Combining Fractional Order Operators and Adaptive Passivity-Based Controllers: An Application to the Level Regulation of a Conical Tank

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Abstract: In this paper, fractional order (FO) operators and adaptive passivity-based controllers (APBC) are combined. The main aim is to explore possible advantages of combining FO operators and APBC over the use of integer order APBC. The proposed technique is experimentally applied to the level regulation of a conical tank. For comparison purposes, integer order (classical) Proportional Integer (PI) controller and FO-PI controller are also experimentally applied and studied. Results showed that the design of FO-PI controller is simpler than classical PI controller design, since it uses a single set of parameters for the whole operational range of the plant; whereas three different set of parameters were adjusted for the classical PI controller. The design of FO-APBC and APBC is similar, and simpler than FO-PI and classical PI controllers design, not needing to know the plant parameters. Moreover, the proposed FO-APBC is more robust than APBC under plant parameter variations.

Keywords: Fractional order controller, adaptive passivity-based control, conical tank.

1. INTRODUCTION

Conical tanks are non-linear systems used in industrial applications for settlement and storage due to its shape that facilitates drainage. Different control techniques have been reported in the technical literature to control conical tanks.

From the integer order controllers viewpoint applied to conical tanks, a controller that combines fuzzy logic (Fuzzy) and proportional integer (PI) control strategies is used in (Madhubala T.K. et al., 2004; Betancor C.S. et al., 2013; Arivalahan R. et al., 2012; Ganesh Ram A. and Abraham Lincoln S., 2013). In these papers the values of the PI settings depend on the level range. An artificial neural network (ANN) PI controller, following similar PI setting ideas as the previous works, was used in (Dhanalakshmiand Vinodha R., 2013; Srivignesh N. et al., 2012). Experimental validation of an adaptive PI controller is described in (Vijayalakshmi S. et al., 2014).

To level control of conical tanks, a perceptron ANN controller was proposed in (Nandhini E. and Balaji M., 2015), a sliding mode controller (SMC) was applied in (Teena T. and Hepsiba, 2014) and in (Beena N. et al., 2015), and a non-adaptive passivity-based controller (PBC) in (Chandrasekar P. and Ponnusamy L., 2013, 2014).

From the FO controller viewpoint, in (Priya C. and Lakshmi P., 2011) a FO-PI controller was applied to a conical tank of a total height of 0.4 m for the level regulation at 0.3m and 0.37 m, obtaining better results than classical PI controllers. In (Djari A. et al., 2014), the use of FO-SMC was explored, but applied to two nonlinear systems, and better results than classical SMC for certain sliding surfaces were obtained. In (Rebai A. et al., 2015) a FO-Fuzzy PID controller was proposed for a piezoelectric actuator.

In this paper, a comparative experimental study between FO-APBC and APBC applied to the level regulation of a conical tank is performed. The main aim is to explore for possible advantages of combining FO operators with APBC as a simple and adaptive technique for this application. The whole operational range was studied, including a low level operating range (under 0.3 m), medium level (from 0.3 to 0.45 m) and high level (over 0.45 m), for a a 0.80 m high conical tank with maximum diameter of 0.34 m

The APBC design proposed here is based on (Travieso-Torres J.C. et al., 2007) for tracking and regulation. This was an extension of results from (Travieso-Torres J.C. and Duarte-Mermoud M.A., 2004) for stabilization of nonlinear systems, dealing with an Error Model of type 2 according to (Narendra K.S. and Annaswamy A.M., 2005).

In Section 2 of this paper, the plant is described through a mathematical model detailed in Appendix A. The applied controllers are presented in Section 3, using the design procedure shown in Appendixes B to E. The experimental results obtained are presented, compared and discussed in Section 4. Finally, in Section 5 some conclusions are drawn.

2. PLANT DESCRIPTION

2.1 Conical Tank Pictures and Control Diagram

A picture of the conical tank plant used in this study, located in the Automática Laboratory at the Electrical Engineering Department of the University of Chile, is shown in Fig. 1.

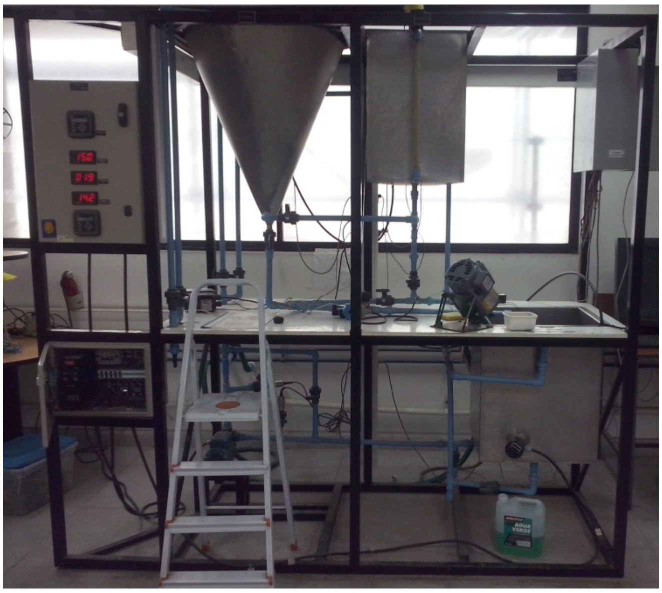

Fig. 1. Picture of the conical tank plant.

