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An application of Rolling chaos 0-1 test on Stock Market

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Abstract

In this paper we apply a rolling 0-1 test for chaos on different stock market indices returns in the world, considering different time period windows to capture the effects of adding new information. A rolling sample is defined for each index and at the same time, wavelet denoising has been employed since approximately 1995 to the end of 2012. Empirical evidence of continuous chaotic behavior for all indices is found.

Keywords: Hinish Test, Rolling Method, Stock Indices.

Resumen

En el presente artículo se aplica un modelo *rolling* 0-1 para caos respecto al retorno de diferentes índices del mercado accionario en el mundo, considerando diversas ventanas de tiempo para capturar los efectos de la adición de nueva información. Una muestra *rolling* es definida para cada índice y simultáneamente una metodología *wavelet* de reducción de ruido es definida para cada uno de los índices, considerando datos desde aproximadamente 1995 hasta fines del 2012. Los resultados muestran evidencia empírica que sostiene un comportamiento caótico continuo para todos los índices utilizados.

Palabra clave: Test de Hinich, Modelo Rolling, Índices Accionarios.

1. Introduction

Largest Lyapunov exponent (LLE) is the main tool employed for testing behavior chaos in financial time series (Hsieh, 1991; Parisi, Espinosa and Parisi, 2007). This method requires, however, the reconstruction of a phase space which implies accepting certain biases so as to determine the immersion dimension, mean period and time of delay. Test 0-1 has recently been used to solve these inconveniences so as to determine chaos in financial series (Webel, 2012).

The 0–1 test for chaos is based on a Euclidean extension instead of a phase space reconstruction. In theory, it yields one with probability one if the latter is chaotic and zero otherwise. Webel (2012) has evidence of chaotic behavior of the German stock market. The author concludes that the fluctuations of stock returns are not entirely caused by random shocks, but that they are also partially caused by some deterministic chaotic motion.

In this paper we present the results of our investigation on how permanent in time this chaotic behavior, which is present in financial series, is. To achieve this, we applied the rolling method, which allows us to observe how test 0-1 changes as new information is added. The rest of the paper is structured as follows: Section II presents the methodology which will be used, Section III presents the data to be used in this study, and Section IV presents the most relevant results. The final conclusions are presented in Section V.

2. Methodology

A. Returns calculations

As usual, the returns for each index are calculated as first differences in natural logarithms according to the following expression:

$$R_t = Ln\left(\frac{P_t}{P_{t-1}}\right) \tag{1}$$

where,
R_t is the return period t,
P_t and P_{t-1} are the daily closing prices index.

B. The zero-one (0-1) test for chaos

To determine whether a given deterministic nonlinear dynamic system is chaotic, Gottwald and Melbourne (2004, 2005) proposed a novel test approach. According to Ke-Hui *et al*. (2010) the definition of chaos is given if the test result is less than 0.1 −which indicates that the dynamics is not chaotic− or more than 0.1 −which indicates that the dynamics is chaotic− making this distinction clear. Moreover, this test has two advantages over LLE: The phase space reconstruction is not needed because it is applied directly to time series data; and it is a binary test. The most powerful aspect of this method is that it is independent of the nature of the data under consideration. In particular, the equations of the underlying dynamical system does not need to be known, and there is no practical restriction of the dimension of the underlying data.

The 0-1 test considers a set of discrete data $\phi(n)$ which given an observation $\phi(j)$, where j = 1,2,…,N, represents a one dimensional observable data set as defined by Gottwald and Melbourne (2009). Chosen a constant $c \in (0,\pi)$ and defined the translation variables:

$$p_c(n) = \sum_{j=1}^{n} \phi(j)\cos(jc), \qquad q_c(n) = \sum_{j=1}^{n} \phi(j)\sin(jc) \tag{2}$$

for $n = 1, 2,\ldots, N$.

