Space–Time Network Coding With Antenna Selection

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***Abstract*—In this paper, we investigate the space–time network coding (STNC) with antenna selection (AS) in the cooperative multiple-input–multiple-output (MIMO) network, where U users communicate with a common destination D with the aid of R decode-and-forward (DF) relays. In this network, the best transmit/receive antenna pair with the highest signal-to-noise (SNR) ratio is selected to perform the signal transmission and reception over the user–destination and relay–destination links. To quantify the performance degradation of STNC with AS due to time delay between instants of channel state information (CSI) estimation and data transmission, we derive the symbol error rate (SER) and capacity expressions over flat Rayleigh fading channels. The asymptotic SER expression reveals that STNC with AS guarantees full diversity order of $(N_u+\sum_{r=1}^{R} N_r)N_D$, where N_u, N_r, and N_D are the antenna numbers of user u, relay r, and the destination D, respectively, when the AS is performed based on perfect CSI, and outdated CSI degrades the full diversity order to $R+1$. We also examine the effect of spatial correlation on the performance of STNC with AS by obtaining new closed-form expression for the asymptotic SER, which indicates that STNC with AS achieves the full diversity order over flat correlated Rayleigh channels, regardless of the value of spatial correlation coefficient. Numerical and simulation results are provided to demonstrate the accuracy of our theoretical analysis and evaluate the performance of STNC with AS.**

***Index Terms*—Antenna selection (AS), outdated channel state information (CSI), space–time network coding (STNC), spatial correlation, symbol error rate (SER).**

I. Introduction

RECENTLY, various cooperative diversity schemes have been proposed to satisfy the requirements of emerging wireless applications [1]–[3]. However, integrating cooperation between different nodes usually requires perfect timing and/or frequency synchronization [4], [5], which may be difficult or impossible in practical multinode systems as it is very challenging to align all the signals, particularly when the nodes are spatially distributed. When synchronization is imperfect, the performance of cooperative communications would be severely degraded.

Time-division multiple access (TDMA) is regarded as the most commonly used technique to overcome the imperfect synchronization issues in multinode systems [6]. However, TDMA causes relatively large transmission delays as only one user is served in each time slot. To maintain the spatial diversity while reducing the total required time slots, the TDMA-based space–time network coding (STNC) scheme was proposed in [7]. Compared with the traditional TDMA scheme, the STNC scheme improves the throughput by $(U(R+1))/(U+R)$ times. This is due to the fact that the total required time slots are reduced from $U(R+1)$ to $U+R$, where U and R are the numbers of users and relays, respectively. STNC jointly exploits the benefits of both network coding and space–time coding via combining information from different sources at each relay and forwarding the combined signals to a destination in dedicated time slots. Consequently, STNC achieves the full diversity order. Through allowing each relay to exploit the overheard signals transmitted from not only the sources but also the previous relays, a new STNC scheme with overhearing relays was proposed in [8]. When there is no dedicated relay in the systems, clustering-based STNC and optimal node selection-based STNC schemes were presented to exploit the cooperative diversity gain in [9] and [10], respectively.

In STNC, spreading codes are used to protect one symbol against the interference from other symbols [7]. Compared with a rake receiver with maximal ratio combining (MRC) in code-division multiple-access systems [11]–[13], the decoding errors at relays shall be taken into account when evaluating the end-to-end system performance of STNC. By considering the relaying error, the symbol error rate (SER) and outage probability (OP) of STNC were investigated in [7], [14], and [15], respectively.

It is noticed that all the nodes in the network studied in [7]–[10], [14], and [15] are equipped with a single antenna. Since multiple-input–multiple-output (MIMO) technology, in which nodes are equipped with multiple transmit and/or receive antennas, can significantly increase communication reliability through the use of spatial diversity [16], we integrated transmit antenna selection with MRC (TAS/MRC) into STNC in our previous work [17]. It is noticed that TAS/MRC requires multiple radio-frequency chains, which inevitably increases the complexities and costs of the transceivers [18], [19]. Motivated by this, we integrate antenna selection (AS) into STNC here to circumvent this drawback since AS preserves the diversity gain while significantly reducing hardware complexity [18]. With

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AS, the transceiver can be easily implemented with a single front end and an analog switch.

In this paper, we consider STNC with AS in the cooperative MIMO network over flat Rayleigh fading channels, where U users communicate with a common destination D with the assistance of R decode-and-forward (DF) relays and all nodes are equipped with multiple antennas. We adopt AS over user–destination (u–D) and relay–destination (r–D) links, where the optimal transmit/receive antenna pair is selected to provide the highest received signal-to-noise ratio (SNR) based on the channel state information (CSI) estimated at the receiver. For the user–relay $(u{-}r)$ link, only the receive antenna at the relay is selected to maximize the received SNR as the transmit antenna of the user is determined by the CSI of the u–D link.

Compared with single-antenna STNC whose receive node only needs to estimate the CSI of a single-input–single-output channel, each receive node in STNC with AS needs to estimate the CSI of a MIMO channel. After obtaining the CSI of the MIMO channel, the receive node selects the transmit/receive antenna pair with the highest SNR and then feeds back the index of the selected transmit antenna to the transmit node. Obviously, the complexity of STNC with AS relative to single-antenna STNC lies in estimating the CSI of the MIMO channel, selecting the transmit/receive antenna pair, and feeding back the index of the selected transmit antenna. The larger the antenna number, the higher the complexity of STNC with AS relative to single-antenna STNC. It is noticed that the complexity of MIMO channel estimation and that of transmit/receive antenna pair selection are proportional to the product of the number of transmit antennas and the number of receive antennas. Compared with our previous work [17], we adopt AS to preserve the full diversity while further reducing hardware complexity and cost. In STNC with TAS/MRC, outdated CSI has no impact on the transmission over the user–relay link [17], whereas in STNC with AS, the effect of outdated CSI on the user–relay link shall be taken into account. Compared with [17], we also derive the capacity of STNC with AS and quantify the detrimental effect of spatial correlation on the SER of STNC with AS. The primary analytical contributions of this paper are summarized as follows.

1) We integrate AS into STNC as a solution to preserve full transmit and receive diversity with low hardware cost and reduced feedback overhead.
2) We derive new closed-form SER expressions for STNC with AS to quantify the effect of outdated CSI on the system performance by taking the delay correlation coefficients into account and also derive the capacity for STNC with AS. The perfect CSI is a special case of outdated CSI when the delay correlation coefficient is equal to 1. It is shown that outdated CSI vanishes the spatial diversity offered by multiple antennas and reduces the full diversity order of $(N_u + \sum_{r=1}^{R} N_r)N_D$ to $R + 1$, where N_u, N_r, and N_D denote the antenna numbers of user u, $1 \le u \le U$; relay r, $1 \le r \le R$; and the destination D, respectively.
3) To examine the performance degradation of STNC with AS due to spatial correlation, we derive a new compact expression for the asymptotic SER over flat spatially correlated Rayleigh fading channels. It is demonstrated that the spatial correlation exerts no effect on the diversity order but imposes a detrimental effect on the array gain, which worsens the SER.

The remainder of this paper is organized as follows. The system model is presented in Section II. The effects of outdated CSI and spatial correlation on the performance of STNC with AS are quantified in Sections III and IV, respectively. Numerical and simulation results that confirm our analysis are presented in Section V, which are followed by the conclusions in Section VI.

II. System Model

Consider a cooperative MIMO network composed of U users sending information to a common destination D with the aid of R DF relays over flat Rayleigh fading channels. We use $\mathbf{H}_{\nu\mu}$ to denote the channel coefficient matrix between transmitter μ and receiver ν, and the (j,i)th element of $\mathbf{H}_{\nu\mu}$, $h_{ji}^{\nu\mu} \sim \mathcal{CN}(0, d_{\nu\mu}^{-\alpha})$ is the channel coefficient between the ith antenna of transmitter μ and the jth antenna of receiver ν. Here, $(\mu,\nu) \in \{(u,r),(u,D),(r,D)\}$, and $d_{\nu\mu}$ and α denote the distance between μ and ν and the path-loss exponent, respectively. The transmission of STNC takes place over $U + R$ time slots, which are partitioned into two consecutive phases [7]. We now outline the STNC with AS in the cooperative MIMO network as follows.

