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NETWORK EFFECTS OF HIERARCHICAL INFORMATION

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Network effects of hierarchical information

Esta tesis analiza la formación de redes como consecuencia de la búsqueda de los agentes por información valiosa para ellos. La contribución de este trabajo es la separación de la información en tipos. No sólo importa el volumen o cantidad que un agente tenga sino que también qué tipo de información tiene. Como resultados, se tiene la formación de redes en que agentes con niveles y tipos similares de información tienden a estar en los mismos grupos. Agentes pueden actuar en forma competitiva o colaborativa dependiendo del tipo de información que posean.

A María Del Pilar

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A mi familia por todo el apoyo que siempre me han entregado. Mis padres, mi hermanito, Mayi y María del Pilar son la causa de mi felicidad.

Contents

List of Figures	v
1 Introduction	1
1.1 Motivation	1
1.2 Literature review	2
1.3 Notation	3
1.4 Organization	3
2 The model	4
2.1 The setting	5
2.2 Examples	5
2.3 Equilibrium networks	10
2.4 Efficient networks	15
2.5 Equilibrium networks and efficiency	16
Conclusion	20
Bibliography	21

List of Figures

2.1	Example 2.2, one connection.	6
2.2	Example 2.3 initial state.	6
2.3	Example 2.3 equilibrium network $\frac{1}{3}2^{\alpha+1} \cdot 10^{3\alpha+1} \leq c \leq 10^{3\alpha+1}$	7
2.4	Example 2.5 initial state.	8
2.5	Example 2.5 equilibrium network $\frac{1}{3}2^{\alpha+1} \cdot 10^{\alpha+1} \leq c \leq 10^{\alpha+1}$	9
2.6	Example 2.6 initial state.	9
2.7	Example 2.6 equilibrium network $5 \cdot 12^{\alpha} \leq c \leq 10^{\alpha+1}$	10
2.8	PNE networks example proposition 2.9.	13
2.9	Pairwise Nash Equilibrium Network. I_n complete network. Example 2.10. . .	14
2.10	PNE network. Example 2.11.	14
2.11	PNE networks, first example efficiency. Example 2.17.	17
2.12	PNE concentrated information. Example 2.18.	18
2.13	PNE bipartite network. Example 2.18.	19

Chapter 1

Introduction

1.1 Motivation

People form relationships among them because they expect some kind of benefit when networking with each other. For instance, in labor market, information about job opportunities may spread through contacts. All else equal, if an agent is working with a relatively high salary, one might expect that almost all job vacancies that he or she hears about, will be of no interest for him or her. On the other hand, if an agent has a salary low enough, almost all information that arrives to him or her, will be about a job vacancy with better salary perspectives. Therefore, individuals with higher salaries will pass the information through the network more frequently than those with lower wages.

To what extent may having more contacts be beneficial? With whom will an agent be better off when forming a link? The strategic interaction of agents trying to increase their futures wages forms a network structure. This structure affects the value that each agent has for the rest of the network when they decide to form or not a link. Initial asymmetries in wages by itself may cause inequality in wages in a dynamic setting, differences in wage growth due to inefficient information propagation through the network.

There is a lot of literature that explore the impacts of networks on different social topics. Specifically people tend to bond with similar others and this may impact social outcomes. The aim of this work is to show the impact of information on the network structure and attempt to get as an endogenous result some of the known facts, such as formation of groups with similar agents. So, in this work agents will be the same regarding their utility function.

It is worth to be noted that agents differ only on their information. So there are no exogenous discrimination effects, such as homophily through increasing return on links formed with similar agents or a higher matching rate with similar agents such as in Currarini et al.

(2009). This is an important aspect because in a dynamic extension everyone could be in any place on the network if they manage to get the right information.

1.2 Literature review

There is a vast literature exploring the importance of network effects on different social interactions, for instance, labor market and education ¹. In general the effects over a given social environment are explored as in Lobel and Sadler (2015a). They study how homophily impacts social learning or the information transmission over the networks.

Calvo-Armengol and Jackson (2004) explore the effects of social network structures on employment inequality in terms of drop out from the labor market. This work states that initial conditions of employment are crucial in determining consistent inequality in employment. Although the network structure is given they explore different settings, in terms of number of connections and their distribution along the network, giving insights in how important the network structure can be in aggregate employment. Another important result is the duration dependence of the probability of employment, the probability of getting hired decreases in the time span of the unemployment state. They use a game where given a network structure, agents have to decide whether to stay or not in the labor market. Assuming that it is costly to stay in. Agents initially worse in terms of employability drop out, letting their neighbors worse off. This may cause a consistent inequality in employment rate within the network's members. The agents connected with members with better initial conditions face better employment rate than groups with worse initial conditions.

