Monopoly regulation under asymmetric information: prices versus quantities

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We compare two instruments to regulate a monopoly that has private information about its demand or costs: fixing either the price or quantity. For each instrument, we consider sophisticated (screening) and simple (bunching) mechanisms. We characterize the optimal mechanisms and compare their welfare performance. With unknown demand and increasing marginal costs, the sophisticated price mechanism dominates that of quantity, whereas the sophisticated quantity mechanism may prevail when marginal costs decrease. The simple price mechanism dominates that of quantity when marginal costs decrease, but the opposite may arise if marginal costs increase. With unknown costs, both instruments are equivalent.

1. Introduction

■   The design of regulatory mechanisms to control monopoly market power when the firm has better information than the regulator is a problem whose practical relevance is irrefutable. On one hand, the fact that the firm probably has better information about its costs than the regulator was well explained by Weitzman (1978): "[...] even the managers and engineers most closely associated with production will be unable to precisely specify beforehand the cheapest way to generate various hypothetical output levels. Because they are yet removed from the production process, the regulators are likely to be vaguer still about a firm's cost function." On the other hand, there are also good reasons for a firm to have or acquire better information than the regulator about

the demand. Lewis and Sappington (1988) argue that a firm might have better information about the quality and reliability of its product. Another reason is that firms usually invest resources to learn about the demand they face, information that is hardly, or partially, shared with the regulator. Moreover, over time, the firm is the one that will learn directly the behavior of its demand, as it is the one interacting with it routinely. A third possibility arises when a firm is at the top of a vertical structure, as in the case of, for example, ports and airports, in which the demand for the good or service is derived from the equilibrium of the downstream market; thus, if the regulator has imperfect information about either the costs or demand of the downstream firms, then this will be reflected in inferior information about the demand of the firm to be regulated.

Many articles have looked into the issue. For instance, Baron and Myerson (1982), Baron and Besanko (1984), Laffont and Tirole (1986), Sappington (1983), and Sappington and Sibley (1988) focus on cases in which the firm has private information about its costs. In this setting, price and quantity regulation are equivalent from a social welfare perspective, as the demand function is known by the regulator; we discuss this issue at the end of Section 5. On the other hand, Riordan (1984) and Lewis and Sappington (1988) focus on price regulation on settings in which the firm has private information about its demand; see Armstrong and Sappington (2007) for a survey. Somewhat surprisingly, quantity-based mechanisms have not been studied as an option for monopoly regulation when there is asymmetric information on the demand. Yet in many real-world settings in which the market demand is likely to be unknown, economists and authorities have considered implementing quantity-based mechanisms to induce a desired outcome. Examples of such mechanisms are the determination of the level of flights (departures/arrivals) per unit of time at an airport, as in four slot-controlled airports in the United States (Chicago O'Hare, New York LaGuardia, New York Kennedy, and Washington National), or the Reliability Must-Run Generation contracts (used in California, the Phillipines, and India, among other places), where the regulatory body mandates some plants to produce a minimum amount of energy at times, in exchange for compensation. It is then natural to ask: What would be the features of optimal quantity regulation of market power? Which instrument would be preferable, prices or quantities?

The way price regulation works is probably more natural to imagine. The regulator either sets a price and a transfer or offers a menu of prices and transfers to/from the monopolist to choose from. Once the price is fixed, the monopolist must sell to everybody willing to buy at the regulated price. The monitoring issue remaining is that the monopolist may prefer not to sell some units after the transfer has been received. Yet, according to Lewis and Sappington (1988), this issue is unlikely to arise, as consumers would report to the regulator whenever the firm denies a purchase.

