

Impact of wind power generation on a large scale power system using stochastic linear stability



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ABSTRACT

The effect of random and sustained disturbances is studied in this paper. The Ornstein–Uhlenbeck stochastic process is proposed with the aim to represent the variations in the power output produced by wind power generation, considering measurements of a wind farm located in the fourth region of Chile as real field data for its parameters calibration. An example is provided considering the study of the exponential stability using the Lyapunov exponent as an indicator, with the aim to determine the maximum size of a wind farm that can be connected in a busbar of one of Chile's interconnected power system (Sistema Interconectado del Norte Grande, SING), considering successive increments from the output power from the connected wind farm.

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1. Introduction

Dynamic and permanent regime studies that are made on electric power systems (EPS) are of vital importance to the electric industry, because they make it possible to determine the adequate operating conditions for supplying the electric power required by society in an economic, reliable and safe manner. In this context, the most important approaches of the EPS studies are oriented at their planning and operation. One of the main problems that concern these topics consists in keeping the system operating in a steady state, i.e., that the system does not lose its balance when it is subjected to perturbations that affect its behavior.

In their normal operation, electric power systems are subjected to a wide variety of random and sustained in time disturbances, which affect all aspects of the real-time operation of the system. The most common examples of random disturbances affecting the interconnected systems are, e.g., variations in the distribution system by the fluctuation in the electrical demand of the customers and random changes in generation that depend of some non-controllable energy sources. In that aspect, electrical generation that uses wind energy as its main source brings itself random variations to the power systems that depend of the statistical characteristics of the wind speed at its location.

Within the context of transient stability, the random behavior of the system has been approached considering different system scenarios and parameters which are associated with an occurrence probability [1–9]. In terms of quantitative

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assessment, [10] presents an index that allows determining the vulnerability when facing a voltage collapse, establishing that consumption has a random behavior. This stability index corresponds to the time for leaving a stable operation zone.

Stability studies of small probabilistic disturbances are proposed in [11–19]. One of the main approaches is to assign a probability distribution of the real parts of the eigenvalues obtained from the linear equivalent model of the electric system, and then determine the probability that the real parts will be located in the left half-plane. In this context, applications are also presented in which the PSS (Power Systems Stabilizer) controllers are used to decrease the effect of the disturbances that affect the operation but occur in a single instant of time.

To account for the random and permanent effect over time, Lyapunov exponent have been used as stability indices to analyze power systems [20]. Reference [21] gives a theoretical description of the calculation of Lyapunov exponents in structures, considering low dimension systems as applications. However, these studies are still in a theoretical stage and no numerical methods for estimating them are shown.

With the purpose of characterizing faithfully the presence of random perturbations self-sustained over time, it is necessary to consider stochastic models that account for the actual dynamics that take place during the operation of the system. This fact shows the need to use stochastic differential equations to describe completely what happens in the system.

Considering the above, the purpose of this paper is to model stochastically the behavior of certain components of the EPS. In particular, a representation will be made of the generating power of a wind farm by means of a model that accounts for the random and self-sustained over time dynamics.

The treatments found in the specialized literature with respect to the kinds of stochastic models for the EPS are summarized in what follows. Reference [10] includes Brownian motion in the system's dynamic model to describe the load component, in order to study the vulnerability of the system to voltage collapse.

In [22] SDE are used as planning tools in power systems, using a stationary Gaussian process with constant spectral density to model the random variations in lines and loads. In [23] they are used to model small perturbations in load systems and transmission line parameters. In [24] they are applied to analyze the dynamics of an EPS, including discrete perturbations produced by the tap changing operation in a transformer. In references [22,25,26] they are used for stability studies, modeling the behavior of the loads by means of the Ornstein–Uhlenbeck (O-U) process and Ito's nonlinear differential equation. Reference [27] models the wind generation and the loads by the O-U process, but it does not present a validity test of the estimated parameters that can ensure a good fit of the model.

In brief, the specialized literature shows important progress in the study of power systems stability considering random self-sustained disturbances and the Lyapunov exponent as an index of the stability behavior of the system. However, considering the previously stated, it is still necessary to study this topic for large scale power systems, and also, the modeling of the random phenomena taking into account the calibration of the stochastic process using real field measurements. This paper advances in both areas, firstly, the mathematical treatment of the random stochastic disturbance is done considering measurement of an actual wind power park, and the Lyapunov exponent is studied for the injection of wind parks in a real large scale power system (SING).

