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A Parametric Description of Cities for the Normative Analysis of Transport Systems

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Abstract Urban transport systems analysis requires some explicit or implicit representation of the network, activity pattern and flows pattern of the city. When dealing with transit design in real systems, detailed descriptions of cities are too complex to allow an analytical formulation that leads to exact results, so heuristics have been used. Alternatively, optimal design of transit systems at a strategic level has been done based on simplified descriptions using regular patterns or small networks to face and solve ad-hoc transit design problems. In this paper we propose a parametric description of cities for the normative analysis of transit systems. This is achieved after a synthesis of different ways to describe a city's urban form that can be found in the literature, with an emphasis on the road network and the role of centers and subcenters. These diverse descriptions are assessed with the help of topological indicators and synthetic information regarding real cities. The parameters characterize the underlying network, the zones involved and the spatial pattern of transport demand, such that the design of public transport systems can be studied normatively for different city shapes. The model is applied to describe three very different real cities.

Keywords Urban form · Transit system · Polycentrism · Topological analysis

1 Introduction

Understanding modern cities is a highly relevant task because it is the space where the great majority of the individuals interact socially, economically and politically. From a normative viewpoint intervening that space by means of models is a very

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challenging problem because of its complexity and because activities and their connectivity evolve dynamically and interactively. There are several ways to describe cities and their respective urban forms, which depend on the objective pursued by the intended research question. It makes a difference if the focus is on the spatial distribution of activities in order to analyze land use patterns, or if the issue under study is the connectivity network and the spatial structure of displacements when the motivation is to understand and solve transport problems. This latter is the subject of this paper.

Transport systems analysis deal with the movements of persons and goods between different points in space at different moments. In the urban case this analysis requires some explicit or implicit representation of the city. The network and the activity pattern are crucial on these descriptions because of their relevance for transport modelling. Typical implicit models follow some type of regularity, as in the case of concentric cities (e.g. Byrne 1975; Tirachini et al. 2010) and cities shaped as a grid (e.g. Newell 1979; Daganzo 2010). These and other models are chosen to introduce simplicity and workability simultaneously, without questioning their properties as a reasonable representation of actual urban forms and properties. We consider this as a limitation for relevant transport modelling from a normative viewpoint, as there is a co-evolution of urban settlements and road systems (Yamins et al. 2003; Sun et al. 2007).

The objective of this paper is to study different ways to describe a city's urban form analytically and to propose a flexible description based on parameters such that the public transport system can be studied normatively. We begin by reviewing previous studies on urban forms emphasizing those papers whose scope is useful for the analysis of the transport system. This means focusing on those elements that give information about the generation and attraction of trips, and on the elements that explain the network structure, i.e. the physical space where the buses, cars and different transport modes move. We will examine how good available representations are through different metrics in order to propose a new model which is coherent with the state of the art in this area and that is useful for modeling transport problems. We will search for simplicity, noting that there exist many regularities that can be taken into account to create a model, but keeping those characteristics of the cities that are relevant to analyze the optimal shape of transport services.

In the next section we review different ways to represent urban forms explicitly or implicitly, looking at cities' descriptions and classifications, with emphasis on those that are useful for transport modeling; the role played by the dominant zones within a city will receive particular attention, because they explain much of the generation and attraction of trips. Then we describe the graph representations of cities that underlie the formulation of transit design problems, in order to understand their connectivity features and their street structure. In section 3 we compare the behavior of some indicators calculated over these graph representations for real cities and for some ideal graphs. Section 4 contains the description of a parametric graph model that incorporates the discussion of the previous sections. The new model is applied in section 5 finding the parameters that represent three cities. In section 6 we summarize and conclude.

2 Representing Urban Form

2.1 Classification of Cities

It is useful to start reviewing different ways of classifying urban forms in order to realize the large diversity of approaches and to decide which elements are relevant for the analysis of transport problems. Some classifications are based on multivariable analysis, while others consider the geopolitical characteristics, the role played by the dominant zones, the shape of the street network or other criteria.

Under the multivariable approach, more than 30 variables are considered in order to classify cities according to the value of a series of variables like density, proximity between the center and the residential zones, the percentage of built land, the porosity (i.e., the presence of empty spaces), the residential use of the built zones and many others. Huang et al. (2007) and Kasanko et al. (2006) use clusterization methods to make the classification, while Schwarz (2010) uses a principal component analysis.

Using the same type of information it is possible to obtain a classification based on geopolitical distinctions. The cities from the so-called developed countries are usually less dense, less compact and have higher porosity than the cities from the developing ones. Kasanko et al. (2006) show this conclusion when comparing the countries from southern Europe with those from central and northern Europe. Huang et al. (2007) deepen this analysis, and show that the Asian cities -excluding those from Japan- are denser (but also have more porosity) than the Latin American cities. They also find differences between the cities from the developed countries: the cities from the USA are less dense, have more open spaces and are more compact than the European and Japanese cities.

There are three classical ways to classify cities and their zones: by homogeneity (i.e., where each zone presents a low dispersion for some previously defined variable), political division (e.g. municipalities), and dominance of a center. A center is a zone of the city that presents a high concentration of economic activity and/or political control. Because the economic activities are the most relevant attractors and generators of trips, center dominance is a most important element for the analysis of transport problems. For this analysis the concept of a *Central Business District* (CBD) is quite important: it is the area of the city, usually located close to the city geometrical center (but not necessarily so), with the highest concentration of labor activity, and thus, the area that attracts most of the trips in the morning peak.

A classical city model referred to this kind of classification is the *monocentric* model, defined by Anas et al. (1998) as the city "envisaged as a circular residential area surrounding a central business district (CBD) of radius *r*, in which all jobs are located". It assumes then that the city functions around the CBD, such that the most important characteristic of any other part of the city is its distance to the CBD (i.e. a radial city). Alonso (1964) was the original developer of this model, but it was refined afterwards by Mills (1967), Muth (1969) and Makse et al. (1998). Clark (2000) shows that some cities may have an eccentric (in a geometrical sense) CBD and still be considered as monocentric cities. The pure version of this model, however, is obsolete. Hamilton and Röell (1982) show, for example, that real travel time in United States is eight times larger than the theoretical time predicted by this model in its pure radial form. Bertaud (2004) argues that the CBD is still present in modern cities, but it does not appear to be enough to understand the city. Nevertheless, the monocentric model is still used to analyze transport problems in space (e.g. Li et al. 2014).

A complement to the pure dominance represented by a CBD arises with the notion of subcenters. The idea of a subcenter is a sector of the city that is very important to the area that surrounds it, being also highly attractive. McMillen (2001) defines a subcenter as any sector whose employment density is considerable higher than its neighbors density, and that has a significative effect over the general employment density function.[1] Bertaud and Malpezzi (1999) and Clark (2000) show that the subcenters attract trips from everywhere in the city. When a city is structured according to its subcenters, it is defined as a *polycentric* city.

McMillen (2001), McMillen and Smith (2003), Garrocho and Campos (2007) and Suárez and Delgado (2009) study several cities in United States and Mexico, finding cities with 2 subcenters as well as cities with 46 subcenters. But not in every city the CBD and subcenters play the same role: while in Mexico D.F. the CBD by itself concentrates 40 % of the labor activity (Suárez and Delgado 2009), in Los Angeles, California, this quantity is lower than 10 % (Clark 2000); in Dallas-Fort Worth, there is a 40 % of employments that are located neither in the CBD nor in any subcenter (Waddell and Shukla 1993). This last example shows that it may be not enough even to consider both the CBD and the subcenters. In these cases, the city is defined as a *dispersed* city. Figure 1 shows the employment density map for Mexico D.F. and for Dallas-Fort Worth.

