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WAGE INEQUALITY IN CHILE: A DIRECTED SEARCH APPROACH WITH HETEROGENEOUS SKILL DISTRIBUTION

TESIS PARA OPTAR AL GRADO DE MAGISTER EN ECONOMIA APLICADA

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RESUMEN DE LA TESIS PARA OPTAR
AL TÍTULO INGENIERO CIVIL INDUSTRIAL
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El presente trabajo es un estudio sobre la desigualdad de salarios en Chile, entendida como la diferencia en salarios totales obtenidos por las personas de altos ingresos comparados con aquellas de bajos ingresos.

Esta tesis propone un modelo de búsqueda dirigida. Estos modelos sun usados para describir mercados con fricciones, en que los mercados no necesariamente son eficientes. En particular, los modelos de búsqueda dirigida asumen que los trabajadores "dirigen" sus esfuerzos hacia ciertos mercados, caracterizados por un salario particular.

Este modelo también considera una distribución de habilidades, con una ley de movimiento que, bajo ciertas condiciones, converge en una distribución estacionaria, que luego es utilizada para analizar el mercado del trabajo para cada nivel de habilidad, analizando al mismo tiempo los salarios y el producto de la economía.

Para ayudar a calibrar, los datos fueron recolectados de fuentes gubernamentales públicas, y el modelo fue computado utilizando métodos numéricos.

Los resultados sugieren la existencia de un *trade-off* entre igualdad y eficiencia, donde los países en que la gente altamente productiva recibe la mayor parte del producto son también aquellos que más producen y son por lo tanto más eficientes, mientras que aquellos mercados con distribuciones de salarios mas equitativas también tienen niveles mayores de desempleo y menos producto total.

Asimismo, el modelo implica que actualmente Chile se asemeja a un modelo más "rígido", y por lo tanto tiene una distribución más desigual de sus ingresos, estando aun lejos de los modelos con mayor movilidad.

Finalmente, el estudio entrega algunas sugerencias de como políticas educativas pueden apuntar a cambiar la distribución de salarios, y sugiere lineas futuras de investigación posibles.

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The following work is a study about wage inequality in Chile. Wage inequality is understood as the difference in total wages earned by the high earners and low earners.

This thesis proposes a directed search model. This models are used to describe markets with frictions, in which markets don't necessarily clear. In particular, directed search models assume that the searchers direct their efforts towards specific markets, characterized by the same posted wage.

The model also considers a distribution of skills, with a law of motion, that under certain conditions converges into a steady state distribution, which is then used to check the job market for each skill level, as well as the wages and output of the economy.

To help calibrate, the data was collected from public government sources, and the model was computed using numeric methods for both the steady states and the wage distribution.

The results point toward the existence of a trade-off between equality and efficiency, where countries where the highly productive are paid the biggest share of output are also the ones that produce more and are thus more efficient, while markets with a more even distribution of wages also have higher levels of unemployment and less total output.

Also, the model implies that currently Chile leans a bit more towards a more "rigid" distribution, and therefore has a higher amount of wage inequality, still far from the more mobile models.

Finally, the study suggests how policy could shape the direction and steady state of the distribution of skills, and suggests further lines of investigation to follow.

A Olga, Carlos y Lula, los extraño.

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Introduction

Income inequality has recently become a prominent topic in the economic and public discourse, highlighted by the recent global crisis, and some empirical findings in the last decade. Wages play an important role in this discussion, as for most individuals it constitutes most of their total income and, in turn, it's a big determinant of their economic fortunes.

Wage stagnation has also been an important trend observed over the last few decades, especially in the United States, and it suggest that the trend of wage inequality might not be reversing quick enough or even at all, with the wage gap actually growing over the last decades in the United States (Shi (2002)).

In the case of Chile, wage inequality is also very high. This country has one of the biggest Gini indexes according to the world bank (18th over 160) and the third highest among the OECD countries. A recent study found that only 5% of the work force earns more than 1.500.000 CLP, while around 70% earn 500.000 or less (Duran and Kremerman (2016)). Other works report similar statistics, suggesting that this country is an interesting case of study.

These global findings have spawned a big amount of empirical work, trying to determine if these trends will remain going forward, representing a "new normal". A problem to tackle this is the lack (for now) of reliable data for periods longer than 20-40 years, specially in less developed countries. This studies also frequently run into a perennial identification problem, as there is a lot of factors that can influence the wages of individuals.

This work attempts to provide a theoretical explanation for chilean wage inequality, by using a search model for the local labor market. This framework could be extended to other countries, or even generalized, although a number of problems arise as data becomes less reliable.

We focused on search models because of the belief that they perform very well in explaining labor markets, specially the directed search models, as one of its biggest weakness (no chance for wage renegotiation) seems less restricting in a stagnated wage world. This models also allow for easy inclusion of heterogeneous agents, which makes it attractive as a source of inequality in both wages and unemployment levels.

At the core of the model is a worker replacement mechanic which is captured by a Markov process. Its function is to provide a steady state distribution of skills in the population, which will be the primary source of wage dispersion in the model. This skills will be represented mostly by the education level. This steady state is highly correlated with the choice of matrix, and the jobs and wages paid are therefore very different for each one. We develop a measure of inequality in the percentage of output paid in wages, to check how much of the actual produce is paid to each group.

Using data from surveys we calibrate this model and see a couple variations for the Markov process to observe how different policies could impact this distribution. As expected, education levels are very important in determining wage inequality, but also in the overall output of the country, offering a trade-off between wage inequality and output, as will be apparent by the end. We also find evidence for aggregate wage inequality (that is, total number of payments to each skill levels) with the current Markov Matrix.

We then turn to see the effects of 2 extreme cases: A rigid matrix, in which there is very low mobility between skill levels, and a random matrix, were there is no correlation between the parents educational level and the children's. The results suggest that we are closer to the random matrix, but also that a lot of potential output is being sacrificed as a result. Also, we find that a particular skill level is disappearing, possibly because of its "stepping step" role.

The rest of this thesis is organized as follows. Chapter 1 does a revision of relevant literature for both search models and wage inequality works.

Chapter 2 specifies the timeline, the model and the equilibrium conditions, which is mostly a specification of a directed search, discrete time model. Chapter 3 deals with data and calibration, focusing on the CASEN survey and the data treatment that was used for the estimates and calibration targets. It also shows that calibration was mostly successful and explains the procedure used to estimate the Markov Matrix used for the steady states.

Chapter 4 deals with the results of the model, as well as alternative matrices to give a context of how it behaves according to different situations. It is followed by a discussion of these results and the relevant implications for policy. Finally, we conclude in the last chapter.

Chapter 1

Literature

This section will go over search models and models with heterogenous agents related to this work also with a focus on wage heterogeneity and empirical studies.

1.1 Search Models in the Labor Market

Search literature first developed in the early seventies, with works by McCall (1970) , Mortensen et al. (1970) and Gronau (1971), although some people consider Stiger (1961) to be the first work related to search models. This models are characterized by an individual searching for a job who seeks to maximize a discounted sum of utilities, and a wage that is yet to be determined. From there, many different characterizations have been developed.

The basic model focuses on a representative worker, who looks for a job on a market, where the probability of finding a job is usually attributed to a random matching function. Once the job is matched, the wage is generally determined by a negotiation, specifically a Nash bargaining process. The solution to this problem is usually a share rule that depends on the workers bargaining power.

Once the wages are found, the equilibrium describes the level of unemployment and wages. The five important parameters are the bargaining power, the discount rate and the interest rate, the cost of a vacancy and the unemployment benefit. There has been a number of empirical works on those values that will be discussed further on the last section.

This workhorse model has been used for many different applications, from job markets to marriage markets, because it's tractable and can be adapted to many fields. It is useful to model markets were frictions are present and difficult to account for otherwise.

1.2 Directed Search

The directed search approach can trace its beginnings to Moen (1997), who worked on a model where firms can announce wages in order to attract skilled workers. His focus was on the social efficiency of the unemployment rate caused by market interactions, and indeed he shows that given a competitive equilibrium (based on a directed effort from the participants), it is possible for the unemployment rate no to be socially inefficient.

Directed search models focus on agents that look in specific submarkets, usually indexed by a posted wage. Central to this models is some sort of ex-ante wage posting, then workers apply to each wage. This of course sets the limiting assumption that wages are non-negotiable. Even if this might seem harsh, there is evidence that such mechanism describe at least some labor markets Rogerson et al. (2005).

This models introduce a component of choice for the agents. They choose wether to look for a job or remain unemployed, according to the wages in the submarket they belong to. This is useful because different characteristics (like education) play an important role in the distribution of submarkets and, in turn, the competitive wage distribution. This makes it easier to work with heterogeneous agents, as we will distinguish them by their different levels of skill.

Directed search models haven seen a rise in popularity in recent years, as it describes some aspects of the labor market in a very intuitive way.

A study by Albrecht et al. (2006) develops a directed search model with multiple applications and see the effect this has on the offered wages. When only one application is allowed per worker, the wage lies between the competitive and monopsony levels, while once the number of application goes over 1, the wages posted are all at the monopsony level, and inefficient, suggesting that single application models present more attractive properties.

This work builds on the initial Moen interpretation, while adding heterogeneity to the skills for each worker, a development he suggested on his work.

1.3 Wage Dispersion

We now turn to works related specifically to wage heterogeneity and dispersion. One of them by Hornstein, Krusell and Violante provided the initial motivation for this work (Hornstein et al. (2011)). This study introduces a measure of frictional wage dispersion (called mean-min wage ratio), that is, a measure of the wage dispersion that's *caused by* a frictional market. They go on to test different models and apply this measure to each.

Unsurprisingly, the standard random matching model doesn't show evidence for wage dispersion. However, once different specifications are tested, frictional wage dispersion starts to show up. One of this specifications is a directed search model. Other include on-the-job search and and sequential auctions among competing employers.

One interesting finding of the study is that the smaller the value of non market time, the larger the cross sectional wage dispersion is attributable to frictions, suggesting a new role for the "Unemployment Benefit" parameter.

Another study by Shi (2002) takes a look at heterogeneous agents skill-biased technology. This is similar to what is considered for this work. The similarities end there though, as in the study there is a symmetric equilibrium were a high-tech and a low-tech firm receive high and low skilled workers, and exclusively low skilled workers respectively. This model generates inequality and wage dispersion even among identical low skilled workers (as some are on the high-tech and others in the low-tech firm). The study is however a more general case and imposes restriction of firm technology.

We take a straightforward approach to wage dispersion, in which wages are the result of heterogeneous productivity on the worker. This productivity will change over the generations until it arrives on a steady state. A difference between a perfect competition model in which workers are paid according to their skill level, a search model allows for an interesting array of comparison, specially in the efficiency implications from different policies. We will see that more rigid distributions can have a significant impact on the rate of unemployment and the steady state output.

1.4 Inequality in Education

There is a great deal of work on the field of education, and the role it plays in the expected jobs and wages a person will be able to opt to. We are particularly interested on the evidence of Education Inequality, seen as the role that a parent's wage play on the expected educational level of her kids.

There is a lot of work in this area, including recent development for the *causes* of this correlation. For a detailed survey, see Black and Devereux (2010). In this work, not only is this correlation shown, but there are also some possible reasonings as to why it develops.

The model we develop explains this correlation as the relationship between skills (which can be signaled by the educational level) and wages.

1.5 Empirical Studies

We review empirical literature regarding 2 main topics: Equilibrium Search model calibration and matching function estimation.

Van den Berg and Ridder (1998) in a rather famous paper calibrated a search model for the Finnish labor market. To do so, they depart from the usual assumption that the wage offer is exogenous, and consider the response of firms to the structure of the reservation wage (that is, the minimum wage at which the worker would accept the job). This separates their model form the partial equilibrium models and allow for a structural one. They estimate and calibrate the model using a Maximum Likelihood estimation and get interesting results that affect especially the younger unemployed, and their response to changes to the minimum wage (because for them the minimum wage is very similar to their reservation wage). They consider different segments that are identical inside group but differ between groups in education among other variables, which will be a feature we will also consider. They do however find that the earnings distribution is of increasing density, which is at odds with the existing evidence.

Before them, Eckstein and Wolpin (1990) tried to estimate a market equilibrium search model but ran into multiple problems, namely, measurement error accounts for almost all of the dispersion on observed wages. Despite that, the paper offers some insight in the way to do policy analysis in such structural models, by checking for changes on the minimal wage.

Regarding the matching function, Petrongolo and Pissarides (2001) made a survey of the most popular and most robust matching functions in the labor market. Their findings include some convincing evidence supporting the concept of the matching function (for instance, estimates of the Beveridge curve at an aggregate level in different countries). They go on to check for microfoundations (finding evidence for the Cobb-Douglas specification as a coordination failure) and also check for empirical findings and conclude that early aggregate studies converge on a Cobb-Douglas matching function, which however has not been thoroughly micro founded, but behaves well enough to be the go-to function in most models.

1.6 Contribution to the Literature

This work hopes to be a meaningful contribution to the developing labor and education inequality literature, seen from the perspective of the labor market and, in particular, from a search model approach, which seems to us as one of the most promising lines of work in the field currently.

Chapter 2

Model

2.1 The environment

We consider a discrete time model, in which there is a continuum of workers with mass 1. These workers have a productivity level from among I different types (which are finite). λ_i represents the share of unemployed workers with i productivity (or skill) level.

These workers can look for a job in different submarkets, each defined by a posted wage w_i, which is offered by a single representative firm, which opens as many vacancies as it needs. Looking for a job is a costly and frictional process, which is represented by a matching process for each market, that depends only on the number of workers looking for a job, and the vacancies open, on each submarket.

Once the workers find a job, they retain it forever, until they exit the market for good. This process is represented by an exit rate δ, where each period there is a δ chance for a worker to retire/stop working forever.

Unemployed workers have the same chance δ to exit the labor market.

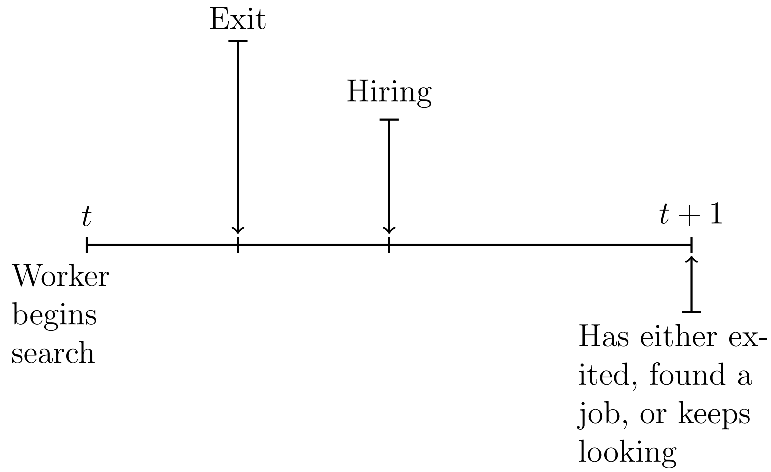
When a person exists the job market, she is replaced by a new entrant (a daughter/son) who enters the unemployed force. The skill level for this new generation is determined by discrete Markov process with probability π_{ij}. The transition matrix for this process is denoted Π.