In the first phase, U users take turns to broadcast their signals to the relays and the destination D in the first U time slots, i.e., user u broadcasts its signal in time slot u. The optimal transmit antenna among the N_u antennas at user u and the optimal receive antenna among the N_D antennas at the destination D are selected to maximize the instantaneous received SNR at destination D. Therefore, the indices of the optimal transmit and receive antennas in the sense of SNR are determined as $\{i^*, j^*\} = \underset{1\le i\le N_u, 1\le j\le N_D}{\arg\max} \|h_{ji}^{Du}\|_F$, where $\|\cdot\|_F$ denotes the Frobenius norm. With optimal transmit/receive antenna pair $\{i^*, j^*\}$, the signals received at the destination D and relay r from user u are

$$y_{Du} = h_{j^*i^*}^{Du}\sqrt{P_{0u}}x_u + n_{Du} \tag{1}$$

and

$$y_{ru} = h_{k^*i^*}^{ru}\sqrt{P_{0u}}x_u + n_{ru} \tag{2}$$

respectively, where P_{0u} is the transmit power of x_u, x_u denotes the signal with unit energy transmitted from user u, n_{Du} and n_{ru} are the additive white Gaussian noise (AWGN) with zero mean and variance of σ^2, and $h_{k^*i^*}^{ru}$, $\|h_{k^*i^*}^{ru}\|_F = \max_{1\le k\le N_r} \|h_{ki^*}^{ru}\|_F$, is the channel coefficient between the i^*th antenna at user u and the optimal receive antenna at relay r.

In the second phase, R relays combine the overheard signals from multiple users during the first phase to a single symbol by assigning a unique spreading code to each signal and then take turns to transmit the combined symbol to the destination D in the last R time slots, i.e., relay r transmits its symbol in time slot $U + r$. The indices of the optimal transmit antenna at relay r and the optimal receive antenna at the destination D in the sense of SNR are determined as $\{k^\star, j^\star\} = \underset{1\le k\le N_r, 1\le j\le N_D}{\arg\max} \|h_{jk}^{Dr}\|_F$, and the signal received at the destination D from relay r is

$$y_{Dr}(t) = h_{j^\star k^\star}^{Dr} \sum_{u=1}^{U} \beta_{ru}\sqrt{P_{ru}} x_u s_u(t) + n_{Dr}(t) \tag{3}$$

where β_{ru} denotes the detection state of relay r on x_u, P_{ru} is the transmit power of x_u at relay r, $s_u(t)$ denotes the spreading code assigned to x_u, and $n_{Dr}(t)$ is AWGN with zero mean and variance of σ^2. With spreading codes, the bandwidth requirement of STNC is T_s/T_c times of that of the traditional TDMA scheme, where T_s and T_c are the symbol and spreading code chip periods, respectively. If relay r correctly decodes x_u, $\beta_{ru} = 1$; otherwise, $\beta_{ru} = 0$. The relay detection state possibly can be done by examining the included cyclic redundancy check digits or the received SNR levels. Here, we assume that the destination D knows the detection states of the relays, which can be obtained through the indicators sent by the relays.

Through matched filtering, the received signal at the destination D from user u' through relay r is

$$\begin{aligned} y_{Dr}^{u'} &= \langle y_{Dr}(t), s_{u'}(t)\rangle \\ &= h_{j^\star k^\star}^{Dr} \sum_{u=1}^{U} \beta_{ru}\sqrt{P_{ru}} x_u \rho_{uu'} + n_{Dr}^{u'} \end{aligned} \tag{4}$$

where $n_{Dr}^{u'}$ is AWGN with zero mean and variance of σ^2, $\rho_{uu'} = \langle s_u(t), s_{u'}(t)\rangle$ is the cross correlation between spreading codes $s_u(t)$ and $s_{u'}(t)$ with $\langle f(t), g(t)\rangle \triangleq (1/T)\int_0^T f(t)g^*(t)dt$ being the inner product between $f(t)$ and $g(t)$ with the symbol interval T, and $g^*(t)$ is the complex conjugate of $g(t)$. Here, we assume that $\rho_{uu} = \|s_u(t)\|_F^2 = 1$ and define the cross-correlation matrix as

$$\mathbf{R}_\rho = \begin{bmatrix} 1 & \rho_{21} & \cdots & \rho_{U1} \\ \rho_{12} & 1 & \cdots & \rho_{U2} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{1U} & \rho_{2U} & \cdots & 1 \end{bmatrix}.$$

Based on $\mathbf{R}_\rho$, we rewrite (4) into matrix form as

$$\boldsymbol{y}_{Dr} = h_{j^\star k^\star}^{Dr} \mathbf{R}_\rho \mathbf{P}_r \boldsymbol{x} + \boldsymbol{n}_{Dr} \tag{5}$$

where $\boldsymbol{y}_{Dr} = [y_{Dr}^1, y_{Dr}^2, \ldots, y_{Dr}^U]^T$, $\mathbf{P}_r = \mathrm{diag}\{\beta_{r1}\sqrt{P_{r1}}, \beta_{r2}\sqrt{P_{r2}}, \ldots, \beta_{rU}\sqrt{P_{rU}}\}$, $\boldsymbol{x} = [x_1, x_2, \ldots, x_U]^T$, $\boldsymbol{n}_{Dr} = [n_{Dr}^1, n_{Dr}^2, \ldots, n_{Dr}^U]^T$ is the AWGN vector with zero mean and variance matrix of $\sigma^2\mathbf{R}_\rho$, and $[\cdot]^T$ denotes transpose. The correlation among the entries of $\boldsymbol{n}_{Dr}$ is from the matched-filtering operation of the inner product between different spreading codes.

Assuming $\mathbf{R}_\rho$ being invertible with the inverse matrix $\mathbf{R}_\rho^{-1}$ [20] and multiplying both sides of (5) with $\mathbf{R}_\rho^{-1}$ yield the soft signal of x_u from relay r as

$$\tilde{y}_{Dr}^u = h_{j^\star k^\star}^{Dr} \beta_{ru}\sqrt{P_{ru}} x_u + \tilde{n}_{Dr}^u \tag{6}$$

where $\tilde{n}_{Dr}^u$ is the AWGN with zero mean and variance of $\sigma^2\theta_u$ with θ_u being the uth diagonal element of matrix $\mathbf{R}_\rho^{-1}$ associated with signal x_u. With orthogonal spreading codes, i.e., $\rho_{uu'} = 0, \forall u \ne u'$, $\mathbf{R}_\rho$ is an identity matrix of size U. In such a case, $\theta_u = 1$, $1 \le u \le U$.

Combining the information on x_u from user u and R relays via MRC yields the end-to-end SNR of x_u at the destination D as

$$\Upsilon_u = \Upsilon_{uD} + \sum_{r\in\mathcal{D}_u} \Upsilon_{rD} \tag{7}$$

where $\Upsilon_{uD} = \|h_{j^*i^*}^{Du}\|_F^2 P_{0u}/\sigma^2$, $\Upsilon_{rD} = \|h_{j^\star k^\star}^{Dr}\|_F^2 P_{ru}/(\sigma^2\theta_u)$, and $\mathcal{D}_u = \{r : \beta_{ru} = 1, r = 1, 2, \ldots, R\}$ is the active relay set associated with user u. When all the relays fail to decode the signal x_u, i.e., $\mathcal{D}_u = \phi$, it is obvious that STNC with AS reduces to the conventional single-hop AS scheme.

III. Performance With Outdated Channel State Information

Due to channel fluctuations, the CSI employed in AS process may differ from the exact CSI in data transmission instant; in other words, the employed CSI is outdated [21]. Here, we examine the detrimental impact of outdated CSI on the performance of STNC with AS. Here, we assume that the transmit/receive antenna pair of the r–D link[1] and the receive antenna of the u−r link is selected based on the outdated CSI with τ_r and τ_{ur} time delays between the instants of channel estimation and data transmission, respectively.

