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Regular Paper Suppressing the effect of ICI power using dual sinc pulses in OFDM-based systems



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ABSTRACT

Sensitivity to frequency offset in orthogonal frequency division multiplexing (OFDM)-based systems allow unwanted interference among the subcarriers, which results in intercarrier interference (ICI). A novel family of Nyquist-I pulses, named dual sinc pulses (DSPs), is proposed herein having three new design parameters, giving additional degrees of freedom to minimize ICI power in OFDM-based systems. Several Nyquist-I pulses have been studied and implemented to deal with the problem of frequency offset in OFDM-based systems. In this paper, we investigate the performance of DSPs in terms of ICI power, signal-to-interference power ratio (SIR), and bit error rate (BER) in OFDM-based systems. The frequency responses of the DSPs are studied by adjusting the values of the new design parameters. Theoretical and simulation results presented herein show that DSPs outperform other existing pulses in OFDM-based systems, in terms of ICI power, SIR power, and BER.

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1. Introduction

Several recent applications require high-data-rate transmission over wireless channels. Orthogonal frequency division multiplexing (OFDM) is the most widely used modulation scheme in wireless communication standards. OFDM-based systems allow splitting of high-rate data streams into multiple lower-rate streams that transmit data over N orthogonal subcarriers. OFDM has been studied and suggested as an important technology to contribute in 5G cellular systems [1,2]. A combination of OFDM and multiple-input multiple-output (MIMO) systems as a single unit has been proposed to be implemented at the physical layer of 5G cellular networks. Furthermore, OFDM systems have been widely used in wireless communication standards such as digital terrestrial TV broadcasting (DVB-T), LTE-Advanced, Wi-Fi, WPAN, and WiMAX [3,4]. This is due to its high-data-rate transmission capability, high bandwidth efficiency, robustness to multipath fading given by the ability to convert a frequency selective fading channel into several nearly flat fading channels, and the guard interval that avoids the effect of delay spread by allowing for simple frequencydomain equalization [5].

On the other hand, these systems are characterized by high sensitivity to frequency offset because imperfect synchronization

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http://dx.doi.org/10.1016/j.aeue.2016.04.013 1434-8411/© 2016 Elsevier GmbH. All rights reserved. occurs mostly in OFDM-based systems. Frequency errors are caused by frequency mismatch between the oscillators at the transmitter and receiver sides, Doppler spread, distortion in the channel [6]; such as channel variation in time [7], and Sampling Frequency Offset (SFO) [8]. Thus, a number of impairments arise due to frequency offset; such as attenuation or rotation of the sub-carriers and inter-carrier interference (ICI) which eventually degrades the performance of a system. Several methods have been proposed to minimize sensitivity to carrier frequency offset (CFO) in OFDM-based systems, including frequency domain equalization [9,10], self-cancelation schemes [11,12], pilot insertion [13], receiver side windowing technique [14], and frequency error correction technique [15].

In this paper, a pulse shaping technique is used to deal with the problem of frequency offset in OFDM systems. The use of pulse shaping involves three major issues. First, the use of a pulse with small magnitude side lobes can greatly reduce the ICI power [16]. Second, a broad main lobe should be used because the use of narrow main lobes degrades system performance and results in high bit error rates (BERs) [17]. Third, the pulses used should comply with the Nyquist-I criterion [6]. We propose a novel family of Nyquist-I pulses called dual sinc pulses (DSPs) that have three new design parameters that provide extra degrees of freedom to minimize ICI power function, and a modified sinc power function. The significance of the proposed pulse shaping function is that

it is able to greatly reduce the ICI power by decreasing the magnitude of the side lobes and can yield low BER by including a broad main lobe. The robustness of the proposed pulse is verified herein by means of adjusting the values of its design parameters.