This paper is organized as follows. Section 2 shows the methodology for the analysis of electrical power systems under stochastic disturbances, considering the Lyapunov exponent concept as an stability index for stochastic linear systems, and the mathematical model of the random perturbation, considering real field measurements for the modeling. In Section 3 the application case is presented, where the characterization of random disturbances from wind power plants using the Ornstein–Uhlenbeck process is shown; an example study case is provided, showing the impact of large scale wind power injection in the SING, considering the analysis of the stability of the interconnected system following stochastic disturbances. This analysis is done taking into account the integration of a wind power plant in one of the busbars of the interconnected system and considering a successive increases in the size of the disturbance, such that the Lyapunov exponent can be regarded as an indicator of the global stability of the system. Finally, in Section 4, conclusions and future work are discussed.

2. Methodology

In this section we describe the general methodology to assess the influence of random disturbances from wind power generation on the steady-state performance of a large scale power system, in this case the SING.

2.1. Lyapunov exponent

To study the stability of power systems subjected to stochastic disturbances, a multiplicative model perturbation is considered. This representation allows us to use the Lyapunov Exponent concept as a stability index for the analysis of the steady-state operation of power systems with random variability. In first place, consider the following non-linear equation system:

$$\dot{y} = f(y, p). \quad (1)$$

The system is defined in the state space \mathbb{R}^N , where $N_1 = 2n$ corresponds to the relative angles of the rotors ($\delta_1, \dots, \delta_n$) and velocities ($\omega_1, \dots, \omega_n$) and N_2 corresponds to the rest of the state variables ($N_1 + N_2 = N$). The vector $p \in \mathbb{R}$ corresponds to the variables that can be fitted to guarantee the optimum operation of the system.

Let us also consider $y_0 \in \mathbb{R}^N$ as a set point (y_0 : operation point) of system (1) and the equivalent linear system at that point, given by

$$\Delta \dot{x} = A(p) \Delta x. \tag{2}$$

The classical small disturbance stability study is based on getting the eigenvalues of the linear system of Eq. (2). If the real part are negative, the system will be stable [28].

When we consider random and self-sustained in time disturbances, the classical approach stops being practical (see [29] and [30]). The linear system Eq. (2), when subjected to random and sustained in time disturbances, can be described by the following model [31].

$$\Delta \dot{x} = A(p, \xi_t) \Delta x, \tag{3}$$

where

- ξ_t is a random process that takes values in some set $U \subset \mathbb{R}^m$ and describes the way in which the disturbance affects the state variables of the system. In this case, ξ_t represents the random variations from wind power generation.
- p is the set of control parameters (the gains of the machine controllers, for example). For this paper, we consider that these parameters are constant.

Denoting the solution of Eq. (3) at time greater than $t > 0$ with an initial value $x_0 \in \mathbb{R}^d$ by $\phi(t, x, \xi_t)$, the exponential behavior is given by the Lyapunov exponents

$$\lambda(x, \gamma) = \limsup_{t \rightarrow \infty} \frac{1}{t} \log(\| \phi(t, x, \xi_t(\gamma)) \|). \tag{4}$$

Here γ is an element that represents a realization of the stochastic process which models the random and self-sustained in time disturbances. In general, the stochastic system of Eq. (3) with ergodic disturbance ξ_t can have a number d of Lyapunov exponents. Under the conditions indicated in [32], there is a unique exponent with probability 1 given by the following expression:

$$\lambda(x, \gamma) = \lim_{t \rightarrow \infty} \frac{1}{t} \log(\| \phi(t, x, \xi_t(\gamma)) \|). \tag{5}$$

According to [31] (Theorem 2.1. and Corollary 2.2.), the stochastic linear system of Eq. (3) would be almost surely exponentially stable if and only if $\lambda < 0$.

As shown in [33], if the linearised system is stable under random and sustained in time disturbances, it is possible to assure that the non-linear system will also be stable.

There are multiple options to estimate the Lyapunov exponent of a stochastic linear system. The numerical method used in this paper is based on the trajectory averages of the linear system, since according to the results presented in [31], this method gives highly reliable and accurate results (see [30] and [31] for details about the other methods).

Let us consider the system of Eq. (3) and the following parameters:

- α : number of initial conditions,
- β : number of realizations of stochastic process ξ_t , which represents the random and self-sustained in time disturbance,
- T : simulation time for calculating the Lyapunov exponent.

For simulation time T , trajectory i of disturbance ξ_t and initial condition x_0^j , we have solution $x_n^j(i)$ of Eq. (3), from which the following calculation is made:

$$\lambda^j(i) = \frac{1}{T} \sum_{n=1}^T \log \| x_n^j(i) \| . \tag{6}$$

Averaging the values obtained in Eq. (6) over the number of realizations and initial conditions α and β , respectively, we get

$$\lambda = \frac{1}{\alpha} \cdot \frac{1}{\beta} \sum_{j=1}^{\alpha} \sum_{i=1}^{\beta} \lambda^j(i). \tag{7}$$

For the stochastic linear system of Eq. (3), the Lyapunov exponent is obtained considering that $T \rightarrow \infty$. However, it is possible to get approximate results in a fixed time T considering a large number of realizations of the disturbance and the initial conditions of the linear system [31]. Furthermore, due to the numerical errors of the first iteration, it is convenient to eliminate the initial time period of the simulation, so Eq. (6) is fitted as follows

$$\lambda^j(i) = \frac{1}{T - T_1} \sum_{n=T_1}^T \log \| x_n^j(i) \| . \tag{8}$$

Here T_1 indicates the simulation period in which the numerical method presents numerical errors.