How would quantity regulation work? A monopolist regulated by quantity would either be told how much to produce and the transfer it will receive (or pay), or it may be offered a menu of quantities and transfers to choose from. Once the quantity is fixed, the monopolist must sell at a price that ensures that all produced units are sold. To ensure this, the regulator could ask the monopolist to conduct a uniform price auction so that the value for each unit equals the market clearing price (with the revenues of the sale going to the firm). In fact, in the airport market, the Federal Aviation Administration (FAA) has been considering auctions to ration slots (Berardino, 2009; Brueckner, 2009). However, if auctions are not feasible, the regulator must monitor whether the regulated quantity is actually sold, otherwise the mechanism would lose much of its power. This may turn out not to be so difficult, as many industries indeed have their production and sales routinely monitored already: those in which what is being produced is a *flow*. This is the case of utilities such as electricity generation, water, and natural gas, for which firms already have to report the per-unit of time consumption of households and companies. It is also the case of many transport markets, such as public transport (buses and subway), trains, air travel and airports, ports, and highways, which must report the flow of passengers and goods to authorities. In any of those cases, the quantity to be produced and sold in a period of time can be easily checked,

and thus deviations from the regulated quantity can be discovered.[1] Also, in some settings, it may be easier for the regulator to name a quantity than a price. Output, for example, can simply be monitored in an airport as the number of landings and takeoffs, yet the total price charged by the airport is the sum of a myriad of charges for different services such as luggage handling, cleaning crews, and runway charges.

The first formalization of the "prices versus quantities" comparison is made by Weitzman (1974) who argues that the comparative advantage of one instrument over the other is far from obvious when there are informational problems.[2] He considered settings in which firms are price takers, and thus his results do not extend to the regulation of market power, as we argue in Section 5. The main goal of this article is to understand which instrument is preferable—prices or quantities—in an environment in which the firm has market power and superior information about its demand or costs.[3] To the best of our knowledge, this might be the first article to study quantity mechanisms as an option for monopoly regulation. We compare price versus quantity regulation for both *sophisticated* mechanisms (which offer self-selecting menus to the monopoly) and for *simple* (bunching) mechanisms (which offer just one price or quantity). Although sophisticated mechanisms are elegant and optimally solve the regulator's problem, there has been some discussion regarding their applicability in practice. Simple mechanisms are obviously less beneficial but may be easier to implement practically. We obtain insights regarding when one instrument is better than another, what drives the relative advantage, and how the regulator's degree of sophistication affects the optimal choice of the instrument.

The case in which the firm has private information about its costs is straightforward, and thus is delayed to Section 5. Briefly, price and quantity regulation are equivalent for all cost structures, independent of the regulator's sophistication.

However, when demand is unknown by the regulator, the analysis is much more subtle, making this the main focus of this article. We find that, for increasing marginal costs, sophisticated price regulation dominates quantity; yet, if the principal is restricted to using simple mechanisms, the comparison depends on primitives. When marginal costs decrease and the regulator is sophisticated, quantity dominates, provided that transfers to the firm are not too (socially) costly; yet, if the regulator restricts itself to simple mechanisms, price dominates.

The fact that price regulation dominates for increasing marginal costs is not in itself surprising, because Lewis and Sappington (1988) showed that the first-best mechanism—namely, price equals marginal cost and zero profits for the firm—is incentive compatible and thus optimal. What is surprising is that it *strictly* dominates quantity regulation. This happens because, as we show, the first-best quantity mechanism is not incentive compatible for quantity regulation. Intuitively, if the decision variable is quantity, a firm hoping to exercise market power would like to restrict output, which means that high demand firms would like to pretend they face low demand, so that the regulator restricts output for them. Full extraction from the regulator's viewpoint does not overcome such incentives, because a firm pretending to face a low demand would have to also pay lower transfers than the ones it would have paid if it had told the truth.

With decreasing marginal cost, we find that the optimal quantity mechanism is still a fully separating menu, in stark contrast with price regulation in which the optimal sophisticated mechanism is, in our terminology, a simple mechanism. This implies that when the regulator has minimal or no aversion to transfers, quantity will outperform price, for it can get close enough to the efficient production level. Yet, when transfers are more costly to the regulator, we find that

the price mechanism may be better. How can a simple mechanism outperform a fully separating menu? Intuitively, suppose that marginal costs were decreasing but fairly flat. Then, the unique regulated price would be almost right for all possible demand realizations, thus being close to the first-best. So from an informational perspective, the firm's private information is not crucial to the regulator, entailing minimal informational rents. However, for a quantity mechanism to work, information about demand is critical, implying large informational rents to the firm. Next, we focus on the case in which transfers are socially costless and provide a sharp characterization of the welfare loss induced by the price mechanism relative to the quantity one, when marginal costs decrease linearly and the demand is linear in its arguments. Finally, we illustrate how and by how much this relative welfare loss increases when extreme demand realizations are more likely to emerge.

