THE GAINS FROM TRADE IN AN SPECIFIC FACTOR GROWTH MODEL*

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ABSTRACT

This paper uses a dynamic model of trade with specific factors of production to analyze the evolution of an economy that opens to international trade. Each period the allocation of labor is determined by previous period investment. The model cannot adapt instantaneously to free trade conditions, even though they improve welfare. Transition to the free trade steady state is costly and, among all feasible trajectories, those that have the longest adjustment paths are optimal. In fact, fast transitions can lead to losses from trade. On the other hand, if autarkic prices are very different from world prices, fast adjustment is preferable, since adjustment costs can be compensated by the large gains from specialization under trade.

SINTESIS

Este trabajo utiliza un modelo dinámico de comercio con factores de producción específicos para analizar la evolución de una economía que se abre al comercio internacional. En cada período, la asignación de trabajo está determinada por la inversión del período anterior. El modelo no se puede adoptar instantáneamente a las condiciones de libre comercio, aún cuando ellas mejoran el bienestar. La transición al estado estacionario del libre comercio es costosa y, entre las trayectorias factibles, aquellas que tienen los caminos de ajuste más largos resultan óptimas. De hecho, las transiciones rápidas pueden llevar a pérdidas del intercambio comercial. Por otra parte, si los precios autárquicos son muy diferentes a los precios del mercado mundial, el ajuste rápido es preferible, ya que los costos de ajuste pueden ser compensados por las grandes ganancias derivadas de la especialización bajo el intercambio comercial

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1. INTRODUCTION

The purpose of this paper is to analyze the transition between autarky and free trade in a small economy growth model. The approach can also be used to study the impact of terms of trade shocks. The model considers three factors and two goods. The factors of production are labor, physical capital and human capital. Human capital, which is employed by both sectors, is the source of long run growth. Trade liberalization causes transition costs because labor and physical capital are sector specific. As we would like to concentrate our analysis on the transition costs of opening the economy to trade, we build a model where trade does not alter the rate of growth. In order to attain this result human capital plays a symmetrical role in the production of goods and consequently the relative price of final goods remains constant.

In our model, the direction of trade depends on the difference in the intertemporal rates of discount between the country and the world. We use the model to extend standard trade theory results to a dynamic framework. First, we show that trade is welfare improving. Trade leads to an increase in welfare due to a reallocation of factors of production and through an increase in consumption utility. Second, the gains from trade that are obtained by comparing autarky to an open economy in steady state growth grossly overstate the benefits of liberalization. Numerical simulations show that if we include the transition period, the gains from free trade are much lower than suggested by steady state results. These results indicate that the inability to adjust without cost eliminates a large fraction of the gains from trade. In fact, if we force the economy to adjust quickly to trade liberalization, the economy may be worse off under trade than under autarky. We also note that most of the gains from trade are attained in the first few periods of adjustment. The economy may continue to adjust for a long time but the effects of later periods on welfare are negligible.

Since there are three factors and only two goods, the country always becomes specialized in the production of one of the two goods in finite time. Our main result is to show that the optimal trajectory is the slowest to specialize among the many feasible trajectories. The reason for this result is that by adjusting slowly to the new environment, consumption does not fall abruptly during the transition, i.e., a case of consumption-smoothing. This result is consistent with Moran and Serra (1993), who show within the context of a dynamic CGE model for Guatemala that the positive impact of trade liberalization on GDP is only felt after sufficient time has elapsed to permit the reallocation of resources.

There is a related literature that considers gradual opening of the economy in the case of adjustment costs or externalities. For instance, Colombatto (1993) states that in most cases protectionist policies may ease adjustment costs. Our paper does not consider the possibility of gradual opening to deal with the adjustment costs. Dewatripont and Roland (1992), in the context of a transition from a planned to a market economy, conclude that gradualism can be better than full immediate reform when the cost of the compensations needed in order to respect the existing political constraints are high compared to the allocative gain of immediate restructuring. Another related issue is the case for trade adjustment assistance programs designated to compensate workers for tariff cuts and facilitate their adjustment out of import competing industries. In a model where factors of production are imperfectly mobile, Feenstra and Lewis (1994) argue that by offering an adjustment subsidy to all individuals willing to move, the government can implement Pareto gains using the Dixit-Norman pattern of commodity taxes.

Finally we have simplified the analysis by assuming an homogeneous distribution of income. In an heterogenous society the income distribution effects further complicate the analysis.

2. THE BASIC MODEL

We study cohorts of agents that live a single period and then die, leaving an inheritance to their descendants. In this model, agents care about the welfare of their descendants so they behave like infinitely lived individuals. Specific production and utility functions are used so that explicit solutions can be obtained.

Production

We consider a two-goods three-factors economy. There are two reproducible factors: physical and human capital; and one non-reproducible factor: labor. Individuals inelastically supply one unit of labor. We normalize the number of individuals to be 1. The production functions for the final goods are:

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$$X_{1t} = L_t^{\alpha} H_{1t}^{1-\alpha} \tag{1a}$$

$$X_{2t} = K_t^{\alpha} H_{2t}^{1-\alpha} \tag{1b}$$

where X_{it}, $i = 1,2$ is the amount of final good i, L_t the amount of labor used in production of good 1, K_t the amount of physical capital used in production of good 2 and H_{it} the amount of human capital allocated to the production of good i, all variables for period t.