Although the concepts of a monocentric, polycentric and dispersed cities are intuitively clear, classification is not trivial. Sometimes the class to which a city belongs emerges neatly while in other cases it is under discussion. For example Gordon and Richardson (1996) attempt to analyze Los Angeles - a city usually seen as “the prototypical polycentric metropolitan region” - as a dispersed city, while Pfister et al. (2000) discuss if Sidney is better modeled as a polycentric city or as a dispersed city. To help identifying the centers structure of a city Louail et al. (2014) propose an approach based on the comparison between the maximum concentration and dispersion of mobile phones; also, Louail et al. (2015) identify residential hotspots and work hotspots (again using mobile phones data), and analyze the proportion of trips between these places relative to the total.

2.2 Transport Networks in the Urban Space and their Graph Representations

The analysis of transport problems requires the representation of cities incorporating clearly their transport networks and the passenger flows from one place to another. The most useful tool to represent urban transport networks are the graphs, as we show in Fig. 2. The links represent the streets and the nodes may represent their intersections and dead ends, zones or bus stops, depending on the type of analysis pursued. Typically, the graph will be undirected and connected. The passenger flows are usually represented by means of an Origin–destination (OD) matrix containing the flows

[1] The most usual way to identify centers and subcenters is based on the activity pattern, but it is also possible to study the trip flows over the spatial network. For example, Zhong et al. (2014) identify the spots in the city that are more attractive when doing a random walk using probabilities associated to the observed flows. Louail et al. (2014) study the spatial concentration of mobile phones at different hours of the day in order to identify the evolution of *hotspots*.

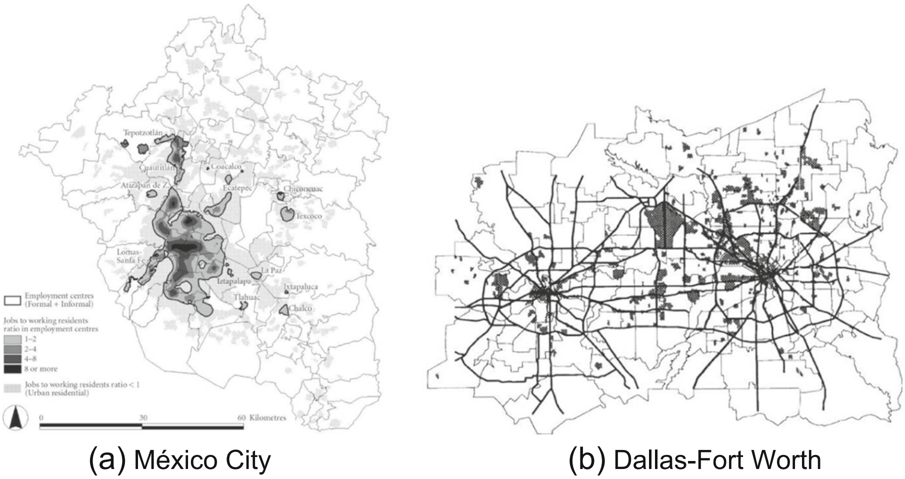


Fig. 1 Monocentric and Dispersed cities. Source: a) Suárez and Delgado (2009) and b) Waddell and Shukla (1993)

between zones. It should be emphasized that these two elements will always have an underlying city pattern, either implicit or explicit.

For the normative analysis of public transport problems, e.g. finding the most appropriate lines structure, different approaches to define the graph can be found in the literature. Following the reviews by Guihaire and Hao (2008) and Kepaptsoglou and Karlaftis (2009) there is a family of papers that deal with any connected graph and where the OD matrix is also generic. This is the approach followed by Dubois et al.

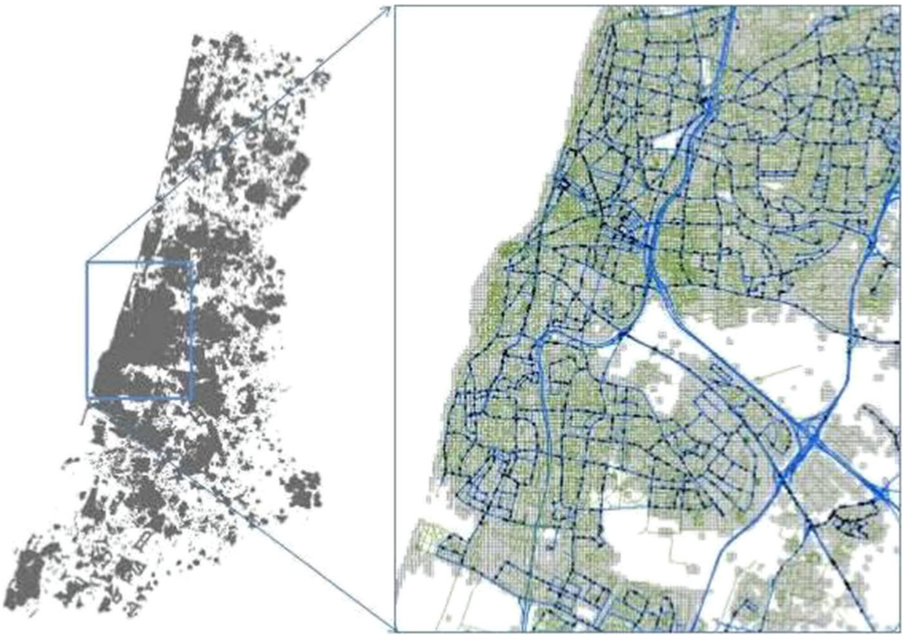


Fig. 2 Tel Aviv (left) and its graph representation (right). Source: Benenson et al. (2013)

(1979), Ceder and Wilson (1986), Pattnaik et al. (1998), Borndorfer et al. (2005), Cenek (2010) and many others. Because in this case the problem is NP-Hard (Quak 2003 and Schöbel and Scholl 2006), these papers use intuitive heuristics or solve some relaxed linear programming problems.

A second approach is to work with some specific graphs assumed - intuitively - to be good and simple representations of the city; this simplicity is useful to obtain results that sometimes are exact, in contrast with those obtained with heuristics. The two basic models that are most commonly used are the grid graph and the concentric model. The grid is used by many authors, e.g. Newell (1979) and Daganzo (2010); in these cases, the demand pattern is uniform. The concentric model with radial demand (not necessarily symmetric) is applied by Byrne (1975) and Tirachini et al. (2010), among others. Also concentric but with uniform spatial demand are the so-called ring-radial models (Badia et al. 2014; Chen et al. 2015). Figure 3 shows graphs for a grid and for a concentric model. In both cases the street design may be part of the model.

As in this paper we will depart from this second approach to construct a parametric city model, it is useful to comment on some characteristics of these intuitive and generic graphs. In its simplest form, all passengers in a concentric city travel only to the center of the city, i.e. a representation of a monocentric city; a dispersed city can also be represented by means of a uniform demand in space in a ring-radial model. On the other hand, in a grid all the nodes are almost equivalent; their only difference is how far each one of them is from the boundaries of the graph (same with the links). As mentioned, the demand pattern for the grid is usually uniform, also representing a dispersed city.