1.1. Related work

Various pulse shaping functions have been studied and compared with the better than raised cosine (BTRC) pulse in terms of the resulting ICI power in OFDM systems [18,19]. In addition, other Nyquist-I pulses have been proposed and implemented, such as the sinc power (SP) pulse [20]. The SP pulse is known for its design simplicity, and also includes an additional degree of freedom that ensures its dominance over the raised cosine pulse and BTRC in minimizing the ICI power. The only drawback of the SP pulse is its high-magnitude side lobes, which may not reduce the ICI power sufficiently in OFDM systems. To address this drawback, the improved sinc power (ISP) pulse [16] has been proposed, including a new design parameter and an exponential expression, and has been shown to yield reductions in ICI power and BER. ISP improves upon SP by increasing its exponential power. A pulse called the phase modified sinc power (PMSP) pulse has performed even better than ISP in terms of ICI power and BER [17]. The PMSP was developed to have smaller-magnitude side lobes and a broader main lobe. However, the smaller side lobes of the ISP and PMSP pulses may not greatly reduce the ICI power at lower values of the parameter *a*, which is used to adjust the amplitude of these pulses. A new windowing function (NWF) was proposed [21] that has better performance in reducing the ICI power, but NWF has the problem that its narrow main lobe may degrade overall system performance by introducing high BER. All the pulses of SP, ISP, PMSP, and NWF were defined in the frequency domain for the roll-off factor $\alpha = 1$. The major idea behind ISP, PMSP, and NWF is to increase the exponential power of the sinc power function or the product of some expression including the sinc power function. However, much research is ongoing to investigate new families of Nyquist-I pulses that can provide a balance in performance between ICI power reduction and improved BER.

On the other hand, other pulses have been recently proposed that are defined for non-unity values of α , in particular smaller values such as 0.22. These include the improved modified Bartlett–Hanning (IMBH) pulse [22] and the improved parametric linear combination pulse (IPLCP) [23,24], which were designed to reduce ICI power and enhance BER in OFDM-based systems. However, because the DSP proposed herein is only defined for $\alpha = 1$, in this work the IMBH and IPLCP are considered for same value of $\alpha = 1$, to enable comparisons among the pulses and to evaluate their performances for the highest value of α .

In this paper, the performance of the DSP is studied and compared with that of other existing pulses in terms of ICI power, SIR, and BER in an OFDM-based system. In addition, the behavior of the DSP is analyzed by adjusting the values of one parameter while fixing the values of the other two parameters. A previously proposed method [18] is adopted to evaluate the performance of different pulse shaping functions in terms of ICI and SIR power. Numerical simulations are performed to model the performance of these pulses when applied in a 64-subcarrier OFDM-based system in the presence of frequency offset over an additive white Gaussian noise (AWGN) channel. Previously derived theoretical expressions [25] are implemented in a BPSK-OFDM system to evaluate the performance of the DSP and other existing pulses over an AWGN channel. Finally, we measure the average elapsed time of a convolution process between the modulated symbols and a pulse shaping function in the OFDM-based system.

The remainder of this paper is organized as follows. Section 2 gives an overview of the OFDM system. Section 3 presents ICI

and SIR evaluation metrics. Section 4 describes the proposed pulse in detail. Section 5 presents performance comparisons between the proposed DSP and other existing pulses in terms of ICI power, SIR power, and BER. Section 6 discusses the differences in time complexity of the convolution process among the various pulse shaping functions. Finally, Section 7 gives concluding remarks.

2. System model

The pulse shaping technique was applied in the N-subcarrier OFDM system illustrated in Fig. 1. In this system, each OFDM symbol is the sum of N data symbols that are modulated using M-PSK, M-QAM or some other modulation scheme. These data symbols are taken as an input to the inverse discrete Fourier transform (IDFT), which is used to assign a particular frequency to each data subcarrier. The transmitted OFDM symbol is converted to a serial data stream, which generates an analog signal in the time domain. Furthermore, a digital-to-analog converter (DAC) is used with the proposed DSP pulse shaping filter to produce a smooth analog signal. The analog signal is then modulated at a particular carrier frequency f_c and transmitted through the channel. The receiver is characterized by the inverse signal processing operations done in the transmitter. In addition, a radio frequency (RF) signal is mixed with the base band signal for processing. An analog-to-digital converter (ADC) is used with the proposed DSP to eliminate all other frequencies above the Nyquist frequency. Moreover, the discrete Fourier transform (DFT) is used at the receiver side to analyze the signal in the frequency domain. The complex envelope of the OFDM symbol transmitted through the channel with the pulse shaping function can be expressed as follows [18]:

$$x(t) = Re\left\{e^{j2\pi f_{c}t} \sum_{k=0}^{N-1} a_{k}p(t)e^{j2\pi f_{k}t}\right\},$$
(1)

where *j* is the imaginary unit $\sqrt{-1}$, f_c is the carrier frequency, f_k is the subcarrier frequency of the *k*th subcarrier, p(t) is the pulse shaping function, and a_k is the data symbol transmitted on the *k*th subcarrier with zero mean and normalized average symbol energy. In this paper, it is assumed that all data symbols are uncorrelated [18] and given as follows:

$$E[a_k a_m^*] = \begin{cases} 1, & k = m \\ 0, & k \neq m, \end{cases}$$
(2)

where * represents the complex conjugate. To ensure the orthogonality of each subcarrier in OFDM systems the following equation needs to be satisfied:

$$f_k - f_m = \frac{k - m}{T}, \quad k, m = 0, 1, \dots, N - 1,$$
 (3)

where (1/T) is the minimum subcarrier frequency spacing that satisfies orthogonality among the *N* subcarriers. In Eq. (4), n(t) is



