Contents

Notations **2**

General Introduction **4**

1 Preliminaries **18**
1.1 Spaces of functions 20
1.2 Measures of noncompactness 24
1.3 Set-valued maps 24
1.4 The Hausdorff distance 26
1.5 Some useful lemmas 27
1.6 Some elements of measure theory 30
1.7 Some classes of sets 32
1.7.1 Uniformly prox-regular sets 32
1.7.2 Uniformly subsmooth sets 33

2 Positively α-far sets **34**
2.1 Relation with other classes 40
2.2 Preservation of positively α-far sets 42
2.2.1 Union of positively α-far sets 42
2.2.2 Intersection of positively α-far sets 43
2.2.3 Inverse images of positively α-far sets 44

3 Galerkin-Like method and applications **52**
3.1 Technical assumptions 53
3.2 Preliminary results 55
3.3 Galerkin-like method 56
3.4 A Generalized Perturbed Sweeping Process 60
3.5 Perturbed state-dependent sweeping process 66
3.6 The perturbed sweeping process 69
3.7 Perturbed second-order sweeping process 72
3.8 The necessity of the compactness assumptions 74

4 A Variant of the Perturbed State-Dependent Sweeping Process **76**
4.1 Technical assumptions 78
4.2 Existence result 79
4.3 Complementarity Dynamical Systems 82

4.4 A control system describing hysteresis effects 85

5 Moreau-Yosida regularization of state-dependent sweeping process 92
5.1 Technical Assumptions 93
5.2 Preliminary lemmas 95
5.3 The concept of Solution 99
5.4 Existence through Moreau-Yosida regularization 100
5.5 The Case of the sweeping process 106
5.5.1 The Finite-Dimensional Case 107
5.6 An Application to Hysteresis 107

6 Regularization of perturbed state-dependent sweeping processes with nonregular sets 110
6.1 Technical assumptions 111
6.2 Preparatory lemmas 114
6.3 An existence result for the perturbed state-dependent sweeping process 116
6.4 Sweeping process and Subdifferentially Perturbed Sweeping Process . 122
6.4.1 The case of the sweeping process 122
6.4.2 Subdifferentially state-dependent perturbed sweeping process . 123

7 Perturbed Sweeping Processes with Nonlocal Initial Conditions 126
7.1 Technical Assumptions 128
7.2 Perturbed Sweeping Process with Nonlocal Initial Conditions 130
7.3 The Case H is Compactly Embedded in a Banach Space E 141
7.4 Tangential conditions 146
7.5 An application to nonlocal differential complementarity systems . . . 148
7.6 An application to Hysteresis 149

8 Existence and Lyapunov pairs for the perturbed sweeping process governed by a fixed set 152
8.1 The Catching-up Algorithm 154
8.2 Lyapunov Pairs and Invariance 156
8.2.1 Weak invariance 160
8.3 Applications 161
8.3.1 Hysteresis 161
8.3.2 Crowd motion 161

9 Lyapunov pairs for Perturbed Sweeping Processes 164
9.1 Lyapunov pairs and invariance 166
9.1.1 Weak invariance 171
9.2 An application to gradient complementarity dynamical systems . . . 171

Conclusions and Perspectives 174

Bibliography 178

List of Figures

2.1 ρ-tube around S. 35
2.2 $S = \{(x, y) \in \mathbb{R}^2 : y \geq -|x|\}$. 37
2.3 Geometrical interpretation of Lemma 2.7. 39
2.4 The union of two convex sets is not positively α-far. 43

3.1 Set S in Example 3.1. 72

4.1 Functions f_* and f^*. 88
4.2 Numerical simulations with $c(w, v) = 1$ and $(v_0, w_0) = (-0.7, -0.8)$. . 89
4.3 Numerical simulations with $c(w, v) = 0.2$ and $(v_0, w_0) = (-1.8, -1.0)$. 89
4.4 Numerical simulations for $c(w, v) = 0.2$, $h_1(w, v) = \sin((v - [v])\pi)$ and $h_2(w, v) = v - w$. 90
4.5 Numerical simulations for $h_1(w, v) = v$ and $h_2(w, v) = v - w$. 90

8.1 Disks, respectively, for $d = \|\cdot\|_1$. $d = \|\cdot\|_2$ and $d = \|\cdot\|_\infty$. 162

9.1 A circuit with an ideal diode, an inductor and a current source. . . . 173