A firm incurs a cost K to open a vacancy, a an unemployed worker receives a benefit b on each period.

We decided to start with a model with a representative firm, with no heterogeneity on firms. This is important, as the results could be different depending on the total vacancies available to each skill level.

In this model each worker is paid according to her's productivity x_i. Therefore she chooses wether she will look for a job or not in a submarket.

We present a brief timeline. We remark that a "exit" event happens before the "job finding" round:



Whenever a worker exits (which happens in between periods), he continues to get paid for the period he exited on.

2.2 Matching

We define the submarket i tightness as the ratio between unemployment and vacancies for each wage:

$$\theta_i \equiv \frac{u_i}{v} \equiv \frac{\lambda_i u}{v}$$

In practice the amount of vacancies will be variable.

Within each submarket the jobs are assigned according to a matching function $m(u, v)$ homogeneous of degree 1, increasing on both arguments. Without loss of generality, we then define the **vacancy filling rate** as follows:

$$\frac{m(u_i, v_i)}{v_i} = m\left(\frac{u_i}{v_i}, \frac{v_i}{v_i}\right) = m(\theta_i, 1) \equiv q(\theta_i) \tag{2.1}$$

Similarly, we define the **job finding rate** as follows:

$$\frac{m(u_i, v_i)}{u_i} = m\left(\frac{u_i}{u_i}, \frac{v_i}{u_i}\right) = m(1, \theta_i^{-1}) \equiv p(\theta_i) \tag{2.2}$$

We notice that this implies that

$$q(\theta) = \theta p(\theta) \tag{2.3}$$

2.3 Utility

At this point it's necessary to check utilities for each party, starting with the firms.

2.3.1 Firms

Each firm posts a vacancy while looking for a job. Each period the cost of keeping that vacancy open is K. Also each period the firm gets a worker with probability $p(\theta_i)$ and he starts working from the next period onwards.

This determines a dynamic optimization problem. The firm decides wether to stop looking (earning 0) or to keep looking. With this we derive the following stochastic bellman equation, as per Pissarides and Mortensen (1999):

$$\max -K + \mathbb{E}\left(\sum_{t=1}^{\infty} \beta^t (V_{t+1})\right) \tag{2.4}$$

Given the probability of finding a job is a stochastic process, the generalized solution to this problem is too keep looking while this value is bigger than 0, rerodering:

$$V_{it}(\theta_i) = -K + \frac{1}{1+r}\left[q(\theta_i)J_{it+1} + (1 - q(\theta_i))V_{it+1}\right] \tag{2.5}$$

Reordering again we get:

$$V_{it}(\theta_i) + rV_{it}(\theta_i) = -K + \left[q(\theta_i)\left(J_{it+1} - V_{it+1}\right) + V_{t+1}\right] \tag{2.6}$$

$$rV_{it}(\theta_i) = -K + \left[q(\theta_i)\left(J_{it+1} - V_{it+1}\right)\right] + \Delta V \tag{2.7}$$

Now, once the firm has found a worker, it begins earning J, which is again identifiable by a bellman equation. In this case, the worker has a chance to exit for good, but does not leave the firm otherwise.

$$J_i t = y_i - w_i + \frac{1}{1+r}\left[(1-\delta)J_{it+1} + \delta V_{it+1}\right] \tag{2.8}$$

Where w_i is the wage for worker of skill i hired and $y_i = yx_i$ is the production (which we will normalize to 1) multiplied by type i skill . Doing the same as last part we get:

$$rJ_{it}(\theta_i) = y_i - w_i + \delta\left(J_{it+1} - V_{it+1}\right) + \Delta J \tag{2.9}$$

2.3.2 Workers

Workers look at posted wages and begin their search on the specified submarket. Much in the same way as the last section, they choose each period wether to look for a job or to abandon the search. The bellman equation for an unemployed worker is therefore:

$$U_{it}(\theta_i) = b + \frac{(1-\delta)}{1+r}\left[p(\theta_i)W_{it+1} + (1-p(\theta_i))U_{it+1}\right] \tag{2.10}$$

reordering again:

$$U_{it}(\theta_i)(1+r) = b(1+r) + (1-\delta)\left[p(\theta_i)W_{it+1} + (1-p(\theta_i))U_{it+1}\right] \tag{2.11}$$

$$U_{it}(\theta_i)(1+r) = b(1+r) + (1-\delta)p(\theta_i)\left[W_{it+1} - U_{it+1}\right] + (1-\delta)U_{it+1} \tag{2.12}$$

We notice that if the worker exits while looking, he doesn't get any utility. As in the firm's case we get:

$$rU_{it}(\theta_i) = b(1+r) + (1-\delta)p(\theta_i)\left[W_{it+1} - U_{it+1}\right] + \Delta U - \delta U_{it+1} \tag{2.13}$$

Finally, once the worker landed a job, he has it until he exits for good, which makes it's valuation rather simple:

$$W_{it} = \sum_{t=0}^{\infty}(1+r)^{-t}(1-\delta)^t w_i \tag{2.14}$$

Which is a known series and yields the following result:

$$W_{it} = w_i\frac{1+r}{r+\delta} \tag{2.15}$$

2.4 Replacement

For this section we focus on what happens when a period as passed. As previously explained, once a person exits the market (which accounts for both natural deaths or retirement), they get replaced by their descendants, into the unemployed force, and with a skill level that's dependent on a markov process. This event leads to a change in the composition of the distribution of skills amongst unemployed population $\boldsymbol{\lambda}$. We now focus on this change and try to find a law of motion for it.

2.4.1 λ's law of motion

We start with a simple equation for the motion of each λ_i. It moves given an inflow and outflow:

$$u_{t+1}\lambda_{it+1} = I - O \tag{2.16}$$

Notice that we weight by the total number of unemployed on period t. We first focus on inflows.

$$I = \left[(1-p(\theta_i))(1-\delta)\right] u_t\lambda_{it} + \delta\sum_{j=1}^{N}\lambda_j\pi_{ji} \tag{2.17}$$

Here the first term represent all the people who didn't exit **and** didn't find a job, while the second term considers every descendant who replaced an exiting person of skill j into skill i. Next, we consider outflows:

$$O = p(\theta_i)(1-\delta)u_t\lambda_{it} + \delta u_t\lambda_{it} \tag{2.18}$$

The first term are all people who survived and got hired, while the second term is every person of that skill level that exited. With this we have the law of motion for every skill level i:

$$u_{t+1}\lambda_{it+1} = \left[(1-p(\theta_i))(1-\delta)\right] u_t\lambda_{it} + \delta\sum_{j=1}^{N}\lambda_j\pi_{ji} - p(\theta_i)(1-\delta)u_t\lambda_{it} - \delta u_t\lambda_{it} \tag{2.19}$$

Or, in vector notation:

$$u_{t+1}\boldsymbol{\lambda_{t+1}} = \boldsymbol{(1-p(\theta))}\cdot\boldsymbol{\lambda_t}(1-\delta)u_t + \delta\Pi^T\boldsymbol{\lambda_t} - \boldsymbol{p(\theta)}\cdot\boldsymbol{\lambda_t}\delta u_t u_t - \boldsymbol{\lambda_t}\delta u_t \tag{2.20}$$

Reordering we finally get:

$$u_{t+1}\boldsymbol{\lambda_{t+1}} = \left[\delta\Pi^T + u_t\left\{1 - 2\delta - 2(1-\delta)\boldsymbol{p(\theta)}\right\}\right]\boldsymbol{\lambda_t} \tag{2.21}$$

Where $\boldsymbol{p(\theta)}$ is the vector containing the individual job finding rates for each wage(skill level) i, anf Π is the Markov matrix that contains the transition probabilities.

2.4.2 u's law of motion

We now turn to the law of motion for u_t, the total amount of unemployment at time t. In a similar way as last section, we decompose it in a difference of flows:

$$u_{t+1} = I - O \tag{2.22}$$

Now, we begin again with the inflow: how many people enter unemployment in a period:

$$I = \sum_{i=1}^{I} (1 - p(\theta_i))\lambda_{it}(1 - \delta)u_t + \delta \tag{2.23}$$

Or, replacing for a dot product into vector notation:

$$I = \boldsymbol{(1 - p(\theta))} \cdot \boldsymbol{\lambda_t}(1 - \delta)u_t + \delta \tag{2.24}$$

As we know we are dealing with a unit mass of workers, and the total amount if new entrants is just the amount that exited. Now onto the outflows:

$$O = \delta u_t + (1 - \delta)u_t \boldsymbol{p(\theta)} \cdot \boldsymbol{\lambda_t} \tag{2.25}$$

With this we get the law of motion for total unemployment:

$$u_{t+1} = \boldsymbol{(1 - p(\theta))} \cdot \boldsymbol{\lambda_t}(1 - \delta)u_t + \delta - \delta u_t - u_t(1 - \delta)\boldsymbol{p(\theta)} \cdot \boldsymbol{\lambda_t} \tag{2.26}$$

Where again we used the dot notation for a sum of probabilities. This actually yields a law that can be used to make λ_{t+1} only dependent of current aggregate unemployment u_t

2.5 Equilibrium

2.5.1 Conditions

We define an equilibrium as a series of conditions such that there is free entry of firms, and we have a steady state for the distribution of λ, and the aggregate unemployment u. The first condition implies that the value of opening a vacancy must be zero. If not, firms would enter the market (as there is no barriers of entry) until this was no longer the case, dragging the value of a vacancy towards 0. If it was less than zero, no vacancies would open and no workers would be hired. Therefore, it must be the case that the value of a job opening is 0:

$$V_{it} = V_{it+1} = 0 \qquad \forall t = 1, 2... \tag{2.27}$$

On the other hand, the second condition implies that thighnesses and hiring rates (and wages) are constant in time. This in turn implies that the value of a job is constant, as is the value of unemployment:

$$J_{it} = J_{it+1} = J_i \qquad \forall t = 1, 2, ... \tag{2.28}$$

$$U_{it} = U_{it+1} = U_i \qquad \forall t = 1, 2, ... \tag{2.29}$$

With this conditions we can solve for the values of θ in equilibrium, and in turn, the value of wages.

2.5.2 Equilibrium Wages

Using this equlibrium conditions, we now derive the wages and equilibrium thighnesses. Using 2.7 and the free entry condition 2.27, we get the following:

$$0 = -K + [q(\theta_i)\,(J_{it+1} - 0)] + 0 \tag{2.30}$$

Which implies then:

$$J_i = \frac{K}{q(\theta_i)} \tag{2.31}$$

On the other hand using 2.28 and 2.9 we get:

$$J_i(r - \delta) = y_i - w_i \tag{2.32}$$

$$J_i = \frac{y_i - w_i}{r - \delta} \tag{2.33}$$

Finally using both 2.33 and 2.31 we can write the wage for each submarket i:

$$\frac{K}{q(\theta_i)} = \frac{y_i - w_i}{r - \delta} \tag{2.34}$$

$$\frac{w_i}{(r - \delta)} = \frac{y_i}{r - \delta} - \frac{K}{q(\theta_i)} \tag{2.35}$$

$$w_i = y_i - \frac{K(r - \delta)}{q(\theta_i)} \tag{2.36}$$

We notice that wages get lower with higher vacancy cost, discount rate, and vacancy filling rates, while higher exit rates makes them rise. With this wages we can move on into the workers.

2.5.3 Directed Search

With the wages we can now write the following value functions for the worker:

$$W_{it} = \left(y_i - \frac{K(r - \delta)}{q(\theta_i)} \right) \frac{1 + r}{r + \delta} \tag{2.37}$$

Which was obtained by replacing 2.36 into 2.15. Now replacing the stationary condition (2.29) in the value of unemployment (2.13) we get:

$$rU_{it}(\theta_i) = b(1 + r) + (1 - \delta)p(\theta_i)\,[W_{it+1} - U_{it+1}] + \Delta U - \delta U_{it+1} \tag{2.38}$$

$$U_i(\theta_i)(r + \delta + (1 - \delta)p(\theta_i)) = b(1 + r) + (1 - \delta)p(\theta_i)\,[W_{it+1}] \tag{2.39}$$

And replacing the value of a job 2.37 in this we get:

$$U_i(\theta_i)(r+\delta+(1-\delta)p(\theta_i)) = b(1+r) + (1-\delta)p(\theta_i)\left[\frac{y_i(1+r)}{r+\delta} - \frac{K(r-\delta)}{q(\theta_i)}\frac{1+r}{r+\delta}\right] \quad (2.40)$$

Finally, directed search implies that the worker, knowing his skill level, and the posted wage (and number of vacancies), decides wether to enter or not in her specific submarket, and this thus determines the tightness of the market. this translates into optimizing the value of unemployment for a representative worker search by picking θ_i

$$\theta_i^* = \arg\max_{\theta}\left(b + (1-\delta)p(\theta_i)\left[\frac{y_i}{r+\delta} - \frac{K}{q(\theta_i)}\frac{r-\delta}{r+\delta}\right]\right)\left(\frac{1+r}{r+\delta+(1-\delta)p(\theta_i))}\right) \quad (2.41)$$

(We remember that $\theta_i = \lambda_i\theta$)

Which is solved by using a logarithmic transformation and differentiation to find the global maximum. This yields the following FOC[1]:

$$
\begin{aligned}
0 = {} & \frac{\lambda\left(-\delta+1\right)}{\left(\lambda\theta+1\right)^{2}\left(\delta+r+\frac{-\delta+1}{\lambda\theta+1}\right)} \\
& + \frac{1}{b+\frac{1}{\lambda\theta+1}\left(-\delta+1\right)\left(-\frac{K\left(-\delta+r\right)\left(\lambda\theta+1\right)}{\lambda\theta\left(\delta+r\right)}+\frac{y_i}{\delta+r}\right)} \\
& \left(-\frac{\lambda}{\left(\lambda\theta+1\right)^{2}}\left(-\delta+1\right)\left(-\frac{K\left(-\delta+r\right)\left(\lambda\theta+1\right)}{\lambda\theta\left(\delta+r\right)}+\frac{y_i}{\delta+r}\right)+\frac{1}{\lambda\theta+1}\left(-\delta+1\right)\left(-\frac{K\left(-\delta+r\right)}{\theta\left(\delta+r\right)}+\frac{K\left(-\delta+r\right)\left(\lambda\theta+1\right)}{\lambda\theta^{2}\left(\delta+r\right)}\right)\right)
\end{aligned}
$$

Which proved difficult to solve in an analytical form, however replacing the values for fixed parameters it is possible to find a solution depending solely on the productivity y_i and distribution λ_i.

Once the tightnesses have been found, as a function of both parameters (y_i) and endogenous variables (λ_i), we can proceed to find the steady state.