When the regulator is restricted to simple mechanisms, both price and quantity regulation switch from menus to fixed values. We find that the ranking of instruments change relative to the case in which the regulator uses sophisticated mechanisms. When marginal costs decrease, the price mechanism always performs better than the quantity mechanism. Although the former is unchanged with respect to the unconstrained case (the optimal sophisticated mechanism is simple), the latter now performs sufficiently worse than before. However, for increasing marginal costs, the comparison depends critically on the slopes of demand and marginal cost, as in Weitzman (1974). In particular, when demand is fairly inelastic so that its slope is larger (in absolute value) than that of the marginal cost, quantity dominates. Thus, for example, a simple quantity mechanism may be appropriate for public transport cases where the demand is rather inelastic (usually the case for subways), and marginal cost increasing due to high traffic volumes. We also explore the benefits of sophistication, in line with Rogerson (2003) and Chu and Sappington (2007) who study which proportion of the welfare gains achievable by sophisticated mechanisms can be secured by using simple ones. We find that in some cases, a simple quantity mechanism secures 75% of the social welfare achieved by a sophisticated quantity mechanism and that moving to a two-options menu—or to a *semi simple* mechanism—can further increase the secured welfare by at least 13%.

Before closing this section, let us illustrate a case in which, we think, sophisticated quantity regulation could be welfare improving: the US Essential Air Service program (EAS). As explained in Tang (2015), following the 1978 deregulation of air travel, a concern grew in the United States that communities with low passenger levels would lose service as carriers shifted their operations to serve larger or more profitable markets. This concern was addressed by the US Congress establishing the EAS program, administered by the Office of the Secretary of the US Department of Transportation (DOT), to ensure that small communities that were served by air carriers before deregulation would continue to receive scheduled passenger service. Subsidies were considered from the beginning. As of June 1, 2015, 159 communities in the United States received subsidized service under EAS; see Tang (2015) for details.

The EAS program works in the following way. First, the DOT establishes minimum levels of service for each community by specifying, for example, the hub airport that will be reached and a minimum number of round trips and available seats that must be provided. Then, air carriers bid on the level of service (frequency and aircraft size) and annual subsidy; the winner signs a two-year contract with the DOT. Two aspects are worth stressing. First, given the low number of enplanements, the service most likely operates with decreasing marginal cost because it has been well established that there are strong economies of aircraft size, which translate into what is known in the airline economics literature as economies of density.[4] Second, one of the current concerns is that the program is too costly, because "The carrier may find it more profitable to charge higher fares to relatively few passengers than to maximize the passenger load with lower

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[4] Decreasing marginal costs for the airline industry have been empirically found by Brueckner and Spiller (1994), Wei and Hansen (2003), and Fageda (2006), among others. Of course, marginal costs start to rise along routes where large aircrafts and frequencies are in place; see Basso and Jara-Díaz (2006) for a discussion.

fares" (Tang, 2015), resulting in larger subsidies. In other words, the problem would be that the firm did not commit or was not obligated to a certain level of output and would find it optimal to ration. Quantity regulation may work here: marginal costs are decreasing and the quantity sold is easy to monitor (it is actually being monitored to date). The carrier would be committed to a passenger load and would then need to price accordingly to meet that flow.

We organize the article as follows. We characterize the optimal sophisticated price and quantity mechanisms when demand is unknown in Section 2 and compare their performance in Section 3. We find the optimal simple price and quantity mechanism and study their differences in Section 4. We provide further results in Section 5, comparing simple and semi simple to sophisticated mechanisms, and tackling the case of unknown costs. We conclude in Section 6. All omitted proofs and calculations are in the Appendix.