The behavior of p_c and q_c, which can be diffusive or non-diffusive behavior, can be investigated by analyzing the mean square displacement $M_c(n)$ and computing the asymptotic growth rate K_c of the mean square displacement. After that, it is necessary to perform for N_c values of c chosen randomly in the interval (0,π). In practice, N_c = 100 is sufficient as Gottwald and Melbourne (2009). When the median of these N_c values of K_c is computed and the final result is K = *median*(K_c). Therefore, a value of $K \approx 0$ indicates regular dynamics, and $K \approx 1$ indicates chaotic dynamics.

The mean square displacement is defined as

$$M_c(n) = \lim_{N\to\infty} \frac{1}{N} \sum_{j=1}^{N} \left[p_c(j+n) - p_c(j)\right]^2 + \left[q_c(j+n) - q_c(j)\right]^2 \tag{3}$$

The limit is assured by calculating $M_c(n)$ only for $\le n_{cut}$, where $n_{cut} << N$. Good results are achieved by setting $n_{cut} = N/10$.

$$M_c(n) = V(c)n + V_{osc}(c,n) + e(c,n) \tag{4}$$

Where
$e(c,n)/n \to 0$ as $n \to \infty$ uniformly in $c \in (0,\pi)$ and

$$V_{osc}(c,n) = (\mathrm{E}\phi)^2 \frac{1-\cos(nc)}{1-\cos(c)}$$

The expectation $\mathrm{E}\phi$ is given by

$$\mathrm{E}\phi = \lim_{N\to\infty} \frac{1}{N} \sum_{j=1}^{N} \phi(j)$$

According to Gottwald (2009), an improved version of the test is achieved when subtracting the $V_{osc}(c,n)$ term

$$D_c(n) = M_c(n) - V_{osc}(c,n) \tag{5}$$

In particular, the slope $V(c)$ of the mean square displacement is identified with the power spectrum.

$$V(c) = \sum_{k=-\infty}^{\infty} e^{ikc} \rho(|k|) = \lim_{n\to\infty} \frac{1}{n} E\left|\sum_{j=0}^{n-1} e^{ijc}\phi(j)\right|^2 \tag{6}$$

The best method that is suggested for calculating Kc (k-median) is the correlation method which is achieved using the calculation of D_c, as follows:

$$K_c = corr(\xi, \Delta) = \frac{\mathrm{cov}(\xi, \Delta}{\sqrt{\mathrm{var}(\xi)\,\mathrm{var}(\Delta)}} \in [-1,1] \tag{7}$$

C. Rolling sample

In this paper, a rolling sample approach is considered. It is used for the first 256 days (1 year of data) and wavelet and chaos are performed. Then, the first observation is dropped and used the next day, also using 256 observations. This sampling approach continues until the last observation is used.

While a static way for the entire series is the usual approach to calculate statistical measures, Webel (2012) e.g. used entire series for German stock markets. Rolling methodology is useful to detect chaos over time.

D. Denoising

To avoid unbiased results, in this paper a wavelet denoising is applied. In this case, four wavelet filters were applied prior to chaos test 0-1 application. A maximal overlap discrete wavelet transforms (MODWT), is used as is usually done in financial time series, instead of discrete wavelet transforms (DWT). Therefore, for each rolling window a denoise filter is performed.

The number of filters applied is determined by the *round (log (length of series)/log (2))* expression. Following the recommendations of Percival and Walden (200) and Gencay *et al.* (2001), four different kinds of wavelets are used in this paper. The wavelets used are: Haar, symmlet, daubechies and coifflet.

Because noise free series are used, the combined use of Test 0-1 and wavelet helps to avoid biased results.

3. Data

This study includes 22 stock market indices from America, Europe, Asia and Oceania, and the data consists of daily log returns for each index (Table 1).