The third and last family of papers includes those models that consider microscopic graphs involving just a few (representative) streets. Perhaps the origins of this view can be found in Mohring (1972), whose city model was a single line. Mohring (1976) kept this city but added complexity incorporating distances between bus stops. Kocur and Hendrickson (1982) and Chang and Schonfeld (1991) studied more complex networks, based on grids but with a single street or point that attracts all the passengers. Jara-Díaz and Gschwender (2003), Jara-Díaz et al. (2012), Gschwender et al. (2016) and Jara-Díaz et al. (2016) studied a cross-shaped network, a single line (but with unbalanced demand), a Y-form network and an extended cross-shaped network, respectively. Some of these papers attempt at a general but simplified representation of an urban area, and others concentrate on specific aspects of transit design for which a few nodes and links strategically ordered are useful. These models share an important property: transport

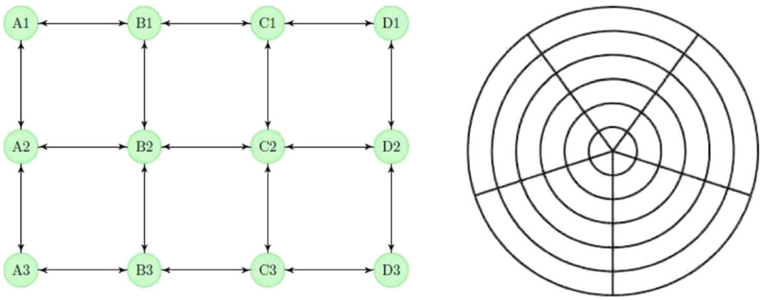


Fig. 3 A grid graph and a concentric city model

problems can be solved analytically; however it is unclear whether the results can be extrapolated. For example, in these simplified graphs there usually exist only one feasible route linking each origin and destination; the problem of route choice is nearly non-existent.

3 Topological Analysis

For synthesis we have urban forms that can be labeled as monocentric, multicentric or dispersed, and networks that respond to a grid, to a concentric graph or to a specific (stylized) toy network. How good are these networks as possible representations of a city in a relative way? In this section we will present some ways to study the graph representation of a city by means of numerical (thus comparable) results that can be used to answer at least partially how good or bad are the city models presented in the previous section, i.e. how close are their numerical indicators to the ones observed in real cities.[2]

There are several ways to apply topological tools for urban studies (Erath et al. 2009). Ducruet and Beauguitte (2014) present an extended review of how network science have been used for spatial analysis. Blumenfeld-Lieberthal (2009), for instance, study connections between cities. Xie and Levinson (2009), study the growth of transportation networks using different approaches including network analysis. Reggiani et al. (2011) study the so-called "commuting network" over different German cities to identify possible hubs and obtain conclusions about the accessibility measures in those cities. Schintler et al. (2007) develop a similar idea to find critical nodes over a transport network and to determine how resilient these networks to failures in those locations are.

This topological analysis considers cities as mathematical objects in order to make some observations and to obtain some conclusions about their general structure. We will consider the primal and dual graph representations of a city shown in Fig. 4.

- In the primal graph, the most common one already described in the previous section, the nodes represent the intersections between the streets and the arcs are the streets linking them. Nodes may also represent the public transport stops or zones.
- In the dual graph, studied in Jiang (2007) and Sun (2013) among others, every street is a node, and two nodes are connected if those streets intersect. This graph is particularly useful to study the connectivity characteristics of the city.

The primal graph is useful to understand the city not only topologically but also geometrically. It also includes explicitly the origins and destinations. The big problem of the primal graph is that a single street is represented by many arcs, one for each block. The dual graph corrects this situation, so it is more appropriate

[2] For the links between city form and the road network and its topology, see Barthélemy and Flammini (2009).

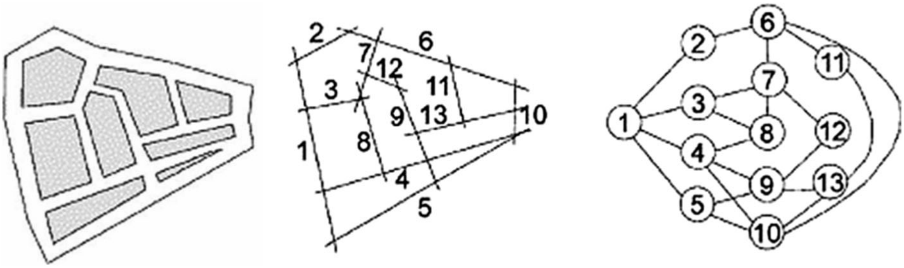


Fig. 4 A city zone with its primal and dual representation. Source: Lin and Ban (2013)

to understand the street structure of a city, although some ambiguity is introduced, because it is not obvious how to define the continuation of a street.[3]

Over these graphs, several indicators can be calculated, some for the primal and some for the dual. Let us describe those that are studied in the literature and that give us relevant information for our purpose. The number of nodes is denoted by n.

- The degree of a node is the number of directly connected nodes (neighbors) it has. The degree of the graph is the average across all nodes.
- Distance between two nodes is the length of the shortest path that links them. For each node X we can find another node that maximizes the distance d_X. The node associated to the minimum d_X is the center of the graph (Y). d_Y is the radio of the graph.
- The cluster coefficient for a single node is the number of connections between each pair of neighbors divided by the total possible connections. The cluster coefficient of the graph is the average across all nodes.
- The face coefficient is defined for the primal graph, which is always planar.[4] It is the ratio between the number of faces and the theoretical bound $2n-5$ (Diestel 2000). Thus, a null value means that the graph is acyclic,[5] while a value equal to 1 implies that it is impossible to add any arc without losing the planarity.
- On the dual graph the grid coefficient for a node (i.e., for a street) is the ratio between the number of cycles of length 4 in which the node is present including a second neighbor (i.e., a node at distance 2), and the total possible cycles of length 4 if all the node's neighbors were connected with all its second neighbors. The grid coefficient is the average across all nodes (1 for the grid).

In order to describe and compare different implicit or explicit city models we will use this quite exhaustive set of relevant indicators which, as a whole, reveal important aspects of the city structure. The degree of the primal graph is indicative of the complexity of the intersections within the city. In the dual graph the degree of a node (street) reflects the number of intersections of that particular street. Therefore, the distribution of the degree of

[3] In order to decide if two consecutive blocks are part of the same street, Porta et al. (2006b) and Lin and Ban (2013) show that a robust alternative is to define an angle close to 0° as the maximum acceptable difference between them.

[4] A graph is planar if it can be drawn in the plane without intersections between the arcs. In these graphs a "face" is an area surrounded by the arcs. A classic result shows that if n is the number of nodes and f the number of faces, then f will never be bigger than $2n-5$ (Diestel 2000).

[5] A cycle in a graph is a path that starts and ends in the same node, without repeating any edge. A graph is called acyclic if it does not have cycles.