[1]Here we already replaced by the eventual matching function from next section, to make it a bit more legible

2.5.4 Steady States

We return to the laws of motion (2.21) and (2.26), which combined with (2.28) give us the following conditions:

$$u\boldsymbol{\lambda} = \left[\delta\Pi^T + u\left\{1 - 2\delta - 2(1-\delta)\boldsymbol{p}(\boldsymbol{\theta})\right\}\right]\boldsymbol{\lambda} \tag{2.42}$$

and

$$u = (\mathbf{1} - \boldsymbol{p}(\boldsymbol{\theta})) \cdot \boldsymbol{\lambda}(1-\delta)u + \delta - \delta u - (1-\delta)u\boldsymbol{p}(\boldsymbol{\theta}) \cdot \boldsymbol{\lambda} \tag{2.43}$$

Solving for the second equation we have:

$$u = \frac{\delta}{1 - (\mathbf{1} - \boldsymbol{p}(\boldsymbol{\theta})) \cdot \boldsymbol{\lambda}(1-\delta) + \delta + (1-\delta)\boldsymbol{p}(\boldsymbol{\theta}) \cdot \boldsymbol{\lambda}} \tag{2.44}$$

Which we then can replace in the first equation:

$$\left(\frac{\delta}{1 - (\mathbf{1} - \boldsymbol{p}(\boldsymbol{\theta})) \cdot \boldsymbol{\lambda}(1-\delta) + \delta + (1-\delta)\boldsymbol{p}(\boldsymbol{\theta}) \cdot \boldsymbol{\lambda}}\right)\boldsymbol{\lambda} = \tag{2.45}$$

$$\left[\delta\Pi^T + \left(\frac{\delta}{1 - (\mathbf{1} - \boldsymbol{p}(\boldsymbol{\theta})) \cdot \boldsymbol{\lambda}(1-\delta) + \delta + (1-\delta)\boldsymbol{p}(\boldsymbol{\theta}) \cdot \boldsymbol{\lambda}}\right)\left\{1 - 2\delta - 2(1-\delta)\boldsymbol{p}(\boldsymbol{\theta})\right\}\right]\boldsymbol{\lambda} \tag{2.46}$$

Which is a vector equation. To solve it, we go back to (2.41), noticing that the equilibrium depends solely on λ_i. Replacing then gets us a system of equation with 4 equations and 4 unknowns. However, this system is non-linear. This means we have to once again use a numeric solution. Adding an additional condition (that the shares add up to one), this system can be solved an return the steady states for $\boldsymbol{\lambda}$ an u, and thus solving the model for a given set of parameters. We now turn to finding these parameters.

Chapter 3

Data and Calibration

To calibrate we need 3 things: a matching function, a set of parameters and the probabilities of the Markov process. We also need data to contrast the final values the model outputs.

3.1 Matching Function

The go-to function here is usually a Cobb-Douglas representation, like this:

$$m(U,V) = U^{\alpha}V^{(1-\alpha)} \tag{3.1}$$

Which mostly fits the data well. However, this representation has a couple of problems:

1. It adds an additional parameter to calibrate, increasing the degrees of freedom and thus making calibration harder.
2. The resulting conditions generated are difficult to solve analytically (and numerically).

Because of this, alternatives were considered. After a short review the selected function was the so called 'Telegraphic' function:

$$m(U,V) = \frac{UV}{U+V} \tag{3.2}$$

This function covers all requisites for a well behaved matching function, and unlike other choices, it doesn't add more parameters to calibrate. Once the function has been chosen, we can find the tightness representations:

$$\frac{m(U,V)}{U} = \frac{V}{U+V} \cdot \frac{\frac{1}{V}}{\frac{1}{V}} = \frac{1}{\theta+1} = p(\theta) \tag{3.3}$$

Conversely, we have:

$$q(\theta) = \theta \cdot p(\theta) = \frac{\theta}{1+\theta} \tag{3.4}$$

Having chosen a function, we now need some data.

3.2 Data

We used the CASEN database for the year 2015, which is the most important socio-economical survey in Chile, and is in charge of the Social Development Department (MIDEPLAN). The 2015 version was applied to 83,887 homesteads, distributed among all the regions, for a total of 266,968 individuals. The data is available to the general public[1].

3.2.1 Data Treatment

This survey is extensive, it has over 700 variables, so we had to treat it to make it more approachable. We kept variables regarding wage, parents education, and current educational level. For the purposes of this work, we kept only non empty data for each of this fields, and set the minimum age at 25, as most people younger would not have had the chance to finish their education. Finally, for normalization purpose, we cut the upper and lower half of the wages to leave 95% of the sample, for wages to keep reasonable numbers. The STATA code can be found in the annex.

It is important to remark that the cut was skewed mostly towards leaving the higher wages in the data, and instead remove most of the zero values that are less prevalent in the model.

3.3 Parameters

For parameters we chose a couple based on direct observation and the rest we let free to help calibrate.

3.3.1 Firm Productivity

We normalized the productivity of our single, homogeneous firm, to a value of 1. This is mostly to normalize the different values as well as fixing a level to calibrate. If there were more than one firm on a subsequent model, the principal change would be to estimate the relative productivity between those firms, by normalizing one of them to a base value, for instance, 1.

3.3.2 Exit Rate

The exit rate considers both the separation rate for jobs and the natural death rate of the country. Using data from the INE (National Institute for Statistics), we calculate the death

[1] `http://observatorio.ministeriodesarrollosocial.gob.cl/casen-multidimensional/casen/docs/casen_2015_stata.rar`

rate to be around 5 deaths per 1000 habitants per year, which translates into an annual rate of 0.005. However, the exit rate also must account for people who "leave" the workforce once the resign (there is no exogenous separation rate besides this rate, which removes the worker from the workforce permanently). This parameter was left free then, to use for calibration, but with a lower bound set at 0.5%

3.3.3 Interest Rate

The interest rate is relatively straightforward, we use the overnight interest rate of the central bank for the last 5 years, at 4% or 0.04.

3.3.4 Unemployment Benefit

The vacancy cost is estimated using data from the "Seguro de Cesantia" or Unemployment Insurance. They report up to 40% reposition rate, but with a mean of 10%. We consider the minimum wage in Chile (250.000 CLP) and get a rough estimate of around 0.015 (maximum wage/output is set at 1).

3.3.5 Vacancy Cost

The vacancy cost was left free to help calibrate.

3.4 Markov Chain

The Markov process represents one of the key concepts of this model. It's interpretation revolves around the idea of how skill levels relate to the skill level of your parents. Specifically, a High Skilled parent will be replaced in the unemployed population and that replacement could be either one of no mobility (Which would be represented by an Identity Markov Chain), equalitarian (Which would be represented by a Markov Chain with equal probability in each transition), or some combination of the two. The Markov Chain elected has strong repercussions in both the wages and the levels of unemployment, as will be shown, and a more equalitarian Chain favors medium income workers and their wages (and gets a more equalitarian steady state distribution), while a more rigid one favors the high skilled, high wage workers and low skilled ones (with a more unequal steady state distribution). Given its importance, the way to estimate it should be carefully considered. To get the estimates we consider an approximation for skill, educational level. This assumes there is some level of correlation between the education and skill level of people. We now need a way to estimate the level of education based on the parents of a given individual. We use the database for the CASEN survey, and a multinomial logit model.

$$logit(\mathbb{E}[Education_i|p,m]) = logit(p_i) = ln\left(\frac{p_i}{1-p_i}\right) = \beta_1 p + \beta_2 m \tag{3.5}$$

With this we can recover the margins for each coefficient and get an estimate of the probability of reaching certain degree of education given a parents educational level. It is important to point out that there is no big difference between the parents in terms of the absolute margin, as shown in the following figure (the rest can be found in the annex):

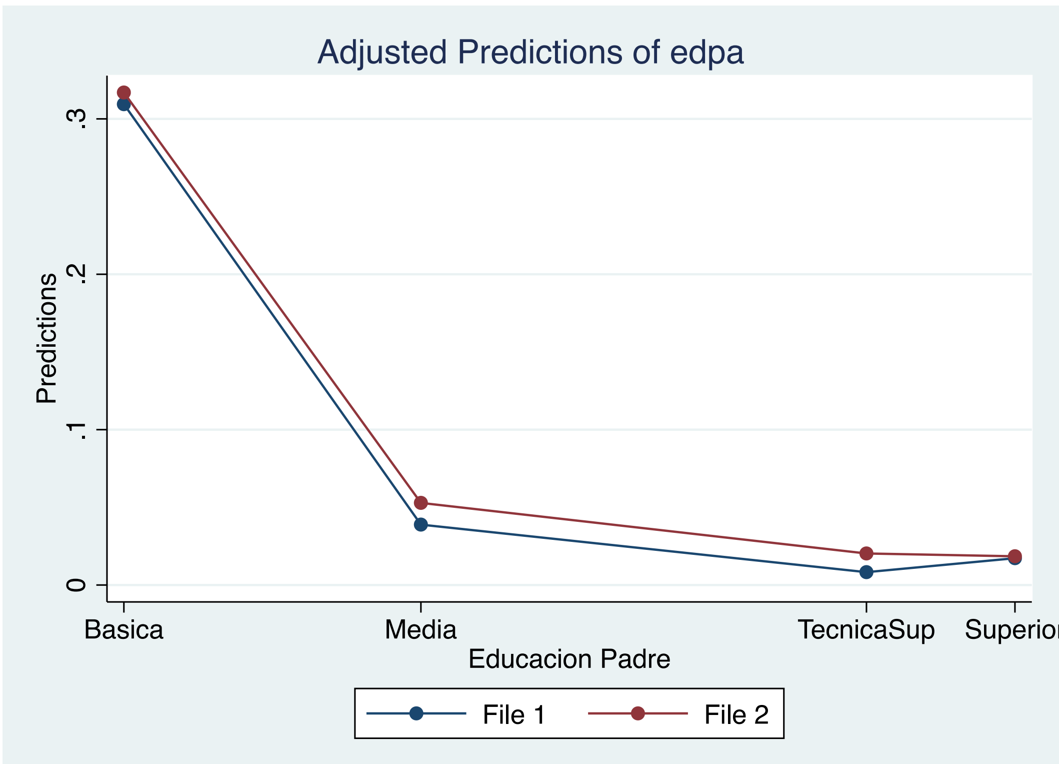


Figure 3.1: Margins for both parents.

3.5 Calibration

Finally for the calibration we used a moments approach. To do this, the wages for our database had to be normalized, which was done using the highest wage (which was 1500000 pesos), and checked for the first two moments. These yielded:

	value
$\mathbb{E}(w_i)$	0.301
$\mathbb{V}(w_i)$	0.045
$sd(w_i)$	0.213

Table 3.1: Estimated moments.

And the following histogram:

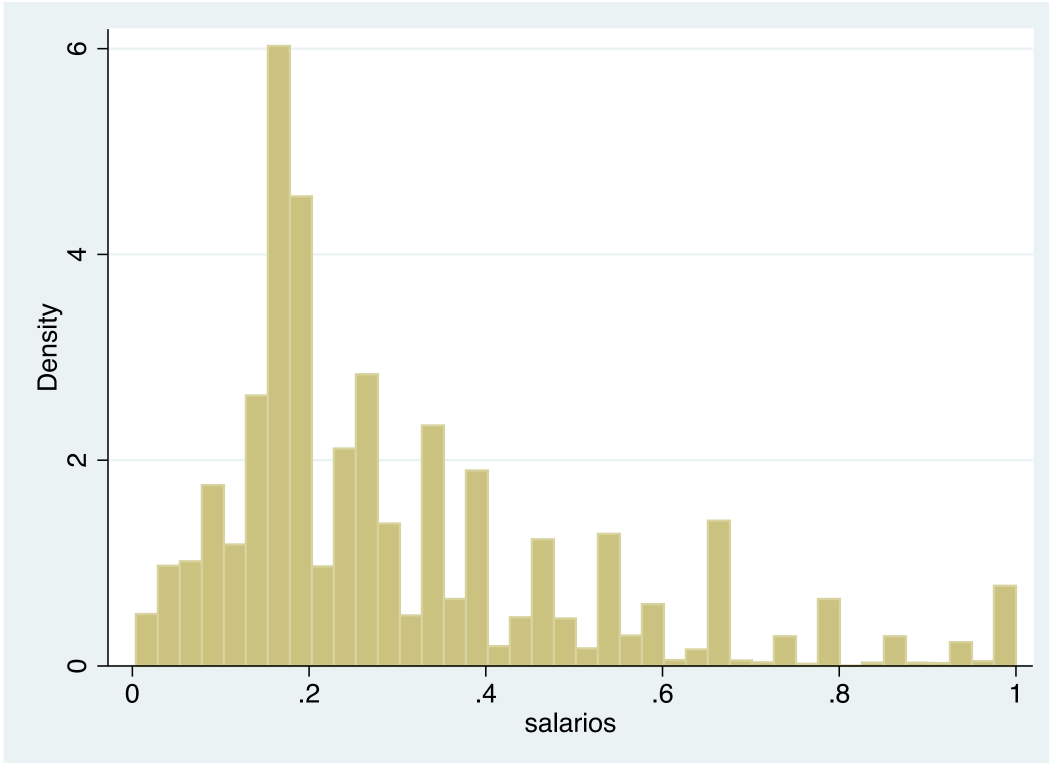


Figure 3.2: Histogram of wages.

We aim to reproduce these values with the model, getting the following results for the generated wage distribution

	value
$\mathbb{E}(w_i)$	0.334
$\mathbb{V}(w_i)$	0.0498
$sd(w_i)$	0.2231

Table 3.2: Estimated moments.

Which is achieved with $k = 0.3$ and $\delta = 0.3$. This values look very similar to the empirical ones, so we will use them for the rest.

Finally we distributed productivity levels equally from 0 to 1, getting the following distribution (which, given that we set output to be 1 at max, completely determines the productivity)

	S	T	M	B
X	1	0.75	0.5	0.25

Table 3.3: Distribution of Productivities per Skill level.

Chapter 4

Results

4.1 Markov Chain

After calibration, the results were found using a *Python* program. The program needs the paramters and outputs both the steady state distribution, as well as the resulting tightnesses, jobs, among others. The main input is the Markov Chain, which we calibrated as described on the previous section. The resulting chain is:

Son's Education

Father Education	S	T	M	B
S	0.69	0.13	0.17	0.01
T	0.55	0.22	0.21	0.02
M	0.28	0.16	0.50	0.06
B	0.08	0.08	0.51	0.33

Table 4.1: Estimated Markov Chain.

We can see a couple of points here. First, Superior Education is the most absorbing state, while both "Basica" and "Tecnica" play more of a "transitional" role. This means that it is highly possible for a kid who was raised by a low educational parent to move on to achieve a Media level of education, or from a kid whose parents have technical degrees to pursue a professional degree.

It is important to remark that "Media" education became mandatory since 2003, which explains the high probability of "landing" there.

Higher education also has a high "arrival" rate, and this has to do with the wide offer of credit for university admission.