2. Sophisticated regulatory mechanisms

■ The monopoly has private information about the demand captured by a parameter $\theta \in \Theta \equiv [\underline{\theta}, \bar{\theta}]$, the firm's *type*. The regulator has a prior belief $G(\theta)$ with a continuous and strictly positive density $g(\theta) \equiv G'(\theta) > 0$, and with a decreasing inverse hazard rate $(1 - G(\theta))/g(\theta)$; a standard property in the literature that is satisfied by any log-concave distribution. For $\alpha \in [0, 1]$, we denote the *weighted inverse hazard rate* by $z_\alpha(\theta) \equiv (1 - \alpha)(1 - G(\theta))/g(\theta)$. Next, we consider a *direct demand function* $Q(p, \theta)$ with $Q_p < 0 < Q_\theta$.[5] For every quantity $q \geq 0$ and type $\theta \in \Theta$, the *inverse demand function* $P(q, \theta)$ obeys $q \equiv Q(P(q, \theta), \theta)$. So, it follows that $P_q = 1/Q_p < 0 < P_\theta = -Q_\theta/Q_p$, that is, in the space of prices and quantities, the demand slopes downward, and higher realizations of θ imply a greater demand. The *gross consumer welfare* of consuming q units given a type θ is $V(q, \theta) \equiv \int_0^q P(q', \theta)dq'$, whereas the *cost* of producing q units is $C(q)$, with marginal costs $C'(q) > 0$.

Assumption 1. $C''(q) > P_q(q, \theta)$ for all $\theta \in \Theta$ and $q \geq 0$.[6]

Assumption 1 states that the demand is steeper than marginal costs. Thus, if marginal costs are decreasing and the regulator cares about maximizing total expected surplus, then "price equals to marginal costs" is optimal when there is no asymmetric information. We further assume that:

Assumption 2. $qP_\theta(q, \theta)$ is convex in q for all $\theta \in \Theta$.

Assumption 3. $P_{\theta q}(q, \theta)q + P_\theta(q, \theta)$ is decreasing in θ for all $q \geq 0$.

Together, Assumptions 1 and 2 ensure, imposing minimal restrictions, a concave optimization problem (see equation (A1) in the Appendix), whereas Assumption 3 secures that the optimal quantity mechanism is a fully separating menu (with no bunching regions[7]). Assumptions 2 and 3 hold if, for example, the (inverse demand) shock is *additive* so that $P(q, \theta) \equiv P(q) + \theta$ (i.e., $P_{q\theta} = 0$), or if the shock is *multiplicative* so that $P(q, \theta) \equiv \theta P(q)$ (i.e., $P_{q\theta} \gtreqless 0$), provided that the elasticity $qP_{qq}/P_q \leq -2$. Although this elasticity inequality seems to be unusual in the literature, it is satisfied, for example, by any isoelastic (inverse) demand $P(q, \theta) \equiv \theta q^{-\epsilon}$ with absolute elasticity $\epsilon \geq 1$. Finally, additive and multiplicative shocks to direct demand functions $Q(p, \theta)$ are defined analogously.

The regulator chooses an assignment rule x that could be a price p or a quantity q and lump-sum transfers T. By the Revelation Principle, we restrict attention to direct and incentive compatible mechanisms without loss of generality. Next, we formally define a sophisticated mechanism.

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[5] For any smooth function $x \mapsto f(x)$ and $x \in \mathbb{R}^n$, we denote its partial derivatives by $f_{x_i}(x) \equiv \partial f(x)/\partial x_i$.

[6] Notice that in terms of prices, Assumption 1 is equivalent to $|C''(Q(p, \theta))| \cdot |Q_p(p, \theta)| < 1$ for all $\theta \in \Theta$.

[7] Bunching occurs when different types are regulated identically, or when they choose the same option in the menu.

Definition 1 (sophisticated mechanisms). A sophisticated regulatory mechanism consists of an assignment rule $x(\theta) \in \mathbb{R}_+$ (where x is either p or q), and a lump-sum transfer rule $T(\theta) \in \mathbb{R}$, such that the menu $\{x(\theta), T(\theta)\}_{\theta \in \Theta}$ induces truthful revelation for all θ.