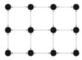
the nodes (streets) in the dual graph is related with the streets' hierarchy; the larger the fraction of streets with degree above the average, the less hierarchical is the street system; we will call this fraction *streets' homogeneity*. The radio of the dual graph has quite an interesting interpretation: it is associated with the number of turns needed to go from any street to another; a small value is known as the "small world property", i.e. only a few turns are needed to go from any origin to any destination. Here the hierarchical concept emerges again as the small world property reflects the existence of long avenues running throughout the city. The cluster coefficient for a given street is high if the streets that share an intersection with it also share intersections among them; this coefficient will be low if parallel streets dominate the city, and will be large if many streets converge into the intersections. This is useful for the comparison against the grid (a pure parallel system with zero cluster coefficient) and against the monocentric model (where all the streets intersect at the CBD, with coefficient one). One has to be careful with the cluster value, as simplified representations of cities imply some degree of spatial aggregation of zones and streets, which diminishes its value because many of the internal connections are lost. The face coefficient increases with the number of cycles in the street system, e.g. adding diagonals in a grid or rings in a radial system; it is closely related with the degree of the primal, as both indicators increase with the ratio of arcs over nodes. Finally, the grid coefficient is designed specifically to compare a particular graph against the grid only. The second column of Table 1 shows ranges for these indicators for more than 200 real cities obtained from different sources.[6]

Columns 3 to 7 in Table 1 contain the topological indices for the most used general networks in transport studies, a theoretical grid sized $a \times b$, the concentric model in two versions (monocentric and ring-radial), and for the two mentioned microscopic models: the Y-shaped network, where two streets converge into one avenue, and the extended cross, where local links converge to or depart from two avenues that cross at the center. For the concentric models we considered one central node and n radial streets; m rings are included in the ring-radial model. To obtain the numerical results, we considered that $4 \leq a, b, n, m \leq 12$.

Before commenting these results, it is worth saying that one of the most important conclusions for the real cities is the extremely hierarchical character of their street structure. Jiang (2007), Masucci et al. (2009) and Sun (2013) obtain the same result when they analyze the distribution of the degree of the nodes in the dual graph, something that cannot be read from the indices above. They realize that a few nodes have a very high degree. That means that a few streets, the large avenues, concentrate most of the intersections. We will name that set of streets the "primary road network". Figueiredo and Amorim (2007) propose that this primary network defines the road network in general, linking different clusters or neighborhoods. These conclusions are reinforced by the fact that the "small world property" is verified in most of the cities dual graphs (Jiang 2007; Figueiredo and Amorim 2007 and Courtat et al. 2011). The existence of this "primary road network" is an important characteristic that should be present in a graph that intends to represent a city model built to analyze transport

[6] Buhl et al. (2006), Hu et al. (2008) and Chan et al. (2011) study the average degree for several cities; Buhl et al. (2006) study the face coefficient; the average degree in the dual graph is studied by Jiang (2007), while the grid coefficient is studied by Figueiredo and Amorim (2007). The cluster coefficient is studied by Porta et al. (2006a, 2006b).

Table 1 Topological indicators for real cities and some graph representations

Index (graph)	Real cities	Grid 	Monocentric 	Ring-radial 	Y 	Cross 
Degree (primal)	2-3.5	~4	1.6-1.85	~4	1.5	1.85
Degree (dual)	<10	4-12	4-12	5.1-17	2	2.6
Streets homogeneity (dual)	0.2-0.4	0.25-0.5	1	0.25-0.75	1	0.2
Radio (dual)	9-20	2	1	1	1	3
Cluster coefficient (dual)	0.06-0.25	0	1	0.84-0.95	1	0.87
Face coefficient (primal)	0.01-0.25	0.33-0.5	0	0.3-0.44	0	0
Grid coefficient (dual)	<0.3	1	undefined	undefined	undefined	0

Note that the dual of the monocentric model is a graph composed by n nodes directly connected (a complete Kn graph); then all nodes have the same degree n-1.

problems. This provokes a conflict with the most important characteristic of the grid, which is its homogeneity. As we explained, in the grid all the streets are almost equivalent, so it is not possible to define a primary road network. This limitation could be fixed up by assigning different capacities or other traffic characteristics, but in any case it would mean a problem because all the streets will present the same connectivity.

Table 1 provides interesting information. Although it should be read with care it is clear that the indices suggest that none of these models is particularly satisfactory. The monocentric model exhibits small values for the radio and high values for the cluster coefficient, showing that collecting nodes indeed reduces the complexity of the network. Furthermore,

- Real cities do not exhibit a grid coefficient close to 1; grid indices perform quite badly in general, which is caused by its homogeneity, i.e. the absence of centers and subcenters and the equal role played by all its streets.
- The pure monocentric model also has a bad topological behavior and has been shown as obsolete to represent the activity pattern of real cities. The ring-radial improves on the topological indices but is not superior to either the grid or the cross.
- Although the Y-shaped network and the extended cross perform as bad as the others, they were not intended to be complete representations of a city. Nevertheless it is interesting to verify how this gets reflected by the topological indicators.

4 The Proposed Model

To propose a model that incorporates the elements discussed in the previous sections, we would like to represent three fundamental features:

- flexibility to represent a monocentric, polycentric or dispersed city,
- the main qualitative conclusions from the topological analysis regarding the structure of the city surrounding its primary road network, and

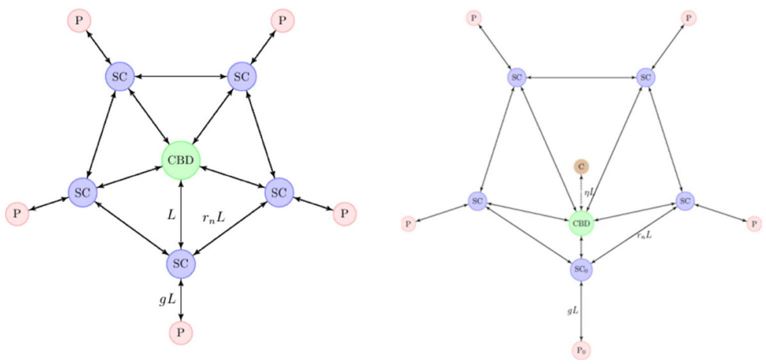
- balance between the complexity of urban interactions with the simplicity of elements like zones and links; symmetry will play a role here.

What we want to achieve is a spatial setting that permits facing transit design from a normative viewpoint, incorporating different transport phenomena and making it possible to face the challenges analytically. The great diversity of cities suggests from the beginning that many of the mentioned features should be represented through different parameters. Let us describe the proposed graph and the demand pattern.

The graph is defined by a CBD and n surrounding zones, each one represented by a subcenter and a peripheral node such that the graph has $2n+1$ nodes. Each subcenter is connected to the CBD, to its own periphery and to its neighbor subcenters. The distance from any subcenter to the geometrical center C of the city is L. The CBD is not necessarily in C; it is located on an imaginary segment connecting C and a subcenter 0 (because of symmetry, it is irrelevant which subcenter), at a distance ηL such that if $\eta=0$, the CBD is in the center of the city. The resulting graph is shown in Fig. 5.

Using the cosine theorem, the distance from the subcenter q (i.e., a subcenter that is q zones away from the subcenter 0) to the CBD is $L_q = L\sqrt{1+\eta^2-2\eta cos\frac{2\pi q}{n}}$. The distance between a periphery and its subcenter is gL. The cosine theorem also tells us that the distance between consecutive subcenters is r_nL, where $r_n = \sqrt{2-2cos\frac{2\pi}{n}}$. Therefore, the main spatial parameters are η, L, g and n. The first one measures the degree of eccentricity of the CBD, the second and the third one define the spatial relevance of the CBD and the subcenters, while the last one may be used to represent spatial dispersion. The primary road network is composed by the streets that connect each subcenter with the CBD. The streets connecting the subcenters themselves also play a relevant role. The fact that every zone is structured around its subcenter may suggest that we are considering a polycentric city, but as we will show later, this analysis will depend on some other parameters that control the demand pattern.