To model the relationship between $h_{jk}^{Dr}(t)$ and $h_{jk}^{Dr}(t-\tau_r)$, we employ the time-varying channel feedback error model to express the channel coefficient as $h_{jk}^{Dr}(t) = \rho_r h_{jk}^{Dr}(t-\tau_r) + \sqrt{1-|\rho_r|^2} e_r(t)$ [22]–[24], where $e_r(t) \sim \mathcal{CN}(0, d_{Dr}^{-\alpha})$ and ρ_r is the normalized delay correlation coefficient between $h_{jk}^{Dr}(t)$ and $h_{jk}^{Dr}(t-\tau_r)$. For Clarke's fading spectrum, $\rho_r = \mathcal{J}_0(2\pi f_r \tau_r)$, where f_r is the Doppler frequency and $\mathcal{J}_0(\cdot)$ is the zeroth-order Bessel function of the first kind [25, eq. (8.402)]. Similarly, for the u−r link, $h_{ki}^{ru}(t) = \rho_{ur} h_{ki}^{ru}(t-\tau_{ur}) + \sqrt{1-|\rho_{ur}|^2} e_{ur}(t)$, where $e_{ur}(t) \sim \mathcal{CN}(0, d_{ru}^{-\alpha})$ and $\rho_{ur} = \mathcal{J}_0(2\pi f_{ur}\tau_{ur})$ is the normalized delay correlation coefficient between $h_{ki}^{ru}(t)$ and $h_{ki}^{ru}(t-\tau_{ur})$, with f_{ur} being the Doppler frequency.

Defining $\tilde{\Upsilon}_{rD}$ and $\tilde{\Upsilon}_{ur}$ as the actual received SNRs of r–D and u−r links when AS is performed based on outdated CSI, the probability density functions (PDFs) of $\tilde{\Upsilon}_{rD}$ and $\tilde{\Upsilon}_{ur}$ are given by [26]

$$\begin{aligned} f_{\tilde{\Upsilon}_{rD}}(\tilde{\gamma}) = \frac{N_r N_D}{\bar{\gamma}_{rD}} \sum_{m=0}^{N_r N_D - 1} \binom{N_r N_D - 1}{m} \\ \times \frac{(-1)^m e^{-\frac{(m+1)\tilde{\gamma}}{(m(1-\rho_r^2)+1)\bar{\gamma}_{rD}}}}{m(1-\rho_r^2)+1} \end{aligned} \tag{8}$$

$$f_{\tilde{\Upsilon}_{ur}}(\tilde{\gamma}) = \frac{N_r}{\bar{\gamma}_{ur}} \sum_{m=0}^{N_r-1} \binom{N_r-1}{m} \frac{(-1)^m e^{-\frac{(m+1)\tilde{\gamma}}{(m(1-\rho_{ur}^2)+1)\bar{\gamma}_{ur}}}}{m(1-\rho_{ur}^2)+1} \tag{9}$$

[1]Here, the r–D link with $r = 0$ represents the u–D link.

respectively, where $\bar{\gamma}_{0D} = E[\|h_{ji}^{Du}\|_F^2]P_{0u}/\sigma^2$; $\bar{\gamma}_{rD} = E[\|h_{jk}^{Dr}\|_F^2]P_{ru}/(\sigma^2\theta_u)$, $1 \le r \le R$; $\bar{\gamma}_{ur} = E[\|h_{ki}^{ru}\|_F^2]P_{0u}/\sigma^2$; and $E[x]$ denotes the expectation of x.

In the presence of perfect CSI, by substituting $\rho_r = 1$ and $\rho_{ur} = 1$ into (8) and (9), respectively, the PDFs of Υ_{uD}, Υ_{rD}, and Υ_{ur} can be expressed using a generic expression given by

$$f_{\Upsilon_\varpi}(\gamma) = \frac{N_\varpi}{\bar{\gamma}_\varpi} \sum_{m=0}^{N_\varpi - 1} \binom{N_\varpi - 1}{m} (-1)^m e^{-\frac{m+1}{\bar{\gamma}_\varpi}\gamma} \tag{10}$$

where $(\Upsilon_\varpi, N_\varpi, \bar{\gamma}_\varpi) \in \{(\Upsilon_{uD}, N_u N_D, \bar{\gamma}_{uD}), (\Upsilon_{rD}, N_r N_D, \bar{\gamma}_{rD}), (\Upsilon_{ur}, N_r, \bar{\gamma}_{ur})\}$, and $\Upsilon_{ur} = \|h_{k^*i^*}^{ru}\|_F^2 \cdot P_{0u}/\sigma^2$ is the instantaneous received SNR of x_u at relay r. For the extreme case of fully outdated CSI, i.e., $\rho_r = \rho_{ur} = 0$, $\tilde{\Upsilon}_{rD}$ and $\tilde{\Upsilon}_{ur}$ reduce to exponentially distributed variables, which is in line with the fact that the transmit/receive antenna is actually selected randomly.

A. SER

1) Exact SER: In the presence of outdated CSI, the SER of STNC with AS associated with x_u is given by

$$\tilde{P}_{s,u} = \sum_{|\tilde{\mathcal{D}}_u|=0}^{R} \sum_{v=1}^{\tilde{\vartheta}} \tilde{P}_{s,u|\tilde{\mathcal{D}}_{u,v}} \Pr(\tilde{\mathcal{D}}_{u,v}) \tag{11}$$

where $\tilde{\mathcal{D}}_u$ is the active relay set associated with x_u in the outdated CSI scenario; $\tilde{\mathcal{D}}_{u,v}$ is the vth, $1 \le v \le \tilde{\vartheta}$ with $\tilde{\vartheta} = \binom{R}{|\tilde{\mathcal{D}}_u|}$, possible choice of $|\tilde{\mathcal{D}}_u|$ active relays from the R relays; and $\tilde{P}_{s,u|\tilde{\mathcal{D}}_{u,v}}$ denotes the SER of detecting x_u conditioned on $\tilde{\mathcal{D}}_{u,v}$. After renumbering the indices of relays belonging to $\tilde{\mathcal{D}}_{u,v}$, the corresponding received SNR of x_u conditioned on $\tilde{\mathcal{D}}_{u,v}$, i.e., $\tilde{\Upsilon}_{u|\tilde{\mathcal{D}}_{u,v}}$, is

$$\tilde{\Upsilon}_{u|\tilde{\mathcal{D}}_{u,v}} = \sum_{r=0}^{\tilde{p}} \tilde{\Upsilon}_{rD} \tag{12}$$

where $\tilde{p} = |\tilde{\mathcal{D}}_u|$ and $\tilde{\Upsilon}_{0D} = \tilde{\Upsilon}_{uD}$. In (11), the probability $\Pr(\tilde{\mathcal{D}}_{u,v})$ is

$$\Pr(\tilde{\mathcal{D}}_{u,v}) = \prod_{r \in \tilde{\mathcal{D}}_{u,v}} (1 - \tilde{P}_{s,ur}) \prod_{r \notin \tilde{\mathcal{D}}_{u,v}} \tilde{P}_{s,ur} \tag{13}$$

where $\tilde{P}_{s,ur}$ is the SER of detecting x_u at relay r in the presence of outdated CSI with τ_{ur} time delay. In the following, we proceed to derive $\tilde{P}_{s,u|\tilde{\mathcal{D}}_{u,v}}$ and $\Pr(\tilde{\mathcal{D}}_{u,v})$ to give the SER for the outdated CSI scenario based on the PDF expressions (8) and (9).

From (8) and (12), the PDF of $\tilde{\Upsilon}_{u|\tilde{\mathcal{D}}_{u,v}}$ is presented in the following lemma.