Jackson and Wolinsky (1996) studies pairwise stability and efficiency in network formation models such as the co-author model and connections model. Such models are used by many authors in later literature to explore new consequences of their research. In the co-author model utility increases with the effort of the co-author in the project but decreases with the time co-authors use on other research. In the connections model, players benefit from connecting to other agents directly and indirectly, but they benefit the most from direct connections, there is a cost for direct links.

Other works that shows the relevance of social networks include Morris (2000) which study how an agent may affect the behavior of the rest of the network. Ballester et al. (2006) study the effect of removing an important agent from the network over the remaining players. Kranton and Minehart (2003) and Corominas-Bosch (2004) use networks to explain interactions between buyers and sellers. Chwe (2000) studies how communication influence information transmission, initial adopters start the process of communication that spreads through the network. Moulin and Sethuraman (2013) show how the problem of resources

¹Calvó-Armengol (2004), Calvó-Armengol and Jackson (2009)

division can be addressed using network analysis. Bramoullé and Kranton (2007) explore the incentives of providing public goods using network analysis and Bramoullé et al. (2014) study in a broad scope how a player's action is amplified when there is a network involved.

The topic explored in this work is most similar to that of Bala and Goyal (2000) (because the model in this work is superficially studied), they explore network formation when agents contact to share information, they benefit linearly on the number of player they observe and the same behavior for the costs. Here there is no competition for information and there is no difference in the information players have, also they all have the same volume of information (one unit). Jackson (2005) offers a survey on network formation and applications to games in networks.

1.3 Notation

We use the same notation as in Calvó-Armengol and İlkılıç (2009). A link between two players is denoted l_{ij} , meaning that player i is connected to player j . In the case of undirected network $l_{ij} = l_{ji}$. A network is denoted g , $l_{ij} \in g$ means that player i is connected to player j in the network g . The network is the set containing the links between players. A network is complete if all player are connected. Given a set of player N , g_N denotes a complete network, and G the set of all possible networks.

Given a network g , a player i , utility function u_i and a link l_{ij} $mu_i(g, l_{ij})$ is defined as follows:

$$mu_i(g, l_{ij}) = u_i(g) - u_i(g - l_{ij})$$

1.4 Organization

Firstly the model is presented with some examples. Then some results regarding equilibrium networks and efficient networks are proposed.

Chapter 2

The model

In this model every agent is endowed with a set of information. Information can be classified in two types. Players have levels for each of the two types of information they could have. The intuition behind this can be stated with the following tale:

Each agent has a page of a book, the book represents the universe of information. With each page agents can deduct more information but some pages allow agents to know more about the universe than other pages. Agents can connect with others and share the pages they have. Adding more pages will increase the level of knowledge but there are some pages that will increase the utility of all the other pages due to key words on them.

We use the static game of network formation due to Myerson (1991): there is a set of players $N = \{1, \dots, n\}$, strategies $S_i = \{0, 1\}^{n-1}$. Then a strategy profile $(s_1, \dots, s_n) \in \times_{i \in N} S_i$ defines a network $g(s)$.

We take the equilibrium concept of Pairwise-Nash equilibrium from Calvó-Armengol and İlkılıç (2009) citing that paper we use the following definition of equilibrium network:

Definition 2.1 (Pairwise-Nash equilibrium) *A network g is a Pairwise-Nash equilibrium (PNE) network if $g = g(s^*)$ with s^* a pure Nash equilibrium strategy profile and $\forall i, j \in N, l_{ij} \notin g$ then $mu_i(g + l_{ij}) > 0$ implies $mu_j(g + l_{ij}) < 0$.*

Mutually beneficial links are always formed.

2.1 The setting

Agents have a utility function $u(h, n)$ where h stands for the amount of information of type H , and n for that of type N , $h \in \mathbb{R}, n \in \mathbb{R}$. This utility function has the following properties

- u is increasing on both h and n .
- $u(\cdot, n)$ is strictly concave.
- $u(h, \cdot)$ is convex.
- u is supermodular.
- $u(\cdot, 0) = u(0, \cdot) = 0$.

The set of players is I and consider the following subsets of I ,

$$I_n \equiv \{i \in I : i \text{ has information of type } N\}$$

$$I_h \equiv \{i \in I : i \text{ has information of type } H\}$$

Players in each set have different types, which are the amounts of information they have. As an assumption, players have different information.