Table 1
Sample details

N°	INDEX	NAME	COUNTRY	FROM	TO	DATA
1	IPSA	CHILE STOCK MKT SELECT	CL	03-Ene-95	28-Dic-12	4488
2	IBVC	VENEZUELA STOCK MKT	VZ	03-Ene-95	28-Dic-12	4351
3	MERVAL	ARGENTINA MERVAL	AR	03-Ene-95	28-Dic-12	4448
4	IBOV	BRAZIL IBOVESPA	BZ	03-Ene-95	28-Dic-12	4452
5	IGBVL	PERU LIMA GENERAL	PE	03-Ene-95	31-Dic-12	4482
6	COLCAP	COLOMBIA COLCAP	CO	16-Jul-02	28-Dic-12	2554
7	INDU	DOW JONES INDUS. AVG	US	04-Ene-95	31-Dic-12	4531
8	MEXBOL	MEXICO IPC	MX	03-Ene-95	31-Dic-12	4528
9	SPX	S&P 500	US	04-Ene-95	31-Dic-12	4531
10	CCMP	NASDAQ COMPOSITE	US	04-Ene-95	31-Dic-12	4531
11	DAX	DAX	GE	03-Ene-95	28-Dic-12	4562
12	IBEX	IBEX 35	SP	03-Ene-95	31-Dic-12	4543
13	CAC	CAC 40	FR	04-Ene-95	31-Dic-12	4574
14	UKX	FTSE 100	GB	04-Ene-95	31-Dic-12	4546
15	MCX	FTSE 250	GB	04-Ene-95	31-Dic-12	4546
16	FTSEMIB	FTSE MIB	IT	02-Ene-98	28-Dic-12	3808
17	OMX	OMX STOCKHOLM 30	SW	03-Ene-95	28-Dic-12	4518
18	SMI	SWISS MARKET	SZ	04-Ene-95	28-Dic-12	4528
19	NKY	NIKKEI 225	JN	05-Ene-95	28-Dic-12	4427
20	HSI	HANG SENG	HK	04-Ene-95	31-Dic-12	4444
21	AS51	S&P/ASX 200	AU	04-Ene-95	31-Dic-12	4554
22	NYA	NYSE COMPOSITE	US	04-Ene-95	31-Dic-12	4531

4. Results

Considering the k-median mean throughout all the windows, the results show that all of the indices are ruled by a chaotic behavior as is shown in Table 2. Four different wavelets were used prior to the elimination of noise. Because of this, *n-w* k-median are obtained for each index, where the amount of data and *w* is the size of the rolling window.

The number of windows that the test detected without chaotic behavior is shown in Table 3. This number is very small compared to the number of rolling windows. Therefore, the episodes of non-chaotic behavior are very isolated, which can be appreciated in Figure 1, which corresponds to the IPSA index. In 17 years, 11 non-chaotic windows show up from a total of 4232. This behavior is similar to the other analyzed series.

Table 2

Chaos Test

INDEX	WINDOWS	K-MEDIAN CHAOS TEST			
		HAAR	SYMMLET	DB8	COIFFLET
IPSA	4232	0.9931	0.9836	0.9818	0.9809
IBVC	4095	0.9928	0.9841	0.9820	0.9834
MERVAL	4192	0.9924	0.9808	0.9813	0.9794
IBOV	4196	0.9933	0.9870	0.9849	0.9838
IGBVL	4226	0.9921	0.9850	0.9845	0.9839
COLCAP	2298	0.9951	0.9868	0.9863	0.9867
INDU	4275	0.9939	0.9856	0.9863	0.9831
MEXBOL	4272	0.9938	0.9871	0.9828	0.9807
SPX	4275	0.9940	0.9871	0.9870	0.9848
CCMP	4275	0.9927	0.9846	0.9853	0.9849
DAX	4306	0.9932	0.9841	0.9822	0.9832
IBEX	4287	0.9939	0.9883	0.9861	0.9875
CAC	4318	0.9934	0.9871	0.9843	0.9844
UKX	4290	0.9939	0.9884	0.9848	0.9861
MCX	4290	0.9908	0.9787	0.9764	0.9767
FTSEMIB	3552	0.9932	0.9860	0.9837	0.9849
OMX	4262	0.9933	0.9857	0.9844	0.9858
SMI	4272	0.9934	0.9869	0.9832	0.9823
NKY	4171	0.9933	0.9865	0.9865	0.9865
HSI	4188	0.9931	0.9833	0.9800	0.9814
AS51	4298	0.9937	0.9867	0.9859	0.9835
NYA	4275	0.9935	0.9856	0.9847	0.9816