2.2. Disturbance modeling of wind generation using the Ornstein–Uhlenbeck process

Specifically, due to the random and non-controllable nature of the wind power generation, the common study of eigenvalues in linear systems is not able to incorporate the random variations in the stability analysis. This methodology considers the entry of a real wind power park disturbance in the dynamic model of the system.

Among the diffusion processes there is the Ornstein–Uhlenbeck process, widely used for the description of physical phenomena, and according to [14,32,34], this is the stochastic process that can best describe random phenomena present in the electric power systems. Based on the above, we model ξ_t according to

$$\xi_t = \rho \cdot Y_t. \quad (9)$$

Here $\rho \geq 0$ is the size of the perturbation and $Y_t = f(U_t)$ is the stochastic process that describes wind power generation, who is function of the Ornstein–Uhlenbeck process U_t . The relation between Y_t and U_t will be given in the next section.

Now, the focus will be the properties of Ornstein–Uhlenbeck process which will be used for the application case. This process satisfy the following stochastic differential equation:

$$dU_t = \eta U_t dt + \nu dW_t. \quad (10)$$

Here $\eta < 0$ and $\nu > 0$.

This stochastic process can captures the dynamics of the disturbances affecting the operation of the system, which in this particular case is the variability of electrical generation coming from wind power plants.

The Ornstein–Uhlenbeck process is Gaussian and it is shown that its expectation is $U_0 e^{\eta t}$ and its variance is $\frac{\nu^2 (e^{2\eta t} - 1)}{2\eta}$. It is also verified that the variance is bounded and admits a stationary probability distribution.

In the present work it will be the fundamental component of the models to be proposed, and use will be made of its good mathematical properties for the fitting of the parameters and the later validation of the models.

In the Ornstein–Uhlenbeck process (10), a technique for estimating parameter η is the use of quasi maximum likelihood [35], whose estimator turns out to be

$$\tilde{\eta} = \frac{1}{\Delta} \log \frac{\sum_{i=1}^n U_{(i-1)\Delta} U_{i\Delta}}{\sum_{i=1}^n U_{(i-1)\Delta}^2}. \quad (11)$$

Here the random variable $U_{i\Delta}$ corresponds to an observation of the O-U process at instant $i\Delta$. For this fitting, these observations will correspond to the actual data provided by the farm, under a certain transformation that will be indicated later; n is the number of available observations, and Δ is the time interval between observations.

Parameter ν , of Eq. (10) can be estimated by equating the quadratic variation of the O-U process with its discretization, from which the following estimator is obtained:

$$\tilde{\nu} = \sqrt{\frac{1}{t} \sum_{i=1}^n (U_{i\Delta} - U_{(i-1)\Delta})^2}. \quad (12)$$

Here t is the total time in which the process was observed, in the time scale considered for η .

3. Application case

This section presents an application case concerning both main topics from this paper. First, the assumptions made for modeling the random phenomena associated with wind generation is presented, considering actual field data measured for the parameters estimation. With the aim to show an example of the effect of random disturbances in a large scale power system, an individual evaluation for the injection of wind power in a single busbar of the SING interconnected system is presented, using the Lyapunov exponent as an index of the stability of the system.

3.1. Characterization of wind generation model

In general wind generation is subjected to various factors, like the wind speed and the installed capacity of the wind farm. According to these elements the following aspects are distinguished [27]:

- The irregularity of the wind speed results in a generation that lacks a definite trend and seasonality.
- It should be noted that the trajectories of the measurements of wind generation are bounded in the interval $[0, P]$ (where P corresponds to the farm's total installed capacity). Moreover, the wind generation samples do not change discontinuously.
- It is often considered that wind speed has a Weibull distribution. However, it is mathematically advantageous to consider that it has a logarithmic normal distribution. This makes it possible to use stochastic differential equations, based on Brownian motion, to get a dynamic representation of the power delivered by the wind farm.
- The square of the mean wind generation cannot increase during many consecutive hours, and this is associated with the property of reversion to the mean. This implies that the trajectories of the process will be around the mean value.

Table 1
Parameters estimated from the sum of active power observations of the 11 generators.