We assume that information flows both ways and the cost must be paid by both sides. For each link, there is a constant cost c , so the expenditure function is the following,

$$e_i(h, n) = c \cdot \#P_i(h, n)$$

With P_i the set of players whose information agent i is observing in network g in order to gather the set of information (h, n) . For player i we denote the information as (h_i, n_i) .

2.2 Examples

For the sake of illustration it is assumed in the examples below that the utility function have the following form,

$$u_i(h, n) = nh^\alpha, \quad \alpha \in (0, 1)$$

This function clearly satisfies the assumptions required.

We may think of this as information of type N as directions or contact information of clients. H may be any service or good that clients are willing to buy.

Example 2.2 *The agents within a box represent the initial state while the graph outside the box represents the equilibrium network. Both players want to connect if $c \leq 10^{\alpha+1}$.*

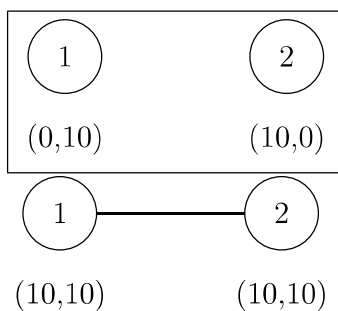


Figure 2.1: Example 2.2, one connection.

Example 2.3 *Consider the following example, where we have relatively richer agents endowed with hierarchical information,*

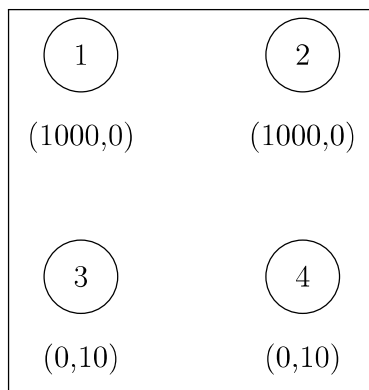


Figure 2.2: Example 2.3 initial state.

For players 1 and 2 we have the following

Table 2.1: Agent 1 and 2, expenditure, information and utility.

e	(h,n)	u
0	1000,0	0
c	1000,10	$10^{3\alpha+1}$
2c	1000,20	$2 \cdot 10^{3\alpha+1}$
3c	2000,20	$2^{\alpha+1} \cdot 10^{3\alpha+1}$

Agents 3 and 4 have the following

Table 2.2: Agent 3 and 4, expenditure, information and utility.

e	(h,n)	u
0	0,10	0
c	1000,10	$10^{3\alpha+1}$
2c	1000,20	$2 \cdot 10^{3\alpha+1}$
3c	2000,20	$2^{\alpha+1} \cdot 10^{3\alpha+1}$

In this example there are different cases, if $c > 10^{3\alpha+1}$, the equilibrium network is the empty network, if $c < \frac{1}{3}2^{\alpha+1} \cdot 10^{3\alpha+1}$ the complete network is the equilibrium. If $\frac{1}{3}2^{\alpha+1} \cdot 10^{3\alpha+1} < c < 10^{3\alpha+1}$ the equilibrium network is the following graph

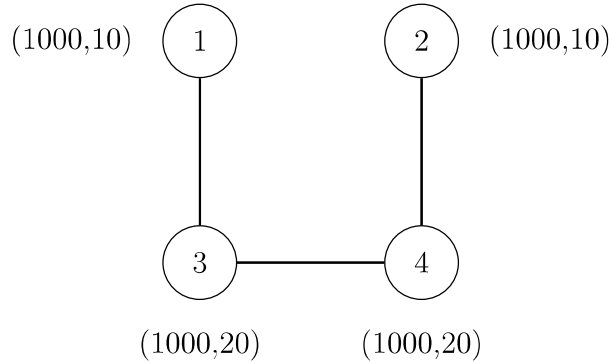


Figure 2.3: Example 2.3 equilibrium network $\frac{1}{3}2^{\alpha+1} \cdot 10^{3\alpha+1} \leq c \leq 10^{3\alpha+1}$.

Remark 2.4 Note the fact that in the equilibrium network, initially richer agents in terms of hierarchical information are poorer regarding total utility relative to initial poorer agents but with non hierarchical information.