The control diagram of the plant is described in Fig. 2. It is composed by a lower level storage tank receiving water from the conical tank of total height *H* and maximum radius *R*. A recirculation pump fed by a variable speed drives (VSD), sends the water back to the conical tank allowing to control the level. The discharge valve opening is adjusted, modifying the discharge flow.

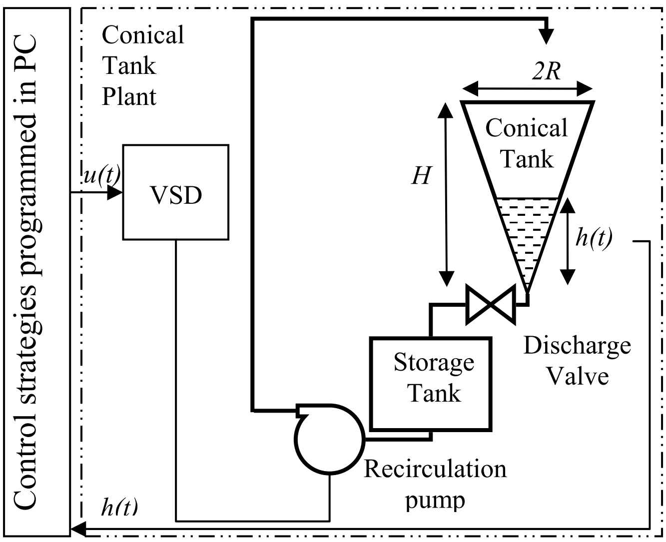

Fig. 2. Control diagram of the conical tank plant.

Control strategies are programmed in a PC reading the tank level *h(t)* and setting the control input *u(t)* through a SNAP-UP1-ADS programmable controller from an Opto 22 device with a sampling time of *50 ms*. The level range is measured using an hydrostatic level sensor, series HDP-8 by Indian Instruments Co., wired to the interface. The control input is applied to the reference analog-input of the VSD, model VLT 2800 by Danfoss, feeding fluid through a recirculation pump of 336 Watts.

The control strategies studied herein were implemented in Simulink/Matlab, version R2013, running on a PC.

2.2 Mathematical Model of the Plant

The nonlinear mathematical model of the conical tank plant from Fig. 2 is described by:

$$\dot{h}(t)=\frac{H^2}{\pi R^2}C_2 h(t)^{-2}-\frac{H^2}{\pi R^2}C_3 h(t)^{-\frac{3}{2}}+\frac{H^2}{\pi R^2}h(t)^{-2}C_1 u(t) \tag{1}$$

A detailed derivation of the model plant (1) is performed in Appendix A, together with a description of their variables and parameters.

3. CONTROLLERS DESIGN

All four controllers studied; IO-PI, FO-PI controller, APBC and FO-APBC, are presented in this section based on designs contained in Appendixes B to E, respectively.

3.1 Classical PI Controller Design

The classical PI controller designed in Appendix B was first used for the conical tank plant. The following design methodology was used in this case: The model (1) was first linearized; Then, experimental tests were done to calculate plant parameters; Based on these parameters the linearized model was evaluated in three different operating points (low, medium and high); Then, three different PI initial settings were obtained using the Root Locus Method (RLM); Finally the PI parameters were fine-tuned experimentally, obtaining the final structure that follows

$$u(t)=K_P e(t)+K_I\int e(t)dt$$

$$\text{with}\begin{cases} K_P=35, K_I=0.6 & \text{for } h(t)\geq 0.45\text{m} \\ K_P=25, K_I=0.45 & \text{for } 0.3\text{m}\leq h(t)<0.45\text{m} \\ K_P=21, K_I=0.3 & \text{for } h(t)<0.3m \end{cases} \tag{2}$$

3.2 FO- PI Controller Design

The following version of a FO-PI controller was designed and applied to the level regulation of the conical tank having the form:

$$u(t)=K_P e(t)+\frac{K_I}{\Gamma(\alpha)}\int_0^t \frac{1}{(t-\tau)^{1-\alpha}}e(\tau)d\tau \tag{3}$$

with K_P*=50*, K_I*=1*, and α=0.19. See Appendix C for details.

The design methodology is now simplified as compared to the classical PI since only one setting is obtained for the whole operational range, as explained in Appendix C. Here a completely known nonlinear model (1), with the plant parameters experimentally calculated, is controlled by the FO-PI controller; Using the Particle Swarm Optimization (PSO) technique with starting points of $P=0.35$, $I=0.6$ and $\alpha=0.02$ and a fitness function of integral of the squared error between $h(t)$ and h^*, the initial values of K_P=40, K_I=0.6 and α=0.30 are obtained; and after an experimental fine tuning, taking some ideas from (Bhaskaran et al., 2007), the values K_P=50, K_I=1.0 and α=0.19 were set for a faster response.