Physical capital is produced with constant returns to scale using labor alone. We assume that physical capital totally depreciates in one period, hence:

$$K_{t+1} = 1 - L_t \tag{2}$$

In general we denote with lower case variables referring to individual agents; if necessary we distinguish individual agent variables with the superscript u. The production of human capital has the general form

$$h_{t+1} = f(h_t, g_t, \bar{h}_t), \tag{3}$$

where h_t is the stock of human capital of an agent, g_t is the amount of human capital he invests in order to produce new human capital and $\bar{h}_t$ is the average level of human capital, all variables dated at time t.

Let G_t denote the aggregate expenditure in education, then the human capital balance condition implies that:

$$H_{1t} + H_{2t} = H_t - G_t. \tag{4}$$

We chose good 2 to be the numeraire and p_t to designate the price of good 1. The marginal value of human capital must be the same in both sectors, i.e.,

$$p_t L_t^{\alpha} H_{1t}^{-\alpha} = K_t^{\alpha} H_{2t}^{-\alpha} \tag{5}$$

which can be rewritten as:

$$\frac{H_{1t}}{H_{2t}} = \frac{L_t}{K_t} p_t^{1/\alpha} \tag{6}$$

This equation, in conjunction with (4) and (1), implies:

$$H_{1t} = \frac{L_t}{\mu_t}(H_t - G_t) \tag{7a}$$

$$H_{2t} = \left(1 - \frac{L_t}{\mu_t}\right)(H_t - G_t) \tag{7b}$$

where:

$$\mu_t = L_t + p_t^{-1/\alpha} K_t \tag{8}$$

From (1) and (7) factor prices can be written

$$w_t = \alpha p_t \left(\frac{H_t - G_t}{\mu_t}\right)^{1-\alpha} \tag{9a}$$

$$r_t = \alpha (p_t)^{1-1/\alpha} \left(\frac{H_t - G_t}{\mu_t}\right)^{1-\alpha} \tag{9b}$$

$$v_t = (1-\alpha)p_t \left(\frac{H_t - G_t}{\mu_t}\right)^{-\alpha} \tag{9c}$$

where w_t denotes the wage, r_t the return on capital and v_t the return on human capital.

Equation (8) can be rewritten .

$$\mu_t = \frac{w_t L_t + r_t K_t}{w_t} \tag{10}$$

thus μ_t represents the value of past and current labor employed in the production of final goods, in this sense it plays a role similar to (H-G).

Consumption

All agents are assumed to have identical preferences which are represented by:

$$Max \; V(h_t, k_t) = u(c_{1t}, c_{2t}) + \gamma \, V(h_{t+1}, k_{t+1}) \tag{11}$$

where k_t denotes the holdings of physical capital of an individual at the beginning of period t, c_{it} his consumption of good i at time t and γ is a parameter that

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expresses the value of the future. In each period an individual spends $g_t l$ units of human capital educating his descendant and leaves an inheritance of k_{t+1} units of physical capital. Let e_t denote the expenditure on consumption in period t. The budgetary constraint implies that

$$e_t = w_t (1 - k_{t+1}) + r_t k_t + v_t (h_t - g_t) \qquad (12)$$

For simplicity we assume that

$$u (c_{1t}, c_{2t}) = a \log c_{1t} + (1-a) - \log c_{2t} \qquad (13)$$

Maximization within each period implies that:

$$c_{1t} = \frac{a \, e_t}{p_t} \qquad (14a)$$

$$c_{2t} = (1-a) \, e_t \qquad (14b)$$

The corresponding single period indirect utility function is:

$$v (e_t, p_t) = A + \log e_t - a \log p_t \qquad (15)$$

where $A = a \, loga + (1-a)log(1-a)$. From (9) and (12), individual expenditures can be expressed as

$$e_t = p_t (H_t - G_t)^{1-\alpha} \, \mu_t^{\alpha} \left[\alpha \frac{l_t + k_t p_t^{-1/\alpha}}{\mu_t} + (1-\alpha) \frac{h_t - g_t}{H_t - G_t} \right] \qquad (16)$$

where $l_t = 1 - k_{t+1}$. It is immediate from (16) that total expenditure in this economy can be written as

$$E_t = p_t (H_t - G_t)^{1-\alpha} \, \mu_t^{\alpha}, \qquad (17)$$

thus the expression between brackets in (16), which we denote ε_t, represents the fraction of total expenditure corresponding to an agent and consequently $\sum_u \varepsilon_t^u = 1$.

In the remainder of this paper we assume that all agents have equal endowments. It follows that the ε_t are the same for all agents and remain constant over time.