Example 2.5 The following example shows a situation where segregation of an agent occurs,

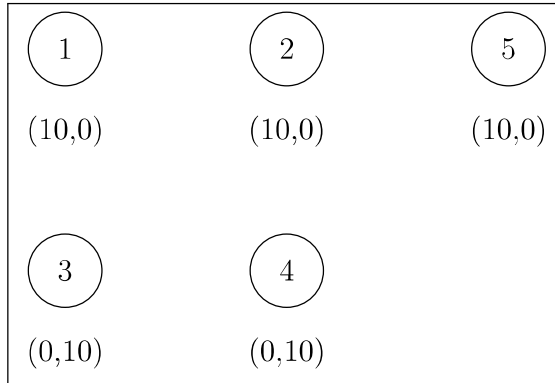


Figure 2.4: Example 2.5 initial state.

Table 2.3: Agent 1,2 and 5, expenditure, information and utility.

e	(h,n)	u
0	10,0	0
c	10,10	$10^{\alpha+1}$
2c	10,20	$2 \cdot 10^{\alpha+1}$
3c	20,20	$2^{\alpha+1} \cdot 10^{\alpha+1}$
4c	30,20	$2 \cdot 3^{\alpha} \cdot 10^{\alpha+1}$

Table 2.4: Agent 3 and 4, expenditure, information and utility.

e	(h,n)	u
0	0,10	0
c	10,10	$10^{\alpha+1}$
2c	10,20	$2 \cdot 10^{\alpha+1}$
3c	20,20	$2^{\alpha+1} \cdot 10^{\alpha+1}$
4c	30,20	$2 \cdot 3^{\alpha} \cdot 10^{\alpha+1}$

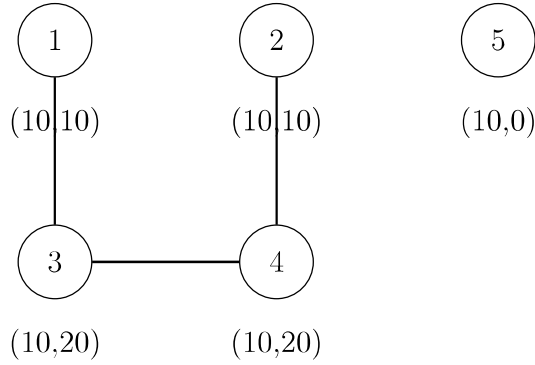


Figure 2.5: Example 2.5 equilibrium network $\frac{1}{3}2^{\alpha+1} \cdot 10^{\alpha+1} \leq c \leq 10^{\alpha+1}$.

Example 2.6 *The following example shows the effect of non homogeneous amounts of information among agents,*

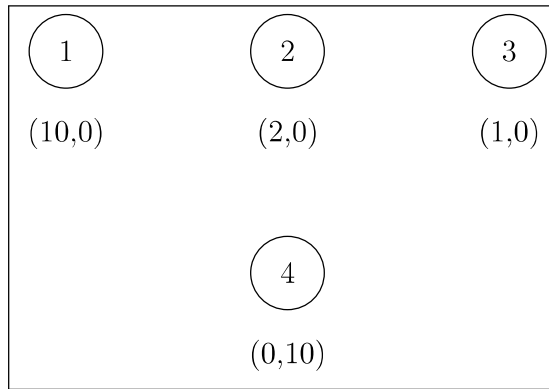


Figure 2.6: Example 2.6 initial state.

Table 2.5: Example 2.6: Agent 4 expenditure, information and utility.

e	(h,n)	u
0	10,0	0
c	10,10	$10^{\alpha+1}$
2c	12,10	$10 \cdot 12^\alpha$
3c	13,10	$10 \cdot 13^\alpha$

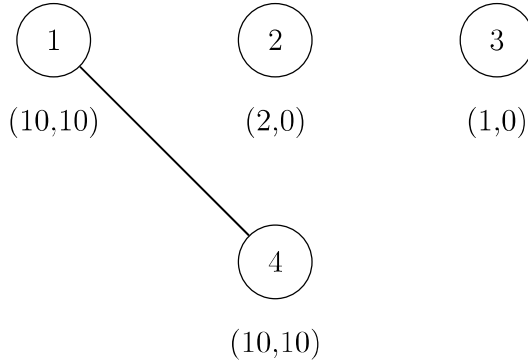


Figure 2.7: Example 2.6 equilibrium network $5 \cdot 12^\alpha \leq c \leq 10^{\alpha+1}$.