The demand pattern has to consider the CBD as a natural attractor of trips, but not the only one. So we will assume that a fraction $\alpha<1$ of all the trips generated in each periphery will go to the CBD. The subcenters also attract trips but not exclusively from its surrounding zones - represented in this model by its periphery-, so a portion β of the



(a) $\eta = 0$ (b) $1 > \eta > 0$

Fig. 5 The urban form with $n=5$.

trips generated in a periphery will go to its own subcenter; the rest (γ) will go to the other (foreign) subcenters such that $\alpha + \beta + \gamma = 1$. For simplicity, each of the foreign subcenters will be assumed to attract the same amount of trips. This could be easily relaxed, but it is also useful to reinforce the parametric intention of the model as α represents the tendency to a monocentric model, β to a polycentric model and γ to a dispersed model, as shown below.[7] We will denote the total patronage by Y.

The subcenters will also generate trips. Let us define $\tilde{\alpha}$ and $\tilde{\gamma}$ as the portion of the trips generated on the subcenter that go to the CBD and to the other subcenters, respectively, such that $\tilde{\alpha} + \tilde{\gamma} = 1$. To preserve the idea that α represents monocentrism and γ dispersion, we will impose $\frac{\tilde{\alpha}}{\alpha} = \frac{\tilde{\gamma}}{\gamma}$ (implying $\tilde{\alpha} = \frac{\alpha}{1-\beta}, \tilde{\gamma} = \frac{\gamma}{1-\beta}$), leaving only two free parameters for the demand pattern so far.

Regarding trip origins, the peripheries will be the most important trip generators but not the only ones – as explained above –, because in that case we would have no nodes being origin and destination simultaneously, and the streets connecting the periphery with the subcenter would have less traffic than those connecting the subcenters with the CBD (defined as the primary road network). So we will assume that a fraction a of the trips will be generated in the peripheries and a fraction $b = 1 - a$ in the subcenters with $b < a$ usually. Although there are no subcenters with different levels of attraction and generation, the most important features are considered in the model, as shown below. The demand pattern is represented in Fig. 6. Tables 2 and 3 summarize the OD matrix and the model parameters respectively.

The mathematical relations among parameters are summarized in Table 4. This helps showing the relation between the values of α, β and γ and the type of city behind them as represented in Fig. 7, where the monocentric, polycentric and dispersed cities are shown to be particular cases of this parametric description (note that only the trips emerging from a specific zone are shown). In these figures we impose $\eta = 0$.

The model as presented reflects the fundamental features introduced at the beginning of this section, including symmetry. These, however, might prevent the representation of some cities that exhibit important irregularities. This can be faced by introducing appropriate parameters in the model in order to allow a more precise representation of some cities, sacrificing regularity and simplicity. The most important assumption that could be relaxed is the equal role played by the zones and the respective subcenters, which has geometric and travel demand dimensions. This can be done by defining new parameters $G_i, \theta_i \geq 0$, such that $\sum_i \theta_i = 1$. $G_i L$ represents the distance between the subcenter i and the geometrical center of the city; to preserve the shape of each zone, the distance from the periphery to the subcenter will be $gG_i L$ in this case. Regarding travel demand, we only modify the trip generation, so we preserve the relationship between α, β and γ with the center structure shown in Fig. 7. The portion of trips generated at the zone i will be $\theta_i Y$ instead of $\frac{1}{n} Y$. The city shape that includes these changes is shown in Fig. 8 using $G = (0.7, 1.0, 1.4, 1.0, 1.1)$ starting with the "southern" zone and moving counterclockwise; the O-D matrix is shown in Table 5.

[7] With these definitions, the value of α for Mexico D.F. is 0.4 and less than 0.1 for Los Angeles, and the value of γ for Dallas-Fort Worth is 0.4 (see Fig. 1).

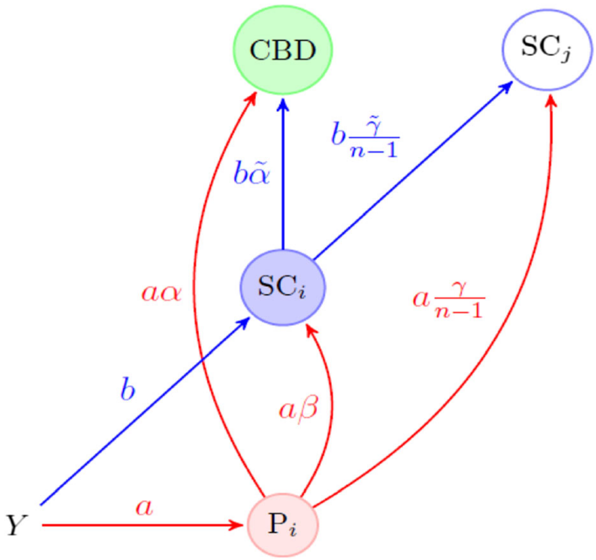


Fig. 6 The demand pattern of the city model

This expanded version of the model provides enough flexibility to represent very different types of cities, by simply changing the values of the parameters. For instance, a port city typically has its CBD close to the sea (zone 0), i.e., close to the boundary of the city. This case can be represented with η close to 1, and G_0 close to zero.

The model proposed responds to the analysis presented in section 2. It is flexible enough to allow and represent different urban forms; it has a natural concentric look, (but it allows an eccentric CBD); and it has a primary road network that determines the city. As we criticized other models precisely because of its weak topological indicators, i.e. very different from those of the real cities, let us calculate and discuss the topological characteristics of the new model, as discussed in section 3.

The fraction of nodes of the dual graph whose degree is higher than the average deserves a particular analysis, because it also shows how relevant is the primary road network. It is easy to realize that the arcs that we defined as the primary road network, i.e. those that connect the CBD with a subcenter, have a degree of $n+2$. The arcs connecting subcenters have a degree of 6, while the arcs connecting a periphery with its subcenter have a degree of 3. This means that the average

Table 2 OD matrix corresponding to the demand pattern of the city model

O/D	CBD	Subcenter i	Subcenter $j(\neq i)$	P i	Pj	Total
Periphery i	$\frac{a\alpha}{n}Y$	$\frac{a\beta}{n}Y$	$\frac{a\gamma}{n(n-1)}Y$	0	0	$\frac{a}{n}Y$
Subcenter i	$\frac{b\tilde{\alpha}}{n}Y$	0	$\frac{b\tilde{\gamma}}{n(n-1)}Y$	0	0	$\frac{b}{n}Y$
Total	$(a\tilde{\alpha}+b\tilde{\gamma})Y$	$\frac{(a\beta+a\gamma+b\tilde{\gamma})Y}{n}$	$\frac{(a\beta+a\gamma+b\tilde{\gamma})Y}{n}$	0	0	Y

Table 3 Summary of the parameters present in the model

Parameter	Definition	Interpretation
n	Number of zones	Reduces importance of each subcenter
Y	Total patronage	Magnitude of the system
L	Distance from any subcenter to the geometrical center C of the city	Size of the city
g	Distance periphery-subcenter/ distance subcenter-CBD	Spatial concentration of the city
η	Portion of displacement of the CBD from the center of the city in an axis CBD-subcenter	Eccentricity of the CBD
α	Trips proportion from periphery that go to the CBD	Large value for monocentric city
β	Trips proportion from periphery to own subcenter	Large value for polycentric city
γ	Trips proportion from periphery to foreign subcenters	Large value for dispersed city
$\tilde{\alpha}$	Trips proportion from subcenter to the CBD	Correction of α for the trips generated at the subcenters
$\tilde{\gamma}$	Trips proportion from subcenter to other subcenters	Correction of γ for the trips generated at the subcenters
a	Trips proportion that depart from the periphery	Dispersion on trip generation
b	Trips proportion that depart from a subcenter	Concentration on trip generation