Externalities in Human Capital Production

We believe that education is partly acquired informally from parents, acquaintances and the rest of society (media, cultural activities, etc.) and partly from formal expenditures in education by parents. We want a technology for education that corresponds to the notion that there is a decrease in the cost of acquiring additional human capital, the higher the average level of human capital in society. This dependency on the average level of human capital in society creates an externality in the production of human capital. Moreover, we would like the level of human capital of the parent to have a positive influence in the stock of the descendant, reflecting the effects of rearing on education. A simple specification satisfying these conditions is

$$h_{t+1} = \rho h_t^{\delta} \bar{h}_t^{1-\delta} + g_t, \quad 0 < \delta < 1 \tag{18}$$

thus

$$h_t - g_t = h_t + \rho h_t^{\delta} \bar{h}_t^{1-\delta} - h_{t+1}, \quad 0 < \delta < 1 \tag{19}$$

The Euler Equations

Agents are assumed to have perfect foresight on the trajectory of prices. Consequently, assuming an interior solution each individual's Euler equations are given by:

$$\frac{\partial u(c_t)}{\partial k_{t+1}} = -\gamma \frac{\partial u(c_{t+1})}{\partial k_{t+1}} \tag{20}$$

$$\frac{\partial u(c_t)}{\partial h_{t+1}} = -\gamma \frac{\partial u(c_{t+1})}{\partial h_{t+1}} \tag{21}$$

From (15) and (16) and recalling that $l_t = 1 - k_{t+1}$ and that the ε_t remain constant over time, equation (20) becomes

$$\mu_{t+1} = \gamma p^{-1/\alpha} \mu_t \tag{22}$$

From equations (15, (16) and (19), equation (21) becomes

$$H_{t+1} - G_{t+1} = (1 + \rho\delta)\gamma (H_t - G_t) \tag{23}$$

The transversality conditions are

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$$\lim_{t \to \infty} \gamma^t \frac{\delta u(c_t)}{\delta k_t} k_t = 0 \qquad (24)$$

$$\lim_{t \to \infty} \gamma^t \frac{\delta u(c_t)}{\delta h_t} h_t = 0 \qquad (25)$$

Using (15 and (16), equations (24) and (25) become

$$\lim_{t \to \infty} \gamma^t \frac{\alpha}{\epsilon_t} \frac{p^{-1/\alpha}}{\mu_t} k_t = 0 \qquad (26)$$

$$\lim_{t \to \infty} \gamma^t \frac{1-\alpha}{\epsilon_t} \frac{1+\delta\rho}{H_t - G_t} h_t = 0 \qquad (27)$$

Given the properties exhibited by function u,[1] the solution of each agent's optimization problem can be characterized by the Euler and transversality conditions.

3. THE CLOSED ECONOMY

Let C_{it} denote the aggregate consumption of good i in period t, i.e., $C_{it} = \Sigma_u c_{it}^u$, then from (14)

$$\frac{C_{1t}}{C_{2t}} = \frac{a}{(1-a)} P_t \qquad (28)$$

From (1) and (13) the ratio of final goods production may be written

$$\frac{X_{1t}}{X_{2t}} = \left(\frac{L_t}{K_t}\right)^\alpha \left(\frac{H_{1t}}{H_{2t}}\right)^{1-\alpha} = \frac{L_t}{K_t} p_t^{1/\alpha - 1} \qquad (29)$$

From (28) and (29) the closed economy equilibrium condition is:

$$\frac{a}{1-a} = \frac{L_t}{K_t} p_t^{1/\alpha} \qquad (30)$$

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[1] Function u satisfies the conditions of Theorem 4.15 in Stokey, Lucas and Prescott (1989).

From (8) and (30) it follows that in autarky $\mu_t = L_t/a$. This implies that equation (22) may be rewritten:

$$L_{t+1} = \gamma p^{-1/\alpha} L_t \tag{31}$$

Recalling (2) and (30), the amount of labor allocated to production of the first good remains constant:

$$L_t = \frac{a}{\gamma(1-a)+a} \tag{32}$$

consequently the closed economy relative price of good 1 satisfies $p = \gamma^\alpha$. Because of the way in which the model was set the final goods price ratio does not depend on either tastes or the relative factor abundance.

Closed Economy Dynamics

Aggregating equation (19) over individuals we obtain:

$$H_t - G_t = H_t + \rho\Sigma_u(h_t^u)^\delta(\bar{h}_t)^{1-\delta} - \Sigma_u h_{t+1}^u \tag{33}$$

recalling that all individuals are identical, equation (33) becomes:

$$H_t - G_t = (1+\rho)H_t - H_{t+1} \tag{34}$$

so that equation (23) becomes:

$$H_{t+2} - [(1+\rho\delta)\gamma + (1+\rho)] H_{t+1} + (1+\rho)(1+\rho\delta)\gamma H_t = 0 \tag{35}$$

with roots:

$$\lambda_1 = (1 + \rho) \tag{36a}$$

$$\lambda_2 = (1 + \rho\delta)\gamma \tag{36b}$$

If the solution to the difference equation contains a non-zero constant corresponding to $\lambda 1$, as $t \rightarrow \infty$ the rate of growth of human capital tends to $\lambda_1-1 = \rho$. This implies that $G_t = H_t$ in the limit (see equation (34)), so that all human capital is dedicated to the production of human capital and there is no production of final goods. This is inconsistent with utility maximization so the constant corresponding to λ_1 is zero. Formally, if $G_t = H_t$, when t tends to infinity the transversality condition (27) is not satisfied. Therefore the growth rate is $\lambda_2-1 = \gamma(1+\delta\rho)-1$. This implies that