2.3 Equilibrium networks

Proposition 2.7 *If there is only one type for I_h and I_n and $I_h \cap I_n = \emptyset$, then they are willing to connect with all agents in I_n or with no one.*

PROOF. Let us take an agent initially endowed with only H information, so this agent starts with information $(h_0, 0)$. Suppose that this agent wants to connect with at least one agent with N information (each one with n as volume of N information), so we have

$$u(h_0, n) \geq c$$

Note that the assumption regarding agents having the same volume of information can be stated for fixed h_0 as

$$e_i(h_0, P_i n) = P_i c$$

$P_i \in \mathbb{N}$ the number of connections with agents with N information.

Given the fact that $u(h, \cdot)$ is convex and that the available volumes of N information are multiples of n we have, we restrict the proof to a countable set $\{n, 2n, \dots, Nn\}$. We can think of $u(h, \cdot)$ being defined as a convex function over a discrete space using the definition on Yüceer (2002).

$$\begin{aligned}
u(h_0, 2nt) &\leq u(h_0, 0)(1-t) + u(h_0, 2n)t && , t \in [0, 1] \\
&\Rightarrow \\
u(h_0, 2nt) &\leq u(h_0, 2n)t && , t \in [0, 1] \\
&\Rightarrow \\
u(h_0, n) &\leq \frac{1}{2}u(h_0, 2n) \\
&\Rightarrow \\
2u(h_0, n) &\leq u(h_0, 2n) \\
&\Rightarrow \\
2c &\leq u(h_0, 2n)
\end{aligned}$$

Note that if the agent is not indifferent, $u(h_0, n) > c$ then

$$2c < 2u(h_0, n) \leq u(h_0, 2n) \Rightarrow 2c < u(h_0, 2n)$$

So if the agent is willing to accept the first connection, he or she wants a second connection. Using the same argument we get the inductive step,

$$\begin{aligned}
u(h_0, N) &\geq Nc \\
&\Rightarrow \\
u(h_0, N+1) - u(h_0, N) &\geq c \\
&\Rightarrow \\
u(h_0, N+1) &\geq (N+1)c
\end{aligned}$$

So if an agent is willing to connect with one agent, they are willing to connect with everyone with N information. Analogously, if an agent does not want to connect with one agent with N information, he or she does not want to connect with any one.

Take an agent initially endowed with only type N information, it is clear that if no agent with H information (each one with h as volume of H information) want to connect with him or this first connection is not optimal for the agent with N information, then the agent with N information will not make any connection with other agents with this type of information, given that $u(0, \cdot) = 0$ and $u(\cdot, n)$ is concave. If this agent wants to connect with one agent of type H , then we are in the same problem as in the agent of type H . Note that in the case of indifference (if in the dimension of N the function is linear), having more N does not matter itself, but it increase the marginal value of type H "for free" given the supermodularity. \square

Proposition 2.8 *If there is only one type for I_h and I_n and $I_h \cap I_n = \emptyset$, there is $\bar{H} \in \mathbb{R}$ such that if $\sum_{i \in I} h_i > \bar{H}$ then every Pairwise Nash Equilibrium Network is a disconnected graph. \bar{H} is non decreasing on $\sum_{i \in I} n_i$.*

PROOF. The fact that every agent has the same volume of information or have only one type (also in the same volume for all agents) implies that the expenditure function has the following form,

$$e_i(P_i^h h + h_0, P_i^{nh} n + n_0) = (P_i^h + P_i^{nh})c, \quad P_i^{nh}, P_i^h \in \mathbb{N}$$

$\{I_n\}_{n \in N}$ is a finite family of sets implies there is a maximum amount of information an agent can get, let us call this value (h_{max}, n_{max})

Given the fact that for each $n \in N$ $u_i(\cdot, n)$ is strictly concave, we have that there is $\bar{P}_i^h(n)$ such that for $P_i^h > \bar{P}_i^h(n)$ we have

$$u_i((P_i^h + 1)h + h_0, n) - u_i(P_i^h h + h_0, n) < e_i((P_i^h + 1)h + h_0, n) - e_i(P_i^h h + h_0, n)$$

Therefore, clearly no agent is willing to have more than $\bar{P}_i^h(n)$ links with H information owners and the same for $\bar{P}_i^h(n_{max})$, then take $\bar{H} = \sum_{i \in I} \bar{P}_i^h(n_{max}) \cdot h$. If $\sum_{i \in I} h_i > \bar{H}$ there would be more H information than any possible demand for this on any network and therefore some H information owners would not be linked to any agent.