3.3 Classical APBC Design

Based on the technique proposed in (Travieso-Torres J.C et al., 2007) the following control law was designed to preserve the stability of the level error of model plant (1)

$$u(t) = h(t)^2 \theta(t)^T \omega(t) \tag{4}$$

with the information vector $\omega(t) \in \Re^3$ defined as $\omega(t) = \left[h(t)^2 \quad h(t)^{-3/2} \quad u(t)^p \right]^T$, the auxiliary control variable $u(t)^p = -Ke(t) \in \Re$, and $\theta(t) = \left[\theta_1(t) \quad \theta_2(t) \quad \theta_3(t) \right]^T \in \Re^3$ the controller parameters adjusted through the adaptive law

$$\dot{\theta}(t) = -\delta e(t)\omega(t) \quad \in \Re^3 \tag{5}$$

The values $K=0.001$ and $\delta=0.25$ were finally obtained according to Appendix D.

The design methodology of the APBC is simpler than previous PI controllers, avoiding the computation of the plant parameters H, R, C_1, C_2 and C_3 since they are treated as unknown.

3.4 FO-APBC Designed

A version FO-APBC was also considered in this study, using the same control law (4) with its information vector and auxiliary control variable but with an adaptive law given by:

$$D^\alpha \theta(t) = -\delta e(t)\omega(t) \in \Re^3 \tag{6}$$

where $D^\alpha \theta(t)$ denotes the Caputo derivative of order α. (See Aguila-Camacho et al., 2013). The controller parameters $K=0.001$, $\Gamma=0.15$, and $\alpha=0.05$ were used and chosen as explained in Appendix E.

4. EXPERIMENTAL RESULTS

In this Section the experimental results obtained by applying the four controllers previously designed are presented. The set point curve used to compare the performance considers step changes every 600 seconds having values of: 0.15, 0.25, 0.35, 0.45, and 0.55 m.

4.1 Comparative Results Analysis

Experimental results obtained considering the discharge manual valve opening set to 45°, and the controller setting from Section 3, are discussed in this Section.

In Fig. 3 the results obtained after applying the classical PI controller to the conical tank are shown. The steady state error tends to zero at *300 s* with a maximum overshoot of *10%*. A deviation can be observed when changing the PI setting from middle level to high level range at *2250 s*.

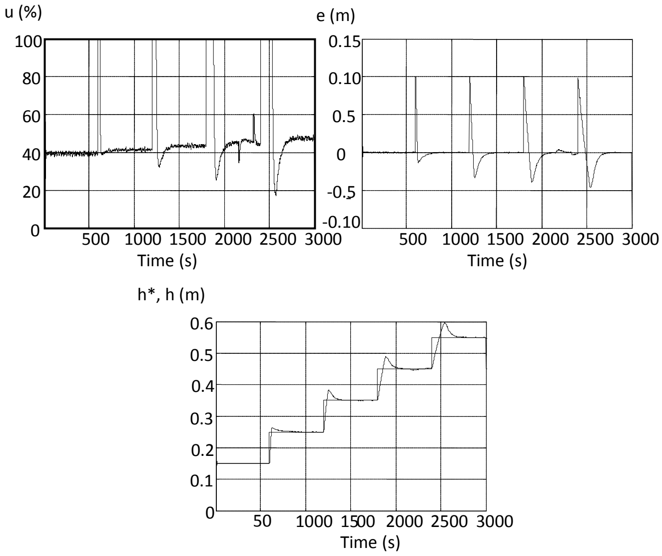

Fig. 3. Results when applying the classical the PI controller.

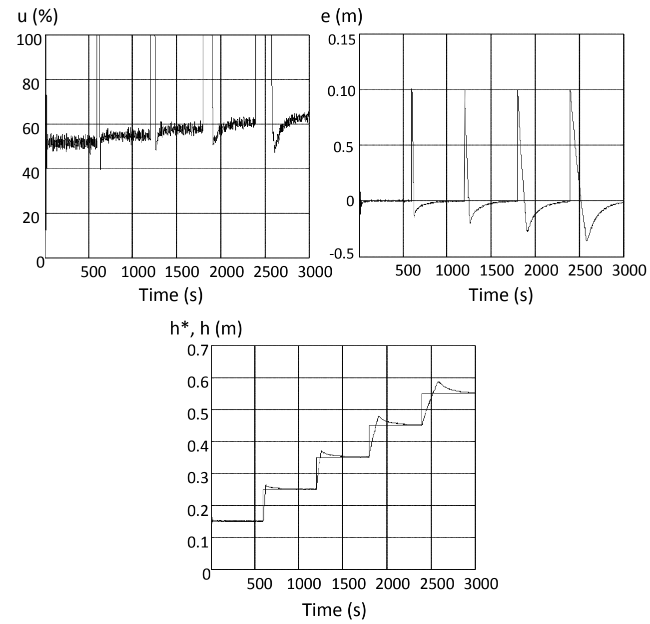

Fig. 4. Results when applying the FO-PI controller.

After applying the FO-PI controller to the conical plant, the results described in Fig. 4 were obtained. Here the steady state error tends to zero at *400 s*, slower than in Fig. 3, with a similar maximum overshoot of 10%.

The results obtained using the APBC to the conical tank are shown in Fig. 5. In this case the steady state error tends to zero at *150 s* and without overshoot. The result obtained for the level reference of *0.15 m*, which is the closer to zero, is slower than the results for higher reference levels.