We further want to discuss a couple of metrics from this Chain. We consider the following diagram for the steady states:

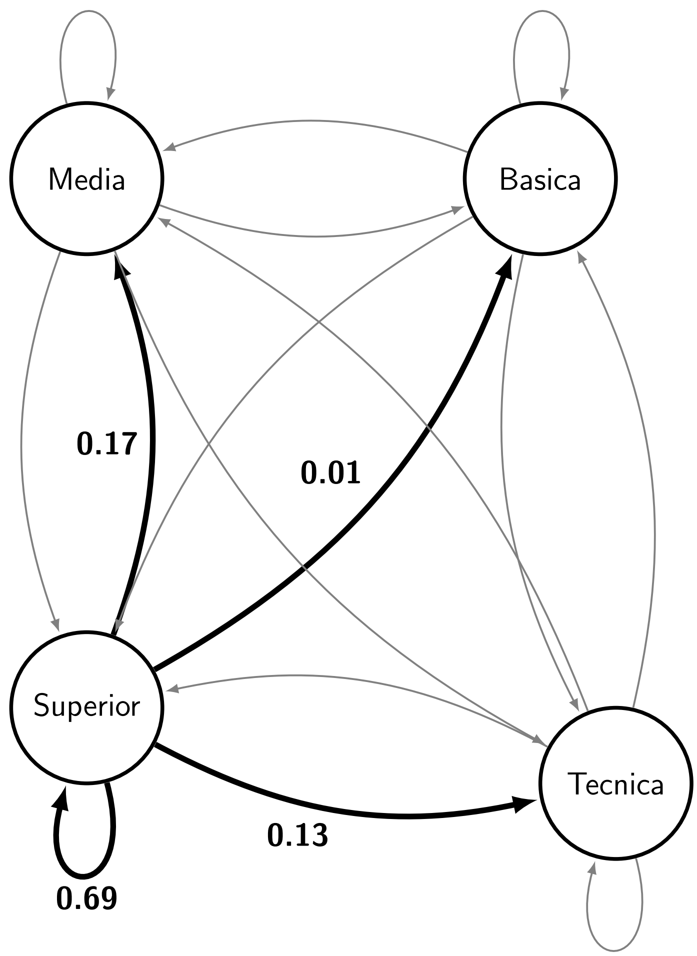


Figure 4.1: Markov Chain Representation.

Figure (4.1), for clarity, emphasizes only one of the states with its output (the different probabilities for her kids to achieve up to each degree of education). We can see a high degree of persistence here. Actually "Superior Rate" is the most "absorbing" state, and if we were to calculate the markov chain steady state, we would get the following values:

	S	T	M	B
SS	0.53	0.15	0.28	0.03

Table 4.2: Markov Steady State.

However, we argue that the calibrated markov chain is a good measure for steady state probabilities, because they were calculated on a 15 year time frame.

Moreover, the education process is not the only effect in the dynamic system. We also consider the actual effects from the directed search model.

4.2 Directed Search

For directed search models, the number of vacancies changes along the number of people who decide to look for a job. In this thesis, this dynamic plays a central role in the way the distribution λ evolves towards a steady state. There are 2 principal elements that affect the steady state, as can be seen in 2.21, one is the markov chain, discussed above, the second one is the probability of finding a job, which depends on those vacancies. A higher number of vacancies means its harder to find a job, so it's more difficult to exit from a given state. The number of vacancies that the firm opens is mostly dependent on its expected value and, in particular, the productivity asociated with the wage it posts. This implies that the productivity *multiplier*. For this work we assumed the following multipliers:

	S	T	M	B
x	1	0.75	0.5	0.25

Table 4.3: Productivity Multipliers

This is a crude approach, and for the purposes of calibration works well, but has a series of implications in the vacancies, which will lead to very skewed results, that we will see next:

4.3 Steady State

We use the markov chain and compute the steady state values for λ, which we display below:

	S	T	M	B
λ	0.062	0.098	0.029	0.810

Table 4.4: Estimated Steady State λ.

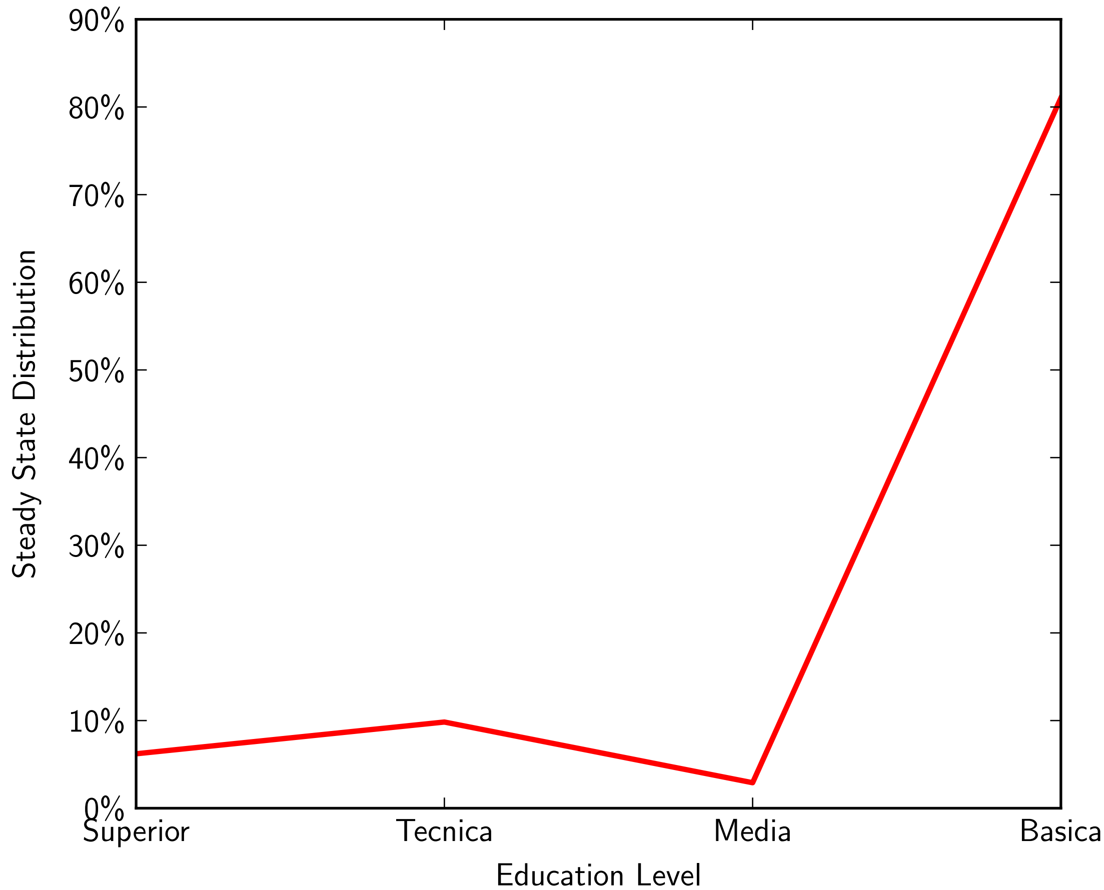


Figure 4.2: Percentage of total output by educational level.

As shown, the steady state implies that given the probabilities of transition, the death rate and cost of vacancy and benefits of unemployment, in the long run (assuming agents optimize) the distribution leans heavily towards "Basica" levels of education (which are mandatory) To shed a bit more light to this, lets see each market.

4.4 Tightness

Using this distribution, we proceed to check the tightnesses for each submarket. Recalling that there is an actual "aimed" tightness and a realized one when the distribution is supplied, we find the following results:

	S	T	M	B
θ_i	1.090	0.799	3.335	0.173

Table 4.5: Estimated Sectorial θ.

However, this is just the "aimed" tightness. This is because the optimization problem solved by the agent only lets him control θ, while λ is already fixed. The actual "realized" tightness is this:

	S	T	M	B
$\theta_i \lambda^{ss}$	0.067	0.078	0.097	0.140

Table 4.6: Estimated Sectorial θ.

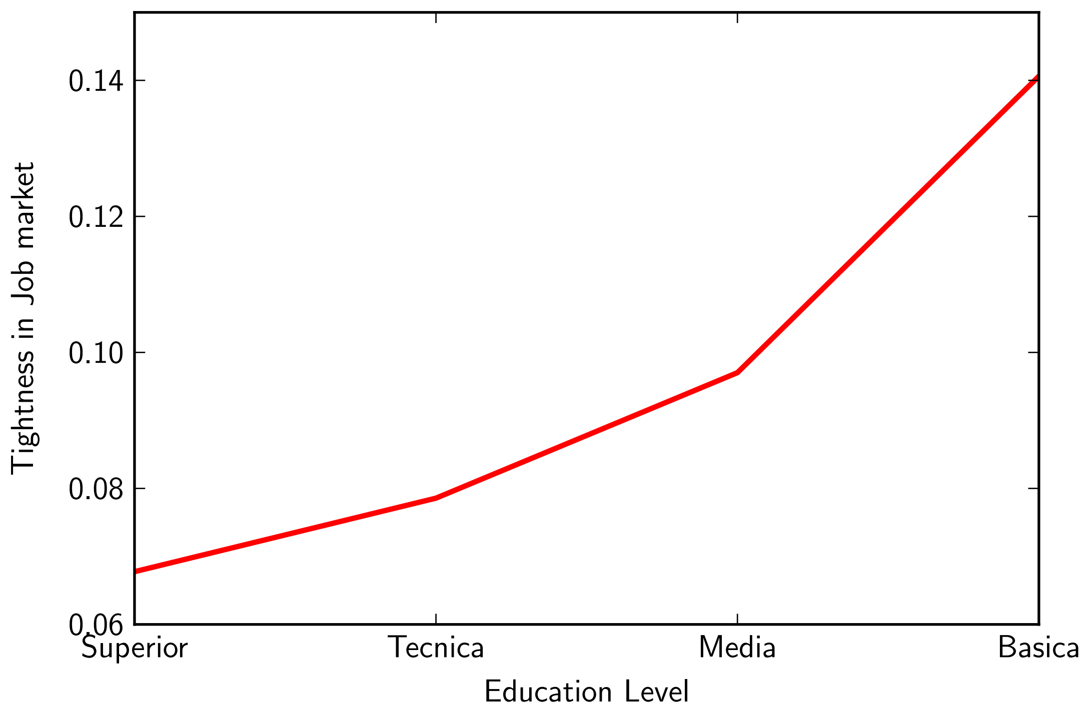


Figure 4.3: Tightness by sector.

We see that tightness is decreasing in education, recalling that $\frac{dp(\theta)}{d\theta}$ is increasing in θ, this means that the lower the educational level, the easier it is to get a job. In this case we have a lot of people in the low skilled market, which amounts to a low unemployment rate of only 1.5 %.

Considering that one of the ways to leave the low skilled unemployment sector is by getting a job, these tightnesses imply a hard to leave unemployed market, and by checking we see a low ammount of vacancies in that market, 0.11, the lowest in all markets.

4.5 Jobs

Once tightnesses have been calculated it's relatively straightforward to compute the jobs and vacancies.

We recall that there is no population growth, therefore the unit mass of workers remains constant, i.e. $u+e=1$. Moreover, the distribution of skills on the population as a whole *only* depends on the markov process that describes it, thus, if we call the steady state distribution of the whole population S

$$(1+\delta)S_t + \delta S_t \Pi = S_{t+1}$$

It can be shown that the steady state for this equation is the same as the steady state for the markov chain. Using the fact that $\boldsymbol{\lambda} + \boldsymbol{e} = \boldsymbol{S}$ (vector equation) on the steady state, we compute the jobs, which are:

	S	T	M	B
Jobs	0.52	0.14	0.27	0.03

Table 4.7: Jobs by sector.

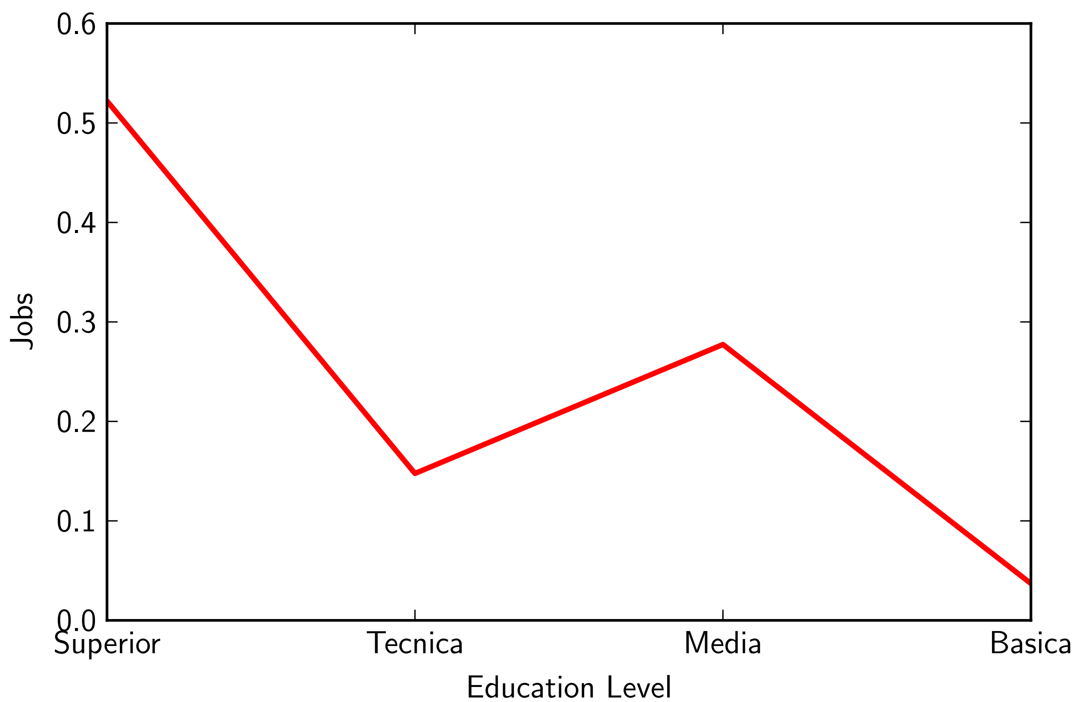


Figure 4.4: Jobs by sector.

With this we see the outlook of the labor market. It is focused mostly on the "Basica" educational sector, which could be predicted by both it high tightness and high λ. Recalling that the unit mass of workers is normalized to 1, nearly 98.5% of people have a job, mostly in the "Superior" productivity sector. This sector also presents a high tightness, so we will also check for the vacancies, as the distribution here is different to the one for tightness. We also note an increase in both "Media" and "Superior" education compared to "Tecnica".

	S	T	M	B
V	0.237	0.204	0.165	0.114

Table 4.8: Vacancies by sector.

We see a rather different outlook here. Even though people are concentrated in the "Basica" skill market, the high skilled sector presents a large group of vacancies. This suggests that firms want more high skilled workers, however they are not getting them, as people concentrated more around the low productivity.

4.6 Wages

Computing the wages we get:

	S	T	M	B
w_i	0.995	0.745	0.496	0.247

Table 4.9: Wages by sector.

Which means wages are very close to the actual output produced, as shown in the next table.

