Contents

1	Intr	oduction	1
	1.1	Bioprocesses and bioremediation	1
	1.2	Mathematical models of bioreactors and classical results	5
		1.2.1 Mathematical models	5
		1.2.2 Classical results on chemostats	7
		1.2.3 Classical results on SBRs	12
		1.2.4 Stochastic models of bioreactors	14
	1.3	Model of inhomogeneous lake	16
	1.4	Singular perturbations	20
	1.5	Contributions of the thesis	24
		1.5.1 Deterministic optimal control for continuous bioremediation processes	24
		1.5.2 Study of stochastic modeling of sequencing batch reactors	28
2	Bior	remediation of natural water resources	
	via (Optimal Control techniques	31
	2.1	Introduction	31
	2.2	Mathematical model	33
		2.2.1 Description of the dynamics	33
		2.2.2 Preliminary results on the dynamics	35
		2.2.3 Minimal time optimal control problem	36
	2.3	Application of Pontryagin maximum principle	37
	2.4	The effect of recirculation	39
	2.5	Numerical Simulations	43
	2.6	Conclusion	44
3	Opt	imal feedback synthesis and minimal time function for the bioremediation of wa-	
•	ter r	resources with two patches	46
	3.1	Introduction	46
	3.2	Definitions and preliminaries	48
	~ ~	Study of the releved problem	50
	3.3		32
	3.3 3.4	Synthesis of the optimal strategy	52 57

	3.6	Numerical illustrations	65
	3.7	Conclusion	68
4	Min	imal-time bioremediation of natural water resources with gradient of pollutant	71
	4.1	Introduction	71
	4.2	Definitions and preliminaries	73
	4.3	Optimal control problem	75
	4.4	Numerical simulations	81
	4.5	Conclusions	83
5	Stoc	hastic modelling of sequencing batch reactors for wastewater treatment	85
	5.1	Introduction	85
	5.2	Stochastic SBR model	86
	5.2 5.3	Stochastic SBR model Existence of solutions of the controlled stochastic model	86 95
	5.2 5.3 5.4	Stochastic SBR model Existence of solutions of the controlled stochastic model Existence of solutions of the controlled stochastic model The optimal reach-avoid problem Existence Existence	86 95 101
	5.2 5.3 5.4 5.5	Stochastic SBR model	86 95 101 107
6	5.2 5.3 5.4 5.5 Con	Stochastic SBR model	86 95 101 107 110

List of Tables

2.1	Comparison between optimal treatment time and the time given by the best con-	
	stant control, from initial condition $z = (5, 8)$	43
2.2	Comparison between optimal treatment time and the time given by the best con-	
	stant control, from initial condition $z = (6, 7)$	43
2.3	Comparison between optimal treatment time and the time given by the best con-	
	stant control, from initial condition $z = (2, 10)$	43
2.4	Comparison of optimal times with respect to diffusion parameter D from initial	
	condition $z = (5, 8) \ldots \ldots$	44
2.5	Comparison of optimal times with respect to diffusion parameter D from initial	
	condition $z = (6,7)$	44
2.6	Comparison of optimal times with respect to diffusion parameter D from initial	
	condition $z = (2, 10)$	44
3.1	Time comparisons (in hours) for $r = 0.3$ and target value $\underline{s} = 1$ (top), 0.1 (bottom) [g/l] (initial condition s(0) and diffusion parameter d are given in [g/l] and [1/h],	
	respectively)	67
41	Comparison of treatment times for the optimal strategies	82
4.2	Comparison of optimal times with respect to maximum recirculation parameter $\bar{Q}_{\rm R}$	52
	for different initial conditions	82

List of Figures

1.1	Scheme of water treatment	3
1.2	Industrial bioreactor	3
1.3	Scheme bioreactor-settler	3
1.4	Typical growth functions. On the left, the Monod uptake function; on the right, the	
	Haldane uptake function.	6
1.5	Trajectories on the phase portrait, depending on the diffusion parameter D. In	
	green, the washout equilibrium; in red, the nontrivial equilibrium	9
1.6	Equilibrium points for the Monod growth function. We see that $D_1 < \mu_{max}$, and	
	then the concentration of substrate at equilibrium is $s_r^{\dagger} = \lambda_1$. For D_2 , the equilib-	
	rium is the washout	9
1.7	Equilibrium points for the Haldane growth function. We see that for $D_1 < \mu(s^{\dagger})$	
	there exist two equilibrium points λ_1 (locally asymptotically stable) and λ_2 (unsta-	
	ble) besides the washout (unstable). For $D_2 = \mu(s^{\dagger})$ the equilibrium concentration	
	is $s_{\rm r}^{\star} = s^{\dagger}$ (locally asymptotically stable), and the washout is unstable. For D_3 , the	
	equilibrium is the washout (stable)	10
1.8	Relation between fictitious time τ and real time t	14
1.9	Behavior of the inhomogeneous representation of a lake. From the homogeneous	
	initial distribution of pollutant (on the left), the system evolves up to a point in	
	which two zones are clearly differentiated (on the right). Taken from [4]	18
1.10	First model of inhomogeneity: the active-dead zones configuration	19
1.11	Second model of inhomogeneity: the model with two patches	19
1.12	Third model of inhomogeneity: the configuration in series with recirculation	20
1.13	Total pollutant concentration in the resource of the full dynamics (3.6) with the	
	strategy (3.25), for different values of ϵ .	23
1.14	Velocity set of the dynamic (1.30) when $s_1 \neq s_2 \dots \dots \dots \dots \dots \dots \dots \dots \dots$	27
2.1	Connections between active zone, dead zone and bioreactor.	32
2.2	Connections between active zone, dead zone and bioreactor, with the recirculation	
	pumps	40
31	Modeling scheme of the treatment of two interconnected natches (definitions of	
5.1	the variables and parameters are given in Section 3.2)	47
32	Graphs of $u(\cdot)$ and corresponding $\gamma(\cdot)$	65
2.2	$\mathcal{L}_{\mathcal{F}}$ or $\mathcal{F}(\mathcal{F})$ and conceptonian $\mathcal{F}(\mathcal{F})$	55

3.3	Optimal paths for $d = 0.1[h^{-1}]$ (left) and $d = 10[h^{-1}]$ (right) with $r = 0.3$ 65
3.4	Level sets (in hours) of V_0 (left) and V_∞ (right) for $r = 0.3.$
3.5	Trajectories and controls generated by the two-pumps and one-pump optimal feed-
	back, for r=0.3, $\underline{s}=1$ [g/l] and $\underline{s}(0)=(3,10)$ [g/l]. On the left d=0.1[1/h], and on the
	right d=10[1/h]
3.6	Total pollutant concentration in the resource of the full dynamics (3.6) with the
	strategy (3.25), for different values of ϵ
4.1	Modeling scheme of the nonhomogeneity of concentrations in the resource as two
	zones, and a recirculation pump
4.2	Backward integration of the extremals, with $\bar{Q}_{R} = 0.$
4.3	Backward integration of the extremals, with $\bar{Q}_{R} = 1$
4.4	Backward integration of the extremals, with $\bar{Q}_{R} = 10.$
5.1	Plot of the function \tilde{v} for $\tilde{\gamma} = 1$
5.2	Plot of the function \tilde{v} for $\tilde{\gamma} = 5$
5.3	Plot of the function \tilde{v} for $\tilde{\gamma} = 10$
6.1	First extension: model with three patches and two pumps
6.2	Second extension: problem with two pollutants