In order to ensure truthful revelation, we need to write incentive compatibility constraints, for which we require the profits associated with untruthful type reports. If a firm of type θ declares $\hat{\theta}$, then its respective profits for price and quantity regulation are:

$$\pi(\hat{\theta}, \theta) = p(\hat{\theta})Q(p(\hat{\theta}), \theta) - C(Q(p(\hat{\theta}), \theta)) + T(\hat{\theta}) \quad \text{and}$$

$$\pi(\hat{\theta}, \theta) = P(q(\hat{\theta}), \theta)q(\hat{\theta}) - C(q(\hat{\theta})) + T(\hat{\theta}).$$

In both cases, incentive compatibility (IC) requires:

$$\Pi(\theta) \equiv \max_{\hat{\theta} \in \Theta} \pi(\hat{\theta}, \theta) = \pi(\theta, \theta) \quad \forall \theta \in \Theta. \tag{1}$$

A common assumption in adverse selection problems is that the single crossing property (SCP) holds, that is, that the slope of an indifference curve in the allocation-transfer space of the profit function is monotone in θ. This allows us to replace (IC) constraints by a monotonicity constraint on the allocation rule, which fixes (up to a constant) the transfer rule; see Baron and Myerson (1982). Indeed, characterization of optimal mechanisms without the SCP remains an open question.[8] Henceforth, we assume the SCP for both mechanisms and offer a precise description of what it entails in Lemma 1.

Lemma 1 (single crossing property). (i) For price regulation, the SCP holds iff $Q_\theta(1 - C''Q_p) \geq -Q_{p\theta}(p - C')$ for all p, θ. (ii) For quantity regulation, the SCP holds iff $q P_{q\theta}/P_\theta \geq -1$ for all q, θ.

Assumption 4. The SCP holds for both price and quantity regulation, as stated in Lemma 1.

The SCP holds for quantity regulation if, for example, shocks are additive, or multiplicative, provided the elasticity $q P_q/P \geq -1$. For instance, an inverse demand $P(q, \theta) = \theta q^{-1}$ satisfies the SCP and Assumptions 2 and 3. On the other hand, the SCP for price regulation is more complicated, as it depends also on the cost technology. Observe that additive demand shock ($Q_{p\theta} = 0$) is not necessary for the SCP to hold, but it is a simple sufficient condition.

Lemma 2 (incentive compatibility). A price regulatory mechanism is incentive compatible iff:

(i) $p(\cdot)$ rises in θ, and (ii) $\Pi'(\theta) = [p(\theta) - C'(Q(p(\theta), \theta))]Q_\theta(p(\theta), \theta)$.

A quantity regulatory mechanism is incentive compatible if and only if:

(iii) $q(\cdot)$ rises in θ, and (iv) $\Pi'(\theta) = P_\theta(q(\theta), \theta)q(\theta)$.

Next, a regulatory mechanism must be individually rational as well, that is,

$$\Pi(\theta) \geq 0 \quad \forall \theta \in \Theta, \tag{2}$$

where we normalize the firm's outside option to zero. Note that, in principle, the regulator may want to shut down the firm for some demand realizations. Sophisticated mechanisms already account for this issue, as for some types the assignment rule may end up being zero; however, this is not necessarily true for simpler mechanisms. Still, throughout this article, we assume for simplicity that there is no shutdown, or that the regulator finds it too costly to bear the risk of having no output.

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[8] The problem is that without the single crossing property, local incentives for truth-telling do not imply global incentives for it; see Araujo and Moreira (2010) and the references therein.