	S	T	M	B
w_i	99.52 %	99.45%	99.32%	99.02%

Table 4.10: Wages by sector.

However, we are interested in actual wages paid, so we need to weight these wages by the amount of actual jobs. Once we do we get the "paid" wages.

	S	T	M	B
w_iJ	0.51	0.11	0.13	0.01

Table 4.11: Actual Wages by sector.

Here it's interesting to check the amount paid to each educational level. We can check that even though there is more people working on the low skilled "media" market, the total amount paid is similar that for those with high productivity.

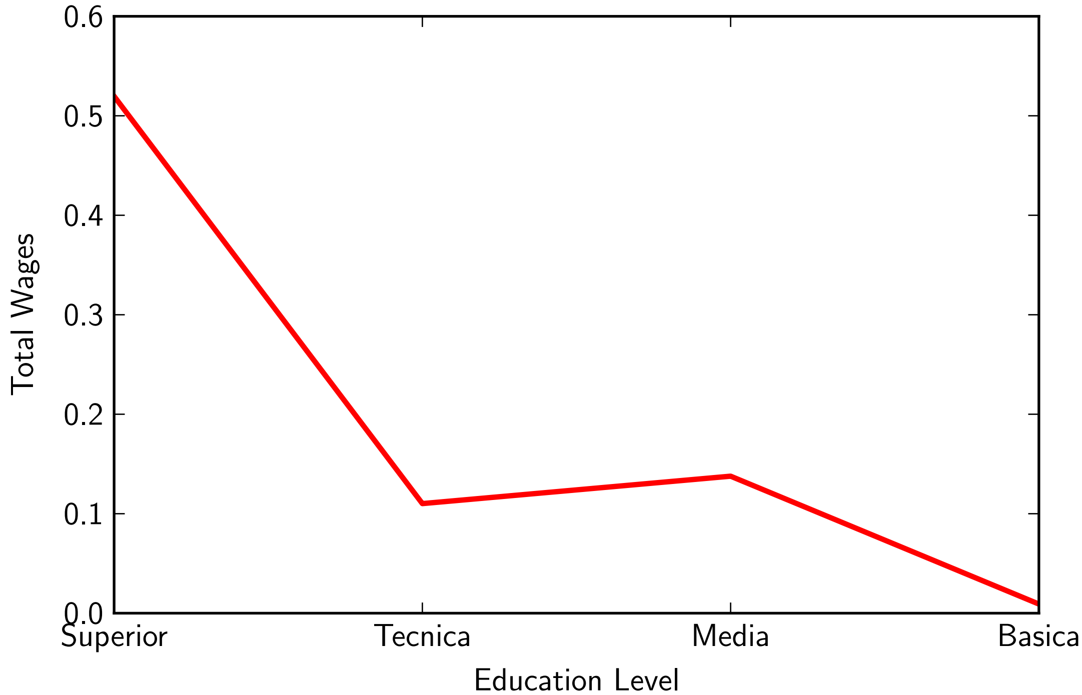


Figure 4.5: Total Wages Paid.

This graph is somewhat similar to the histogram of wages presented in 3.2. Of course, given that the model only has 4 productivities, the adjustment is a bit awkward, however, given the calibration, it is still a very close representation of the observed data.

Once we got the wages, however, we are interested in a different metric: How much of the actual output do these wages represent? To do this we need the total output, which we get by multiplying the wages by the jobs as if we had equal amounts of each firm. With this we get the total output level: 0.780.

We use this value and check the paid wages (that is, wages weighted by the actual amount of people working for that wage) as a fraction of actual output (table 4.12).

	S	T	M	B
Percentage	66.56 %	14.11 %	13.72 %	1.17 %

Table 4.12: Percentage of Output by sector.

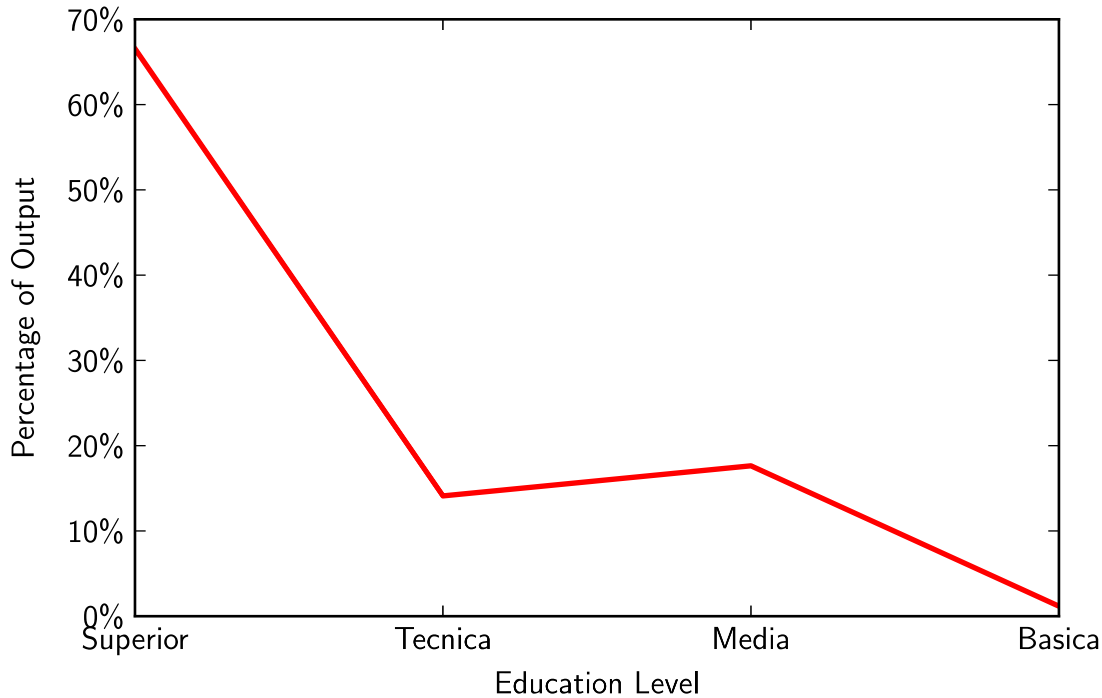


Figure 4.6: Percentage of Output.

Which means high skilled workers get paid a in total output more than any other productivity levels save for low skilled jobs, which account for about an 80% of the labor market and get around 60% of total wages, while the high skilled sector (and the "Tecnica" level) only represent a 5% of the market and each earns around 16% of the output. This suggest an important level of wage inequality.

Also it is important to remember that most of the unemployment is located in the "Basica" level.

We now check additional cases to compare the model to, and also as ground work for comparative statistics.

4.7 Other Chains

For discussion purposes, we will calculate results for 3 additional Markov chains: The same chain with no productivity spread. For a low-mobility chain, with more spread in productivity and a fully random one, with a small productivity spread. Additional tables and graphs can be found in the annex.

4.7.1 Less productivity Spread

Results for this case are found in the Annex, however the highlight is that the steady state lambda is much less skewed:

	S	T	M	B
λ	0.03	0.35	0.02	0.57

Table 4.13: Estimated Steady State λ.

Which translates into a higher productivity, and very different wage to output ratio, that looks a lot like the λ^{ss} distribution.

This suggest that a policy that would increase the productivity for low skilled workers should improve the distribution of income relative to population size, which could therefore be a focus to decrease inequality.

We now turn into the more extreme cases for comparison purposes.

4.7.2 Semi-Rigid Distribution

To represent this we use a low mobility Markov Chain, with a high spread of productivity (1,0.6,0.3,0.1) and the results:

	S	T	M	B
S	0.8	0.2	0	0
T	0.1	0.8	0.1	0
M	0	0.1	0.8	0.1
B	0	0	0.2	0.8

Table 4.14: Estimated Markov Chain.

We check then the more important statistics, shown below:

	S	T	M	B
λ	.22	0.19	0.17	0.40

Table 4.15: Estimated Steady State λ.

We can see that this steady state is much more skewed towards extreme cases, which makes sense given the low mobility. Lets check the jobs

	S	T	M	B
Jobs	0.16	0.32	0.32	0.16

Table 4.16: Jobs by sector.

Unemployment in this situation is 1.4 %, with a mostly even distribution of unemployment across sectors. Checking now for the wages as a percentage of production:

	S	T	M	B
w_i/x	99.52 %	99.38%	99.11%	98.45%

Table 4.17: Wages as % of productivity by sector.

These didn't change, as the distribution doesn't depend on the jobs, instead it depends on the "realized" tightness, which comes from the optimization problem that doesn't depend on λ.

Checking also for total wages:

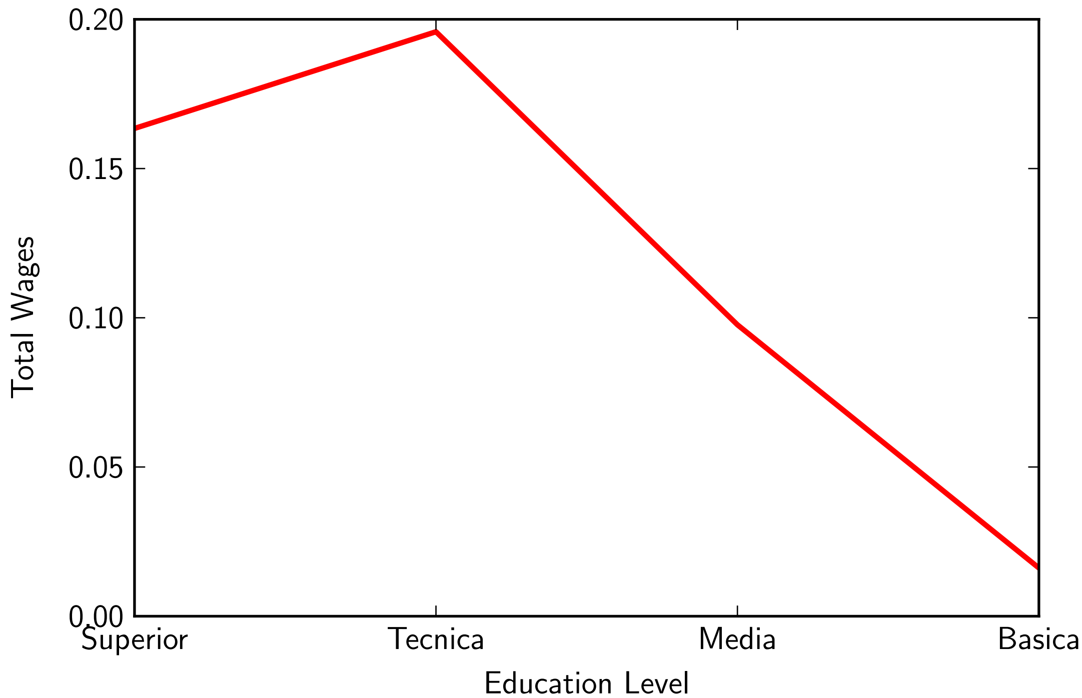


Figure 4.7: Total Wages Paid.

And finally checking for wages as a percentage of output (which is 0.425):

	S	T	M	B
w_i/Y	34.32 %	41.12 %	20.5 %	3.3 % %

Table 4.18: Wages as % of total output by sector.

We see a shift from "Superior" education towards "Tecnica" in the amount of wages, this mostly has to do with the markov chain's steady state, and productivity multipliers, that make the "Tecnica" workers more numerous, and the pay not much lower than the higher skilled jobs.

	value
$\mathbb{E}(w_i)$	0.429
$\mathbb{V}(w_i)$	0.066
$sd(w_i)$	0.256

Table 4.19: Estimated moments for Semi-rigid.

As can be seen, the overall expected wage is higher, however so is the variance, which can be interpreted as a more unequal distribution.

Also, the output is slightly higher, while the unemployment is slightly lower.

4.7.3 Random Chain

For the last we consider a random walk process, which once made discrete gets a matrix with identical values for each transition, with a low productivity spread (1,0.9,0.8,0.7):

	S	T	M	B
S	0.25	0.25	0.25	0.25
T	0.25	0.25	0.25	0.25
M	0.25	0.25	0.25	0.25
B	0.25	0.25	0.25	0.25

Table 4.20: Estimated Random Markov Chain.

We check then the more important statistics, shown below:

	S	T	M	B
λ	0.007	0.007	0.008	0.994

Table 4.21: Estimated Steady State λ.

As seen, there is a high concentration at the low skill level.

	S	T	M	B
Jobs	0.24	0.24	0.24	0.24

Table 4.22: Jobs by sector.

Unemployment in this situation is 1.6 %, with most of it coming from "Basica" skilled unemployed. This is slightly higher than the more rigid models, and the calibrated one.

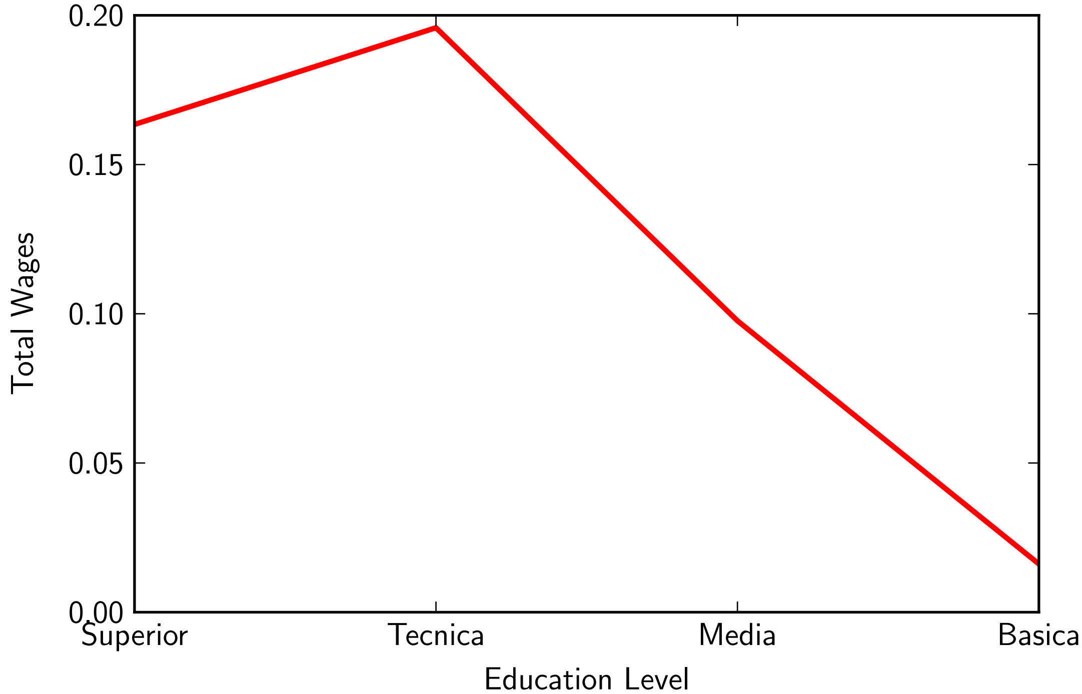


Figure 4.8: Total Wages Paid.

checking for wages as a percentage of output (which is 0.83, but since the overall productivity is higher, it doesn't mean much):

	S	T	M	B
w_i/Y	29.27 %	26.33%	24.59 %	24.59 % %

Table 4.23: Wages as % of total output by sector.