Lemma 1: The PDF of $\tilde{\Upsilon}_{u|\tilde{\mathcal{D}}_{u,v}}$ is derived as

$$f_{\tilde{\Upsilon}_{u|\tilde{\mathcal{D}}_{u,v}}}(\tilde{\gamma}) = \tilde{\Phi} \sum_{m_0=0}^{N_0 N_D - 1} \cdots \sum_{m_{\tilde{p}}=0}^{N_{\tilde{p}} N_D - 1} \tilde{\varphi} \sum_{r=0}^{\tilde{p}} \tilde{\mu}_r e^{-\tilde{\xi}_r \tilde{\gamma}} \tag{14}$$

where

$$\tilde{\Phi} = \prod_{r=0}^{\tilde{p}} \frac{N_r N_D}{\bar{\gamma}_{rD}}$$

$$\tilde{\varphi} = (-1)^{\sum_{r=0}^{\tilde{p}} m_r} \prod_{r=0}^{\tilde{p}} \frac{\binom{N_r N_D - 1}{m_r}}{m_r (1 - \rho_r^2) + 1}$$

$$\tilde{\xi}_r = \frac{m_r + 1}{(m_r (1 - \rho_r^2) + 1) \bar{\gamma}_{rD}}$$

$$\tilde{\mu}_r = \prod_{r'=0, r' \ne r}^{\tilde{p}} \left(\tilde{\xi}_{r'} - \tilde{\xi}_r\right)^{-1}.$$

Proof: The proof is presented in Appendix A. ■

Integrating (14) with the aid of [25, eq. (3.351.1)], we obtain the cumulative distribution function (CDF) of $\tilde{\Upsilon}_{u|\tilde{\mathcal{D}}_{u,v}}$ as

$$F_{\tilde{\Upsilon}_{u|\tilde{\mathcal{D}}_{u,v}}}(\tilde{\gamma}) = 1 - \tilde{\Phi} \sum_{m_0=0}^{N_0 N_D - 1} \cdots \sum_{m_{\tilde{p}}=0}^{N_{\tilde{p}} N_D - 1} \tilde{\varphi} \sum_{r=0}^{\tilde{p}} \frac{\tilde{\mu}_r e^{-\tilde{\xi}_r \tilde{\gamma}}}{\tilde{\xi}_r}. \tag{15}$$

Based on the CDF of the received SNR, the closed-form expression for SER is given by [27]

$$P_s = \frac{a}{2} \sqrt{\frac{b}{\pi}} \int_0^\infty \frac{F(\gamma)}{\sqrt{\gamma}} e^{-b\gamma} d\gamma \tag{16}$$

where parameters a and b are up to a specific used modulation scheme, which encompasses a variety of modulations such as binary phase-shift keying (BPSK) $(a = b = 1)$ and M-ary quadrature amplitude modulation $(a = 4(\sqrt{M} - 1)/\sqrt{M}; b = 3/(2(M - 1)))$ [28].

Substituting (15) into (16) and applying the identity [25, eq. (3.371)] to solve the resultant integral, the exact SER of x_u at the destination D conditioned on $\tilde{\mathcal{D}}_{u,v}$, i.e., $\tilde{P}_{s,u|\tilde{\mathcal{D}}_{u,v}}$, is given by

$$\tilde{P}_{s,u|\tilde{\mathcal{D}}_{u,v}} = \frac{a}{2} - \frac{a\sqrt{b}\tilde{\Phi}}{2} \sum_{m_0=0}^{N_0 N_D - 1} \cdots \sum_{m_{\tilde{p}}=0}^{N_{\tilde{p}} N_D - 1} \tilde{\varphi} \sum_{r=0}^{\tilde{p}} \frac{\tilde{\mu}_r}{\tilde{\xi}_r} (\tilde{\xi}_r + b)^{-\frac{1}{2}}. \tag{17}$$

For the extreme case of perfect CSI with $\rho_r = 1$, it is easy to verify that (17) is reduced as

$$P_{s,u|\mathcal{D}_{u,v}} = \frac{a}{2} - \frac{a\sqrt{b}\Phi}{2} \sum_{m_0=0}^{N_0 N_D - 1} \cdots \sum_{m_p=0}^{N_p N_D - 1} \varphi \sum_{r=0}^{p} \frac{\mu_r}{\xi_r} (\xi_r + b)^{-\frac{1}{2}} \tag{18}$$

where $\Phi = \prod_{r=0}^{p} (N_r N_D/\bar{\gamma}_{rD})$, $\varphi = (-1)^{\sum_{r=0}^{p} m_r} \prod_{r=0}^{p} \binom{N_r N_D - 1}{m_r}$, $\xi_r = ((m_r + 1)/\bar{\gamma}_{rD})$, $\mu_r = \prod_{r'=0, r' \ne r}^{p} (\xi_{r'} - \xi_r)^{-1}$, $p = |\mathcal{D}_{u,v}|$, and $\mathcal{D}_{u,v}$ denotes the active relay set in the perfect CSI scenario.

To calculate $\Pr(\tilde{\mathcal{D}}_{u,v})$, we now derive the SER of the $u-r$ link, $\tilde{P}_{s,ur}$, in the presence of time delay τ_{ur} between the instants of CSI estimation and data transmission. Based on (9), the CDF of $\tilde{\Upsilon}_{ur}$ is given by

$$F_{\tilde{\Upsilon}_{ur}}(\tilde{\gamma}) = 1 - N_r \sum_{m=0}^{N_r-1} \binom{N_r-1}{m} \frac{(-1)^m}{m+1} e^{-\frac{(m+1)\tilde{\gamma}}{(m(1-\rho_{ur}^2)+1)\bar{\gamma}_{ur}}}. \tag{19}$$

Substituting (19) into (16) with the aid of identity [25, eq. (3.371)], we have

$$\tilde{P}_{s,ur} = \frac{a}{2} - \frac{a\sqrt{b}N_r}{2} \sum_{m=0}^{N_r-1} \binom{N_r-1}{m} \frac{(-1)^m}{m+1} \times \left(\frac{m+1}{(m(1-\rho_{ur}^2)+1)\bar{\gamma}_{ur}} + b\right)^{-\frac{1}{2}}. \tag{20}$$

By substituting $\rho_{ur} = 1$ into (20), the SER of the $u-r$ link in the presence of perfect CSI, i.e., $P_{s,ur}$, is given by

$$P_{s,ur} = \frac{a}{2} - \frac{a\sqrt{b}}{2} \sum_{m=1}^{N_r} \binom{N_r}{m} (-1)^{m-1} \left(\frac{m}{\bar{\gamma}_{ur}} + b\right)^{-\frac{1}{2}}. \tag{21}$$

Substituting (20) into (13), the probability of $\tilde{\mathcal{D}}_{u,v}$ in the presence of outdated CSI with time delay τ_{ur} over the $u-r$ link, $\Pr(\tilde{\mathcal{D}}_{u,v})$, is obtained. Substituting (13) and (17) into (11) yields the exact closed-form SER expression $\tilde{P}_{s,u}$ in the outdated CSI scenario. In the extreme case of fully outdated CSI with $\rho_r = \rho_{ur} = 0$, we can conclude that the STNC with AS reduces to the single-antenna STNC, which agrees with the fact that the optimal transmit/receive antenna in the sense of SNR selected based on the fully outdated CSI actually corresponds to a random transmit/receive antenna.

For the perfect CSI scenario, we can derive the closed-form SER expression based on (18) and (21). Obviously, the SER of STNC with AS in the outdated CSI scenario encompasses that of STNC with AS in the perfect CSI scenario as a special case with $\rho_r = 1$ and $\rho_{ur} = 1$. It is noticed that our result encompasses the SER expression for single-antenna STNC in [7] as a special case.

2) Asymptotic SER: Here, we derive the asymptotic SER expression to characterize the diversity order and array gain to offer useful insights into the behavior of STNC with AS in the high-SNR regime for both perfect and outdated CSI scenarios.

