Contents

| | Intr | roduction | | | | | | |
|---|---------------------|--|----|--|--|--|--|--|
| 1 | Conceptual review | | | | | | | |
| | 1.1 | Equilibrium and stability | | | | | | |
| | 1.2 | Bifurcations | | | | | | |
| | | 1.2.1 Saddle-node bifurcation | | | | | | |
| | | 1.2.2 Transcritical bifurcation | | | | | | |
| | | 1.2.3 Pitchfork bifurcation | 1 | | | | | |
| | 1.3 | Bifurcation in extended systems | 1: | | | | | |
| | | 1.3.1 Spatial instability and pattern formation | 1 | | | | | |
| | 1.4 | Variational systems | 1 | | | | | |
| | 1.5 | Fredholm alternative | 1 | | | | | |
| | 1.6 | Normal forms and amplitude equations | 1 | | | | | |
| | 1.7 | Fronts | 1 | | | | | |
| | 1., | 1.7.1 Fronts connecting unstable states | 2 | | | | | |
| | | 1.7.2 Fronts connecting stable states | 2 | | | | | |
| | 1.8 | Liquid Crystals | 2 | | | | | |
| | 1.0 | 1.8.1 Liquid crystal cells and anchoring | 2 | | | | | |
| 2 | Zig-zag instability | | | | | | | |
| | 2.1 | Harnessing liquid crystals | 2 | | | | | |
| | 2.2 | Frank-Ossen free energy | 2 | | | | | |
| | | 2.2.1 Dynamical equation | 3 | | | | | |
| | | 2.2.2 Fréedericksz transition | 3 | | | | | |
| | 2.3 | Interface dynamics | 3 | | | | | |
| | 2.0 | 2.3.1 Instability analysis | 3 | | | | | |
| | | 2.5.1 Histability analysis | J | | | | | |
| 3 | Asy | emmetric front propagation in systems with reflection symmetry | 39 | | | | | |
| | 3.1 | Variational systems and reflection symmetry | 3 | | | | | |
| | 3.2 | Non-variational systems | 4 | | | | | |
| | | 3.2.1 An example of permanent dynamics | 4 | | | | | |
| | 3.3 | Bistable model with nonlinear gradient | 4 | | | | | |
| | 3.4 | Kink interaction | 4 | | | | | |
| | - | 3.4.1 Asymmetric kink interaction | 4 | | | | | |
| 4 | Pho | oto-isomerization fronts | 4 | | | | | |
| | | Experimental | 4 | | | | | |

| | | 4.1.1 | Setup | 47 |
|--------------|-------|------------------------------|--|-----------|
| | | 4.1.2 | Observation of nematic–isotropic fronts | 48 |
| | 4.2 | An ord | der parameter for liquid crystals | 48 |
| | 4.3 | Landa | u-de Gennes theory | 50 |
| | 4.4 | Nemat | tic-Isotropic interface | |
| | | 4.4.1 | Exact velocity | |
| | | 4.4.2 | Approximated method | |
| | 4.5 | | ian forcing | |
| | 4.6 | | nt effects | |
| | | 4.6.1 | Linear analysis | |
| | | 4.6.2 | Adiabatic approximation | 62 |
| | Con | clusio | n | 63 |
| Bi | bliog | raphy | | 65 |
| A | _ | zag wa igurat | all lattice in a nematic liquid crystal with an in-plane switchir ion | ng 72 |
| В | Asy | mmeti | ric counterpropagating fronts without flow | 81 |
| \mathbf{C} | Asy | $\mathbf{mmet}_{\mathbf{i}}$ | ric counter propagation of domain walls | 87 |
| D | - | | all dynamics induced by the coexistence between monostable are | nd 100 |

List of Figures

| 1 | Robust phenomena in nature (a) Wind induced patterns in sand dunes [51]; (b) Patterned pigmentation in mammals; (c) Hydrodynamic soliton [18]; (d) Oscillon in vibrated grains [79] | 2 |
|-----|---|----|
| 2 | Interfacial phenomena in different contexts: (a) Dendrites in a snowflake [32, 44]; (c) Burned paper; (c) Solidification front of supercooled water [6]. (d) Fronts connecting an homogeneous state with a patterned state in sand under an oscillatory water flow [11]. Red arrows indicate the direction of front propagation | 3 |
| 1.1 | Straight (left) and bent (right) states of a vertical sheet of paper under a gravitational field. h is the distance from the top of the paper sheet to the fingers and $h^* > h_c$ | 8 |
| 1.2 | Saddle–Node bifurcation: two states $u_+ = \sqrt{\varepsilon}$ (blue line) and $u = -\sqrt{\varepsilon}$ (red line) emerge. The solution u_+ (solid line) is stable and u (dashed line) is unstable | 9 |
| 1.3 | Transcritical bifurcation: two states $u_0 = 0$ (blue line) and $u_1 = \varepsilon$ (red line) exchange their stability. Solid line represents a stable state while dashed line represents an unstable state | 10 |
| 1.4 | Pitchfork bifurcation: the state $u_0 = 0$ is stable for $\varepsilon < 0$ and, for $\varepsilon > 0$, it becomes unstable while two states $u_+ = \sqrt{\varepsilon}$ (blue line) and $u = -\sqrt{\varepsilon}$ (red line) emerge. Solid lines represent stable and dashed lines represent unstable. | 11 |
| 1.5 | Imperfect Pitchfork bifurcation or Cusp catastrophe. The orange corresponds to the curve of the equilibrium states. At the bottom of the plot, the bistable region is colored purple | 12 |
| 1.6 | Spatial instability in the Swift-Hohenberg model. The plot shows the eigenvalue curve as function of k just at the point of the instability $\varepsilon = 0$. Note that the local maxima are at $k_c = \pm q$ | 14 |
| 1.7 | Discrete front propagation in a domino toppling chain. The vertical and the toppled states are connected by a moving front | 18 |
| 1.8 | Example of a monotonous front solution. The solution connects asymptotically two homogeneous states. The front position accounts for the spatial location that has maximum spatial variation and its core is given by the spatial size around of front position | 20 |
| 1.9 | FKPP front propagation. The figure shows a spatio-temporal diagram obtained from numerical simulation of model (1.46) | 21 |