Here we see a much more homogeneous wage distribution. This is impacted both by the markov chain and the lower productivity spread, suggesting the 2 aspects that could be a focus for policy recommendations.

	value
$\mathbb{E}(w_i)$	0.700
$\mathbb{V}(w_i)$	0.693
$sd(w_i)$	0.832

Table 4.24: Estimated moments for random.

Which again means less as the overall productivity is higher, so we focus instead on the wages paid as fraction of total output, which was shown to be skewed towards the more numerous workforce.

4.8 Discussion

Checking for all the results, there are a number of findings to discuss here.

First of all, the model presents a decent amount of wage inequality, as the amount paid to the high skilled workers (who represent only about 6% of the working force) is very big considering the amount paid to the low skilled types.

Second, the model is more similar to the semi-rigid model, which has less mobility, but higher output and lower unemployment.

Third, the model presents a disappearing 'Media' skill level among unemployed people, which is caused by both the estimated Markov Chain, and the distribution of productivity used, which makes it easier to get out of that state.

Fourth, conversely, the rigid chain gets a more even steady state distribution, but at the price of a higher share of output paid in wages to a smaller amount of people. This distribution also has a lower unemployment rate, and a higher total output, but exhibits the highest level of inequality, compared to the random distribution and the calibrated model.

Fifth, given the parameters, it can be seen that more rigid Markov processes make for a more 'equal' distribution of skills in the steady state, acting like a 'brake' against the tendency towards a wages distribution heavily focused on the the highest skill level, because its harder for those in that skill level to find a job, and therefore moving out of there is difficult when compared to higher productivity wages.

Interestingly, in the model, higher educational values have more 'absorbing' probabilities, which explains the higher steady state distribution when compared to a random distribution model. If this became even more absorbing (for instance increasing the transition from 'Superior' to superior by 10%) we could have a higher number of steady state 'Superior' workers, a higher productivity, and an increased level of wage inequality.

Moreover, more rigid Markov Chains present lower levels of unemployment, which is also more evenly distributed, while random Markov processes tend to focus unemployment on the lower skills level.

All of this suggest the existence of a trade-off in this model, between equality and efficiency, which will be further discussed further in the concluding remarks.

It is also important to remark that this model is a single, homogeneous firm, and some of the implications could very well change given a high level of productivity spread between firms, offering more (or less) vacancies in different submarkets, and therefore altering the unemployed and employed distributions.

0.

Conclusion

In this study we used a steady state and directed search approach to the question of wage inequality. We show that under our assumptions there is a steady state for the distribution of skills that leads to a more unequal wage distribution, when compared to an economy were the education of your parents has no influence on your own. While it's true that no such country exists today, it suggests that the more 'mobile' the education levels are, the wages of the population are more equally distributed among the working force, however, it presents higher levels of unemployment, which is also more focused on the low skilled level, which concentrate almost all of it. Also, productivity plays an important role here, as it translates to more inequality the higher the spread of productivities is.

On the other hand, we noticed a trade-off between efficiency and equality, understood as the difference between total produce and the way it is shared between each worker. More rigid societies could be perceived as more 'fair', because the ones that receive the biggest share of output are also those who contribute more to aggregate output. In this case there is also less unemployment, which is more evenly distributed, and a higher total output.

The way to move from one kind of distribution to the other is mainly through education policy. In this sense, it's important to see the effect that individual policies could have on the relation between the parents educational level and their sons. With this model, the more correlated these two are, the more rigid the resulting distribution will be. Another crucial role is played by educational policies that aim to improve the productivity of low levels of education, so as to diminish the productivity spread.

The policy implications should take into account the stated trade-off, as well as the long term objectives for the policy makers. An efficiency-equality trade-off has been observed in many areas, from economic matters to operational management, and it remains a very contingent topic, which should be discussed at large and with as much evidence as possible. This study provides another piece of information to consider.

Most countries currently strive for equal opportunities for education for their citizens. Under our model, and if we assume innate talent to be essentially uniformly distributed, this could eventually lead to a more mobile society. If this were the case, policies would lead to a more uniform wage distribution, but also a less efficient society with more unemployment, which would also be more concentrated on the lower skill levels.

In Chile in particular, recent legislation has made "Media" levels of education mandatory. This would imply the disappearance of the "Basica" skill level, but the obtained results suggests that this would not change the core trade-off observed in this study.

As more and more data becomes available, this theoretic results can be tested, for instance checking how the probabilities calculated here change with educational policies, then checking the effect on wage inequality. The relation between education and wage inequality (and indeed income inequality) has been studied from many angles, and this could be a further line of inquiry into this matter.

Future lines of investigation could focus more on the dynamics (around the steady states) or a more specific search model, including for instance endogenous job destruction, or allowing for on-the-job search. Other models could include a different specification for unemployment benefit (as a fraction of wage income, for instance), or a changing death rate. Also there is the strong assumption of no wage increase once it was accepted (i.e. no renegotiation or raises). Simpler specifications could also be useful to dig deeper in the partial effects of each parameter.

Finally, search theory and directed search models are still developing fields, with plenty of interesting lines of investigation still growing. It is entirely possible that further works and studies open the door for a more thorough specification, or a more realistic structural approach. It is the hope of the author that this study will be useful as a stepping stone for further lines of work on this topic.

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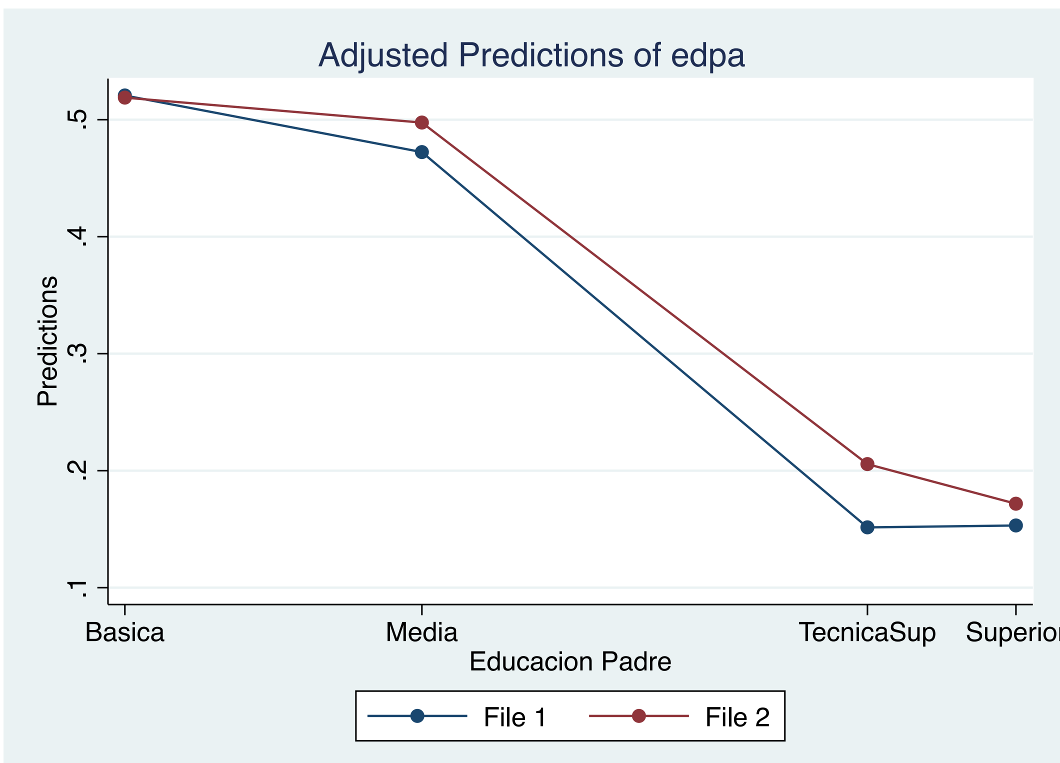
Annex

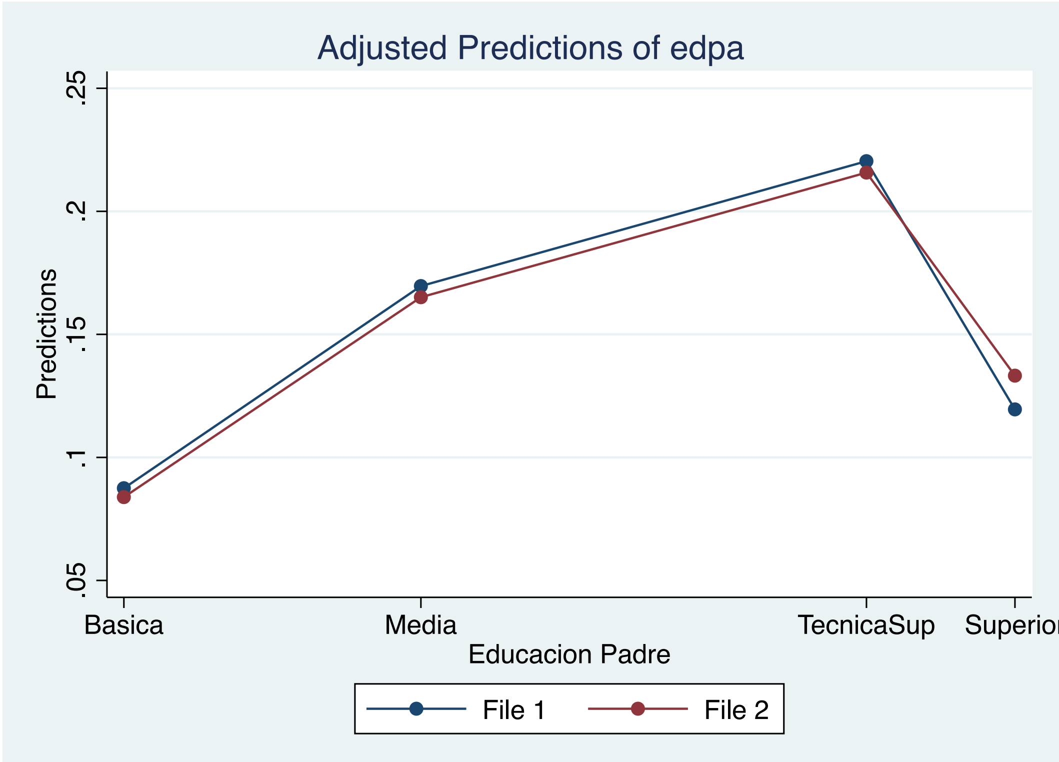
Here we include additional graphs and tables, plus the code which was used in this work.

Graphs

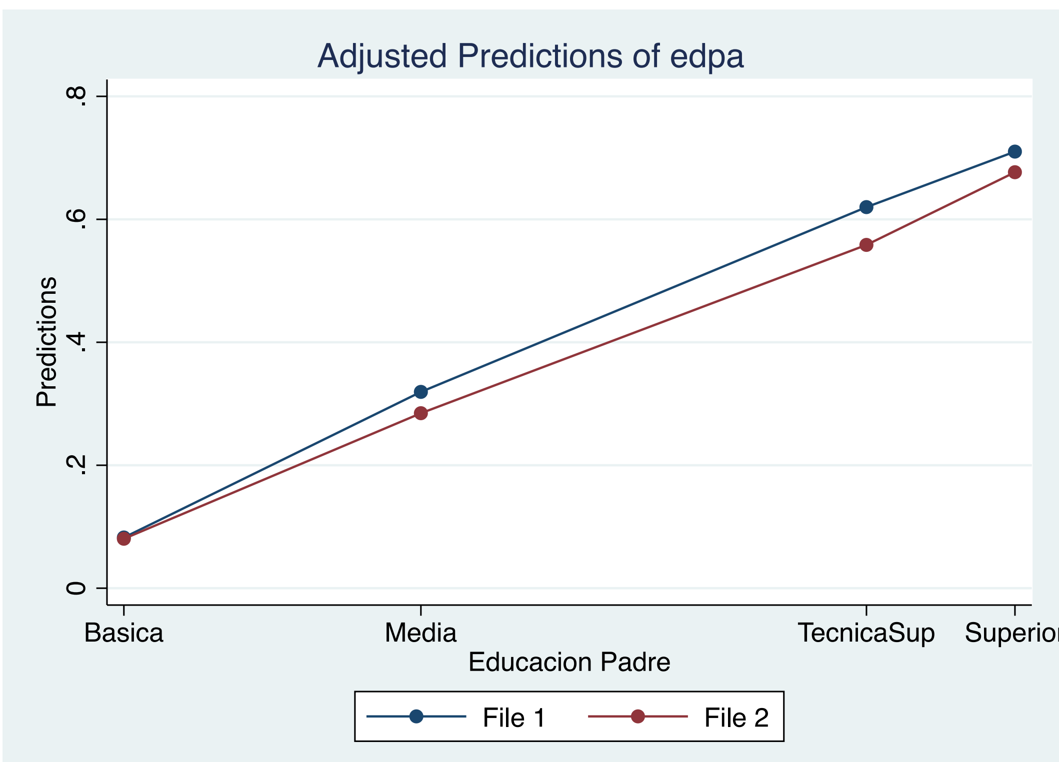
Comparing the margins for both parents



Margins for both parents for ED=Media.

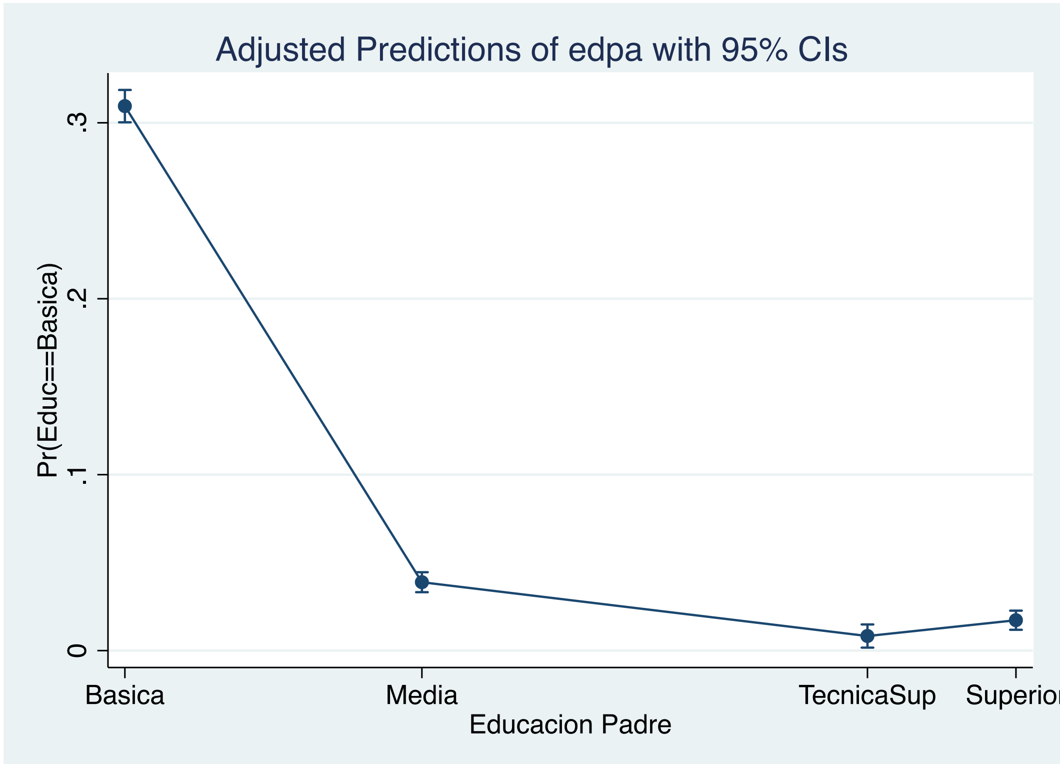


Margins for both parents for ED=Tecnica.