\bar{H} non-decreasing on $\sum_{i \in I} n_i$ comes from u being supermodular and Topkis's monotonicity theorem . □

Proposition 2.9 *If there is only one type for I_h and I_n and $I_h \cap I_n = \emptyset$, there is a link cost \bar{c} such that for every cost $c < \bar{c}$ in every Pairwise Nash Equilibrium network agents in I_n form a complete network between them. Also there are cost levels $c_j < \bar{c}$ such that every agent in I_n is connected to j agents in I_h .*

PROOF. From proposition 2.7 we know that agents want to connect with all agents with type N information or no one, the limit \bar{c} could be taken as the cost that makes an agent to be indifferent regarding a connection. If they are willing to have some H information then all agents in I_n want the same amount and because I_h are willing to connect with all or no one in I_n , agents on the latter set get the same amount of connections to I_h . If agents in I_n are willing to connect then all agents with information of type N will be linked, creating a complete network between them. □

The basic unit in this network may look like the figure below, where $I_n = \{5\}$, $I_h = \{0, 1, 2, 3, 4\}$ for different cost levels we will have a different number of I_n connected to the agents on I_n , but the same for all I_n .

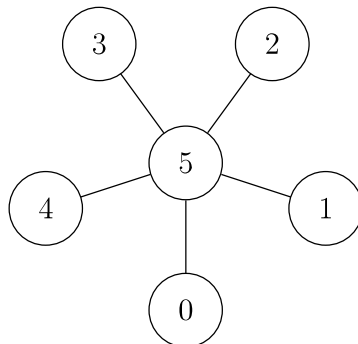


Figure 2.8: PNE networks example proposition 2.9.

Example 2.10 Consider the following example of PNE where $I_n = \{0, 1, 2, 3, 4\}$ and $I_h = \{5, 6\}$

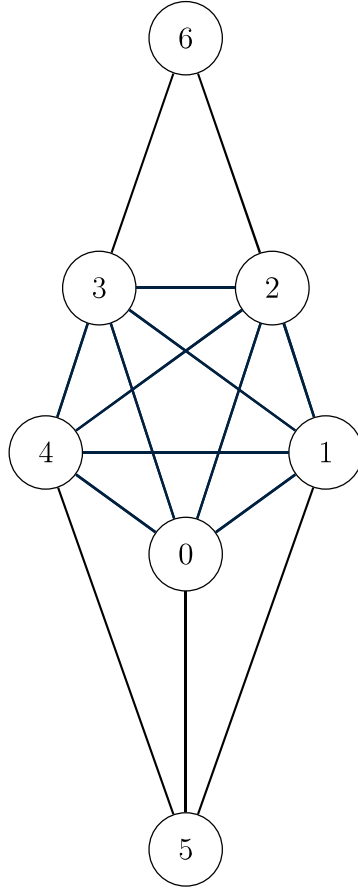


Figure 2.9: Pairwise Nash Equilibrium Network. I_n complete network. Example 2.10.

Example 2.11 *The following network may be Pairwise-Nash, $I_n = \{2, 3\}$, $I_h = \{1\}$.*

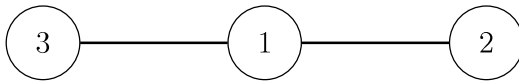


Figure 2.10: PNE network. Example 2.11.

Proposition 2.12 *If $I_h \cap I_n = \emptyset$ in any Pairwise Network Equilibrium $g \subseteq g_N$, $\forall i \in I_n \cup I_h \exists \underline{h} \in \mathbb{R}$ such that $l_{ij} \in g \Rightarrow h_j \geq \underline{h}$*

PROOF. In order to be Pairwise Nash it should be Nash, so $mu_i(g, l_{ij}) \geq 0$ this implies that $u_i(h_i + h_j, n_i) - u_i(h_i, n_i) \geq c$ given the fact that u is increasing we know there is a lower bound for h_j so the last inequality is satisfied. \square

Proposition 2.13 *If $I_h \cap I_n = \emptyset$ in any Pairwise Network Equilibrium $g \subseteq g_N$, $\forall i, j \in I_h$*

such that $\exists k \in I_n, l_{ik}, l_{jk} \in g, \exists \delta_k \in \mathbb{R} |h_i - h_j| \leq \delta_k$. δ_k is non-increasing on n_k .

PROOF. Suppose not, then the difference between h_i and h_j may be very large, but one can take a difference high enough so $mu_k(g, l_{ik}) < 0$ or $mu_k(g, l_{jk}) < 0$. Thus information of type H can not be highly concentrated on equilibrium networks. Given supermodularity differences are amplified with n_k , therefore δ_k is non-increasing on n_k . \square

Remark 2.14 *Agents in I_h compete for information of type N , the agents with the greatest amount of H set the connections of other agents from I_h because agents with much less volumes of H can not be connected to the same agent from I_n as the endowed with highest values of H (if there is enough variance on types).*

2.4 Efficient networks

Given the convexity on the dimension of type N information, it may be efficient to concentrate this information. Given the fact that the utility function is strictly concave on the information of type H , it may be inefficient to do the same with this.