Finally, in Fig. 6, the results obtained applying the FO-APBC to the conical tank are shown. From Fig. 5 and Fig. 6, FO-

APBC and APBC show similar results amongst them, with the adaptive parameters $\theta(t)$ changing smoothly from one operating point to another.

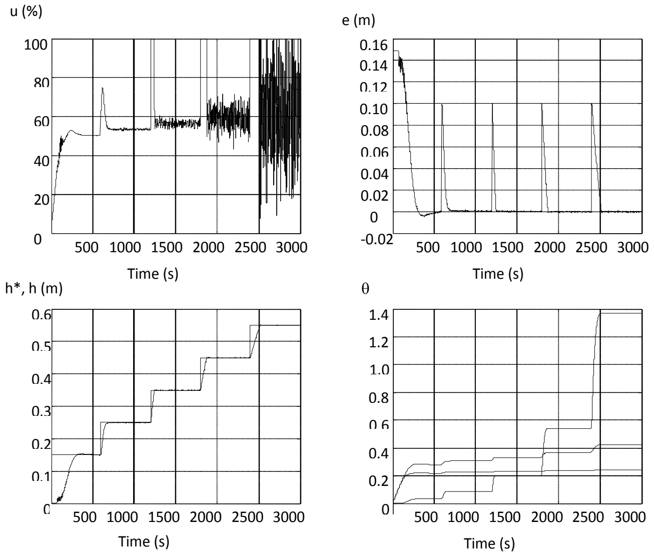

Fig. 5. Results when applying the APBC.

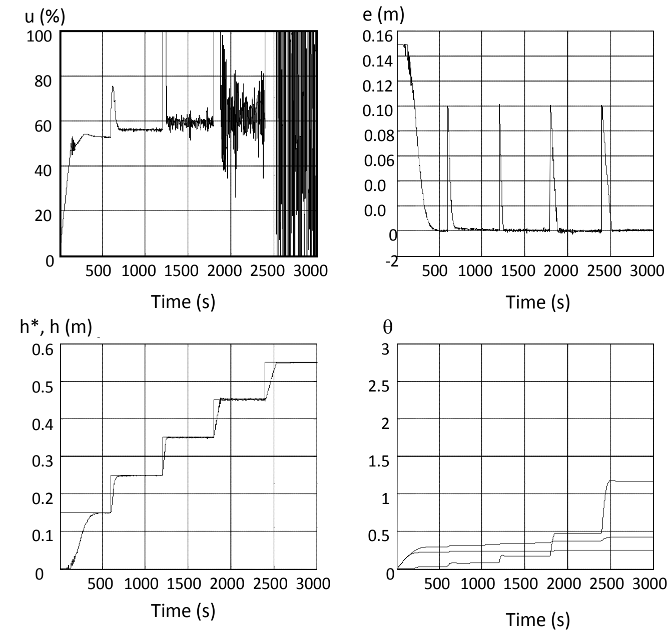

Fig. 6. Results when applying the FO-APBC.

The responses from APBC and FO-APBC shown in Fig. 5 and Fig. 6 respectively are faster than the results from the classical PI and FO-PI controllers shown in Fig. 3 and Fig. 4, and did not exhibit overshoot.

4.2 *Robustness Analysis under Parameter C_3 Variation*

Experimental results when the parameter C_3 was increased in *52.5%* (actual $C_3=21.84$), are discussed in this Section. This variation was obtained after changing the set of the discharge manual valve opening from *45°* to *37°,* which are the two usual operating positions. The four controller settings are the same as in Section 3 and the same set point as in Section 4.1 is used.

The robustness results obtained when using the classical PI controller under parameter C_3 variation are shown in Fig. 7.

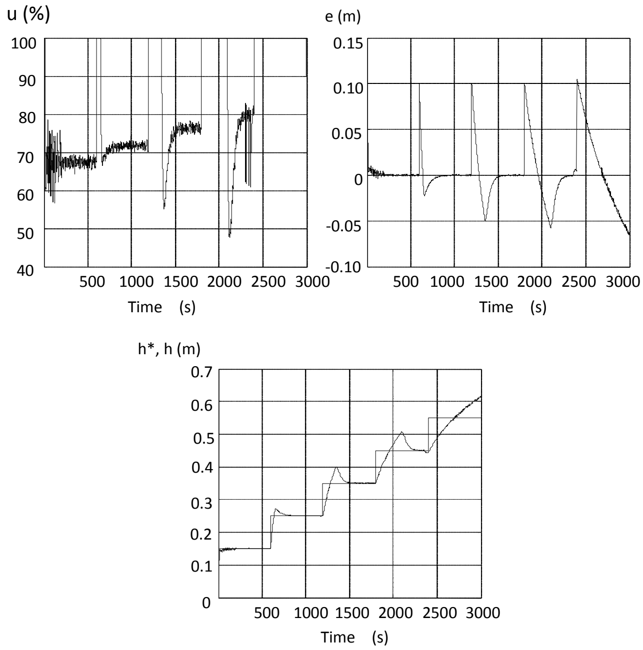

Fig. 7. Robustness of the Classical PI controller.