$$H_t = [\gamma (1 + \delta\rho)]^t H_0 \qquad (37)$$

Hence the condition for positive growth is $1/(1+\rho\delta) > \gamma$. The growth rate λ_2-1 is not the optimal feasible rate of growth[2]. This suggests there is scope for a planner that would take into account the externality implicit in (18) to achieve the social optimum[3]. The growth rate falls with the value of $1-\delta$, which measures the extent of the externality. When the externality is present, only a fraction δ of future increases in human capital is appropriated by the agent, so (22) holds. The conditions for positive growth are more stringent, the larger the externality. When $\delta = 0$ growth is not possible.

4. THE SMALL OPEN ECONOMY

Trade could be caused either because there are technical differences (α) or time-preference differences (γ). Consider the case of a small autarkic economy that opens its economy. It will be shown that in finite time the economy specializes. This result is a consequence of the fact that the number of factors exceeds the number of goods. We show that the optimal trajectory is the slowest to specialize among feasible trajectories. The reason for this result is that by adjusting slowly to the new environment, society does not have to abruptly reduce consumption in the path to specialization: a case of consumption-smoothing.

In what follows p denotes the world price of good 1. If the rest of the world has the same technology and preferences but a different time preference γ^w, consequently $p = (\gamma^w)^\alpha$. In this model trade is driven by differences in the value different societies put on the future. If the country has a lower discount rate than the rest of the world, it specializes in good 2 (which uses capital), otherwise it specializes in good 1.

Euler Equations

Since the second Euler equation leads to the same condition as in autarky, the opening of the economy does not change the rate of growth. We study the optimal trajectory before specialization in more detail. Substituting equation (8) in (22), and recalling that $K_{t+1} = 1 - L_t$, leads to the second-order difference equation:

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[2] The optimal growth rate corresponds to the case of no-externality, i.e., $(1+\rho)\gamma-1$, which is the one a social planner would achieve.

[3] When $\delta=1$, i.e., there is no externality in the production of human capital, agents may have different human capital stocks and the previous results continue to hold. When $\delta=0$ the externality affects all agents the same way so even if all agents have different stocks of human capital the previous results hold.

$$L_{t+1} + p^{-1/\alpha}(1-L_t) = \gamma p^{-1/\alpha}(L_t + p^{-1/\alpha}(1-L_{t-1})) \tag{38}$$

Therefore

$$L_{t+1} - (p^{-1/\alpha} + \gamma p^{-1/\alpha})L_t + \gamma p^{-2/\alpha}L_{t-1} = \gamma p^{-2/\alpha} - p^{-1/\alpha} \tag{39}$$

This second-order equation has two roots: $p^{-1/\alpha}$ and $\gamma p^{-1/\alpha}$. The existence of bounds to the values of L_t, imply that we cannot use a transversality condition in order to eliminate one of the roots of the associated polynomial. Hence the solution will be a linear combination of a polynomial in the two roots. Thus, a solution will have the general form:

$$L_t = M_1(p^{-1/\alpha})^t + M_2(\gamma p^{-1/\alpha})^t + M_3 \tag{40}$$

where M_1 and M_2 are constants to be determined and M_3 denotes the particular solution:

$$M_3 = p^{-1/\alpha}/(p^{-1/\alpha} - 1) > 1 \tag{41}$$

In what follows we assume that the domestic relative price of the labor intensive good 1 is above the world price, i.e., $p^a = \gamma^\alpha > p = (\gamma^w)^\alpha$, where p^a denotes the autarky price of good 1 and γ^w the time discount factor in the rest of the world.[4] Hence $\gamma p^{-1/\alpha} > 1$. We will show that in this case 1 falls over time and in a finite number of periods the economy becomes specialized in the production of the second good.

Two boundary conditions are needed: L_0, which we already know, and L_1. Although the value of L immediately following liberalization is free to jump, among the paths for L_t that satisfy equation (40), the one that maximizes welfare will be chosen. Equivalently, instead of selecting L_1, agents choose the number of periods before they specialize and feasible solutions can be determined by assuming $L_T = 0$ for different values of T.

In the following expressions the constants $M_1(T)$ and $M_2(T)$ depend on the number of periods before specialization T. The solution to (39) can be written in terms of these constants as:

$$L_t = M_1(T)(p^{-1/\alpha})^t + M_2(T)(\gamma p^{-1/\alpha})^t + M_3. \tag{42}$$

and the boundary conditions are

$$L_0 = M_1(T) + M_2(T) + M_3 \tag{43a}$$

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[4] The case where trade goes the other way is solved similarly except for the differences we will point out.

$$L_T = 0 \tag{43b}$$

When the country is already specialized in the production of good 2 $(t \geq T)$, then $L_t = 0$, $K_t = 1$, and $\mu_t = p^{-1/\alpha}$. The second Euler equation remains the same as before, while the first equation becomes an inequality stating that $\gamma p^{-1/\alpha} > 1$.[5]