Fig. 1. OFDM system model.

assumed to be an additive complex Gaussian noise process with zero mean and variance of $(N_0/2)$ per dimension in the channel. To allow investigation of the particular impact of the frequency offset upon ICI, an ideal channel is assumed; namely, $h(t) = \delta(t)$. A similar approach has been considered in previous reports [16–18,20,21]. In addition, we aimed to evaluate the performance of different Nyquist-I pulses in an OFDM system with respect to the ICI power, so it became necessary to include the effect of frequency offset, for which reason the frequency offset Δf is added, and an ideal channel is assumed. Noise is added to the OFDM system as given in Eq. (5). The received signal is given as follows:

$$r(t) = x(t) \otimes h(t) + n(t), \tag{4}$$

where \otimes represents convolution, h(t) is the impulse response of the channel, and n(t) is the additive complex Gaussian noise.

3. ICI evaluation metric

Distortion in the channel and frequency differences between the transmitter and receiver oscillators introduces frequency offset Δf and phase error θ . Previously reported notations [18] are used to evaluate the performance of the proposed pulse in terms of ICI power and SIR power for OFDM-based systems. At the receiver side, the signal r(t) given in [18] becomes the following:

$$r(t) = e^{j(2\pi\Delta ft+\theta)} \sum_{k=0}^{N-1} a_k p(t) e^{j2\pi f_k t} + n(t) e^{j(2\pi(-f_c+\Delta f)t+\theta)}.$$
(5)

The decision variable \hat{a}_m for the transmitted symbol a_m in [18] can be expressed as follows:

$$\hat{a}_m = \int_{-\infty}^{\infty} r(t) e^{-j2\pi f_m t} dt, \tag{6}$$

From Eq. (6), the decision variable \hat{a}_m can be decomposed as follows:

$$\hat{a}_m = a_m e^{j\theta} P(-\Delta f) + e^{j\theta} \sum_{k=0, k \neq m}^{N-1} a_k P\left(\frac{m-k}{T} - \Delta f\right).$$
⁽⁷⁾

The power of the desired signal and of the ICI in [18] is defined as follows:

$$\sigma_m = |a_m|^2 |P(\Delta f)|^2, \tag{8}$$

$$\sigma_{ICI}^{m} = \sum_{k=0, k \neq mn=0, n \neq m}^{N-1} a_{k} a_{n}^{*} P\left(\frac{k-m}{T} + \Delta f\right) P\left(\frac{n-m}{T} + \Delta f\right).$$
(9)

The average ICI power depends on the frequency offset Δf and on the Fourier transform of the pulse p(t), denoted as P(f). Therefore, the average ICI power across different sequences is defined as follows [18]:

$$\overline{\sigma_{ICI}^{m}} = \sum_{k=0,k\neq m}^{N-1} \left| P\left(\frac{k-m}{T} + \triangle f\right) \right|^{2}.$$
(10)

The signal-to-interference power ratio (SIR) is given in [18] as follows:

$$SIR = \frac{|P(\Delta f)|^2}{\sum_{k=0, k \neq m}^{N-1} \left| P\left(\frac{k-m}{T} + \Delta f\right) \right|^2}.$$
(11)

4. Nyquist pulse shaping function

An OFDM-based system consists of N orthogonal subcarriers where signal analysis is made in the frequency domain [5]. The superposition of the subcarriers in OFDM makes it difficult to study the exact behavior of the received data. Moreover, we need to ensure that the received data satisfies the condition given in Eq. (3), or else interference will exist between the carriers. The most reliable way to ensure data robustness in the frequency domain is by evaluating the ICI power for OFDM-based systems. In addition, the OFDM signal spectrum is formed by placing P(f) on each subcarrier as shown in Fig. 2.

