for the optimization problem (2.6) for *C. reinhardtii* with the same parameter values in regular time intervals. . . . 42

2.7 Surface and level curves of the net rate of production for the optimization problem (2.6) for *C. reinhardtii* with the same parameter values in winter period. . . . . 43
2.8 Biomass trajectories associated to the optimal values of initial concentration $x_0^*$ and terminal time $T^*$ in different light environments, with turnaround times $t_a = 1$ (left) and $t_a = 2$ (right). . . . . 44
2.9 The figure shows the results of the table 2.4. It may be noted that the higher the incidencial light, higher the final biomass concentration, while the terminal batch time decreases and the mean volumetric productivity increases, the initial concentration needed to achieve optimal productivity decreases slightly. . . . . 46
2.10 Linear regression and zero intercept for the estimation of the light attenuation parameter $a$. . . . . 50
2.11 Approximation of functions $\mu_1(.)$ and $\mu_2(.)$ by nonlinear least squares method. . . . . 51

3.1 In this setting $\bar{\mu}_n = 2, \bar{\mu}_s = 1, K_n = 1, K_s = 0.12, K_x = 0.5, s_{in} = 1, D = 0.5, k_1 = 1$ and considering the growth functions in the application section, the graph show the existence of the three equilibrium points that was proven in lemma above. . . . . 59
3.2 Graph of the function $\mu_3(.)$ for $k_2 = 1$ and the parameter values in table 3.5. . . . . 65
3.3 Graph of the non-competitive inhibition function $\phi_2(.)$ for the parameter values in table 3.5. . . . . 73
3.4 Top: Growth functions with parameter values of table 3.5 (from [1]) with a pond depth $h = 0.1\ m$ (left) and $h = 0.5\ m$ (right). Down: zoom near the origin . . . . . 73
3.5 Solutions of the system (3.6) with initial condition $(x_1(0), x_2(0), s_1(0), s_2(0)) = (10, 40, 30, 50)$ and a pond depth $h = 0.1\ m$, considering a dilution rate $D = 0.1$ (left) and $D = 0.4$ (right). In the left figure the equilibrium $E_a = (0, 1496, 0.045, 0)$ is globally stable, however in the right figure the positive equilibrium $E_c = (11.2461, 1034, 0.388, 36.88)$ is globally stable. . . . . 74
3.6 Solutions of the system (3.6) with initial condition $(x_1(0), x_2(0), s_1(0), s_2(0)) = (10, 40, 30, 50)$ and a pond depth $h = 0.5\ m$, considering a dilution rate $D = 0.9$ (left) and $D = 1.2$ (right). In the left figure the equilibrium $E_c = (26.11, 95.34, 2.86, 109.5)$ is globally stable, however in the right figure the positive equilibrium $E_a = (0, 68.74, 114, 0)$ is globally stable. . . . . 74

4.1 Example of a network with one *mobile* zone . . . . . 78
4.2 Example of a MRMT network . . . . . 84
4.3 Example of a MINC network . . . . . 85
4.4 Example of a network with one mobile and four immobile zones . . . . . 95
4.5 Nyquist diagrams (black: original system, blue: reduced MRMT, green: reduced MINC) . . . . . 97
4.6 Structure of the example . . . . . 98
4.7 Simplified equivalent structure of the example . . . . . 99
4.8 The MINC configuration with two inmobile compartments. . . . . 101
4.9 The MRMT configuration with two inmobile compartments. . . . . 102

6.1 The Compartmental System Connectivity Diagram (source [2]). . . . . 126
6.2 An $n$-Compartment Catenary System (source [2]). . . . . 127
6.3 An $n$-Compartment Mammilary System (source [2]). . . . . 127