As noted above, the growth rate of human capital does not change with trade opening. We might as well concentrate on the evolution of individual expenditure. Using the indirect utility function (14) and the definition of e_t (equation (16)), welfare changes (Δ indicates change) due to the opening of trade are proportional to

$$\Delta v(e_t, p_t) = \Delta(\log (\mu_t)^\alpha + (1-a)\Delta(\log p) \tag{44}$$

Omitting those variables that are not affected by the opening of the economy and recalling the definition of μ_t, welfare is given by

$$W = \alpha \sum_1^\infty \gamma^{t-1} \log\left(L_t + (1-L_{t-1})p^{-1/\alpha}\right) + \frac{1-a}{1-\gamma} \log p \tag{45}$$

Now in period T the economy becomes specialized in the production of good 2, i.e., $L_t = 0$, for $t \geq T$. Hence equation (45) can be rewritten

$$W = \alpha \sum_1^T \gamma^{t-1} \log\left(L_t + (1-L_{t-1})p^{-1/\alpha}\right) + \frac{1-a-\gamma^T}{1-\gamma} \log p \tag{46}$$

Rearranging terms

$$W = \alpha \sum_1^T \gamma^{t-1} \log\left(L_t - L_{t-1}p^{-1/\alpha}\right) - \frac{a}{1-\gamma} \log p$$

From (41) and (42) it follows that

$$L_t - L_{t-1}p^{-1/\alpha} = (\gamma p^{-1/\alpha})^{t-1}(\gamma - 1)M_2(T)) \tag{48}$$

Replacing (48) in equation (47) results in

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[5] An analogous case occurs when the country specializes in the production of good 1. Here $L_t = 1$, $K_t = 2$, $\mu_t = 1$. The second Euler equation remains as before, while the first Euler equation becomes the condition $\gamma p^{-1/\alpha} < 1$.

$$W = \alpha \sum_1^T \gamma^{t-1} \log\left((\gamma p^{-1/\alpha})^{t-1}(\gamma-1)M_2(T)\right) - \frac{a}{1-\gamma}\log p \qquad (49)$$

The last term in (49) corresponds to the discounted utility that would be achieved if the country could jump to the open economy steady state. The first term represents the transition cost in moving to the open economy steady state. The transition cost is always negative, as the expression $(\gamma p^{-1/\alpha})^{t-1}(\gamma-1)M_2(T)$ is less than 1. Rearranging terms leads to

$$W = \alpha \log(\gamma p^{-1/\alpha})\sum_1^{T-1} t\gamma^t + \alpha\frac{1-\gamma^T}{1-\gamma}\log((\gamma-1)M_2(T)) - \frac{a}{1-\gamma}\log p \qquad (50)$$

In the appendix we show that $M_2(T)$ is negative and that its single minimum corresponds to the longest feasible trajectory (Lemma 3). Using this lemma we prove our main result.

Theorem: The longest feasible trajectory to specialization is optimal.
Proof: The proof uses the welfare expression (50). Given that $M_2(T)$ is negative, it follows that welfare is an increasing function of both T and the absolute value of $M_2(T)$. Since Lemma 3 in the appendix shows that the longest feasible trajectory corresponds to the minimum of M_2, the theorem follows.

<div align="right">Q.E.D.</div>

It is well known that in these models free trade is welfare enhancing. We will confirm this result with the following proposition.

Proposition: Trade is welfare increasing.
Proof: Let W_0 denote welfare if the allocation of labor is kept fixed at the autarkic level $L_0 = a/(\gamma(1-a)+a))$ after the opening of the economy. From (49)

$$W_0 = \alpha \sum_1^\infty \gamma^{t-1}\log(L_0 + (1-L_0)p^{-1/\alpha}) + \frac{1-a}{1-\gamma}\log p \qquad (51)$$

then

$$W_0 = \left[\alpha \log(L_0 + (1-L_0)p^{-1/\alpha}) + (1-a)\log p\right]\frac{1}{1-\gamma} \qquad (52)$$

Differentiating (52) results in

$$\frac{dW_0}{dp} = \frac{1}{1-\gamma}\left[\frac{-(1-L_0)p^{-(1/\alpha+1)}}{L_0 + (1-L_0)p^{-1/\alpha}} + \frac{1-a}{p}\right] \qquad (53)$$

rearranging terms

$$\frac{dW_0}{dp} = \frac{1}{1-\gamma}\left[\frac{- a(1-L_0)p^{-1/\alpha} + (1-a)L_0}{pL_0 + (1-L_0)p^{1-1/\alpha}}\right] \tag{54}$$

Since the denominator is positive,

$$sgn(dW_0/dp) = sgn(-a(1-L_0)p^{-1/\alpha} + (1-a)L_0) \tag{55}$$

since $L_0 = a/(\gamma(1-a)+a)$ and simplifying we obtain $sgn(dW_0/dp) = sgn(1 - \gamma p^{-1/\alpha})$, which shows that when $\gamma p^{-1/\alpha} > 1$ (so that the country specializes in good 2) and trade lowers price. If p falls, trade is welfare improving (obviously welfare after trade is larger than W_0). Conversely, when prices rise and $\gamma p^{-1/\alpha} < 1$, a rise in price is beneficial.