In the case of the FO-PI controller with a change in the plant parameter C_3, the results of Fig. 8 were obtained.

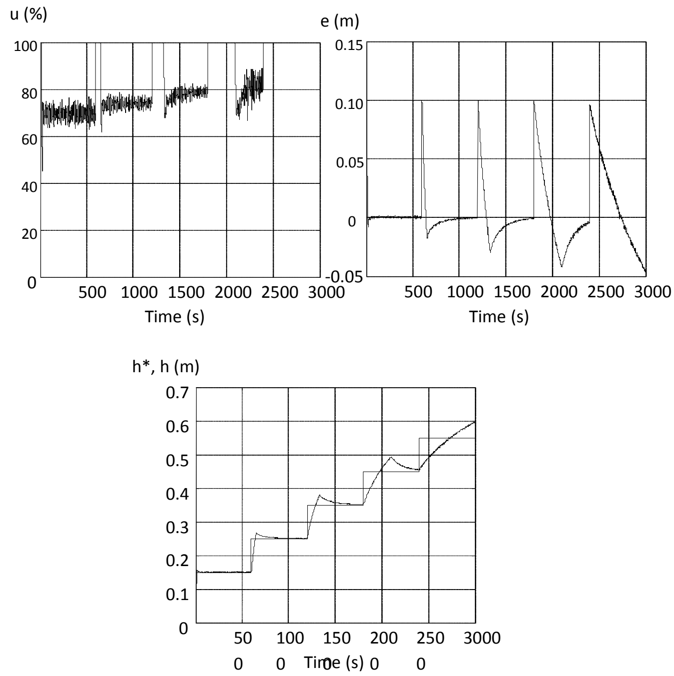

Fig. 8. Robustness for the FO-PI controller.

In Fig. 7 it can be seen that for the level range under *0.5 m* the steady state error tends to zero at *450 s* with a maximum overshoot of *12%,* slower than in previous test shown in Fig. 3. It can be seen in Fig. 8 that the controlled plant behaves slower than the previous case shown in Fig. 4. FO-PI has a kept the maximum overshoot of 10%. Also, for the highest reference step of *0.5 m*, the classical PI and the FO-PI controllers were not capable of regulating the level in less than *600 s* with the control output *u(t)* completely saturated.

The results obtained applying the APBC with the change in the plant parameter C_3 are shown in Fig. 9. The performance is slower than the results shown in Fig. 5.

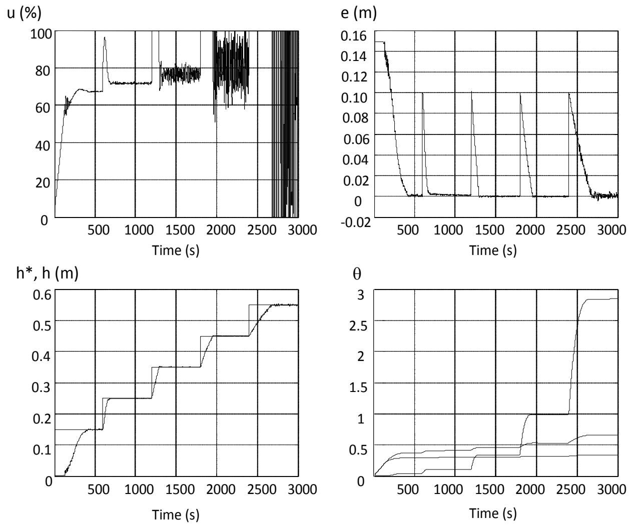

Fig. 9. Robustness of the APBC

Finally, the results obtained with the FO-APBC under the change in the plan parameter C_3, are shown in Fig. 10, having a similar behaviour to the results from Fig. 6.

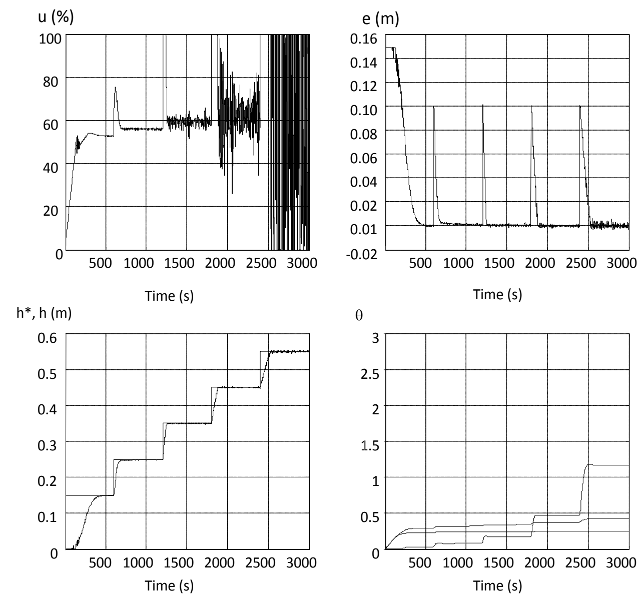

Fig. 10. Robustness of the FO-APBC.