In the high-SNR regime, the probability that relay r decodes the signal x_u correctly approaches one, i.e., $\lim_{\bar{\gamma}_{ur}\to\infty} \beta_{ru} = 1$. As such, the received SNR of x_u in the presence of perfect CSI is given by

$$\Upsilon_u^\infty = \sum_{r=0}^{R} \Upsilon_{rD}^\infty \tag{22}$$

where $\Upsilon_{0D}^\infty = \Upsilon_{uD}^\infty$. The asymptotic CDF of Υ_u^∞ is given by

$$F_{\Upsilon_u^\infty}(\gamma) \approx \frac{\Theta}{\Gamma(G_d+1)\bar{\gamma}_{uD}^{G_d}} \gamma^{G_d} \tag{23}$$

where $\Theta = \prod_{r=0}^{R} \kappa_r^{N_r N_D} \Gamma(N_r N_D + 1)$, $\kappa_r = \bar{\gamma}_{uD}/\bar{\gamma}_{rD}$, $\kappa_0 = 1$, and $G_d = (N_u + \sum_{r=1}^{R} N_r) N_D$.

Proof: The proof is presented in Appendix B. ■

Based on (16) and (23), the asymptotic SER of STNC with AS in the high-SNR regime can be easily derived by substituting (23) into (16) and calculating the resultant integral as

$$P_{s,u}^\infty \approx (G_a \bar{\gamma}_{uD})^{-G_d} \tag{24}$$

where the array gain $G_a = b((a\Theta(2G_d-1)!!)/(\Gamma(G_d+1)2^{G_d+1}))^{-(1/G_d)}$.

From (24), we demonstrate that STNC with AS achieves the full diversity order of $(N_u + \sum_{r=1}^{R} N_r)N_D$ in the cooperative MIMO network, where the contributions of the u–D and r–D links to the diversity order are $N_u N_D$ and $N_r N_D$, respectively. In the special case where all the nodes are equipped with a single antenna with $N_u = N_r = N_D = 1$, the diversity order reduces to $R+1$, which is consistent with the result given in [7] and [15].

In the presence of outdated CSI, the received SNR of x_u in the high-SNR regime is given by

$$\tilde{\Upsilon}_u^\infty = \sum_{r=0}^{R} \tilde{\Upsilon}_{rD}^\infty \tag{25}$$

where $\tilde{\Upsilon}_{rD}^\infty$ is the received SNR of x_u in the high-SNR regime from relay r, and $\tilde{\Upsilon}_{0D}^\infty = \tilde{\Upsilon}_{uD}^\infty$. The asymptotic CDF of $\tilde{\Upsilon}_u^\infty$ is given by

$$F_{\tilde{\Upsilon}_u^\infty}(\tilde{\gamma}) \approx \frac{\tilde{\Theta}}{\Gamma(R+2)} \frac{\tilde{\gamma}^{R+1}}{\bar{\gamma}_{uD}^{R+1}} \tag{26}$$

where

$$\tilde{\Theta} = \prod_{r=0}^{R} \left(\sum_{m_r=0}^{N_r N_D - 1} \binom{N_r N_D - 1}{m_r} \frac{N_r N_D \kappa_r (-1)^{m_r}}{m_r(1-\rho_r^2)+1} \right).$$

Proof: The proof is presented in Appendix C. ■

Based on $F_{\tilde{\Upsilon}_u^\infty}(\tilde{\gamma})$, we now derive the asymptotic SER by substituting (26) into (16) and calculating the resultant integral as

$$\tilde{P}_{s,u}^\infty \approx (\tilde{G}_a \bar{\gamma}_{uD})^{-\tilde{G}_d} \tag{27}$$

where the diversity order $\tilde{G}_d = R+1$ and array gain $\tilde{G}_a = b((a\tilde{\Theta}(2\tilde{G}_d-1)!!)/(\Gamma(\tilde{G}_d+1)2^{\tilde{G}_d+1}))^{-(1/\tilde{G}_d)}$.

Comparing (27) with (24), it is evident that outdated CSI has a severely detrimental effect on the SER and degrades the full diversity order of $(N_u + \sum_{r=1}^{R} N_r)N_D$ to $R+1$, which indicates that the contribution of the multiple antennas to the diversity vanishes due to outdated CSI.

B. Capacity

1) OP: The OP is an important quality-of-service measure as it characterizes the probability that the instantaneous capacity falls below a predetermined threshold R_{th}, which corresponds to the SNR threshold $\Upsilon_{\text{th}} = 2^{R_{\text{th}}} - 1$.

With outdated CSI, from (7), the OP associated with user u is given by

$$\begin{aligned}\tilde{P}_{\text{out},u} &= \Pr(C_u < R_{\text{th}}) \\ &= \sum_{|\tilde{\mathcal{D}}_u|=0}^{R} \sum_{v=1}^{\tilde{\vartheta}} \tilde{P}_{\text{out},u|\tilde{\mathcal{D}}_{u,v}} \Pr(\tilde{\mathcal{D}}_{u,v}) \end{aligned} \tag{28}$$

where C_u is the capacity of user u, $\Pr(\tilde{\mathcal{D}}_{u,v})$ is given by (13), and $\tilde{P}_{\text{out},u|\tilde{\mathcal{D}}_{u,v}}$ is the OP of C_u conditioned on $\tilde{\mathcal{D}}_{u,v}$, which can be derived from (15) directly as

$$\tilde{P}_{\text{out},u|\tilde{\mathcal{D}}_{u,v}} = F_{\tilde{\Upsilon}_{u|\tilde{\mathcal{D}}_{u,v}}}(2^{R_{\text{th}}} - 1) \tag{29}$$

where $F_{\tilde{\Upsilon}_{u|\tilde{\mathcal{D}}_{u,v}}}(2^{R_{\text{th}}} - 1)$ is the CDF of $\tilde{\Upsilon}_{u|\tilde{\mathcal{D}}_{u,v}}$ evaluated at $\tilde{\gamma} = 2^{R_{\text{th}}} - 1$. Substituting (13) and (29) into (28) yields the OP associated with user u. For the perfect CSI scenario, the OP can be obtained by substituting $\rho_r = 1$ and $\rho_{ur} = 1$ into (28).

2) Ergodic Capacity: In the presence of outdated CSI, the ergodic capacity of user u is

$$\tilde{C}_u = \sum_{|\tilde{\mathcal{D}}_u|=0}^{R} \sum_{v=1}^{\tilde{\vartheta}} \tilde{C}_{u|\tilde{\mathcal{D}}_{u,v}} \Pr(\tilde{\mathcal{D}}_{u,v}) \tag{30}$$

where $\tilde{C}_{u|\tilde{\mathcal{D}}_{u,v}}$ denotes the ergodic capacity of user u conditioned on $\tilde{\mathcal{D}}_{u,v}$, and $\Pr(\tilde{\mathcal{D}}_{u,v})$ is given by (13). Based on (14), the ergodic capacity of user u conditioned on $\tilde{\mathcal{D}}_{u,v}$ is mathematically formulated as

$$\begin{aligned}\tilde{C}_{u|\tilde{\mathcal{D}}_{u,v}} &= \int_0^\infty \log_2(1+\tilde{\gamma}) f_{\tilde{\Upsilon}_{u|\tilde{\mathcal{D}}_{u,v}}}(\tilde{\gamma}) d\tilde{\gamma} \\ &= -\log_2(e)\tilde{\Phi} \sum_{m_0=0}^{N_0 N_D - 1} \cdots \sum_{m_{\tilde{p}}=0}^{N_{\tilde{p}} N_D - 1} \tilde{\varphi} \sum_{r=0}^{\tilde{p}} \frac{\tilde{\mu}_r}{\tilde{\xi}_r} \\ &\quad \times e^{\tilde{\xi}_r} \text{Ei}(-\tilde{\xi}_r) \end{aligned} \tag{31}$$

where $\text{Ei}(x) = \int_{-\infty}^{x} (e^t/t) dt$, $x < 0$ is the exponential integral function. We note that (31) can be obtained directly by using the definite integral of the exponential function, which is given in [25, eq. (4.337.2)].

Substituting (13) and (31) into (30), we obtain the ergodic capacity of user u in the presence of outdated CSI directly. For the perfect CSI scenario, the ergodic capacity of user u can be easily derived by substituting $\rho_r = 1$ and $\rho_{ur} = 1$ into (30).