6. RESULTS AND CONCLUSIONS

We have shown that trade is beneficial. However, during the transition to the specialized economy there exist adjustment costs. This is why this economy selects the longest feasible transition, distributing this cost over many periods. The transition costs depend on the specific parameters of the model, which is simulated in Table 1.

Figure 1 shows the evolution of the labor devoted to sector 1 production for different adjustment periods.[6] Table 1 presents simulation results for selected parameter values of total discounted welfare after liberalization.[7] Some results become clear from the simulations. First, transition costs are high. Hence steady state welfare comparisons overstate the benefits from trade. Second, the smallest transition costs correspond to cases when the differences between autarkic and world prices are lowest. In relative terms, however, the transition costs are smaller when the differences between autarkic and world prices are highest and the gains from trade are largest. Third, the larger the difference between autarkic and world prices, the shorter the transition. The intuition is that the benefits in the free trade steady state are so large compared to those under autarky, that agents are willing to achieve the transition in a short time. In doing this they sacrifice the consumption-smoothing effect of longer transitions. Finally, note from Figure 2 that most of the benefits of trade liberalization take place in the first few periods, even though the economy continues to evolve (in the example of Figure 2, for 20 periods).

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[6] The figures are based on the case in which $\rho=0.4$ and $p_{1/\alpha} = 0.296$.
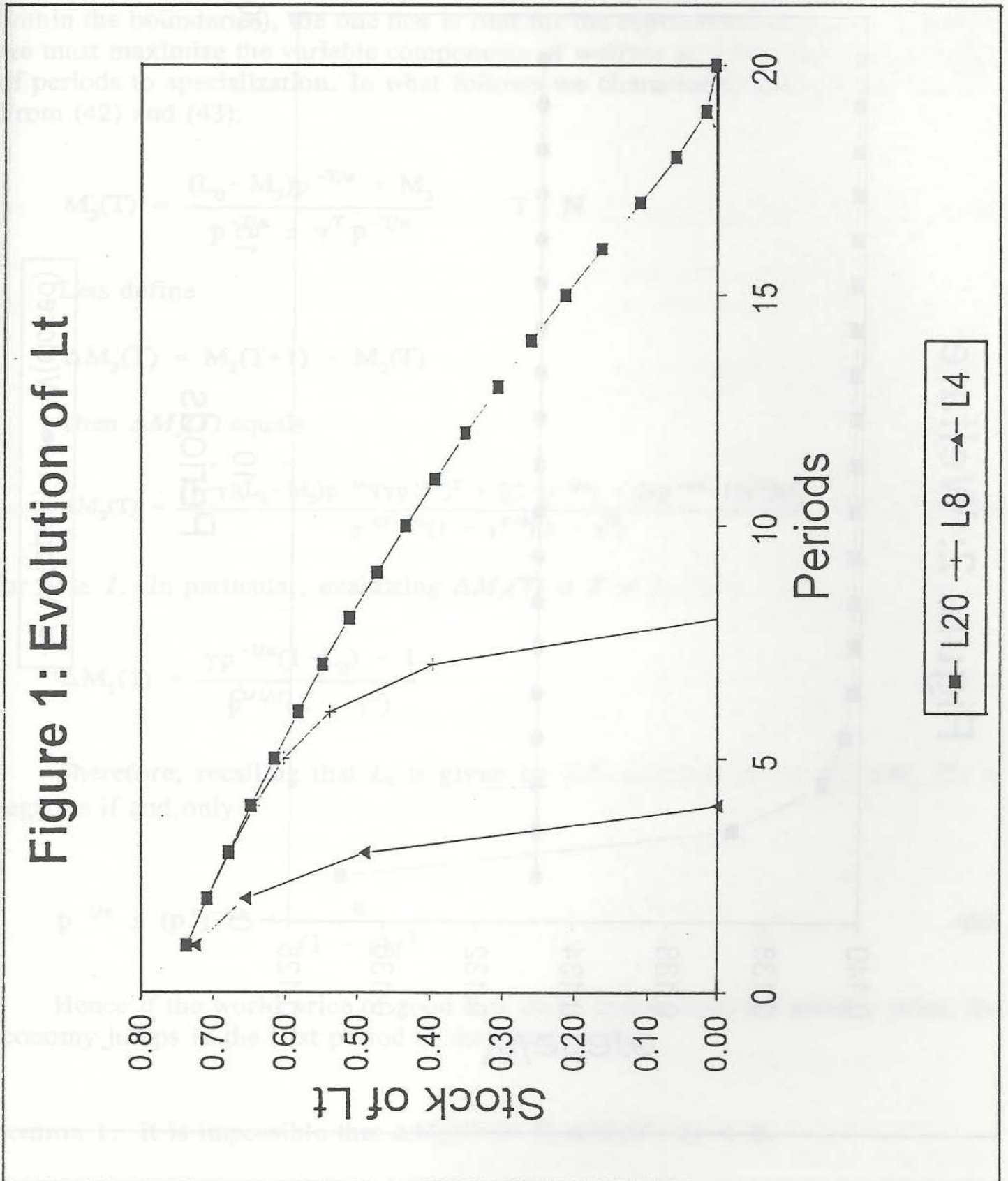[7] In the simulations, $\alpha = 0.5$, $a = 0.5$.