The APBC, gets slower under the increasing of the plant parameter C_3, whereas the FO-APBC exhibits a more robust performance than APBC. In contrast to Fig. 7 and Fig. 8, in the APBC and FO-APBC cases the steady state error tends to zero for the whole operational range, and with no overshoot.

5. CONCLUSIONS

In this paper, a comparative experimental study between FO-APBC and APBC applied to the level regulation of a conical tank, was performed exploring the whole level operational range. Also classical PI controller and FO-PI controller were considered for comparison purpose and to obtain experience in the adjustment of fractional order controller.

Results showed that the design of FO-PI controller is simpler than the design of classical PI controller using a single set of parameter for the whole operational range; whereas three different set of parameters were adjusted for the classical PI controller for the level regulation at the operational range low, medium, and high. In both cases, an overshoot was always presented. Under variation of the plant parameter C_3, for the highest reference step of 0.55 m, these PI controllers were not capable of regulating the level in less than 600 second with the control output *u(t)* completely saturated.

An adjustment in the design of FO-APBC and APBC was made to avoid division by zero near h(t)=0, allowing the application of the APBC theory from Travieso-Torres, J.C.,et al (2007). After applying these adjusted adaptive controllers a faster response without overshoot and a better degree of robustness under parameter C_3 variations compared to both PI controllers, was obtained for a level range over 0.2 m. In both cases the adaptive parameters $\theta(t)$ changed smoothly from one operating point to another.

With the exception of PI controllers under parameter C_3 variation and the highest operating point, all controllers assure that steady state error tends to zero. This happens even when control signal is saturated in its upper and lower levels.

After increasing the plant parameter C_3, the results from APBC were slower than its performance test considering the original parameter value. Instead, a more robust result was obtained from FO-APBC under this variation of the plant parameter, improving APBC. Thus, advantages of combining FO controllers and APBC strategies were found, and the main aim of this research was achieved.

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APPENDIXES

A. Mathematical Model of the Conical Tank Plant

The mathematical model of the conical tank plant was derived by performing a mass balance of the water inside the tank and the inflow and the outflow. For further details the reader is referred to (Travieso-Torres et al., 2016; Jáuregui et al., 2016).

Assuming a constant water density ρ and performing a water mass balance for the whole system, we can write $\dot{V}(t) = Q_{in}(t) - Q_{out}(t)$, where $\dot{V}(t)$ is the speed of the water volume variation at time t, $Q_{in}(t)$is the volumetric input flow at time t (inflow) and $Q_{out}(t)$is the volumetric output flow at time t (outflow) both expressed in cm^3/s.

From Fig. 2 we see that the inflow is driven by the pump rotation velocity $u(t)$ (considered as the control signal in this problem) and can be modelled as $Q_{in}(t) = C_1 u(t) + C_2$. The velocity $u(t)$ is related to the speed adjusted through the variable frequency drive feeding the pump, expressed as a percentage in the range $u(t) \in [0\% , 100\%]$.

With respect to the outflow $Q_{out}(t)$, this is produced just for gravity action, and using the Torricelli's law it is proportional to the square root of the pressure difference before ($P_{in} = P_0 + \rho g h(t)$) and after ($P_0$) the discharge valve, where P_0 is the atmospheric pressure. Thus, the outflow will be modeled as $Q_{out}(t) = C_3 \sqrt{h(t)}$ finally obtaining the process mass balance equation $\dot{V}(t) = C_1 u(t) + C_2 - C_3 h(t)^{\frac{1}{2}}$. Constants C_1, C_2, and C_3 depend on density, gravity acceleration, geometry, valve shape and materials, and they are expressed in cm^3/s, cm^3/s and $cm^{5/2}/s$, respectively.

Based on the geometry of the conical tank the water volume inside the tank can be expressed as $V(t) = \pi/3 \cdot r(t)^2 h(t)$, in cm^3, where the radius $r(t)$ and the height $h(t)$ of the water level are related through $r(t) = (R/H)h(t)$, both expressed in cm. H is the total tank height and R is the tank maximum radius expressed both in cm. Then, the volume can be re-expressed as $V(t) = (\pi/3) \cdot (R/H)^2 h(t)^3$ and its time derivative is $\dot{V}(t) = \pi \cdot (R/H)^2 h(t)^2 \dot{h}(t)$. Replacing this equation in the process mass balance equation previously obtained the following equation is obtained $\pi \left(\frac{R}{H} \right)^2 h(t)^2 \dot{h}(t) = C_1 u(t) + C_2 - C_3 h(t)^{\frac{1}{2}}$ which can be rewritten as

$$\dot{h}(t) = \frac{H^2}{\pi R^2} C_2 h(t)^{-2} - \frac{H^2}{\pi R^2} C_3 h(t)^{-\frac{3}{2}} + \frac{H^2}{\pi R^2} h(t)^{-2} C_1 u(t)$$

This last equation represents the dynamical model of the conical tank plant, which has a highly nonlinear nature.