Parameter	Estimated value
η	-8.707802
ν	0.230935
h	9.633223

Use was made of observations measured in the field from 11 windmills that make up the Canela wind farm, located in the Fourth Region of Chile. In this case, two stochastic models were obtained that allow the dynamic representation of the power generation of the complete farm. The model corresponds to a one-dimensional representation of the total output.

The treatment to which the data were subjected is presented below, with the aim to get the parameters of the stochastic processes using the data from the real wind farm.

3.1.1. Presentation of data and estimation model

The data delivered from the wind farm correspond to measurements of active power in kW from 11 windmills, made every second during three hours.

The set of observations will be denoted by $\{y_{i\Delta}^n\}_{i=0,\dots,10800}$, where $\Delta = 1$ s represents the time between observations, n is the identifier for each mill, and i is the number of samples considered. The sum total of the power generation of the mill every second will be denoted by $\{\tilde{y}_{i\Delta}\}_{i=0,\dots,10800}$.

If it is desired to consider directly process (10) as a representation of wind generation, it would have to be assumed that the wind follows a normal distribution, but as already stated, it is considered that the wind follows a normal-logarithmic distribution, suggesting that the process that describes the wind generation, which will be denoted by $\{Y_t\}_{t \in T}$, complies with:

$$\ln Y_t - h = U_t \tag{13}$$

$$\Rightarrow Y_t = e^{U_t+h}. \tag{14}$$

Here h is the mean of $\ln Y_t$. This ensures that $\ln Y_t - h$ has a zero mean and a normal distribution. So (14) is the candidate model and with transformation (13) one can work directly with the O-U process.

According to Ito's formula [36], Eq. (14) satisfies the following SDE:

$$dY_t = \left(\eta \ln Y_t - \eta h + \frac{\nu^2}{2} \right) Y_t dt + \nu Y_t dW(t). \tag{15}$$

The numerical resolution schemes will be useful at the time of analyzing the behavior of the stochastic process that is being studied and then simulating the trajectories for later applications.

For a stochastic differential equation of form (15), the Milstein numerical scheme would be:

$$Y_{i+1} = Y_i + \nu Y_i (W_{i+1} - W_i) + \left(\eta Y_i \ln(Y_i) - \eta Y_i h + \frac{1}{2} Y_i \nu^2 \right) (t_{i+1} - t_i) + \frac{\nu^2 Y_i}{2} ((W_{i+1} - W_i)^2 - (t_{i+1} - t_i)). \tag{16}$$

For further details of the just mentioned numerical schemes, see [37].

3.1.2. Parameter fitting

Since the estimators respond well under small pass sizes, a time scale greater than that of seconds (actual sampling time), i.e., instead of considering that a time interval of 10800 s was observed with a Δ of 1 s, it will be said that a time interval of 3 h was observed with $\Delta = 3/10800$. According to this:

- η is estimated from Eq. (11) considering as observations of the process the $\{\tilde{y}_{i\Delta}\}_{i=1,\dots,10800}$ data transformed according to (13).
- ν is obtained from Eq. (12), with $t = 3$ h. and considering the same transformation for η .
- h is estimated using the average of the natural logarithm of the actual observations of the farm, i.e., the following estimator is used $\tilde{h} = \frac{1}{n} \sum_{i=0}^n \ln(Y_{i\Delta})$.

Table 1 shows the results obtained from the process that considers the random component of the sum of the 11 generators of the wind farm.

To observe the behavior of the mathematical model, trajectories were simulated, discretizing the SDEs by means of the Milstein scheme shown in (16). Fig. 1 shows a simulated trajectory in black, and the original data trajectory is shown in red for comparison.