P(f) has to satisfy the Nyquist-I criterion to achieve zero interference among the subcarriers. This concept is described in [18] and can be implemented by using Eqs. (1) and (2), yielding the following expression:

$$\int_{-\infty}^{\infty} p(t)e^{j2\pi(f_k - f_m)t}dt = \begin{cases} 1, & k = m\\ 0, & k \neq m \end{cases}$$
(12)

The orthogonality of the subcarriers can be ensured by requiring spectral nulls in P(f) at the frequencies $\pm 1/T$, $\pm 2/T$, ..., $\pm n/T$. This approach can be expressed as follows:

$$P(f) = \begin{cases} 1, & f = 0\\ 0, & f = \pm 1/T, \pm 2/T, \dots, \end{cases}$$
(13)

The above Eq. (13) mathematically describes the Nyquist-I criterion as defined in the frequency domain.

4.1. Dual sinc pulse

The proposed DSP is a novel family of Nyquist-I pulses designed to reduce the ICI power in OFDM-based systems; it is characterized by three new design parameters β , γ , and n. The constant β is used to keep the upper part of the main lobe wide and smooth, γ is used to control the phase of the pulse, and n is used to reduce the size of the tail. Moreover, DSP is only defined for roll-off factor $\alpha = 1$, similar approach was used for ISP [16], PMSP [17], and NWF [21] pulses. Other pulses included in comparisons with DSP, are IMBH [22] and IPLCP [23,24], which are pulses defined for all values of α with range $0 \le \alpha \le 1$ unlike the proposed one. The proposed pulse is the product of a constant, a sinc power function, and a modified sinc power function. The frequency-domain expression of the DSP is denoted as $P_{DSP}(f)$ and is expressed as follows:

$$P_{DSP}(f) = \left\{\beta sinc(fT) \times \left[(1-\beta)sinc(\gamma fT)\right]^2\right\}^n,$$
(14)

The new design parameters β , γ , and *n* are defined for all real numbers. These parameters provide additional degrees of freedom to minimize ICI power, increase SIR power, and improve BER in OFDM systems. To verify that DSP meets the Nyquist-I criterion given in Eq. (13), we examined the DSP given in Eq. (14) at f = 0, and for any value of the parameters i.e. β , γ and *n*, it is always equal to one. Moreover, if the proposed pulse is evaluated for



Fig. 2. OFDM signal spectrum.

 $f = \pm 1/T, \pm 2/T, \dots$, it is always equal to zero. Thus, we can say that the DSP satisfies the Nyquist-I criterion which is expressed in frequency domain in Eq. (13).

It has been observed that a pulse with the small magnitude side lobes can reduce the ICI power and increase the SIR power [16]. However, having a broader main lobe results in low BER, which in turn enhances the performance of OFDM systems [17]. Thus, our aim is to develop a pulse that has small magnitude side lobes to minimize the ICI power, and a wide main lobe to achieve low BER.

Fig. 3 compares the frequency functions of the ISP, PMSP, NWF, and DSP pulses. It is obvious that the side lobes of the proposed DSP and NWF have the smallest amplitude, almost vanishingly small compared to those of the ISP and PMSP. Due to the sufficient decrease in its tail size, it can be assumed that DSP will greatly reduce the ICI power. Although the NWF pulse has the smallest side lobes, it may not be effective in achieving low BER because it has a narrow main lobe. The DSP provides a broader main lobe for a particular value of β , without affecting the side lobes.

The role of β is to keep the upper part of the main lobe broader and smooth at a particular value i.e. $\beta = 1.76$. In Fig. 3, different values of β are taken as examples to show the difference in the performance of the DSP. Moreover, these values were considered to be suitable in reducing the ICI power after running extensive computer simulations, as discussed in detail in the next section. The use of β equal to 0.35 and 1 respectively yielded DSP with narrow and sharp-edged main lobes, whereas $\beta = 2$ is violating by exceeding the required amplitude. Among the values shown, only $\beta = 1.76$ presents a broad and smooth main lobe. The use of this value was supported by further investigations presented in the next section, including numerical simulations demonstrating its ICI power reduction and theoretical simulations demonstrating its low BER. Note that $\beta = 1.76$ is used throughout the remainder of this manuscript owing to its unique performance.

Fig. 4 shows that when the value of γ is increased, the main lobe of the DSP becomes narrower without affecting the side lobes. The role of γ is to control the phase of the DSP, with reductions in γ broadening the main lobe. However, the side lobes of the DSP remain small as the γ value changes. In addition, DSP shows smaller side lobes than ISP and PMSP for all values of γ except $\gamma = 0.5$, which verifies that DSP can perform better in reducing ICI power and increasing SIR power. The use of $\gamma = 0.5$ yields a broader main lobe, but disrupts the conditions by increasing the amplitude of the



Fig. 3. Comparison of frequency functions of different pulses for $\alpha = 1$, and β varies for DSP with $\gamma = 1, n = 1$.