IV. Performance With Spatial Correlation

Correlated fading occurs in many practical scenarios due to the limited antenna separation or the lack of local scatters [29]. Here, we examine the impact of spatial correlation on the performance of STNC with AS.

According to the common Kronecker structure, the spatially correlated channel matrix $\mathbf{H}_{Dr}^{\text{cor}}$, $0 \leq r \leq R$, between relay r and the destination D can be decomposed as [27], [30]

$$\mathbf{H}_{Dr}^{\text{cor}} = \mathbf{R}_D^{\frac{1}{2}} \mathbf{H}_{Dr} \mathbf{R}_r^{\frac{1}{2}} \tag{32}$$

where $\mathbf{R}_D$ and $\mathbf{R}_r$ denote the spatial correlation matrices at the destination D and relay r, respectively. Here, user u is assumed as the zeroth relay for the sake of simplicity. The (p, q)th elements of $\mathbf{R}_D$ and $\mathbf{R}_r$, i.e., $R_{p,q}^D$ and $R_{p,q}^r$, satisfy [31]

$$R_{p,q}^D = \begin{cases} \varrho_p^D \varrho_q^D, & p \neq q \\ 1, & p = q \end{cases} \tag{33}$$

$$R_{p,q}^r = \begin{cases} \varrho_p^r \varrho_q^r, & p \neq q \\ 1, & p = q \end{cases} \tag{34}$$

respectively, where $0 \leq \varrho_p^D < 1$, $1 \leq p \leq N_D$, $0 \leq \varrho_q^r < 1$, and $1 \leq q \leq N_r$. Consequently, $\mathbf{R}_D$ and $\mathbf{R}_r$ can be parameterized by column vectors $\boldsymbol{\varrho}_D = [\varrho_1^D, \varrho_2^D, \ldots, \varrho_{N_D}^D]^T$ and $\boldsymbol{\varrho}_r = [\varrho_1^r, \varrho_2^r, \ldots, \varrho_{N_r}^r]^T$, respectively.

Based on the Kronecker product of the transmit and receive spatial correlation matrices, we have [27], [32]

$$\text{vec}(\mathbf{H}_{Dr}^{\text{cor}}) = \text{vec}(\mathbf{H}_{Dr}) \mathbf{R}^{\frac{1}{2}} \tag{35}$$

where $\text{vec}(\cdot)$ denotes the matrix vectorization operation, $\mathbf{R} = \mathbf{R}_r \otimes \mathbf{R}_D$, and $\otimes$ denotes the Kronecker product. Based on the mixed-product property of Kronecker product, $\mathbf{R}$ is rewritten as

$$\mathbf{R} = \boldsymbol{\varrho} \boldsymbol{\varrho}^T \tag{36}$$

where the column vector $\boldsymbol{\varrho} = \boldsymbol{\varrho}_r \otimes \boldsymbol{\varrho}_D$ with the ith element $\varrho_i = \varrho_m^r \varrho_n^D$, $1 \leq i \leq N_r N_D$, $m = \lceil i/N_D \rceil$, $n = i - (\lceil i/N_D \rceil - 1)N_D$, and $\lceil x \rceil$ is the ceiling function of x. In addition, the (p, q)th element of $\mathbf{R}$, $R_{p,q}$ satisfies

$$R_{p,q} = \begin{cases} \varrho_p \varrho_q, & p \neq q \\ 1, & p = q. \end{cases} \tag{37}$$

With AS, we would like to select the transmit/receive antenna pair that corresponds to the highest channel coefficient out of the $N_D \times N_r$ correlated channel coefficients between relay r and destination D. With the selected transmit/receive antenna pair, the single polynomial expansion of the asymptotic CDF of $\Upsilon_{rD,\text{cor}}^\infty$ in the high-SNR regime is given by [26], [33]

$$F_{\Upsilon_{rD,\text{cor}}^\infty}(\gamma) \approx \omega_r \left(\frac{\gamma}{\bar{\gamma}_{rD}} \right)^{N_r N_D} \tag{38}$$

where

$$\omega_r = \left(\sum_{i=1}^{N_r N_D} \frac{\varrho_i}{1 - \varrho_i} + 1 \right)^{-1} \prod_{i=1}^{N_r N_D} \frac{1}{1 - \varrho_i}.$$

Taking the first derivative of (38) yields the asymptotic PDF of $\Upsilon_{rD,\text{cor}}^\infty$ in the high-SNR regime. Based on the PDF of $\Upsilon_{rD,\text{cor}}^\infty$, we follow the similar procedure specified in Section III-A2 to derive the asymptotic SER of STNC with AS over flat spatially correlated Rayleigh fading channels as

$$P_{s,u,\text{cor}}^\infty \approx (G_a^{\text{cor}} \bar{\gamma}_{uD})^{-G_d} \tag{39}$$

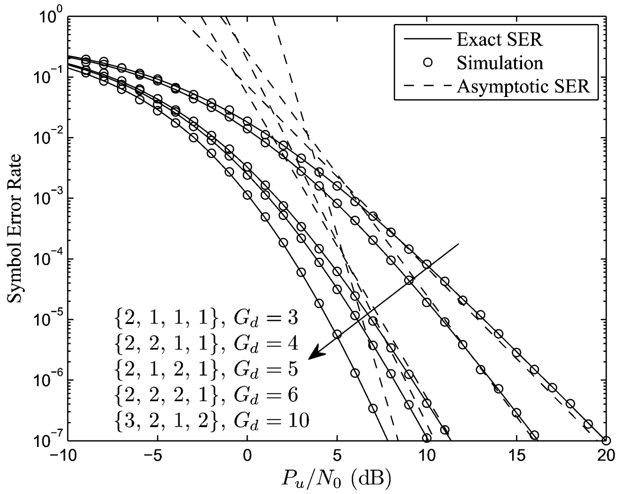

Fig. 1. SER of BPSK modulation of STNC with AS for perfect CSI scenario with different network configurations. (Tuple $\{a, b, c, d\}$ indicates that $R = a$, $N_u = b$, $N_r = c$, and $N_D = d$, respectively.)

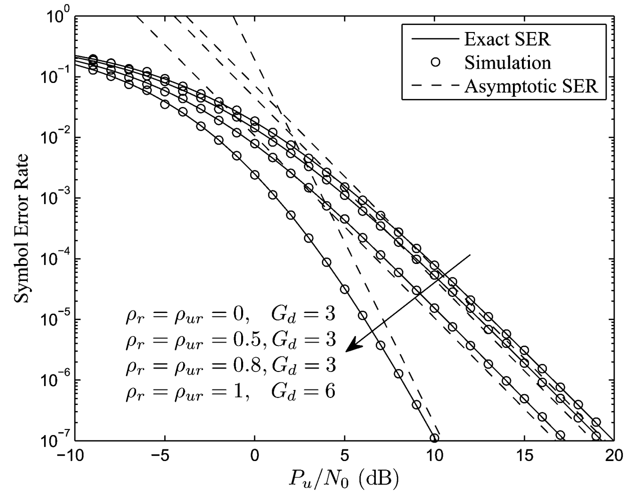

Fig. 2. SER of BPSK modulation of STNC with AS for outdated CSI scenario with $R = 2$, $N_u = N_r = 2$, and $N_D = 1$.

where array gain $G_a^{\rm cor} = b(a\Theta\Omega((2G_d - 1)!!)/\ (\Gamma(G_d + 1)2^{G_d+1}))^{-(1/G_d)}$, and $\Omega = \prod_{r=0}^{R} \omega_r$. It is easy to demonstrate that (39) is equal to (24) by setting $\varrho_i = 0$ in (39).

From (39), we observe that STNC with AS achieves the full diversity order of $(N_u + \sum_{r=1}^{R} N_r)N_D$ over flat spatially correlated Rayleigh fading channels regardless of the values of spatial correlation coefficients and the impact of spatial correlation lies in the array gain. It is expected that the higher the value of the spatial correlation coefficient, the lower the array gain, and therefore the higher the SER.