| 1.10 | Example of a monotonous front solution. The solution connects asymptotically two homogeneous states. The front position accounts for the spatial location | |
|------|--|----------|
| | that has maximum spatial variation and its core is given by the spatial size | |
| | around of front position | 22 |
| 1.11 | Potential (1.52). The states u_{\pm} are local minima of the potential. Changing | 0.0 |
| 1.12 | the value of η changes the relative stability of both states | 23 25 |
| | Nematic, smectic and cholesteric phases are represented | 25 27 |
| 2.1 | (a) Splay deformation in a nematic liquid crystal; (b) Twisted deformation; | |
| 2.2 | (c) bend deformation [39] | 31 34 |
| 3.1 | (a) Stationary solution of bistable model (3.13) for $\eta=0$ and $\nu>0$. (b) Spatiotemporal diagram of counter propagative fronts of the model (3.13) for $\eta=0.3$ and $\nu=0.6$ | 43 |
| 4.1 | Schematic sketch of the experimental set-up. P_1 and P_2 : polarizers L : $DDLC$: dye-doped liquid crystal cell, plano-convex lenses, $DDLC$: dye doped liquid crystal cell, P : polarizer along x axis, L : imaging lens, CDD: charge-coupled device camera | 48 |
| 4.2 | Experimental temporal sequence of snapshots at 55, 65, 125 and 500 s. $P_0 =$ | |
| 4.3 | $350 mW$ and $\omega = 3.4 mm$ | 49 50 |
| 4.4 | Curve of the free energy density F as function of the order parameter S considering no spatial variations ($\nabla S = 0$). The homogeneous states satisfy $\partial F/\partial S = 0$ | 52 |
| 4.5 | Bifurcation diagram of homogeneous states given by equation (4.12). The equilibria are plotted as function of the bifurcation parameter A ($B = 1$). Stable states appears in blue and the unstable states in red. The equilibrium $S_{iso} = 0$ represents the isotropic phase and upper blue branch represents the nematic phase S_+ | 53 |
| 4.6 | The blue curve given by $ \eta < \eta_c$ encloses the bistable region. The red curve is a parametric curve given by the relations (4.17) and (4.18) for $B = 2$. The arrow indicates the direction in which A increases. | 54 |
| 4.7 | Front velocity (Eq. 4.28 with a negative sign) as a function of A plotted in | J. |
| | green color over the bifurcation diagram. The Maxwell Point A_M , where $v=0$ | 56 |

| to an experimental snapshot of rolls. The right panel correspond to a simulation of the system (4.53) with the following values of the parameters: $\varepsilon = 0.2$ $D = 0.5$, $\lambda = 1$, $\delta_x = 1$, $\delta_y = 0.8$ and $\alpha = -4.8$ | nd | |
|--|---------------------|----|
| $D=0.5, \ \lambda=1, \ \delta_x=1, \ \delta_y=0.8 \ {\rm and} \ \alpha=-4.8. \ldots$ 4.9 Growth rate as a function of wave number. The values of the parameters ar $\varepsilon=0.24, \ D=0.5, \ \lambda=1, \ \delta_x=1.$ The dashed curve corresponds to $\alpha=-1$ | la- | |
| 4.9 Growth rate as a function of wave number. The values of the parameters ar $\varepsilon = 0.24, D = 0.5, \lambda = 1, \delta_x = 1$. The dashed curve corresponds to $\alpha = -1$ | 24, | |
| $\varepsilon = 0.24, D = 0.5, \lambda = 1, \delta_x = 1$. The dashed curve corresponds to $\alpha = -1$ | | 61 |
| , | re: | |
| just below the instability, and the $\alpha = -4.8 \dots \dots \dots \dots \dots$ | -1, | |
| | | 62 |