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TABLE 1

ρ	γ	$p^{1/\alpha}$	W_c	W_s	W	T
.30	.410	.120	.1286	.7830	.5152	1
"	"	.350	"	.3877	.1649	4
"	"	.380	"	.3570	.1451	8
"	"	.400	"	.3383	.1342	20
.35	.356	.080	.1310	.8520	.5617	1
"	"	.270	"	.4410	.1899	4
"	"	.323	"	.3801	.1486	8
"	"	.342	"	.3591	.1371	20
.40	.308	.070	.1330	.8348	.5209	1
"	"	.240	"	.4480	.1804	4
"	"	.275	"	.4050	.1529	8
"	"	.296	"	.3821	.1399	20

Notes:
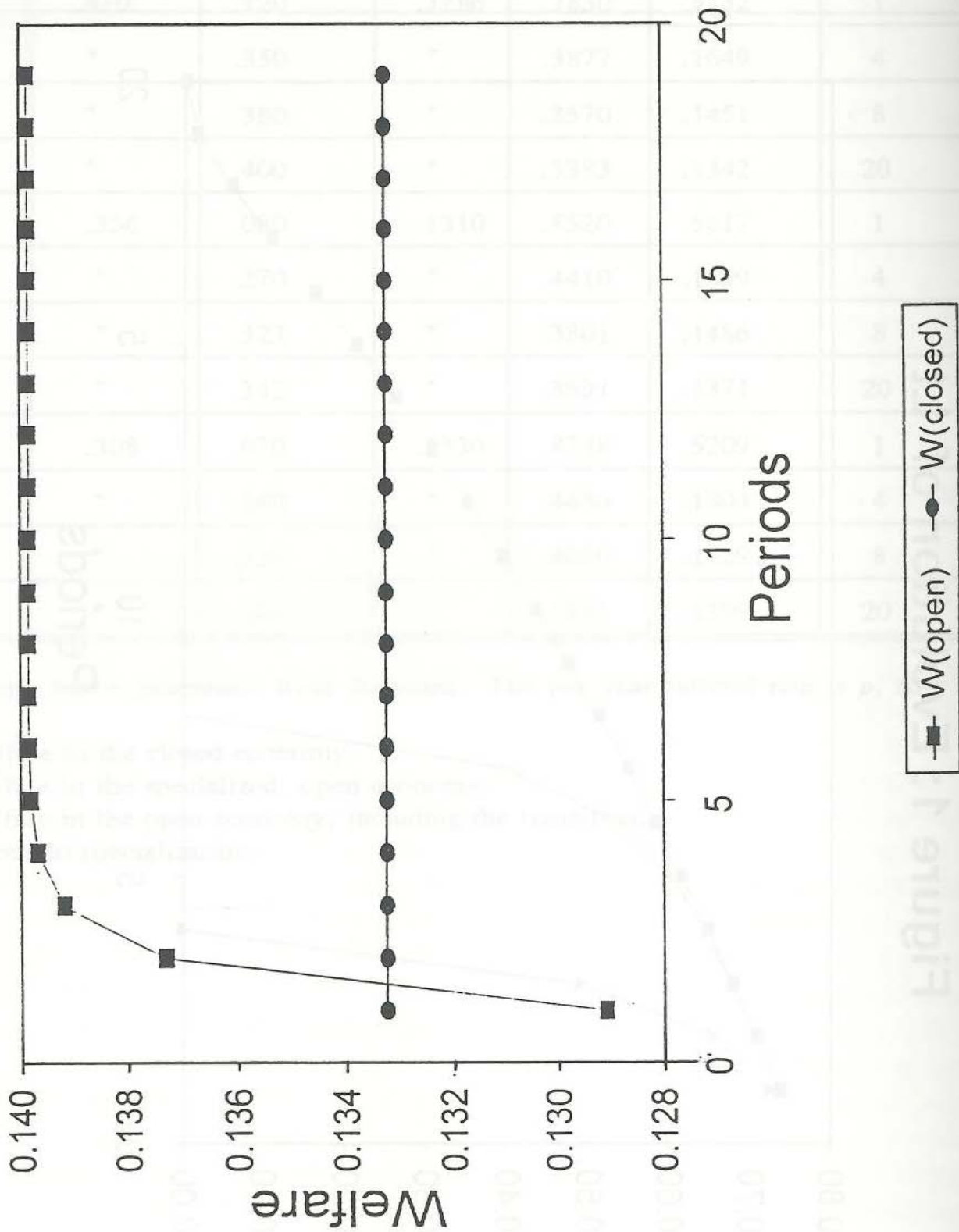
1) We assume that a generation lives 30 years. The per year interest rate is ρ, so $\gamma = (1+\rho)^{30}$.

2) W_c: Welfare in the closed economy.

   W_s: Welfare in the specialized, open economy.

   W : Welfare in the open economy, including the transition.

   T : Periods to specialization.