Fig. 4. Comparison of frequency functions of different pulses for $\alpha = 1$, and γ varies for DSP with $\beta = 1.76$, n = 1.

pulse. The value $\gamma = 1$ is used throughout the remainder of this manuscript as it yields sufficiently reduced side lobes and a wider main lobe among all other values. Contrastingly, $\gamma > 1$ allows the main lobe to become narrower, which is not required, and $\gamma = 2$ yields a narrow main lobe almost similar to that of NWF. In addition, it has been reported that higher values of $\gamma > 1$ should not be used because they may yield a narrow main lobe and thus introduce high BER [17]. Although the ISP and PMSP pulses have broader main lobes than the DSP and NWF pulses, their side lobes are not small enough to greatly reduce the ICI power, as shown in Fig. 3.

Fig. 5 illustrates that when the value of n is increased, the main lobe of DSP becomes narrow whereas the upper part of the main lobe remains smooth and wide, in contrast to the NWF pulse, for which increasing n yields a narrower main lobe with a sharp top edge. The role of n is to ensure that the side lobes are of relatively small magnitude. In addition, n allows the top edge of the main lobe to remain smooth even though its value is increased. It can be observed from the figure that when n = 1, the DSP has the smallest side lobes and a wider main lobe. Values of n > 2 yield small



Fig. 5. Comparison of frequency functions of different pulses for $\alpha = 1$, and *n* varies for DSP with $\beta = 1.76$, $\gamma = 1$.

side lobes but narrow the main lobe, making them unsuitable for achieving low BER. Accordingly, n = 1 is used throughout the remainder of this manuscript because it yields a broad main lobe and small side lobes.

We investigated the frequency-domain closed-form expression of DSP in three different ways, by varying each of the design parameters n, β , and γ while holding the other two constant. First, we considered variations in β while using the fixed values of $\gamma = 1$ and n = 1. This allowed the main lobe to become broader and smoother along its top edge. Second, we considered variations in γ for the fixed values of $\beta = 1.76$ and n = 1. This allowed the main lobe to become wider for small values of γ without affecting the magnitude of the side lobes, and thus may result in low BER. Third, we considered variations in n for the fixed values of $\beta = 1.76$ and $\gamma = 1$. In this scenario, the main lobe became narrow for n > 2 while the magnitude of the side lobes remained small.

Different values of the design parameters are taken herein as examples to demonstrate the robustness of the proposed pulse. In general, for $\beta = 1.76$, $\gamma = 1$ and n = 1, the DSP shows better performance by providing a wide and smooth main lobe as well as small side lobes. Despite of forming important shapes by satisfying the theory of the smallest side lobes [16] and the wider main lobe [17], we cannot guarantee that the DSP yield the best performance in terms of ICI power and BER at $\beta = 1.76$, $\gamma = 1$ and n = 1. Therefore, we need to implement the different pulse shaping functions in a real OFDM system to evaluate and compare their performance in terms of ICI power and BER, by means of both numerical simulations and theoretical investigations. In addition, we have to determine an optimal β for $\alpha = 1$ to minimize ICI power and achieve low BER in OFDM systems via numerical simulations; which require extensive computer simulations, because an analytical solution for the optimal β seem unrealizable at this point. Regarding the other two parameters γ and n, it is observed that they satisfy the requirements of small side lobes and a broad main lobe effectively at $\gamma = 1$ and n = 1. Therefore, we use $\gamma = 1$ and n = 1throughout the remainder of the present manuscript.

In wireless communication systems, it is important to recover a desired signal perfectly after synchronization among the transmitter and receiver oscillators. From the above results and analysis it is observed that the DSP can perform better in minimizing ICI power and achieving low BER for OFDM-based systems.

5. Performance evaluation

The side lobes of the DSP were considerably reduced compared to those of ISP and PMSP, as shown in Fig. 4. It can be seen in this figure that ISP and PMSP have high side lobes for amplitude $a \le 0.5$ as compared to a = 1 given in [16,17], which show that ISP and PMSP have high ICI power at $a \le 0.5$. On the other hand, the side lobes of the DSP and NWF were almost vanishingly small, suggesting that both these pulses may greatly reduce the ICI power. But NWF lags behind due to its narrow main lobe as shown in Fig. 5, which may degrade the performance of the system by introducing high BER. Increasing γ narrows the main lobe of the DSP as depicted in Fig. 4; this can greatly decrease the ICI power but increases the BER of the system [16,17].