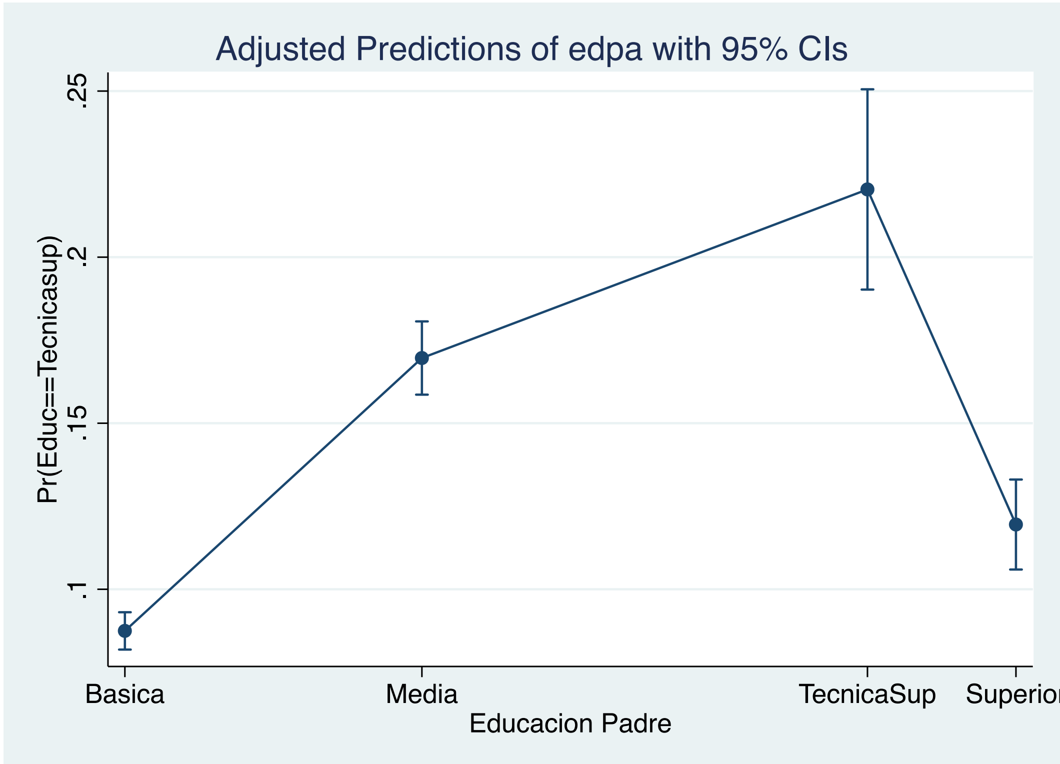
Margins for both parents for ED=Superior.

Margins for father

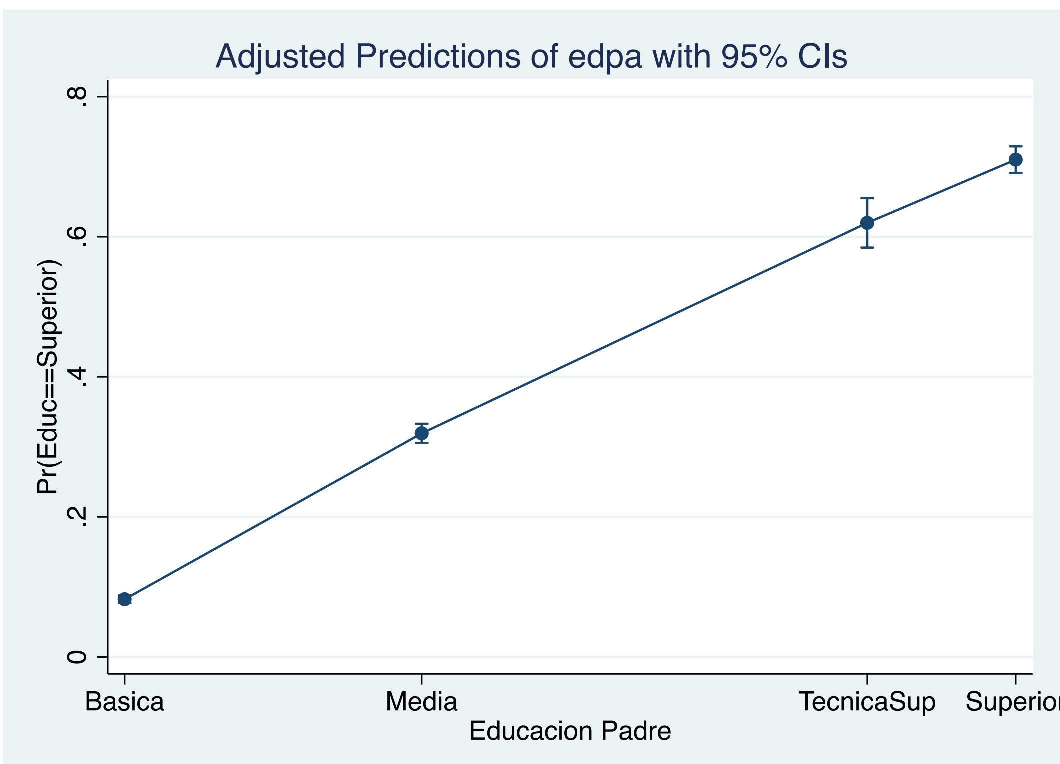


Margins father for ED=Media.

Margins for father for ED=Tecnica.



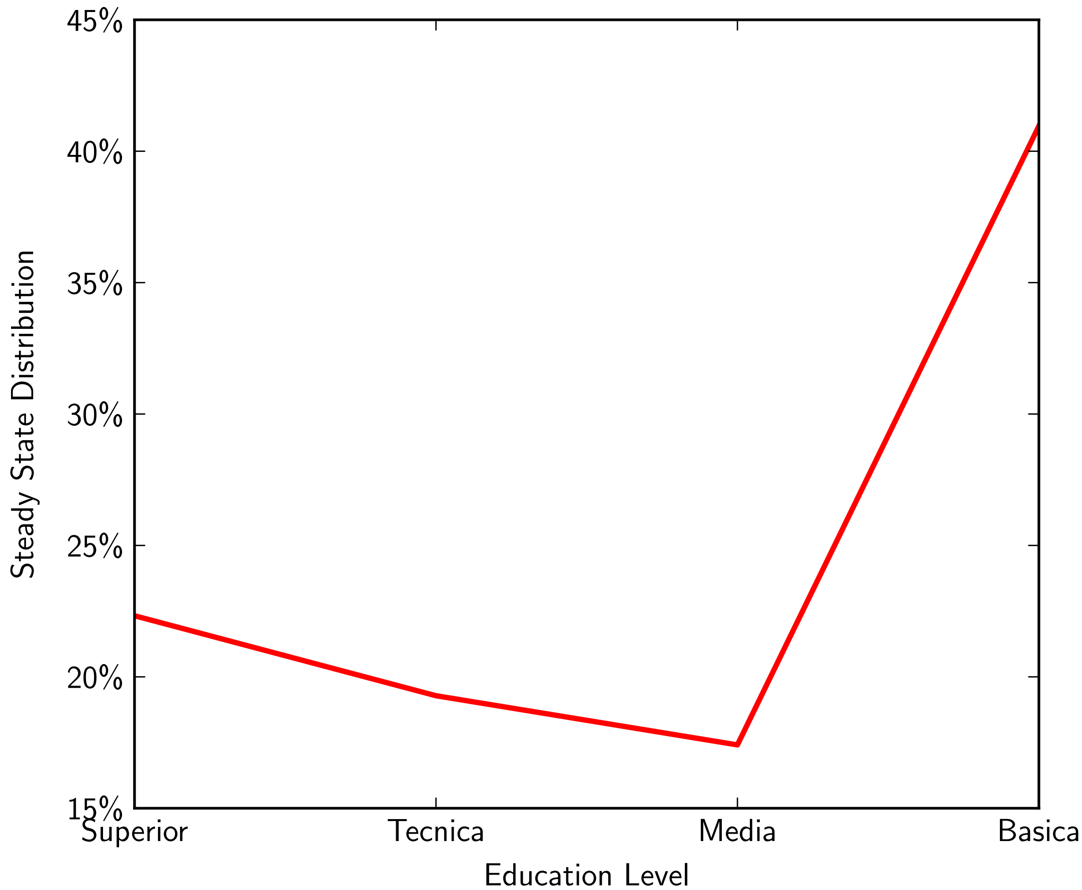
Margins for father for ED=Superior.



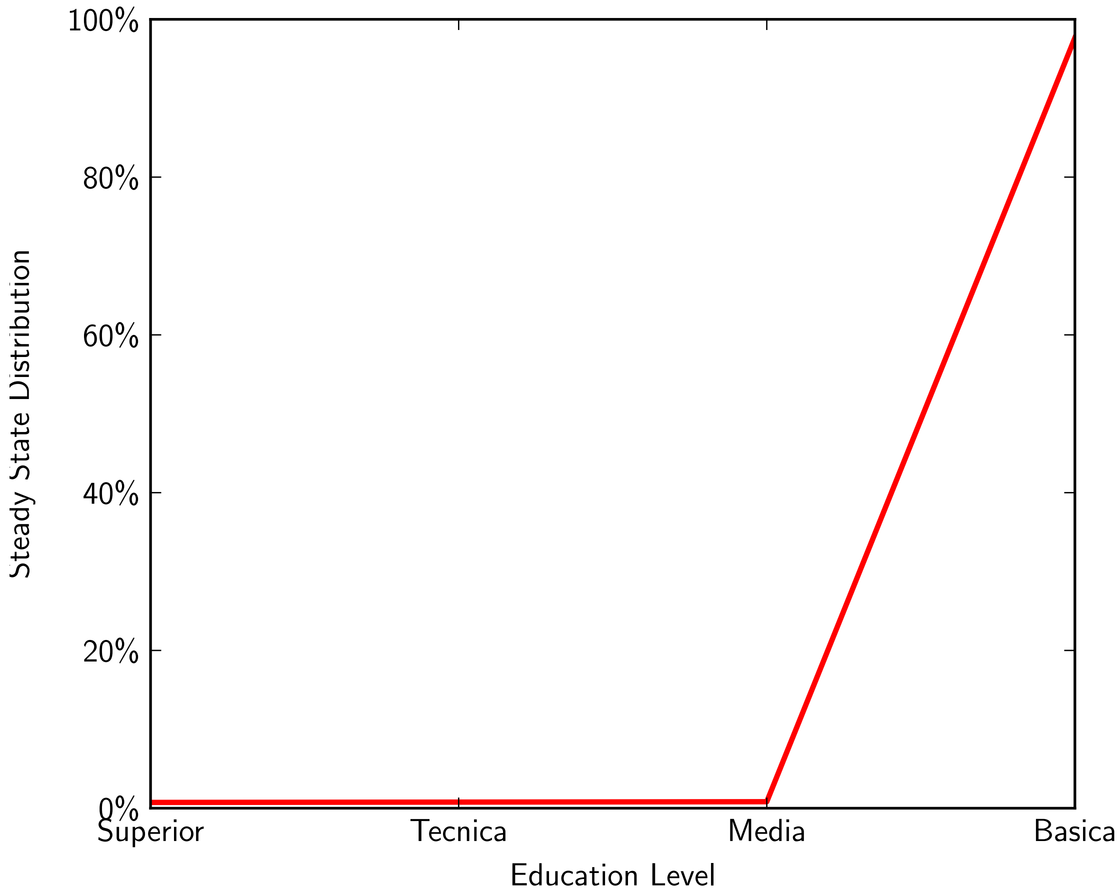
Margins for father for ED=Superior.