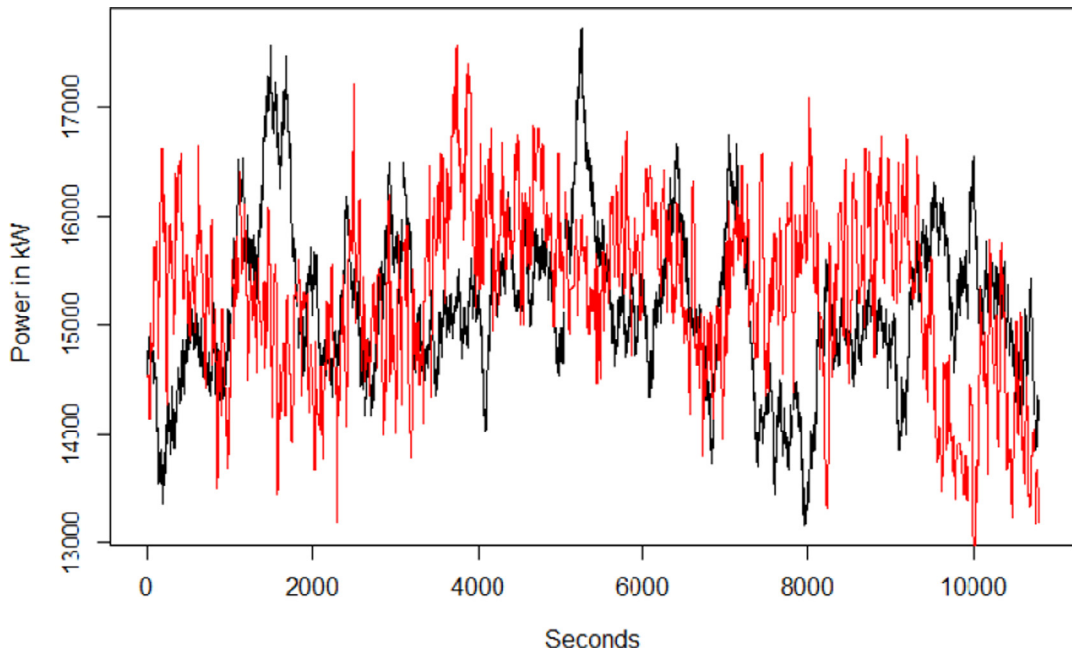


Fig. 1. Simulated trajectories of the active power output generation in kW.

3.1.3. Model residual analysis

In order to validate the parametric model, the uniform residues associated with parameters estimated for the O-U process were analyzed. This method consists basically in determining whether the actual observations obtained come from a process with the chosen model’s characteristics [38]. For this the expectation and conditional variance of the process must be known, and since they can be calculated explicitly for the O-U process, it will be verified if the data under the transformation (13) come from an O-U.

For the O-U process the expectation and conditional variance sought are respectively $E_{\eta,v}(U_{\Delta} | U_0 = U_{t_{i-1}}) = U_{t_{i-1}}e^{\eta\Delta}$ and $V_{\eta,v}(U_{\Delta} | U_0 = U_{t_{i-1}}) = \frac{v^2(e^{2\eta\Delta}-1)}{2\eta}$.

From this we calculate the following random variables:

$$R_{t_i(\eta,v)} = \frac{U_{t_i} - E_{\eta,v}(U_{\Delta} | U_0 = U_{t_{i-1}})}{\sqrt{V_{\eta,v}(U_{\Delta} | U_0 = U_{t_{i-1}})}} = \frac{U_{t_i} - U_{t_{i-1}}e^{\eta\Delta}}{v\sqrt{\frac{e^{2\eta\Delta}-1}{2\eta}}} \tag{17}$$

Eq. (17) is known as the standardized residues. Since this process is Gaussian, the standardized residues must follow a standardized normal distribution, which implies that if they are composed with the normal standard cumulative distribution function Φ , random variables with uniform distribution $U(0, 1)$ must be obtained, i.e., the variable

$$\Psi_{t_i}(\eta, v) = \Phi(R_{t_i}(\eta, v)), \tag{18}$$

where $\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^z e^{-t^2/2} dt$, must comply with $\Psi_{t_i}(\eta, v) \sim U(0, 1)$.

If transformation (13) of the data can be represented by an O-U, it must be true that taking these data as observations of the random variable U_{t_i} we must get that the variables (18) follow a uniform distribution $U(0, 1)$. This can be verified by means of a quantile-quantile graph that compares the quantiles of the random variable (18), with the theoretical quantiles of a variable with uniform distribution $U(0, 1)$.

For the numerical calculation of (17), the original time scale is considered, i.e., $\Delta = 1$ s. After getting $R_{t_i}(\eta, v)$ and $\Psi_{t_i}(\eta, v)$, we got the quantile-quantile graph of Fig. 2 for $\Psi_{t_i}(\eta, v)$:

Since the points follow approximately the identity line, it means that the observations obtained fit the stated type of model, and therefore there are no reasons to doubt the fit.

3.2. Stochastic stability analysis considering wind power injection in busbar 4

3.2.1. Interconnected system and injection location characteristics

In this paper we propose to study the effect of sustained and random disturbances in a large scale and real power system, in this case the SING. According to this, we consider the database of the power grid supplied by the coordinator