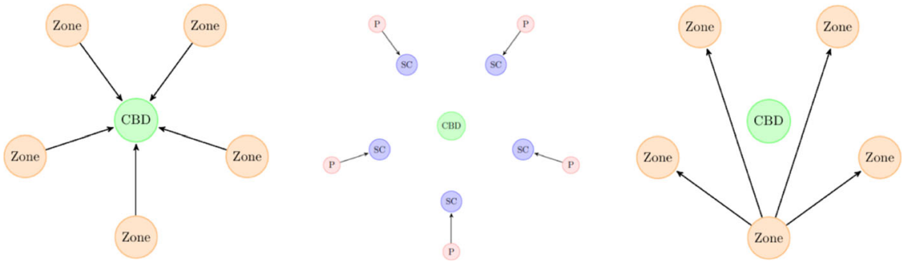
degree in the dual graph is $\frac{(11+n)}{3}$. So if $n \leq 7$, the portion of nodes having a degree higher than the average will be 2/3 (the streets connecting the CBD with the subcenters and the arcs connecting subcenters); if $n > 7$ it will be 1/3. The first quantity is too high for the values found in the literature, while the latter is within the observed range. In any case, note that if we made a "zoom" in the peripheries more streets with small degree would appear, improving the results for this model. This reinforces the idea that the primary road network is formed by the streets with the highest degree, i.e., those that connect the CBD with its subcenters, precisely the streets that define the core of the urban structure. The ranges of the indicators for the more than 200 real cities and for this model are summarized in Table 6, assuming $4 \leq n \leq 12$.

As we can see, the comparison makes our model superior to the grid, to both concentric models and to the specific graphs studied in previous sections. Almost in every topological dimension the indicators of the proposed description are better than all the previous models, even in those dimensions where our model is out of the range of actual values. Let us analyze this further.

The average degree is within the observed range both for the primal and dual graphs of our model. The face coefficient is slightly higher than the maximum observed, but is

Table 4 Summary of the mathematical relations between the parameters

Associated concept	Equation(s)
Trip generation	$a + b = 1$
Trip attraction from the periphery	$\alpha + \beta + \gamma = 1$
Trip attraction from the subcenters	$\tilde{\alpha} + \tilde{\gamma} = 1$
Relations between trip attractions	$\tilde{\alpha} = \frac{\alpha}{1-\beta}, \tilde{\gamma} = \frac{\gamma}{1-\beta}$



(a) Monocentric $\alpha \to 1$ (b) Polycentric $\beta \to 1$ (c) Dispersed $\gamma \to 1$

Fig. 7 Parametric representation of city types.

undoubtedly superior to the coefficients of the grid (0.5) and the monocentric city (0). The cluster coefficient is also higher – although closer than in the other networks –, but this is an expected result, because representing zones as nodes implies collecting many nodes, increasing artificially the number of connections and thus, the cluster coefficient. Something analogous happens with the radio, the only indicator in which one of the other networks is as good as the new proposed model. As we stated earlier, real cities have a very hierarchical road structure, unlike the grid. On the other extreme, the problem with the monocentric model is its extreme simplicity. The clearest example is that the primary road structure in this case corresponds to the whole city!

In summary, we can say that the topological properties of the proposed model fit those of the real cities, with the exception of those indicators where collecting nodes affects the measure directly. In any case, the results are much better than those obtained with the grid, the monocentric model, the Y or the extended cross. The proposed zonal structure defines the network and the urban hierarchy that determines the parametric representation of the travel demand structure. In this way the proposed city model

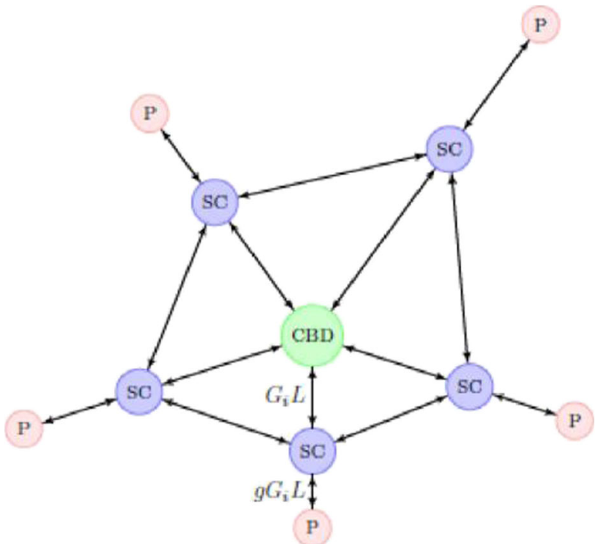


Fig. 8 The urban form with asymmetry

Table 5 OD matrix for the demand pattern of the city model with asymmetries

O/D	CBD	ubcenter i	Subcenter $j(\neq i)$	P $\boldsymbol{i}$	Pj	Total
Periphery i	$a\alpha\theta_i Y$	$a\beta\theta_i Y$	$\frac{a\gamma\theta_i}{(n-1)} Y$	0	0	$a\theta_i Y$
Subcenter i	$b\tilde{\alpha}\theta_i Y$	0	$\frac{b\tilde{\gamma}\theta_i}{(n-1)} Y$	0	0	$b\theta_i Y$
Total	$(a\alpha + b\tilde{\alpha})Y$	$\left[a\beta\theta_i + \frac{(a\gamma+b\tilde{\gamma})(1-\theta_i)}{n-1}\right] Y$	$\left[a\beta\theta_j + \frac{(a\gamma+b\tilde{\gamma})(1-\theta_j)}{n-1}\right] Y$	0	0	Y

verifies the most important characteristics described at the beginning of this section and it is a clear improvement compared with the previous models.

5 Three Examples

Let us see how three actual cities can be represented by this model, a useful exercise to illustrate to which degree the model captures the main elements of a city, keeping in mind that it is aimed at analyzing transit design from a normative viewpoint, which requires to locate the subcenters and to calculate the relative importance of their trip generation and attraction. As intended, a number of simplifications and interpretations will be needed to define the subcenters and both residential and work zones. We will use data available from previous papers that were developed with different purposes. This means that some approximations will be needed to adapt some data, but this is enough to show the potential of this model. Each of the examples will illustrate different types of cities. In Fig. 9 we show the maps of the three cities chosen (different scales): Santiago, Chile, Bordeaux, France and Los Angeles, USA. Dark zones in Santiago represent commercial importance; commercial malls are also shown.

Santiago is the capital and largest city of Chile (6 million people approximately). We will consider 34 municipalities, including Santiago Centro, the historical CBD of the city, as Ortiz and Escolano (2005) do. Each node in our model will represent one or more municipalities. In what follows - in order to construct an aggregate description of the city - we will use geographical elements as well as aggregate knowledge about trips attraction and generation.