Next, we implemented a real OFDM-based system via numerical simulations to evaluate the DSP and other existing pulses in terms of ICI and SIR power. A Binary phase shift keying (BPSK) modulation scheme is used to modulate a 64 subcarriers OFDM system. Table 1 lists the parameters used in the ICI and SIR simulations. Block oversampling is used to sample the signal with a sampling frequency higher than the Nyquist rate to avoid aliasing and phase distortion. The purpose of using BPSK modulation is to validate the comparison results, in particular to implement the

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Simulation	parameters.
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Parameter	Value
Modulation Number of symbols Number of subcarriers Input data block size Block oversampling Roll-off factor, α	BPSK 100 64 52 10 1

expressions given in Eqs. (15) and (16), which use BPSK modulation, as it was done in [17,21,23]. Fig. 6 shows the ICI power of an OFDM-based system using DSPs with different values of β . Investigations regarding the sub-optimum value of β in terms of the ICI power and the BER is based on two key factors that allow us to run extensive computer simulations. First, we need to assure that the sub-optimum value should maintain small magnitude side lobes that sufficiently decreases the ICI power [16]. Second, such sub-optimum value allow the main lobe to be broad enough to achieve low BER [17]. It is observed that for β equal to 0.35 and 1, ICI power reduction is achieved, but also yielded narrow main lobes, as shown in Fig. 3. Such narrow main lobes may not perform well in terms of BER. However, we will discuss investigations on the effect upon BER of using sub-optimum value of the constant β later on. It is observed that β of 1.76 yields a reasonably broad main lobe from Fig. 3 and sufficiently reduces ICI power.

It was noticed that increasing β showed the maximum ICI power, eventually creating distortion in the OFDM system by raising its sensitivity to frequency offset. In general, we can define a particular range that includes only those values which yield greatly reduced ICI power, namely $0 \le \beta \le 2$. Within this range we can easily find a sub-optimum value of the constant β for the DSP in terms of BER as well. Furthermore, in terms of ICI power reduction, the proposed pulse shows better performance at $\beta = 1.76$ in combination with $\gamma = 1$ and n = 1. But the use of these values still needs to be evaluated in terms of their effect upon BER to prove the theory that a broad main lobe results in low BER [17].

Fig. 7 compares the recently proposed pulses IMBH [22] and IPLCP [23,24] with the newly proposed DSP family of Nyquist-I pulses; in terms of ICI power. The only difference between these pulses and other exponential pulses including ISP [16], PMSP [17], and NWF [21] is that the exponential pulses and the proposed



Fig. 6. ICI power of DSP applied in a 64 subcarriers OFDM system at $\gamma = 1$, n = 1 whereas β value varies for $\alpha = 1$.



Fig. 7. ICI power of the DSP and other evaluated pulses applied in a 64 subcarriers OFDM system for $\alpha = 1$.

DSP are defined for $\alpha = 1$, whereas the IMBH and IPLCP are defined for different values of α , with the small value of $\alpha = 0.22$ being of particular interest. However, we consider only $\alpha = 1$ for IMBH and IPLCP in the present work to allow comparison of the performance of these pulses with that of the DSPs. The DSP performed better in minimizing ICI power than the IMBH and IPLCP pulses at $\alpha = 1$, for large and small normalized frequency offsets ΔfT .

Next, we compared the DSP with the three pulses most commonly discussed in the literature, namely ISP [16], PMSP [17], and NWF [21], in terms of ICI power. Another reason for making such comparisons is that these pulses were only defined for $\alpha = 1$. ISP yielded superior performance over other conventional pulses such as rectangular, raised cosine, and BTRC in minimizing the ICI power. It is well known for its design simplicity. On the other hand, important contributions of the PMSP include a broad main lobe and smaller side lobes, which yield better performance than ISP in reducing ICI power and achieving low BER. However, the design and computational complexity of PMSP is greater than that of ISP. Recently, the NWF has been proposed, which yields performance even better than that of ISP or PMSP. It has almost vanishingly small side lobes, but the major problem of a narrow main lobe.