Figure 1

Figure 1: Evolution of (Lt

Stock of Lt (vertical axis): 0.00, 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80

Periods (horizontal axis): 0, 5, 10, 15, 20

Legend: L20, L8, L4

Figure 2: Welfare

Legend: W(open) (square markers), W(closed) (circle markers)

Y-axis: Welfare (0.128, 0.130, 0.132, 0.134, 0.136, 0.138, 0.140)

X-axis: Periods (0, 5, 10, 15, 20)

We need to find a rule that selects among all feasible trajectories of L_t (i.e., those that satisfy the first Euler equation and the boundary conditions and L remains within the boundaries), the one that is best for the representative agent. Therefore we must maximize the variable components of welfare as a function of the number of periods to specialization. In what follows we characterize the optimal solution. From (42) and (43):

$$M_2(T) = \frac{(L_0 - M_3)p^{-T/\alpha} + M_3}{p^{-T/\alpha} - \gamma^T p^{-T/\alpha}} \qquad T \in N \tag{56}$$

Lets define

$$\Delta M_2(T) = M_2(T+1) - M_2(T) \tag{57}$$

Then $\Delta M_2(T)$ equals

$$\Delta M_2(T) = \frac{(1-\gamma)(L_0 - M_3)p^{-1/\alpha}(\gamma p^{-1/\alpha})^T + [(1-p^{-1/\alpha}) + (\gamma p^{-1/\alpha}-1)\gamma^T]M_3}{p^{-(T+1)/\alpha}(1 - \gamma^{T+1})(1 - \gamma^T)} \tag{58}$$

for $T \geq 1$. In particular, evaluating $\Delta M_2(T)$ at $T = 1$,

$$\Delta M_2(1) = \frac{\gamma p^{-1/\alpha}(1-L_0) - 1}{p^{-1/\alpha}(1 - \gamma^2)} \tag{59}$$

Therefore, recalling that L_0 is given by (25) and that $p^a = \gamma^\alpha$, $\Delta M_2(1)$ is negative if and only if

$$p^{-1/\alpha} \leq (p^a)^{-1/\alpha} + \frac{a}{(1 - a)\gamma^2} \tag{60}$$

Hence if the world price of good 1 is much higher than the autarky price, the economy jumps in the first period to the steady state.

Lemma 1: It is impossible that $\Delta M_2(T) > 0$, $\Delta M_2(T+1) < 0$.

Proof: We prove the Lemma by contradiction. Assume that exists T such that $\Delta M_2(T) > 0$ and $\Delta M_2(T+1) < 0$. From equation (58) the above conditions imply

$$(M_3-L_0)(1-\gamma)p^{-1/\alpha}(\gamma p^{-1/\alpha})^T + M_3[(1-p^{-1/\alpha})+(\gamma p^{-1/\alpha}-1)\gamma^T] > 0 \qquad (61)$$

$$(M_3-L_0)(1-\gamma)p^{-1/\alpha}(\gamma p^{-1/\alpha})^{T+1} + M_3[(1-p^{-1/\alpha})+(\gamma p^{-1/\alpha}-1)\gamma^{T+1}] < 0 \qquad (62)$$

Multiplying (61) by $\gamma p^{-1/\alpha}$ and subtracting (62) we have

$$M_3[(p^{-1/\alpha}-1)(1-\gamma p^{-1/\alpha}) + (\gamma p^{-1/\alpha}-1)\gamma^{T+1}(p^{-1/\alpha}-1)] > 0 \qquad (63)$$

Therefore

$$M_3(1-p^{-1/\alpha})(\gamma p^{-1/\alpha}-1)(1-\gamma^{T+1}) > 0 \qquad (64)$$

recalling that M_3 is positive and that $\gamma p^{-1/\alpha} > 1$, equation (64) is a contradiction.

Q.E.D.

Lemma 2: $M_2(T)$ is negative and as a function of T has a single minimum.

Proof: From (58) it follows that $\Delta M_2(T) = \dfrac{L_0 - 1}{1 - \gamma} \, 0$, and $\lim_{T\to\infty} M_2(T) = L_0 - M_3 < 0$. From (56) $\lim_{T\to\infty} M_2(T) = 0$. These results, in conjunction with Lemma 1, imply that $M_2(T)$ is negative and has a single minimum, as shown in Figure 1a. When condition (60) is not satisfied, the minimum occurs at $T = 1$ as shown in Figure 1b.

Lemma 3: The single minimum of $M_2(T)$ corresponds to the longest feasible trajectory.

Proof: Let L^T_t be the labor employed in the production of the first good in the t-th period of a trajectory satisfying the first Euler equation and such that $L^T_T = 0$, i.e., when the economy specializes at time T. Using (42) and (43a) we may write

$$L^T_t = M_2(T)[\gamma^t p^{-t/\alpha} - p^{-t/\alpha}] + (L_0-M_3)p^{-t/\alpha} + M_3 \qquad (65)$$

using (56) equation (65) becomes

$$L^T_t = [M_2(T)-M_2(t)](\gamma^t-1)p^{-t/\alpha} \qquad (66)$$

Equation (66) shows that the longest feasible (i.e., $L^T_t > 0$ for all t) trajectory corresponds to the T where the minimum of M_2 is achieved.

Q.E.D.

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