B. Classical PI Controller Design

B.1 For the design of the PI controller the plant parameters H, R, C_1, C_2 and C_3 need to be known. Furthermore, the nonlinear system is linearized around the equilibrium point given by the ordered pair (h_{ep}, u_{ep}) through the following first-order Taylor approximation:

$$\dot{e}_{ep}(t) = A e_{ep}(t) + B\left(u(t) - u_{ep}\right)$$

where $e_{ep}(t) = h(t) - h_{ep}$ is the level deviation with respect to the equilibrium value h_{ep}, with

$$A = \left.\frac{\partial f(h,u)}{\partial h}\right|_{h_{ep},u_{ep}} = -2\frac{\mathrm{H}^2}{\pi R^2 \mathrm{h}_{ep}^3}\left(C_2 + C_1 u_{ep}\right) + \frac{3}{2}\frac{\mathrm{H}^2}{\pi R^2 \mathrm{h}_{ep}^{\frac{5}{2}}} C_3$$

$$and\ B = \left.\frac{\partial f(h,u)}{\partial u}\right|_{h_{ep},u_{ep}} = \frac{\mathrm{H}^2}{\pi R^2 \mathrm{h}_{ep}^2} C_1$$

with $\dot{h} = f(h,u)$ taken from the right hand of model (1) with u_{ep} the input associated to h_{ep} at the equilibrium point defined by $u_{ep} = (C_3 h_{ep}^{\frac{1}{2}} - C_2)/C_1$.

B.2 The parameters A and B of the linearized model are now calculated. Plant parameters were experimentally determined or measured as explained in detail in (Travieso-Torres et al., 2016; Jáuregui et al., 2016) and briefly described below.

The total tank height H and the tank maximum radius R were measured, giving H=80 cm and R=17 cm, respectively. Later, an experiment is performed when $Q_{out}=0$ (drainage valve closed) and measuring the height $h(t)$ of the tank along the time, while adding a constant inflow Q_{in} by keeping constant the frequency of the drive. This experiment is repeated twice for two different speeds u_1 and u_2 of $u(t)$. Two different set of data are obtained from which $C_1=3.9\ cm^3/s$ and $C_2=-70.8\ cm^3/s$ were obtained.

Parameter C_3 was also experimentally computed by filling the conical tank up to certain level, and then analyzing the discharge without input flow ($Q_{in}=0$) for a fixed position of the discharge valve, while recording the variation of height and time during *485* seconds. This test was done twice, first with the opening of the discharge valve set to *45°* and then set to *37°*. The values obtained for C_3 were $C_3=14.3\ cm^{5/2}/s$ and $C_3=21.8\ cm^{5/2}/s$, respectively.

B.3 Then three equilibrium points; low *(0.225 m, 35.3 %)*, medium *(0.375 m, 40.3 %)* and high *(0.525 m, 44.4%)*, are defined. And the following linearized mathematical model of the conical tank was obtained for these operating points:

$$\dot{e}_{ep}(t) = \begin{cases} \text{-0.0051} e_{ep}(t) + 0.0132(\mathrm{u(t)}- 35.3) & h(t)<0.30 \\ \text{-0.0014} e_{ep}(t) + 0.0048(\mathrm{u(t)}-40.3) & 0.30 \le h(t) < 0.45 \\ \text{-0.0006} e_{ep}(t) + 0.0024(\mathrm{u(t)}-44.4) & 0.45 \le h(t) \le 0.60 \end{cases}$$

B.4 Then, a PI controller is designed to control around each equilibrium point. The PI settings were found using the Root Locus method (RLM) available in Matlab imposing, for the closed-loop response under a step reference change, with a settling time of *200 s* and a *10%* of maximum overshoot (MO) obtaining

$$K_P = 21, K_I = 0.30 \quad \text{for} \quad h(t) < 30 \text{ (low)}$$
$$K_P = 25, K_I = 0.45 \quad \text{for} \quad 30 \le h(t) < 45 \text{ (medium)}$$
$$K_P = 35, K_I = 0.60 \quad \text{for} \quad 45 \le h(t) \le 60 \text{ (high)}$$

B.5 Finally, after using the previous settings, and applying the PI controller to the conical tank plant with $e(t) = h(t) - h^*$ the level error around the desired operating point h^*, a settling time of *350 s* with a MO of *8%* was obtained. Thus an experimental fine-tune was needed to get closer to the required performance reaching a settling time of *250 s* with *10%* of MO with following values

$K_P = 35, K_I = 0.6$ for $0.45m \le h(t) < 0.7m$

$K_P = 25, K_I = 0.45$ for $0.3m \le h(t) < 0.45m$

$K_P = 21, K_I = 0.3$ for $0.1m \le h(t) < 0.3m$

C. FO-PI Controller Design

C.1 It is use the nonlinear model obtained in Appendix A with plant parameters calculated in Appendix B. In contrast to the PI controller, where only two parameters are at disposal of the designer (K_P, K_I), in the case of the FO-PI controller three design parameters are available. Beside parameters K_P and K_I, the order of integration α is considered.

C.2 Then, an optimization procedure based on the PSO optimization technique, similar to that described in (Aguila-Camacho et al., 2013) , was used to determine the values of K_P ,K_I and α, for a single PI controller valid for the whole operating range. For this procedure we used the PSO available from MatlLab/Simulink using as fitness function the integral of the squared error e(t), and the PI settings of the high operational range of the PI controller with *α=0.02* as starting point. The resultant values obtained by this procedure were $K_P=40$, $K_I=0.6$ and *α=0.30*, obtaining a settling time of *500 s* with a MO of *6%*.