Fig. 8 presents an ICI power comparison of ISP, PMSP, NWF, and DSP when applied in a 64-subcarrier OFDM system with normalized frequency offset ΔfT . DSP outperformed the other existing pulses in this system by achieving low ICI power at both small and large ΔfT . ISP showed the greatest ICI power among the pulses studied. Hence, it is proved that high-magnitude side lobes result in high ICI power [20,21]. However, NWF and DSP had the smallest side lobes, and DSP was able to perform better than NWF in reducing the ICI power. Therefore, DSP is the pulse that performed best in minimizing the ICI power, as verified by these numerical simulations.

The DSP achieved the highest SIR among the pulses tested, as depicted in Fig. 9. The ISP, PMSP, NWF, and DSP were employed in a 64-subcarrier OFDM system to evaluate their performances in terms of SIR power. The numerical results are mentioned below which were obtained during simulations. The SIR power of the DSP was 30.53 dB at $\Delta fT = 0.15$, higher than that of ISP, PMSP, and NWF, which were 26.24, 26.70, and 29.18 dB, respectively. At $\Delta fT = 0.35$, the SIR power of the DSP was 17.74 dB, whereas ISP, PMSP, and NWF respectively achieved 16.02, 16.24 and 17.19 dB. Thus, it was verified by means of simulations that DSP performed better in obtaining high SIR power than other existing pulses.



Fig. 8. ICI power of different pulses applied in a 64 subcarriers OFDM system for $\alpha = 1$.



Fig. 9. SIR power of different pulses applied in a 64 subcarriers OFDM system for $\alpha = 1$.

The theoretical BER expressions given in Eqs. (15) and (16) was implemented to evaluate the performance of the DSP and to compare it to that of other existing pulses in terms of BER. These pulses were applied in BPSK-OFDM systems over an AWGN channel. More details regarding the BER expressions are given elsewhere [25]. The average BER is given as a function of Δf , carrier phase noise θ , average ICI power P_{ICI} and the pulse shape P(f); N is the number of subcarriers and $\gamma_b = E_b/N_0$.

$$\overline{BER}_{OFDM} = 1 - (1 - BER_{symbol})^N, \tag{15}$$

$$BER_{symbol} = \frac{1}{2} \left(Q \left(\cos\theta \left[P(-\Delta f) + \sqrt{P_{ICI}} \right] \sqrt{2\gamma_b} \right) + Q \left(\cos\theta \left[P(-\Delta f) - \sqrt{P_{ICI}} \right] \sqrt{2\gamma_b} \right) \right)$$
(16)

Fig. 10 illustrates the BER comparative analysis of the suboptimum value of β in the DSP for $\Delta fT = 0.15$, $\theta = 10^{\circ}$ and $\alpha = 1$. This analysis verifies the selection of $\beta = 1.76$ as a sub-optimum value because this value accomplishes a balance in the



Fig. 10. BER comparison for a sub-optimum value of β for DSP using a BPSK-OFDM system for $\alpha = 1$.

performance of the DSP in terms of ICI power and BER. Contrastingly, $\beta = 0.35$ and 1 showed the worst performance in terms of BER while highly reducing ICI power, as shown in Fig. 6. Hence, it is proved that a narrow main lobe greatly reduces the ICI power but results in high BER in an OFDM system [17]. In addition, broadening the main lobe yields lower BER. Therefore, the sub-optimum value of $\beta = 1.76$, in combination with $\gamma = 1$ and n = 1, yielded small side lobes and a wider main lobe that minimized both ICI power and BER.

A comparative analysis of the BER of DSP, IMBH, IPLCP, and other exponential pulses including the ISP, PMSP, and NWF was carried out for $\Delta fT = 0.2$, $\theta = 30^{\circ}$, and $\alpha = 1$.

Fig. 11 shows that the DSP yielded a lower energy per bit to noise power spectral density ratio (E_b/N_o) for a BER equal to 10^{-3} at $\Delta fT = 0.2$ and so on. The performance of the IMBH and IPLCP was not sufficient in terms of BER, in comparison with DSP at $\Delta fT = 0.2$. This analysis confirmed the superiority of DSP at $\Delta fT = 0.2$ by demonstrating that it yielded the lowest BER at $\alpha = 1$ among the other recommended pulses for OFDM systems.