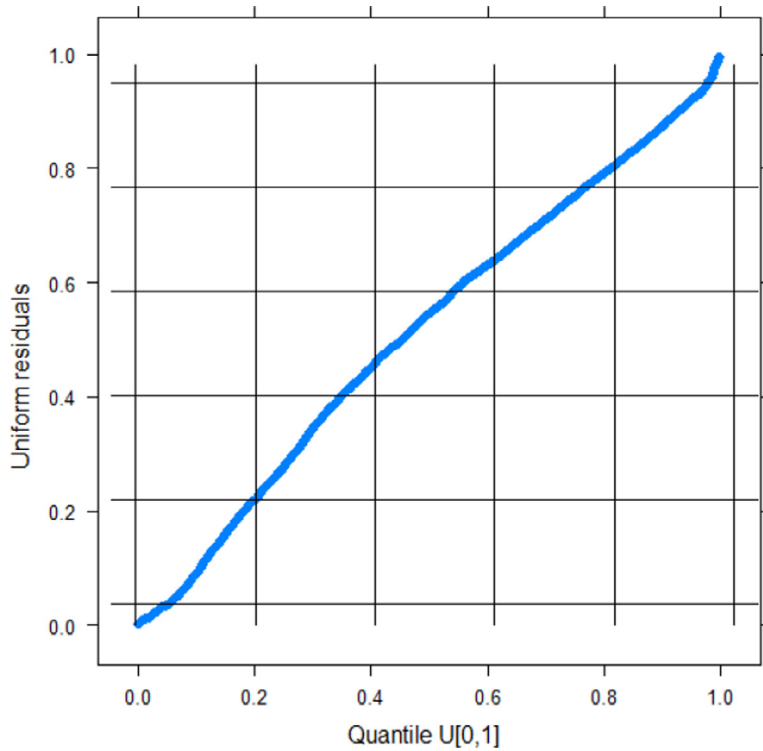


Fig. 2. Uniform residues quantile-quantile graph of the O-U model of the wind generation data.

Table 2

Busbar where wind power stochastic generation variability is studied.

Busbar number	Busbar name	Nominal noltage [kV]
4	Chacaya CTH	15.75

of the system (CDEC-SING) in its website [39], which is provided in a format compatible with DigSILENT PowerFactory. The publication date of the used database is Feb. 14, 2014.

We consider the partial reduction of the power system, keeping the dynamic of the generation machines and the simplification of the sub-systems with nominal voltage below 66 kV. The result of the reduction is a power system with 241 busbars, 23 operating synchronous machines and a total load of 2181.7 MW. The mathematical treatment considers the construction of the linear model of state variables considering the methodology proposed in [40].

In order to present an application example of the methodology, we chose a busbar of the system where the power generated by wind power plants is injected. The details of this busbar are shown in Table 2 and its geographical location can be seen in Fig. 3.

3.2.2. Maximum disturbance size analysis methodology

To determine the maximum size of a wind farm that can be connected in the busbar, we compute the Lyapunov exponents of the system considering successive increments from the disturbance size ρ of the connected wind power park. With the results of the characterization of the wind generation model, it is possible to generate random trajectories whose behavior describes a real wind power plant. For this study, a wind park whose disturbance size is $\rho = 1$ is equivalent to a 40 MW random generated wind power generator trajectory, as shown in Fig. 4 for a wind farm with these characteristics.

Following this methodology, we obtain a curve $\lambda(\rho)$ which relates the Lyapunov exponent and the size of the disturbance, and according to the criteria previously shown, from the viewpoint of small perturbation analysis, the system would be stable for every value of ρ while $\lambda(\rho) < 0$.

Each Lyapunov exponent is calculated considering 20 randomly generated trajectories (β) and 4 initial conditions (α) of the system.

In this case, we consider that a wind power park is connected to busbar 4 and the Lyapunov exponent is calculated for multiple ρ sizes. Table 3 shows the values obtained by calculating the Lyapunov exponent for those disturbance sizes



Fig. 3. Electrical and geographical location of the busbar chosen as injection point of wind power.

and their equivalent injected power, considering that for a disturbance value $\rho = 1$, the wind park has a power output of 40 MW.

Fig. 5 shows graphically the behavior of the Lyapunov exponent while the disturbance size grows, and Fig. 6 shows the Lyapunov exponent considering the injected power in busbar 4. With these results, we can say that the Lyapunov exponent is equal to zero ($\lambda = 0$) for a value of $\rho = 7.647$, that is, the whole system loses its stability for a disturbance size $\rho \geq 7.647$ approximately, which represents a wind power park with a power output of 305.88 MW.

The results from Figs. 5 and 6 shows that for random disturbances, the calculated Lyapunov exponent is practically constant and equal to the value of the real part closer to the origin of the eigenvalues in the deterministic system, that is, until some size of the disturbance ρ has been reached, the Lyapunov exponent has a value close to the real part of the maximum eigenvalue of the system without disruption. This indicates that, until a certain size of ρ , the disturbance introduced in the system by wind power parks is not large enough to generate a perceptible effect on the Lyapunov exponent.