Model Plots

λ^{ss}



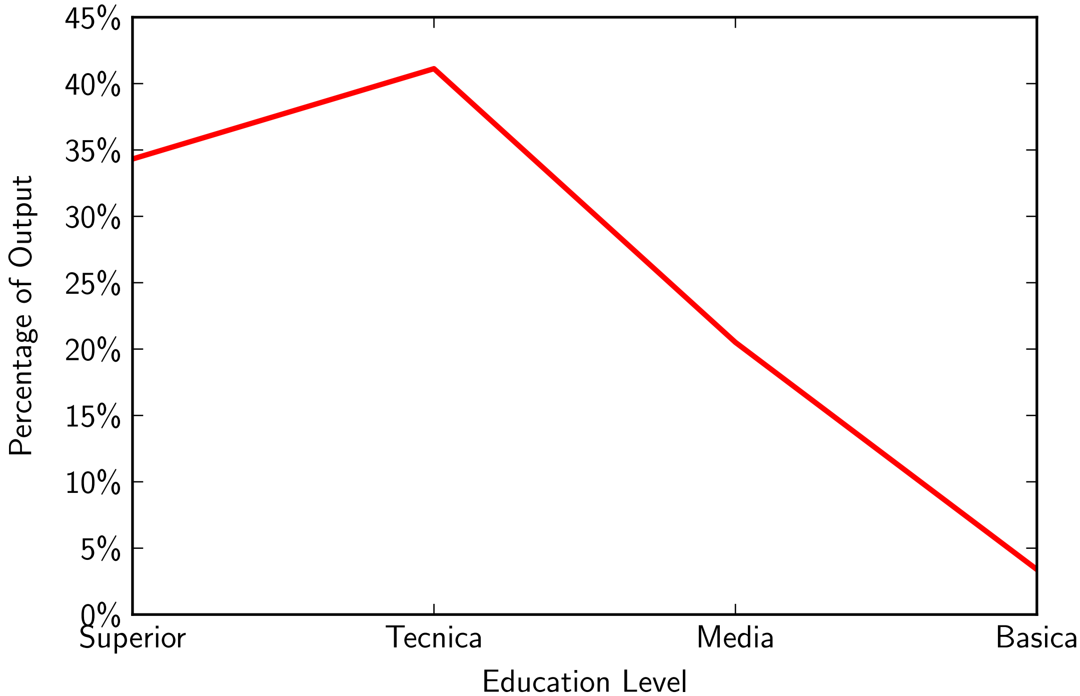
λ for rigid MC



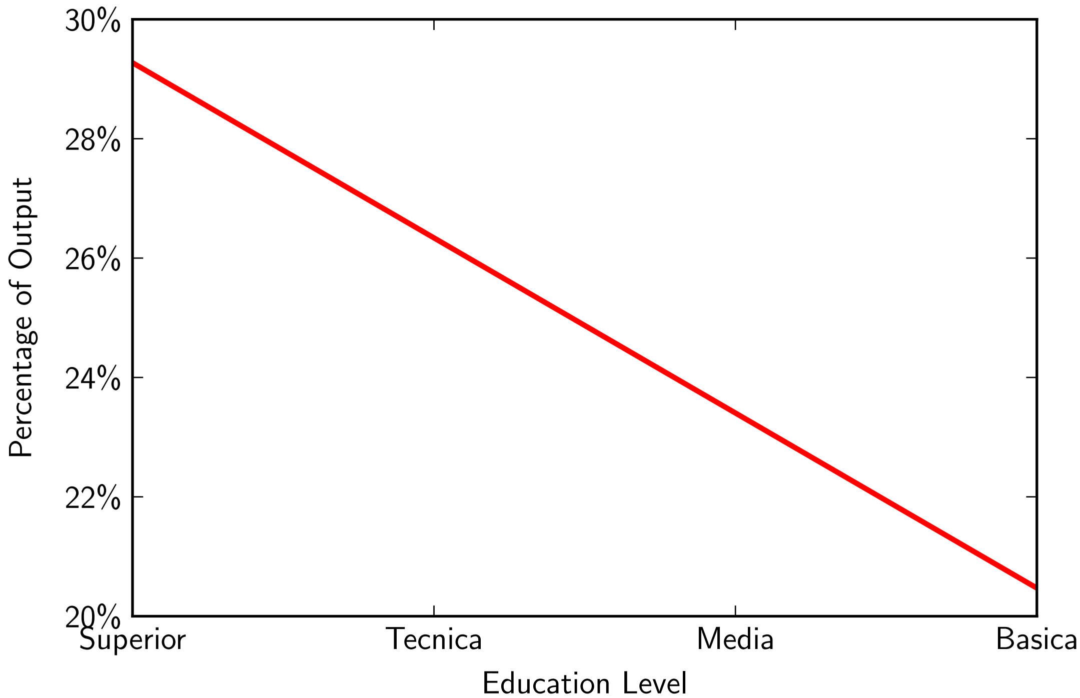
λ for random MC

Model Plots

$w_i/\mathbf{Y}$



Percentage of Output paid in Wages, Rigid MC



Percentage of Output paid in Wages, Random MC

Tables

Code

Python

Main Routine

```
from symbolic import *
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.pyplot import *
import os
from matplotlib.ticker import FuncFormatter


matplotlib.pyplot.show('hold')
np.set_printoptions(precision=5)
```

```
K=0.03 #Mejores resultados con k=0.33,d=0.03,b=0.22
d=0.03 #maximo valor 3,9 por ciento
r=0.04
b=0.02
print('Main')
print('K = '+str(K))
print('d = '+str(d))
print('b = '+str(b))

productividad=np.array([1,.75,.5,.25])
distribucion=np.array([1,.75,.5,.25])
markov=np.array([[.69,.13,.17,.01],[.55,.22,.21,.02],[.28,.16,.5,.06],[.08,
diagon=np.array([.69,.22,.5,.33])
#markov=np.array([[.8,.2,.0,.0],[.2,.6,.2,.0],[.2,.1,.7,.0],[.0,.0,.2,.8]])
#diagon=np.array([.4,.4,.4,.4])
tasaefectiva=np.sum(markov,axis=0)
tasasalida=1-diagon
tasaneta=tasaefectiva-tasasalida
markovt=np.transpose(markov)




print('Productivity = '+np.array_str(productividad))
print('TasaLlegada = ')
print(np.array_str(tasaefectiva))
print('TasaSalida = ')
print(np.array_str(tasasalida))
print('TasaNeta = ')
print(np.array_str(tasaneta))
print('PersistenciaEfectiva = ')
print(np.array_str(diagon/tasaefectiva))

print('Markov = ')
print(np.array_str(markov))
print('Steady Markov = ')
ssmarkov=np.matrix(markov)**10000
ssmarkov=ssmarkov[1,:]
print(ssmarkov)


print('Steady Lambda =')
ssteady=SteadyState(productividad,distribucion,K,d,r,b,markovt)
lass,u=ssteady
lass=np.array(lass.tolist(), dtype=float)
```

```
lass=np.array(lass).reshape(1,-1)
lass=np.squeeze(np.asarray(lass))

print('lambda_ss = '+np.array_str(lass))

thetas=thightness(productividad,lass,K,d,r,b)

print('Thetas_norm = '+np.array_str(thetas))


#plotear
plt.figure(figsize=(6, 5))
plt.plot(lass,color='r', linewidth=2.0)
plt.ylabel('Steady State Distribution')
plt.xlabel('Education Level')
labels = ['Superior', 'Tecnica', 'Media', 'Basica']
plt.xticks(np.arange(4), labels)
plt.tight_layout()
gca().set_yticklabels(['{:.0f}%'.format(x*100) for x in gca().get_yticks()]
plt.savefig('../../IMG/lass.pdf')
plt.savefig('../../IMG/lass.pgf')

realtight=thetas*lass
#plotear
plt.figure(figsize=(6, 4))
plt.plot(realtight,color='r', linewidth=2.0)
plt.ylabel('Tightness in Job market')
plt.xlabel('Education Level')
labels = ['Superior', 'Tecnica', 'Media', 'Basica']
plt.xticks(np.arange(4), labels)
plt.tight_layout()

plt.savefig('../../IMG/tightness.pdf')
plt.savefig('../../IMG/tightness.pgf')






print('Desempleo = '+str(u))






print('Unemployment = '+np.array_str(u*lass))
jobs=(1-u)*ssmarkov
jobs=np.array(jobs.tolist(), dtype=float)
```

```
jobs=jobs.astype(float)
jobs=np.array(jobs).reshape(1,-1)
jobs=np.squeeze(np.asarray(jobs))


print('Jobs = '+np.array_str(jobs))
print('Vacancies = '+np.array_str(np.divide(u,realtight)))

plt.figure(figsize=(6, 4))
plt.plot(jobs,color='r', linewidth=2.0)
plt.ylabel('Jobs')
plt.xlabel('Education Level')
labels = ['Superior', 'Tecnica', 'Media', 'Basica']
plt.xticks(np.arange(4), labels)
plt.tight_layout()

plt.savefig('../../IMG/jobs.pdf')
plt.savefig('../../IMG/jobs.pgf')







print('Tightness = '+np.array_str(realtight))
wages=productividad -K*(r-d)/(realtight*p(realtight))
print('Wages = '+np.array_str(wages))
wagesout=wages/productividad
print('Wages/productividad = '+np.array_str(wagesout))


#expected value of wages:
expct=np.sum(wages*lass)
vari=np.sum(np.power(wages*lass,2))
print('Expected Wage = '+str(expct))
print('Wage Variance= '+str(vari))

# Wages as percent of output
totaloutput=np.sum(jobs*productividad)

perc=wages*jobs/totaloutput

plt.figure(figsize=(6, 4))
plt.plot(perc,color='r', linewidth=2.0)
plt.ylabel('Percentage of Output')
plt.xlabel('Education Level')
labels = ['Superior', 'Tecnica', 'Media', 'Basica']
```

```
plt.xticks(np.arange(4), labels)
plt.tight_layout()
gca().set_yticklabels(['{:.0f}%'.format(x*100) for x in gca().get_yticks()]
plt.savefig('../../IMG/perc.pdf')
plt.savefig('../../IMG/perc.pgf')

twages=wages*jobs
plt.figure(figsize=(6, 4))
plt.plot(twages,color='r', linewidth=2.0)
plt.ylabel('Total Wages')
plt.xlabel('Education Level')
labels = ['Superior', 'Tecnica', 'Media', 'Basica']
plt.xticks(np.arange(4), labels)
plt.tight_layout()
plt.savefig('../../IMG/twages.pdf')
plt.savefig('../../IMG/twages.pgf')
print('Total Wages = '+str(twages))

print('Total Output = '+str(totaloutput))
print('Percentage of output = '+np.array_str(perc))
os.system('say "Done"')
```

Symbolic and Numeric Functions

```
from sympy import *
from sympy.plotting import plot
import numpy as np
import scipy.optimize as opt


# Definiciones
def p(x):
        return 1/(x+1)

def thightness(va,vb,a1,a2,a3,a4):

        theta, la, r, d, b, wi, yi, K, dp, vprob, vlas = symbols('theta lam

        utilidad = ln(b+(1-d)*p(la*theta)*(yi/(r+d) - (K*(r-d)/((r+d)*la*th

        deriv=diff(utilidad,theta).doit()



        res=np.zeros(va.size)
        prod=va
        dist=vb
```

```
        deriv=deriv.subs([(K,a1),(d,a2),(r,a3),(b,a4)])


        #generic theta
        t1=solve(deriv,theta)[1]
        #genereic wage


        #initialization
        expc=0
        vari=0

        for x in range(res.size):

                aux=deriv.subs([(yi,prod[x]),(la,dist[x])])
                sol=solve(aux,theta, simplify=false)
                res[x]=sol[1]




        return res
def SteadyState(va,vb,a1,a2,a3,a4,mk):



        theta, la, r, d, b, wi, yi, K, dp, vprob, vlas = symbols('theta lam


        MK =np.matrix(mk)
        indic=ones(4,1)


        la1,la2,la3,la4= symbols('lambda_1 lambda_2 lambda_3 lambda_4')
        var('la1,la2,la3,la4', positive = True)
        prod=va
        #Utilidad
        utilidad = ln(b+(1-d)*p(la*theta)*(yi/(r+d) - (K*(r-d)/((r+d)*la*th
        deriv=diff(utilidad,theta).doit()
        deriv=deriv.subs([(K,a1),(d,a2),(r,a3),(b,a4)])
        t1=solve(deriv,theta)[1]


        probabilidadfinal=p(t1)*indic
        probabilidadfinal[0]=probabilidadfinal[0].subs([(la,la1),(yi,prod[0
        probabilidadfinal[1]=probabilidadfinal[1].subs([(la,la2),(yi,prod[1
        probabilidadfinal[2]=probabilidadfinal[2].subs([(la,la3),(yi,prod[2
        probabilidadfinal[3]=probabilidadfinal[3].subs([(la,la4),(yi,prod[3
```

```
prob=probabilidadfinal


las = indic
las[0]=la1
las[1]=la2
las[2]=la3
las[3]=la4


suma=Matrix([[la1+la2+la3+la4 -1]])

#print(prob)





steadystate= las*d/(1-(indic-prob).dot(las)*(1-d)+d+(1-d)*prob.dot(
steadystate=steadystate.row_insert(4,suma)

steadystate=steadystate.subs(d,a2)
probando=lambdify((la1,la2,la3,la4),steadystate)
probando2=steadystate.jacobian(las)

probando2lam=lambdify((la1,la2,la3,la4),probando2)


def auxiliar(x):
        return [np.asscalar(probando(x[0],x[1],x[2],x[3])[0]),np.as
def jacobi(x):
        auxi=probando2lam(*x)
        repu=np.array([[auxi[0,0],auxi[0,1],auxi[0,2],auxi[0,3]],[a
        return repu

x0=[.5,.5,.5,.5]
limites=([0,0,0,0],[1,1,1,1])
res=opt.least_squares(auxiliar, x0, jac=jacobi, bounds=limites,gtol
#print(res.x)
res=res.x
u=d/(1-(indic-prob).dot(las)*(1-d)+d+(1-d)*prob.dot(las))
u=u.subs([(la1,res[0]),(la2,res[1]),(la3,res[2]),(la4,res[3]),(d,a2



return (res,u)
```

Stata

```
set more off , perm
clear all
cd "/Users/waio/Library/Mobile Documents/com~apple~CloudDocs/Tesis/Datos"
use"Casen 2015 STATA.dta"
keep edad r10a r10b e6a e3 e6b e6c e6d yoprcorh
keep if (edad >= 25)
keep if (e6a==7 |e6a==9 |e6a==13 |e6a==15 )
keep if (r10a ==3 | r10a==5 | r10a==8 | r10a==9)
keep if (r10b ==3 | r10b==5 | r10b==8 | r10b==9)
label variable e6a "Educacion"
label variable r10a "Educacion Padre"
label variable r10b "Educacion Madre"
label define educ 7 "Basica" 9 "Media" 13 "TecnicaSup" 15 "Superior" , repl
label define educ2 3 "Basica" 5 "Media" 8 "TecnicaSup" 9 "Superior" , repla
label values e6a educ
label values r10a educ2
label values r10b educ2

rename e6a educ
rename r10a edpa
rename r10b edma

mlogit educ i.edpa, base(7)
margins edpa, atmeans predict(outcome(7)) saving(file1 , replace)
marginsplot, name(Basica)
graph export ../IMG/prediccionBasica.pdf , replace
margins edpa, atmeans predict(outcome(9)) saving(file2 , replace)
marginsplot, name(Media)
graph export ../IMG/prediccionMedia.pdf , replace
margins edpa, atmeans predict(outcome(13)) saving(file4 , replace)
marginsplot, name(SuperiorTecnica)
graph export ../IMG/prediccionTecnica.pdf , replace
margins edpa, atmeans predict(outcome(15)) saving(file5 , replace)
marginsplot, name(Superior)
graph export ../IMG/prediccionSuperior.pdf , replace

rename edpa edpa1
rename edma edpa

mlogit educ i.edpa, base(7)
margins edpa, atmeans predict(outcome(7)) saving(file6 , replace)
marginsplot, name(Basica2)
margins edpa, atmeans predict(outcome(9)) saving(file7 , replace)
marginsplot, name(Media2)
margins edpa, atmeans predict(outcome(13)) saving(file9 , replace)
```

```
marginsplot , name(SuperiorTecnica2)
margins edpa, atmeans predict(outcome(15)) saving(file10 , replace)
marginsplot , name(Superior2)

rename edpa edma
rename edpa1 edpa

combomarginsplot file1 file6 ,noci plotdim(_filenumber) name(Comparacion1)
graph export ../IMG/comparacion1.pdf , replace
combomarginsplot file2 file7 ,noci plotdim(_filenumber) name(Comparacion2)
graph export ../IMG/comparacion2.pdf , replace

combomarginsplot file4 file9 ,noci plotdim(_filenumber) name(Comparacion4)
graph export ../IMG/comparacion3.pdf , replace
combomarginsplot file5 file10 ,noci plotdim(_filenumber) name(Comparacion5)
graph export ../IMG/comparacion4.pdf , replace

_pctile yoprcorh, p(95)
sort yoprcorh
gen salarios= yoprcorh/r(r1)

drop if salarios==0 | salarios>1

*graph combine Basica Media Tecnica SuperiorTecnica Superior, ycommon
log close
```