10⁰ 10⁻² 10⁻⁴ BER 10⁻⁶ 10-8 10⁻¹⁰ IMBH, β=1.52, n=2 IPLCP, ε=0.1, γ=1 DSP β=1.76, γ=1, n=1 10⁻¹¹ 8 10 12 14 16 Eb/No, dB

Fig. 11. BER performance comparison of different pulses applied in a 64 subcarriers BPSK-OFDM system with $\Delta fT = 0.2$ and $\alpha = 1$.

Fig. 12 shows the comparison of the DSP with other exponential pulses from the literature that were defined for $\alpha = 1$. The NWF showed worst performance in terms of BER at the high ΔfT of 0.2, because of its narrow main lobe with a sharp top edge, as shown in Fig. 3; the DSP yielded the second-lowest BER and the PMSP yielded the lowest BER. The PMSP is the only pulse that outperformed DSP in terms of BER, and it only did so for the high frequency offset of 0.2. The above analysis and simulation results confirm the superiority of DSP not only in terms of ICI power but also in achieving lower BER than other existing pulses. Due to the wider main lobe, particularly from the top edge, the PMSP showed better performance than the DSP in terms of BER.

6. Complexity evaluation

In OFDM systems, a pulse shaping function having a simple design and expression is extremely desirable because an inverse discrete Fourier transform (IDFT) is used for transforming the amplitudes of the subcarriers into complex-time-domain signals. Subsequently, the modulated symbols are convoluted with a pulse shaping function. A pulse shaping function with a simple design can only reduce the complexity when a convolution process takes place in a system [26]. As a result, DSP reduces the complexity due to its simple expression, given in Eq. (14).

We used the tic and toc commands in MATLAB [27] to evaluate the time complexity by estimating the average elapsed time of the convolution process, according to the simulation parameters given in Table 1. The convolution operation was carried out using different pulse shaping functions: ISP [16], PMSP [17], NWF [21], IMBH [22], IPLCP [23], and DSP. While executing a tic command, a function was used to record the internal time of the computer system according to a stopwatch timer, and the elapsed time in seconds was displayed when the toc command was executed. Further, both of these commands were used to estimate the elapsed time for the completion of a particular set of instructions.

The average elapsed time may vary depending on the specifications of the computer system used to execute MATLAB commands. Table 2 lists the specifications of the computer systems used to execute tic and toc commands. Table 3 presents the average elapsed times of the convolution operations conducted using different pulse shaping functions. The ISP showed the lowest average elapsed time among all pulses tested. Whereas the DSP has the



Fig. 12. BER performance comparison of different pulses applied in a 64 subcarriers BPSK-OFDM system with $\Delta fT = 0.2$ and $\alpha = 1$.

Table 2

Computer system specification.

Title	Technology
Processor	Intel(R) Core(TM)2 Duo CPU E7400
Speed	2.80 GHz
System Type	X86-based PC
RAM	4 GB
Windows	Microsoft Windows 7 Professional

Table 3

Average elapsed time of convolution process.

Pulse	Convolution
ISP	4.5×10^{-5}
DSP	4.7×10^{-5}
NWF	5.4×10^{-5}
PMSP	5.8×10^{-5}
IMBH	$6.1 imes10^{-5}$
IPLCP	6.9×10^{-5}

second lowest average elapsed convolution time, only outperformed by the ISP. Therefore, the DSP is suitable for OFDM-based systems because it reduces the complexity of the convolution operation. The use of IMBH and IPLCP led to the highest average elapsed times due to the complexity of these pulse shaping functions.

7. Conclusion

A novel family of Nyquist-I pulses termed dual sinc pulses (DSPs) is proposed herein to minimize the ICI power and improve BER in OFDM-based systems. The DSP family is characterized by three new design parameters. The constant β keeps the upper part of the main lobe wide and smooth without affecting the side lobes, γ controls the phase of the DSP while allowing the main lobe to remain wide, and the constant *n* keeps the magnitude of the side lobes small. The DSP was studied and compared with the recently proposed pulses IMBH and IPLCP, as well as with other exponential pulses ISP, PMSP, and NWF. After running extensive computer simulations, we found that $\beta = 1.76$, $\gamma = 1$, and n = 1 were sub-optimum values of DSP in terms of reducing ICI power and achieving low BER in OFDM-based systems for $\alpha = 1$. Numerical and theoretical simulations verified that the DSP yields superior performance compared to the other evaluated pulses in terms of ICI power, SIR, and BER in OFDM-based systems. The DSP has great potential to reduce ICI power and enhance BER in OFDM-based systems. The proposed family of Nyquist-I pulses can serve as a key pulse in next-generation wireless communication systems such as 5G.

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