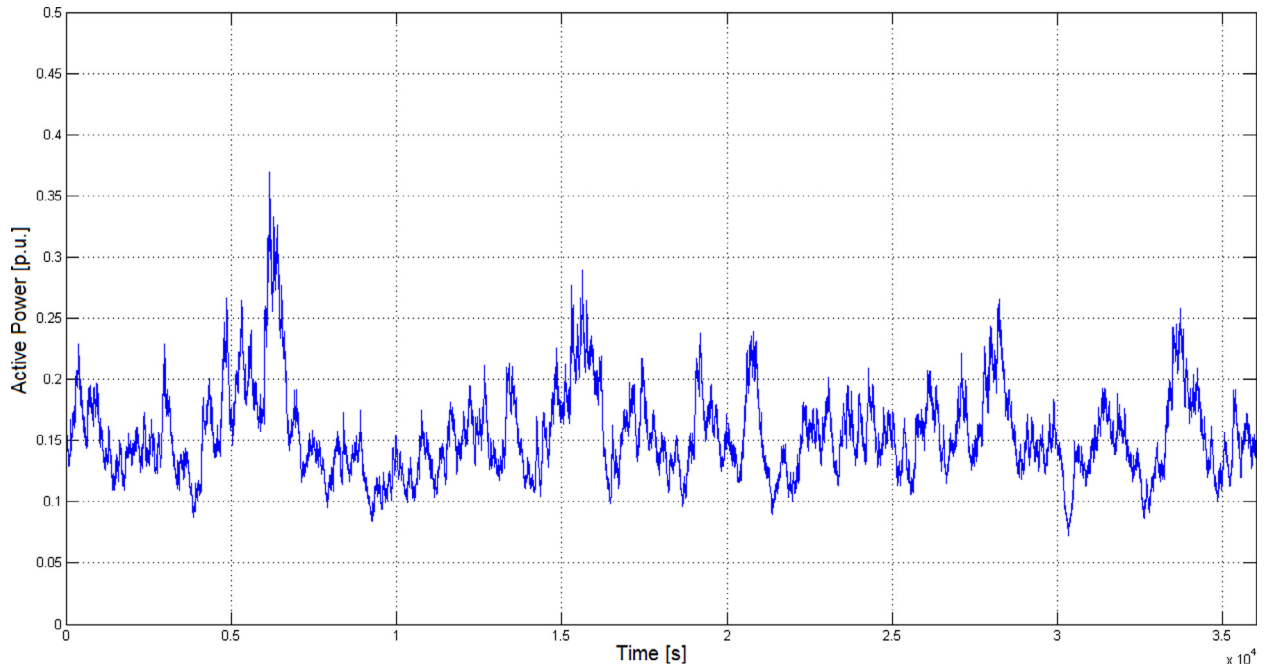


Fig. 4. Trajectory example of wind power generation from the Ornstein–Uhlenbeck process.

Table 3

Lyapunov exponents for wind power generation in busbar 4.

ρ	Active power [MW]	λ
0	0	-0.01648
2	80	-0.01621
4	160	-0.01621
6	240	-0.01634
7	280	-0.01641
7,5	300	-0.01170
7,75	310	0.00816
8	320	0.04607
8,5	340	0.14578
9	360	0.28591
10	400	0.94448
11	440	∞

Fig. 6 gives an importation information for system operation. The EPS will support 300 MW as maximum active power injected from wind power generation. This value represents a restriction for steady state operation and validate the stochastic small signal stability concept presented in this paper. From a practical point of view, the maximum power injected will depend on which bus is affected by wind power or another energy source randomly.

4. Conclusions

This paper develops two topics related to the analysis of stochastic linear systems stability in electrical power systems. Firstly, it shows a methodology that allows characterizing, by means of a stochastic process, the random and self-sustained behavior in time of the output power of a wind farm.

The disturbance modeling was done considering field measurement of the active power from a real wind power plant. Also, the techniques for the estimation and validation of the model is given, and the latter confirms the good fit of the stated equations.

The mathematical model of the disturbance is used to evaluate the impact of wind power penetration on the operation of the systems in permanent regime. This paper uses the Lyapunov exponent with the aim of determine the impact of the fluctuating active power, generated by random energy sources in a large scale interconnected power system (SING).