k> 0.1 implies chaotic behavior

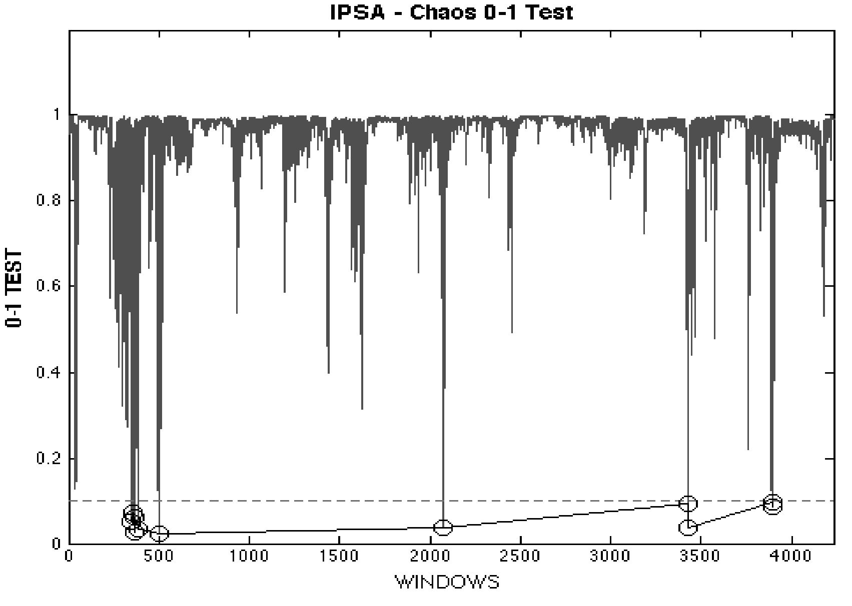
Table 3
Windows with non-chaotic behavior

INDEX	WINDOWS	NON CHAOTIC WINDOWS			
		HAAR	SYMMLET	DB8	COIFFLET
IPSA	4232	0	11	0	3
IBVC	4095	0	5	3	6
MERVAL	4192	0	7	14	6
IBOV	4196	1	12	5	9
IGBVL	4226	1	5	5	3
COLCAP	2298	0	4	2	5
INDU	4275	0	1	5	2
MEXBOL	4272	0	4	2	4
SPX	4275	0	7	6	6
CCMP	4275	0	4	6	3
DAX	4306	0	1	1	1
IBEX	4287	0	9	2	2
CAC	4318	0	11	7	5
UKX	4290	0	4	8	8
MCX	4290	1	12	17	9
FTSEMIB	3552	0	0	0	0
OMX	4262	0	9	7	3
SMI	4272	0	6	3	1
NKY	4171	0	10	10	10
HSI	4188	0	42	15	13
AS51	4298	0	16	3	3
NYA	4275	1	3	3	8

Cantidad de ventanas con k<0.1

Graph 1

Results of the Test 0-1 for Ipsa



5. Discussion

When using Test 0-1 for detecting chaos through a rolling methodology, we find evidence that chaotic behavior in financial series is more permanent and intermittent. This would tend to explain why the prediction models fail when trying to predict the future behavior of stock prices. It also justifies the search for new predictive models that consider the chaotic behavior that these series present. This result is extremely important for portfolio management because it allows for the rapid adjustment of prediction models, disregarding those that are of the linear kind. In the short term, this would lead to forecast improvement.

Because a noise free series is used, the combined use of the Test 0-1 and wavelet avoids finding biased results.

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