We now characterize the performance gap between spatially correlated and uncorrelated scenarios. Specifically, it is determined by the ratio of the average transmit SNR in the correlated scenario and the average transmit SNR in the uncorrelated scenario for the same SER. We derive this ratio as

$$\Upsilon_{\rm gap}^{\infty} = \Omega^{\frac{1}{G_d}}. \tag{40}$$

It is indicated from (40) that, for the same SER, the uncorrelated scenario is superior to the correlated scenario by an SNR gap of $10 \log \Upsilon_{\rm gap}^{\infty}$ dB.

V. Numerical Results

Here, the numerical and simulation results are presented to verify the accuracy of our analysis and examine the impacts of the network parameters on the performance of STNC with AS. We assume that the coordinates of the destination D, relay r, and user u are $(0, 0)$, $((d+r\Delta d)\cos(r\psi), (d+r\Delta d)\sin(r\psi))$, and $(\cos(u\psi'), \sin(u\psi'))$, respectively. In the simulations, $d = 0.4$, $\Delta d = 0.1$, and $\psi = \psi' = \pi/18$. The cross correlations between different spread codes are set to be zero, and the value of the path-loss exponent α is 3.5 [34]. We assume equal transmit power at each node.

Figs. 1 and 2 plot the SER of STNC with AS versus transmit SNR P_u/N_0 for perfect CSI and outdated CSI scenarios, respectively, where $P_u = P_{0u} + \sum_{r=1}^{R} P_{ru}$. We observe excellent agreement between the calculation results using analytical expressions and simulation results marked by "$\circ$." Moreover, the exact curves approach asymptotic curves in the high-SNR regime. This validates the accuracy of our theoretical analysis in Section III. In Fig. 1, the curve with tuple {2, 1, 1, 1} denotes the SER of single-antenna STNC. Fig. 1 shows that increasing the antenna number could improve the system performance in terms of both diversity order and SER. With perfect CSI, STNC with AS provides full diversity order of $(N_u + \sum_{r=1}^{R} N_r)N_D$, as indicated by (24), which implies that it is preferred to distribute more antennas to the destination to improve the network performance.

The asymptotic curves in Figs. 1 and 2 have different slopes. In Fig. 1, the larger the value of antenna/relay number, the steeper the slope of the asymptotic curve, which, in turn, indicates a significant diversity gain. Fig. 2 shows that the diversity gain vanishes due to the outdated CSI. With the decrease in the delay correlation coefficient, the SER of STNC with AS increases. It is noticed that the exact and asymptotic SER curves of STNC with AS for $\rho_r = \rho_{ur} = 0$ in Fig. 2 are identical to those for $N_u = N_r = N_D = 1$ in Fig. 1. This can be explained by the fact that, for extremely outdated CSI of $\rho_r = \rho_{ur} = 0$, the selected antennas are actually random antennas, but not the optimal antennas that maximize the received SNR, and the diversity gain offered by multiple antennas vanishes.

Fig. 3 gives the SER of STNC with varying delay correlation coefficients. It further demonstrates the detrimental effect of outdated CSI on the multiantenna diversity gain. We observe that the multiantenna diversity gain is trivial with the delay correlation coefficient ρ being lower than 0.5, and it almost vanishes with ρ being lower than 0.2. With the same relay number and transmit SNR, STNC with AS associated with different antenna configurations shows the same SER when $\rho = 0$. It is shown that the SER dramatically increases as the delay correlation coefficient decreases from 1 to 0.8. Thus, it is reasonable to increase the delay correlation coefficient, e.g., decreasing the time delay between the instants of channel estimation and data transmission, to decrease the SER and increase the system performance.

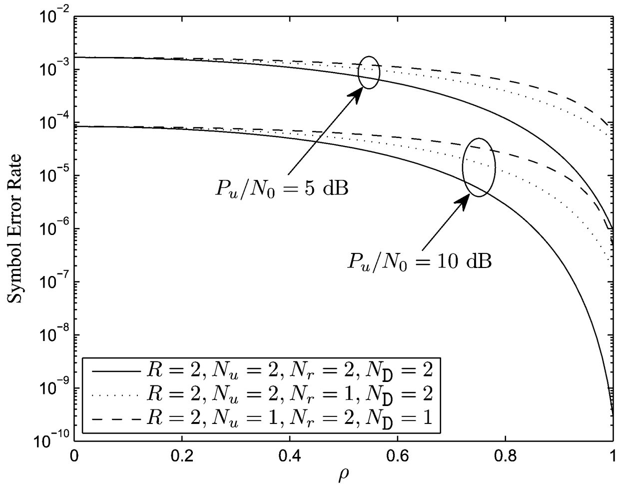

Fig. 3. Detrimental impact of outdated CSI on the SER with variable delay correlation coefficient ρ. ($\rho_r = \rho_{ur} = \rho$).

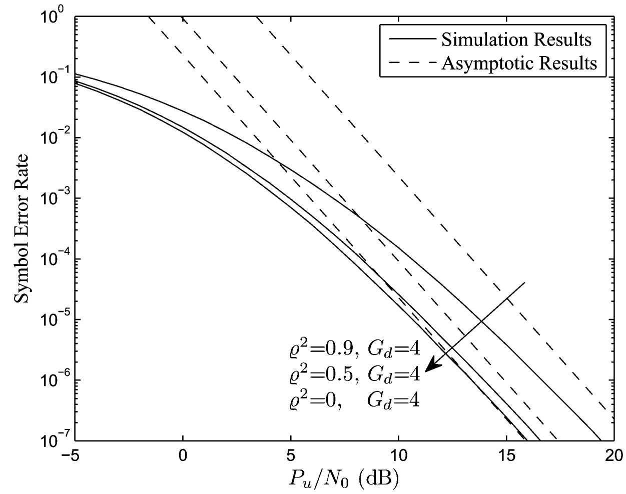

Fig. 5. SER of BPSK modulation of STNC with AS for spatial correlation scenario with $R = 1$, $N_u = N_r = 2$, and $N_D = 1$.

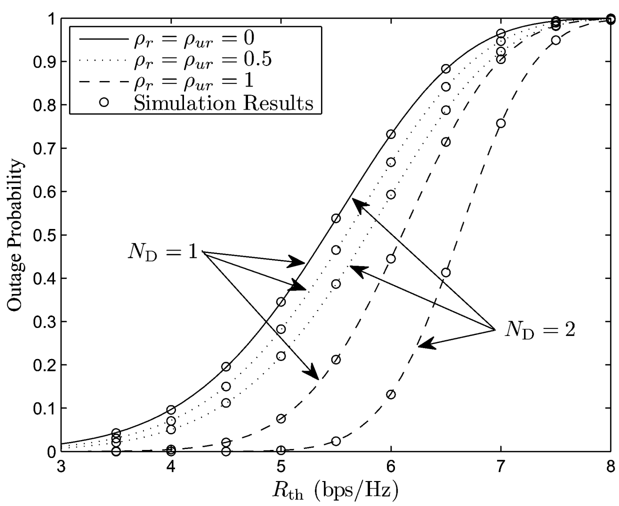

Fig. 4. OP of capacity for the outdated CSI scenario with $R = 2$ and $N_u = N_r = 2$.

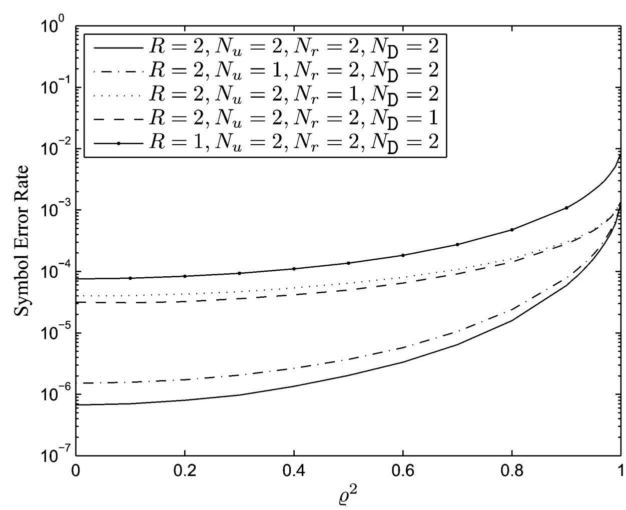

Fig. 6. Detrimental impact of spatial correlation on the SER with variable spatial correlation coefficient ϱ^2.