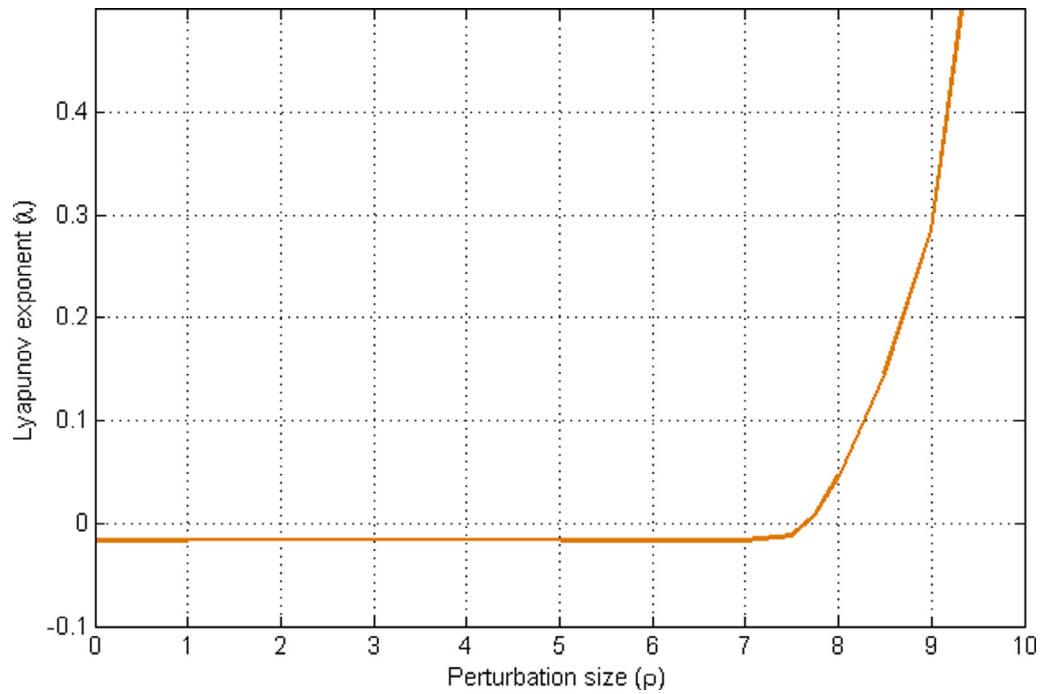


Fig. 5. Lyapunov exponents considering wind power generation in bus 4 and multiple disturbance sizes.

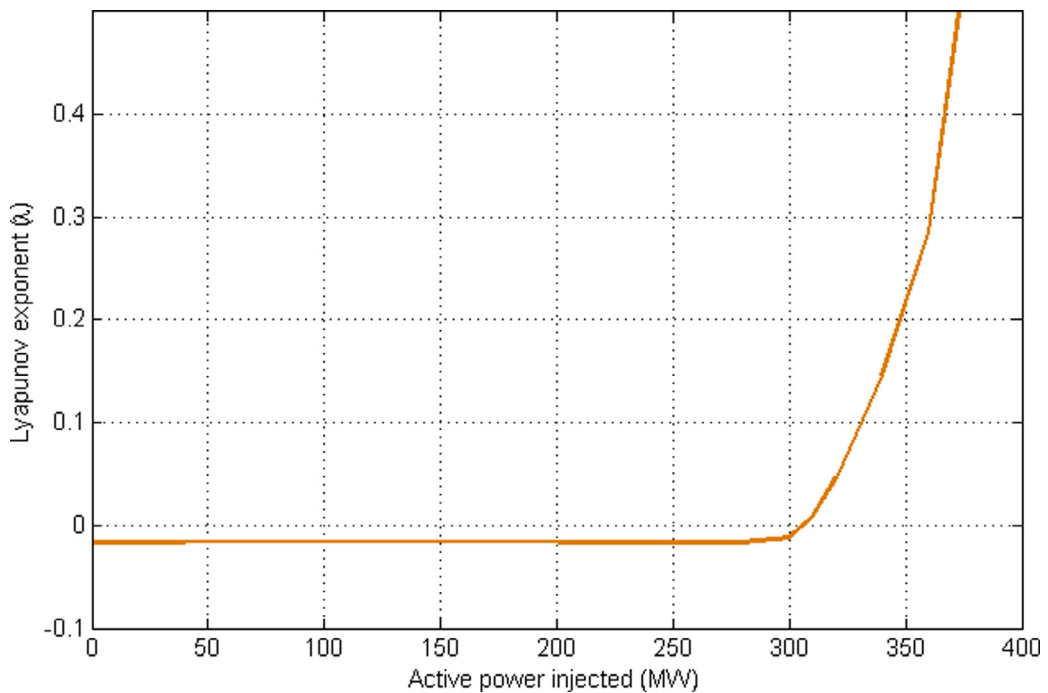


Fig. 6. Power generated by wind power park connected in bus 4 and its Lyapunov exponents.

To maintain steady state operation, power generation by wind sources should be limited to an appropriate value according to the location of the busbar where it connects and considering the magnitude of the power injected by the wind park.

In the future, use will be made of the obtained model to evaluate the impact of wind penetration on the operation of the systems in permanent regime, considering multiple injection points. Additionally, the analysis of the effect produced by random variations resulting from changes in loads will be studied.

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