For each municipality we know how many workers and students (W-S) live there, and how many people work or study there. Besides, in some cases we know

Table 6 Topological indicators for the city model and real cities

Index (graph)	Real cities	This model
Average degree (primal)	2–3.5	~3
Average degree (dual)	<10	5–7.7
Streets' homogeneity (dual)	0.2–0.4	0.33 or 0.67
Radio (dual)	9–20	3
Cluster coefficient (dual)	0.06–0.25	0.33–0.63
Face coefficient (primal)	0.01–0.25	0.27–0.31

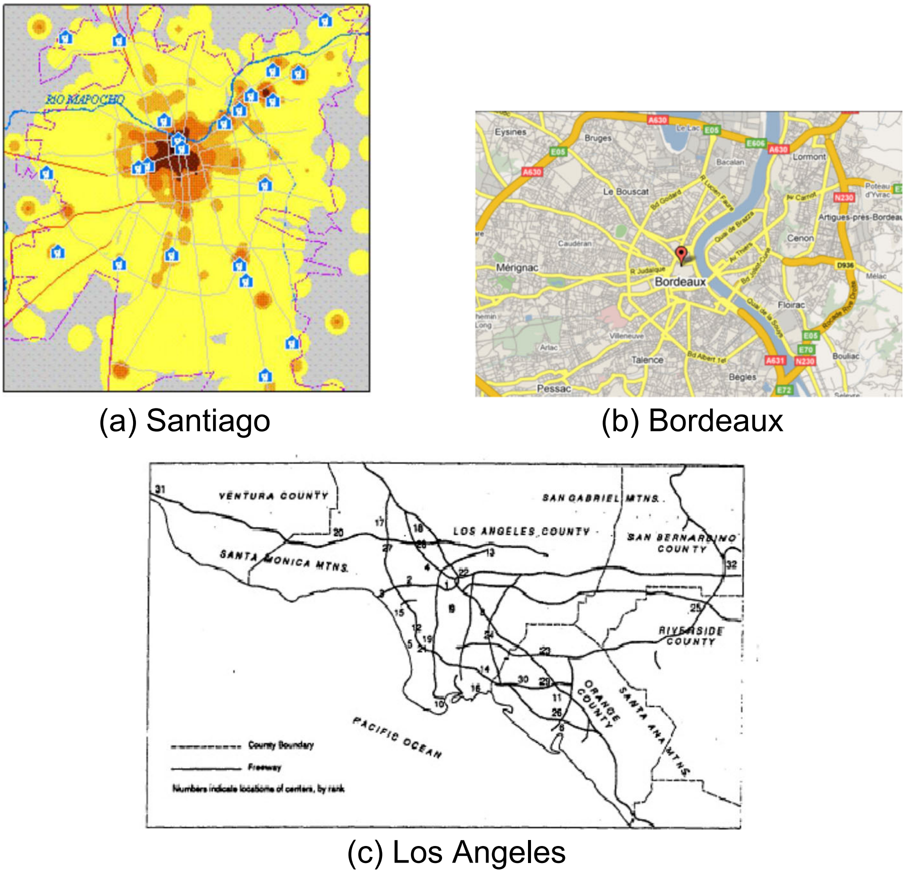


Fig. 9 Maps of the three cities. Source: a) Ortiz and Escolano (2005); b) Google Maps; c) Giuliano and Small (1991)

their most important destinations. Most of the municipalities have at least 20 % of their W-S trips with their origins and destinations within their territory. These trips are not going to be represented in our model. Even though this is not a negligible amount of trips, the consequences for the transport problems are constrained to a local scale. With this assumption, the total patronage Y is 2,565,622. We will consider an extended CBD, including not only Santiago Centro, but also the neighboring Providencia, the second most attractive destination; we will put $\eta = 0$. There are nine municipalities outside the CBD that concentrate more than 2.5 % of the W-S trips attraction each. Two pairs of municipalities are simultaneously neighbors and similar (Las Condes-Vitacura and La Florida-Puente Alto), so we merged them into two zones which makes $n = 7$: West, South-West, South, South-East, East, North-East and North-West. For each zone, some of its municipalities surround the CBD and others are peripheral. We will consider as the subcenter the most attractive municipality out of those surrounding the CBD.

In order to check for symmetry we normalized the population of every zone and obtained that the standard deviation of the vector containing the population of each zone is 0.04. If we only consider the subcenters, the standard deviation increases to 0.086. For some municipalities we have a more disaggregated version of the distribution of the trips generated there. Considering only those municipalities that have both Santiago and Providencia as main attractors and considering their average, we find that $\alpha = 0.25, \widetilde{\alpha} = 0.32$, implying $\beta = 0.22$. The value of b is directly calculated after the population of the subcenters obtaining $b = 0.22$.

If we calculate the distance from the center of each subcenter to the government palace (located in the center of the CBD), we find that the average distance is $L = 10$ km. In each subcenter, we calculate the distance from the municipalities that are not the subcenter, to the subcenter. The average in this case is 8.5 km, so $g = 0.85$.

Although Bordeaux has been understood as a polycentric city (Aguilera and Mignot 2003), this does not prevent the model to be useful, as we may add an auxiliary node as the CBD. Doing so, it exhibits three suburban subcenters (Aguilera and Mignot 2003) and it does not have any outlying subcenters, so it is a very good city to test our model. Its population is about 250.000. In this city 70 % of the people live in the subcenters, thus $b = 0.7$. We also know that 49 % of the total jobs are located in these subcenters, and 82 % of the peripheral jobs. So we have that $\beta + \gamma = 0.82, a(\beta + \gamma) + b\tilde{\gamma} = 0.49$. From this we get $\alpha = 0.72, \beta = 0.18, \gamma = 0.1$.

Let us calculate the spatial parameters. First, we will put $\eta = 0$. We know that the average distance traveled by the people that live in the subcenters is 8.3 km. Assuming that all passengers take the shortest route, average distance may be calculated as $\tilde{\alpha}L + \tilde{\gamma}\sqrt{3}L$, implying $L = 6.6$. The average distance for those living in the rest of the city (in our model, the peripheries) is 15.6 km, and a similar calculation concludes that $g = 1.2$.

Los Angeles is usually considered a quite dispersed city (Gordon and Richardson 1996). By the time Giuliano and Small (1991) reported its analysis, the total working population was approximately 4.5 million people. According to the authors, it had 31 subcenters, but 4 of them composed what they called the "core" (because of their proximity to the CBD), so we are going to consider these 4 inner subcenters and 3 outer subcenters (Riverside, San Bernardino and Oxnard) in order to show the use of the parameters G_i and θ_i. We will assume homogeneity between the inner zones and the outer zones.

The CBD only attracts 3.3 % of the trips. Although we do not have the distribution between local and external workers for each zone, we do know that 68 % of the jobs are not located in any subcenter, which reflects the high degree of dispersion of Los Angeles; accordingly, we will put $\alpha = 0.033$ and $\gamma = 0.68$. Moreover, only 9 % of the inhabitants live in the subcenters, so $b = 0.09$. But the outer subcenters concentrate much less: 0.12 %, so $\theta_o = 0.0004, \theta_i = 0.249$,

Table 7 Parameters describing Santiago, Bordeaux and Los Angeles

Parameter	α	β	γ	a	b	Y (trips)	L (km)	g	n
Santiago	0.25	0.22	0.53	0.78	0.22	2,565,622	10	0.85	7
Bordeaux	0.18	0.72	0.1	0.3	0.7	250,000	6.6	1.2	3
Los Angeles	0.0033	0.287	0.68	0.91	0.09	4,500,000	11.65	0.79	7

where subscripts “o” and “i” stand for outer and inner, respectively. The average distance from the inner subcenters to the CBD is $L = 11.65$ km. The outer subcenters are located at an average distance of 62.3 km, so $G_o = 5.35$. The rest of the high concentration places (the other subcenters reported in the paper) are located at an average distance of 20.88 km, so $g = 1.79$.