C.3 Thus in spite of the use of the PSO technique, the final values used had to be modified by means of a fine tuning, taking some ideas from (Bhaskaran et al., 2007), to end up with values $K_P=50$, $K_I=1.0$ and α=0.19 to obtain 10% of MO.

D. Classical APBC Controller Design

D.1 This design starts by proposing a storage function of the form $V(e,\phi) = \frac{1}{2}e(t)^2 + \frac{1}{2}\delta^{-1}|\beta|\phi(t)^T\phi(t)$, with $V(0,0)=0$, whose time derivative along the trajectories of the system (1) is equals to $\dot{V}(e,\phi) = e(t)\dot{e}(t) + \delta^{-1}|\beta|\phi(t)^T\dot{\phi}(t)$. Here $e(t) = h(t) - h^*$ is the level error around the desired operating point h^*, $\beta = \frac{C_1 H^2}{\pi R^2} \in \Re^+$ thus $|\beta| = \beta$, and $\phi(t) = \left(\theta(t) - \overline{\theta}\right)$ is the error in the controller parameters where $\overline{\theta}$ is the ideal but unknown controller parameter.

D.2 Considering a regulation problem i.e. $\dot{h}^* = 0$, the error derivative $\dot{e}(t) = \dot{h}(t)$, given by equation (1). Evaluating $\dot{V}(e,\phi)$ and regrouping terms we have

$$\dot{V} = e(t)\beta h(t)^{-2}\left({C_2}/{C_1} - {C_3}/{C_1}\, h(t)^{\frac{1}{2}} + u(t) \right) + \delta^{-1}\beta\phi(t)^T\dot{\phi}(t)$$

After conveniently adding the term $\pm\beta^{-1}u(t)^p$ to the right side of equation (to obtain passivity from *e(t) to $u^p(t)$*) and regrouping terms we have

$$\dot{V} = e(t)u^p(t) + e(t)\beta\left(h(t)^{-2}u(t) - \overline{\theta}^T\omega(t)\right) + \delta^{-1}\beta\phi(t)^T\dot{\phi}(t)$$

The ideal and constant controller parameter vector is $\overline{\theta}^T = \begin{bmatrix} -\frac{C_2}{C_1} & \frac{C_3}{C_1} & \frac{\pi R^2}{C_1 H^2} \end{bmatrix} \in \Re^3$ and the information vector is $\omega(t) = \begin{bmatrix} h(t)^{-2} & h(t)^{-\frac{3}{2}} & u^p(t) \end{bmatrix}^T \in \Re^3$.

D.3 Choosing the control law *u(t)* given in (4) we have

$$\dot{V} = e(t)u^p(t) + \beta\phi(t)^T e(t)\omega(t) + \delta^{-1}\beta\phi(t)^T\dot{\phi}(t)$$

Considering $\dot{\phi}(t) = \dot{\theta}(t)$, using the adaptive law given in (5), the last two terms are cancelled, obtaining $\dot{V} = e(t)u^p(t)$ which is the first time derivative of the continuous storage function *V*, with *V(0)=0*. This proves that controller (4) with adaptive law (5) turns the system (1) into a C^1-passive system from the auxiliary control variable $u(t)^p$ to *e(t)* , around the operating point (h^*, u^*) as theoretically proved in (Travieso-Torres J.C. et al., 2007).

Furthermore, after considering $u^p(t) = -Ke(t) \in \Re$, $\dot{V}(e,\phi) = -Ke(t)^2 \le 0 \Rightarrow e(t) \in L^2$ is obtained, and since $V(e,\phi) > 0$ and $\dot{V}(e,\phi) \le 0$ then $e(t), \phi(t), \theta(t), h(t) \in L^\infty$. Finally, since $u^p(t) \in L^\infty$ then $\dot{e}(t) \in L^\infty$. Thus, by Barbalat Lema since $e(t) \in L^2$ and $\dot{e}(t) \in L^\infty \Rightarrow \lim_{t\to\infty} e(t) = 0$, proving that controller (4) asymptotically stabilized the the level error around zero, as shown in (Travieso-Torres J.C. et al., 2007).

D.4. The parameters of the APBC are the adaptive gain δ of the adaptive law together with proportional gain *K*. The method used to tune the parameters of the APBC controller was based in the experience of the research team, with initial values of *K*=0.01 and δ =0.5 to end up with an experimental fine tuning of *K*=0.001 and δ =0.25.

E. FO-APBC Controller Design

E.1 In contrast to the classical APBC technique, here beside the parameters δ and *K* already mentioned in Appendix D, it is necessary to consider the value of the integration order α. Same as in Appendix B, the PSO method together with the fitness function corresponding to the integral of the squared error between *h(t)* and *h** was used to determine the best controller parameters as proposed in (Aguila-Camacho, 2013).

E.2 The resultant controller parameters applying this methodology were *K=0.011,* δ *=0.3* and *α=0.09*. Same as in the Appendix D, an experimental fine tuning was necessary in order to improve a bit the overall response of the system under control to end up with the values *K=0.001,* δ *=0.15* and *α=0.05*.