Proposition 2.15 *If there is only one type for I_h and I_n and $I_h \cap I_n = \emptyset$, there is a link cost \bar{c} such that for every cost $c < \bar{c}$ in every efficient network agents in I_n form a complete network between them. Also there are cost levels $c_j < \bar{c}$ such that every agent in I_n is connected to j agents in I_h and every agent in I_h is connected to all agents in I_n or no agents.*

PROOF. Given the increasing returns on information of type N it is clear that agents in I_n will form a complete network between them for any low enough link cost (such that the network is not empty).

Given the decreasing returns on type H information, there is a maximum amount of links that agents are willing to make with agents on I_h . For each cost there is a maximum of links (we call it j) to make until the marginal utility of H is lower than c_j . Agents on I_h always wants connections with agents in I_n (unless the network is the empty one). This give us that for every cost $c_j < \bar{c}$ agents in I_n make connections to j agents in I_h .

Given an agent in I_h if a connection with an agent in I_n increase the total utility more than twice the link cost, then any other additional connection with an agent in I_n will increase the total utility more than the previous connection and therefore every agent in I_n will connect that agent in I_h in an efficient network. If there is no enough N information such that the network can reach that pivotal agent, then the link cost will remain greater than marginal utility and no connection with agents in I_n will be made. \square

Proposition 2.16 (Meritocracy) *In every efficient network $g \in G$, information of type N is shared firstly with agents with the highest amounts of H type information.*

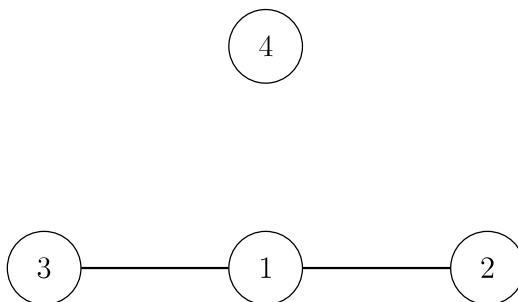
Given $i \in H, j \in N$ such that $l_{ij} \notin g \Rightarrow \exists k \in H, h_i \leq h_k \vee j$ is disconnected.

PROOF. We do not consider the trivial case (empty network). If $l_{ij} \notin g, i \in H, j \in N$ then it is clear that there is a better option for the society (given supermodularity assumption), this is there is an agent with a greater amount of type H information. If not, simply the amount of type H information is not enough to make the total marginal utility greater than twice the link cost. \square

2.5 Equilibrium networks and efficiency

Further research is needed relating equilibrium networks and efficient networks. Some examples are given below.

Example 2.17 *The following network may be Pairwise Nash, $I_n = \{1\}, I_h = \{2, 3, 4\}$ and $h_2 = h_3 < h_4$.*



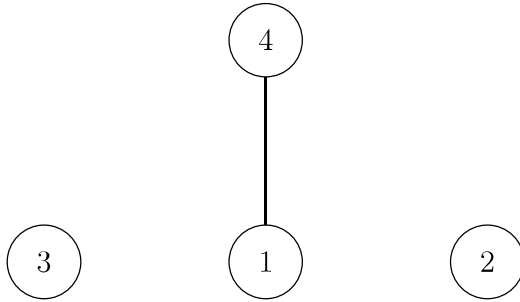


Figure 2.11: PNE networks, first example efficiency. Example 2.17.

Both networks above may be Pairwise Nash equilibrium but the total utility is different, so depending on the link cost and the information endowments one of this networks may be efficient.