Table 7 summarizes the values of the parameters for Santiago, Bordeaux and Los Angeles. They show synthetically that Santiago is a city with some degree of monocentricity but not fully concentrated at the CBD, that Bordeaux is a clear polycentric city and that Los Angeles is a dispersed city. Figure 10 shows the graphs that represent the parametric model of the three cities. As the methodologies used to describe each of these cities are different, the comparison among them has to be taken with care.

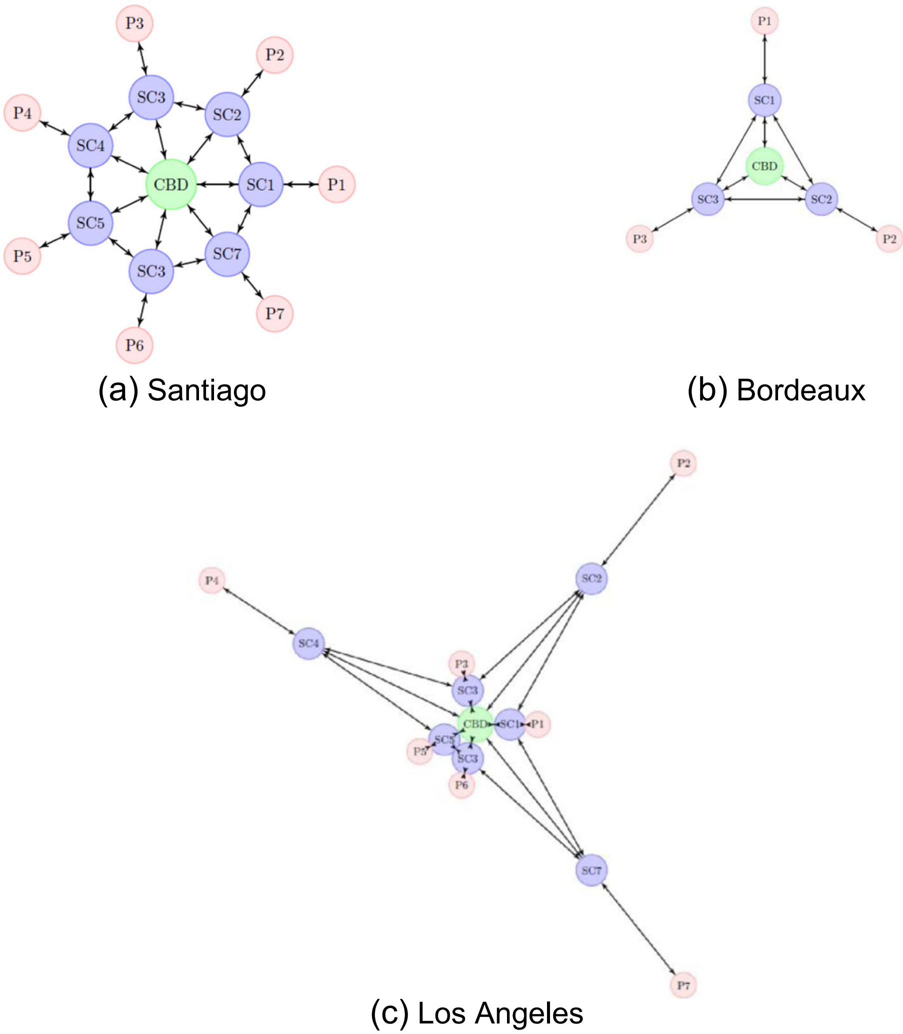


Fig. 10 Graphs for the parametric model.

6 Synthesis and Conclusions

Urban transport systems analysis requires some explicit or implicit representation of the city that includes its network, activity pattern and flows pattern. When dealing with transit design, detailed descriptions of cities are too complex to allow an analytical formulation that leads to exact results, so heuristics have been used. Alternatively, optimal design of transit systems at a strategic level has been done based on descriptions that follow criteria of regularity and simplicity, as the concentric and quadricular (grid) shapes. Besides, different types of small networks have been used to face and solve ad-hoc transit design problems. In this paper we have offered a synthesis of different ways to describe a city's urban form analytically and we have used topological indicators to be able to compare across the different representations and against real cities. After discarding on these grounds the usual representations we proposed a flexible description based on parameters such that the public transport system can be studied normatively. This has been done by paying attention to the underlying network, the zones involved and the spatial pattern of transport demand.

We aimed at a city model that combines the following virtues:

- To be useful for the normative design of transit systems
- To be simple enough as to allow analytic developments
- To be complex enough to represent different urban phenomena
- To be based on research over real cities.

After reviewing different forms of analyzing and classifying cities, with a focus on the idea of dominant zones, we concluded that the classical notion of CBD is not enough to represent modern cities; recognizing the existence of monocentric cities, polycentric cities and dispersed cities was a key element in this discussion as dominant zones explain an important portion of the generation and attraction of trips. Furthermore, the topological analysis helped us conclude that the cities have a very hierarchical street structure, with the existence of a few avenues that determine the structure of the whole city. The need to represent simultaneously a structure of centers and a hierarchy of streets implies a departure from the grid-like representations - where links and nodes are of equal importance - towards a generalized radial structure.

We propose a city model with a CBD and n zones, each one composed by a subcenter and a periphery. The peripheries only generate trips, the CBD only attracts and the subcenters do both. Different parameters define the level of generation and attraction of these different nodes, such that different roles of the centers can be represented. The CBD can be in the center of the city (a symmetric model) but an eccentric CBD can be represented as well.

The topological indicators for the proposed city model show a good behavior, in the sense of being close to those of real cities. The only exception comes from some indicators that are affected directly by the fusion of different micro nodes into a representative one. Our model captures urban characteristics that are relevant for the normative analysis of transit, better than the grid, the monocentric model and other stylized networks used in the literature. We propose this model as a good compromise between simplicity - to allow analytical work towards optimality - and adequate representation of the most important features of real modern cities. In an extension of the model that sacrifices regularity and simplicity, we also provide flexibility to include the possible presence of a hierarchy within subcenters, as well as different distances among them and the CBD. However, there are some other specific phenomena that are not yet incorporated, as a larger number of trips between closer zones, the disaggregation of the nodes in the periphery to incorporate also a local analysis or the possibility of network change and growth (Xie and Levinson 2009).

With this city model many questions related to the structural transport systems of cities may be studied in depth, such as the search for the optimal transit lines structures and the comparative assessment of the heuristics' performance. We are presently working on these topics. To get an idea of the next steps, note that many types of line structures can be defined and tested over this model; as examples we can mention direct lines (from every periphery to every zone), hub & spoke (locating the hub at the CBD or different hubs at each subcenter), or feeder-trunk with connections at the subcenters. Within these structures different types of routes can be envisioned, either through the CBD or touring the subcenters. This last point is important as this model is complex enough to allow passengers to have more than one feasible route, making passenger's choice an interesting component. For each lines structure so defined, the optimal frequencies and bus sizes for each line can be found, i.e. those that minimize the sum of users and operators costs; the best line structure will be the one with the overall minimum. Evidently, the optimal design will depend on the value of the parameters in the model city, such that conclusions on lines structures (as well as frequencies and bus sizes) can be linked to the type of city and the structure of demand.

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