The OPs of capacities with different thresholds are presented in Fig. 4. The perfect match between the theoretical and simulation results demonstrates the correctness of our theoretical analysis in Section III-B. In Fig. 4, the curve with $\rho_r = \rho_{ur} = 0$ corresponds to single-antenna STNC due to the fact that fully outdated CSI vanishes the multiantenna diversity gain completely. It is shown that increasing the antenna number and delay correlation coefficient improves the capacity significantly.

The effect of spatial correlation on the SER of STNC with AS is shown in Figs. 5 and 6. In Fig. 5, the solid curves denote the simulation results. In Fig. 6, the curves are plotted based on the simulation results. We can observe that the spatial correlation deteriorates the SER. The asymptotic curves are parallel with the same slope of -4 in Fig. 5 but are shifted to the right as ϱ^2 increases from 0 to 0.9. This verifies the correctness of our analysis on system diversity order and implies that the spatial correlation has no impact on the achievable diversity order but has impact on the array gain. We observe that the increase in spatial correlation coefficients deteriorates the SER, particularly for large correlation coefficients. Specifically, the effect of correlation is almost negligible when $\varrho^2 < 0.5$ but becomes profound when $\varrho^2 > 0.5$, which is in agreement with the previous results on the effect of spatial correlation [35].

VI. Conclusion

In this paper, we have analyzed the performance of STNC with AS in a cooperative MIMO network over flat Rayleigh fading channels. We have confirmed that STNC with AS preserves the full diversity order of $(N_u + \sum_{r=1}^{R} N_r)N_D$. It is shown that the outdated CSI vanishes the diversity gain offered by multiple antennas and the diversity order degrades to $R + 1$ from full diversity, whereas spatial correlation has no impact on the diversity order. The significant impact of outdated CSI implies that a high channel estimate rate may be required in practice to attain the full benefits of AS.

APPENDIX A
PROOF OF LEMMA 1

Based on (8), the moment-generating function (MGF) of $\tilde{\Upsilon}_{rD}$ is derived by performing the Laplace transform with the aid of the definite integral of the exponential function [25, eq. (3.351.3)] as

$$\begin{aligned}\mathcal{M}_{\tilde{\Upsilon}_{rD}}(s) &= \int_0^\infty f_{\tilde{\Upsilon}_{rD}}(\tilde{\gamma}) \exp\left(-s\tilde{\gamma}\right) d\tilde{\gamma} \\ &= \sum_{m_r=0}^{N_r N_D - 1} \binom{N_r N_D - 1}{m_r} \frac{(-1)^{m_r} N_r N_D (\tilde{\xi}_r + s)^{-1}}{\left(m_r\left(1-\rho_r^2\right)+1\right)\bar{\gamma}_{rD}}.\end{aligned} \tag{41}$$

Since the MGF of the sum of multiple independent random variables is equal to the product of the MGFs of the random variables [28], the MGF of $\tilde{\Upsilon}_{u|\tilde{\mathcal{D}}_{u,v}}$ is

$$\mathcal{M}_{\tilde{\Upsilon}_{u|\tilde{\mathcal{D}}_{u,v}}}(s) = \tilde{\Phi} \sum_{m_0=0}^{N_0 N_D-1} \cdots \sum_{m_{\tilde{p}}=0}^{N_{\tilde{p}} N_D - 1} \tilde{\varphi} \prod_{r=0}^{\tilde{p}} \left(\tilde{\xi}_r + s\right)^{-1} \tag{42}$$

where $N_0 = N_u$.

Expanding (42) in poles and residuals with the aid of partial fraction decomposition [25, eq. (2.102)] yields

$$\mathcal{M}_{\tilde{\Upsilon}_{u|\tilde{\mathcal{D}}_{u,v}}}(s) = \tilde{\Phi} \sum_{m_0=0}^{N_0 N_D-1} \cdots \sum_{m_{\tilde{p}}=0}^{N_{\tilde{p}} N_D - 1} \tilde{\varphi} \sum_{r=0}^{\tilde{p}} \tilde{\mu}_r \left(\tilde{\xi}_r + s\right)^{-1}. \tag{43}$$

The desired result (14) can now be derived directly by performing inverse Laplace transform on (43).

APPENDIX B
PROOF OF (23)

By applying the Taylor series expansion of the exponential function in the PDF of Υ_{rD} in (10) and retaining the first-order term, the first-order expansion of the PDF of Υ_{rD}, i.e., $f_{\Upsilon_{rD}^\infty}(\gamma)$, is derived as

$$f_{\Upsilon_{rD}^\infty}(\gamma) \approx \frac{N_r N_D}{\bar{\gamma}_{rD}^{N_r N_D}} \gamma^{N_r N_D - 1}. \tag{44}$$

Performing the Laplace transform of (44) with the aid of [25, eq. (3.351.3)] yields

$$\begin{aligned}\mathcal{M}_{\Upsilon_{rD}^\infty}(s) &= \int_0^\infty f_{\Upsilon_{rD}^\infty}(\gamma)\exp(-s\gamma) d\gamma \\ &\approx \frac{N_r N_D}{\bar{\gamma}_{rD}^{N_r N_D}} \Gamma(N_r N_D) s^{-N_r N_D}.\end{aligned} \tag{45}$$

As Υ_{rD}^∞ is independent of each other, based on (22) and (45), we have the MGF of Υ_u^∞ as

$$\mathcal{M}_{\Upsilon_u^\infty}(s) \approx \Theta(s\bar{\gamma}_{uD})^{-G_d}. \tag{46}$$

Performing the inverse Laplace transform of (46) and integrating the outcome produce the CDF of Υ_u^∞, as shown in (23).

APPENDIX C
PROOF OF (26)

Applying the Taylor series expansion of the exponential function in (8) and discarding the high-order items yield

$$f_{\tilde{\Upsilon}_{rD}^\infty}(\tilde{\gamma}) \approx \frac{N_r N_D}{\bar{\gamma}_{rD}} \sum_{m=0}^{N_r N_D - 1} \binom{N_r N_D - 1}{m} \frac{(-1)^m}{m\left(1-\rho_r^2\right)+1}. \tag{47}$$

Performing the Laplace transform of (47), $0 \le r \le R$, with the aid of [25, eq. (3.351.3)], we have the MGF of $\tilde{\Upsilon}_{rD}^\infty$ as

$$\begin{aligned}\mathcal{M}_{\tilde{\Upsilon}_{rD}^\infty}(s) &= \int_0^\infty f_{\tilde{\Upsilon}_{rD}^\infty}(\tilde{\gamma})\exp(-s\tilde{\gamma})d\tilde{\gamma} \\ &\approx \frac{N_r N_D}{\bar{\gamma}_{rD}} \sum_{m=0}^{N_r N_D - 1} \binom{N_r N_D - 1}{m} \frac{(-1)^m}{\left(m\left(1-\rho_r^2\right)+1\right)s}.\end{aligned} \tag{48}$$

Based on (25) and (48), we have the MGF of $\tilde{\Upsilon}_u^\infty$ as

$$\mathcal{M}_{\tilde{\Upsilon}_u^\infty}(s) \approx \tilde{\Theta}(s\bar{\gamma}_{uD})^{-(R+1)}. \tag{49}$$

Performing the inverse Laplace transform of (49) yields the PDF of $\tilde{\Upsilon}_u^\infty$, from which we can derive the asymptotic CDF of $\tilde{\Upsilon}_u^\infty$, as shown in (26).

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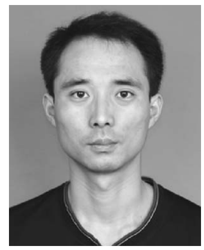

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