The following example may clearly show the point

Example 2.18 *The following network may be Pairwise Nash, $I_h = \{5, 6, 7, 8, 9\}, I_n = \{0, 1, 2, 3, 4\}$.*

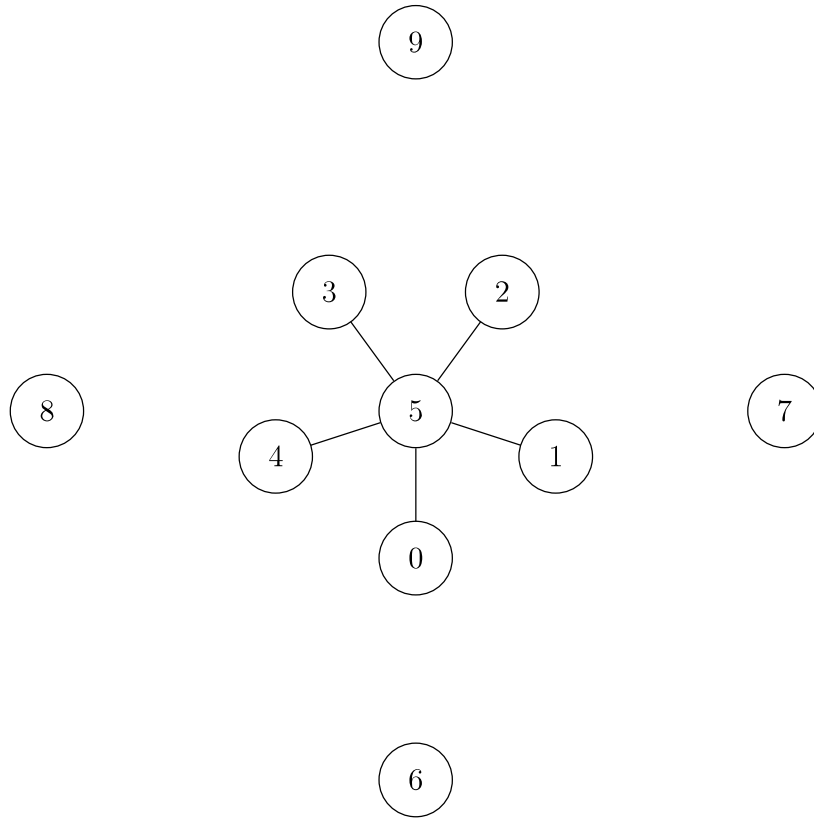


Figure 2.12: PNE concentrated information. Example 2.18.

The following network is a PNE where everyone gets some information

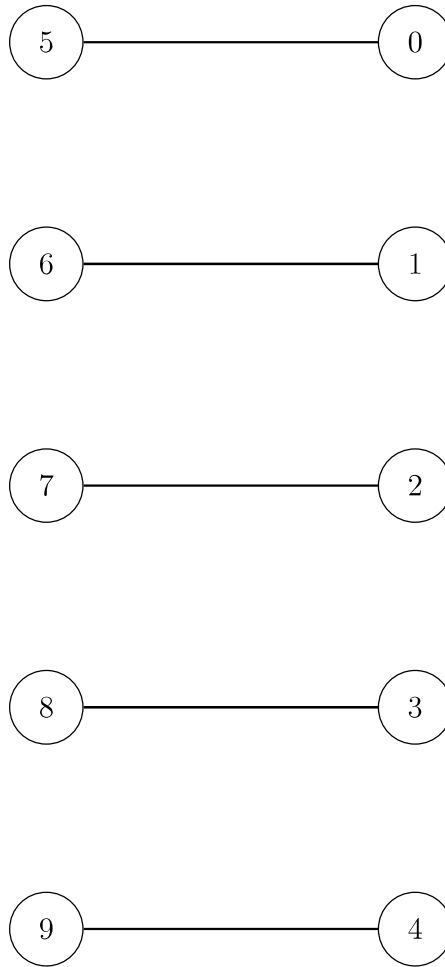


Figure 2.13: PNE bipartite network. Example 2.18.

Again, given the convexity on N dimension, it may be efficient to concentrate the information on one agent from I_h .

Conclusion

In this work we have shown that without deep differences between agents, that is, with agents sharing exactly the same utility function, groups with similar agents arises endogenously. In certain cases, agents with information of type N form groups, and those with information of type H in general are close in the equilibrium networks to agents with similar levels of this information. This is the contribution, the difference on the type of information explains the role on the network. Agents theoretically can be on any role if they get the right information, this could be interesting for a dynamic version because agents share the same utility function.

The convexity of information of type N implies that concentration of this information may increase the marginal value, but on the other hand concentration of information of type H may be inefficient given its strict concavity on this dimension. This is an important trade off in social interactions. In PNE network agents in N play a central role, while in efficient networks agents in H seem to play that central role. We call it meritocracy and gives some interesting first ideas about the difference between equilibrium networks and efficient networks, and the impact on welfare that not meritocracy-based social networks may have.

As continuation of this work it could be interesting to explore the effect of differences on link cost, for instance, agents may agree to pay each one a fraction of the link cost. Also, allowing for more than one link transmission.

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