A regulatory mechanism is *feasible* if it satisfies (1) and (2). The regulator seeks to maximize a weighted sum of consumer surplus and the firm's profit. The social welfare function in terms of prices or quantities is:

$$\mathbb{E}SW(p(\theta), T(\theta), \theta) = \mathbb{E}\left\{ \int_p^\infty Q(x, \theta)dx - T + \alpha[pQ(p, \theta) - C(Q(p, \theta)) + T] \right\} \qquad (3)$$

$$\mathbb{E}SW(q(\theta), T(\theta), \theta) = \mathbb{E}\left\{ \int_0^q [P(x, \theta) - P(q, \theta)]dx - T + \alpha[P(q, \theta)q - C(q) + T] \right\}, \qquad (4)$$

with $\alpha \in [0, 1]$. Observe that when $\alpha < 1$, a dollar out of the consumers' pockets hurts society more than the benefit of a dollar received by the firm. Therefore, the regulator may be willing to accept some output contraction (i.e., *allocative inefficiency*) in order to diminish the size of the subsidy.[9]

Before fully characterizing the optimal sophisticated mechanisms, we define, as a benchmark, the first-best mechanism as the one that maximizes social welfare under complete information.

Definition 2 (first-best mechanisms). The first-best regulatory mechanism consists of transfers $T(\theta)$ and prices $p(\theta)$ or quantities $q(\theta)$ such that for all types $\theta \in \Theta$:

(i) $p(\theta) = C'(Q(p(\theta), \theta))$ or (ii) $P(q(\theta), \theta) = C'(q(\theta))$; and (iii) $\Pi(\theta) = 0$ if $\alpha < 1$.

Notice that the first-best mechanism involves two things. First, it achieves allocative efficiency, because price equals marginal cost. Second, it leaves the firm with zero profits, for consumer surplus is weighted more than the firm's profits whenever $\alpha < 1$. Simple algebra shows that a quantity mechanism $q(\theta)$ that equalizes price with marginal cost is increasing in θ.[10] Thus, allocative efficiency is feasible because it is incentive compatible and individually rational. Nevertheless, as we show in Proposition 1, the optimal sophisticated quantity mechanism is not first–best if $\alpha < 1$. The reason is that allocative efficiency can only be achieved by leaving positive profits to the firm (via large enough transfers), and this is socially costly when $\alpha < 1$. Hence, the optimal quantity mechanism induces allocative distortions (via output contraction) in order to limit the size of transfers, and it always takes the form of a menu regardless of the firm cost technology.

Proposition 1. The optimal quantity mechanism (q^o, T^o) obeys:

$$P(q^o(\theta), \theta) = C'(q^o(\theta)) + z_\alpha(\theta)P_\theta(q^o(\theta), \theta)\left(1 + q^o(\theta)\frac{P_{\theta q}(q^o(\theta), \theta)}{P_\theta(q^o(\theta), \theta)}\right) \qquad (5a)$$

$$T^o(\theta) = C(q^o(\theta)) - P(q^o(\theta), \theta)q^o(\theta) + \int_{\underline{\theta}}^{\theta} P_\theta(q^o(x), x)q^o(x)dx. \qquad (5b)$$

Also, for each type $\theta \in \Theta$, the optimal quantity $q^o(\theta)$ is increasing in the weight α.

This characterization holds for any cost structure and any demand shock (as long as Assumptions 1–4 hold). The optimal quantity mechanism exhibits standard features: all but the highest firm's type charges a price above marginal cost, and all but the lowest type get informational rents. Moreover, as α grows, informational rents can be delivered by making greater use of transfers

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than output contraction, so the optimal quantity for all types below the highest rises. Indeed, the optimal quantity mechanism achieves allocative efficiency only when $\alpha = 1$.

For the price mechanism, the results are starkly different. Now allocative efficiency is feasible if and only if marginal costs are increasing, as only in that case the price schedule $p(\theta)$ is increasing in θ (see Definition 2).[11] Indeed, Lewis and Sappington (1988) showed that the first-best regulatory mechanism is feasible when marginal costs are increasing, and thus optimal.[12] When marginal costs are decreasing and provided the single crossing property holds, Lewis and Sappington (1988) found that the optimal price mechanism involves complete bunching; specifically, the regulator fixes a unique price and transfers (p^*, T^*) independent of the firm's report.

A precise characterization of the optimal price mechanism (p^*, T^*) for decreasing marginal costs follows from Lemma 2 (i). A bunching price $p^* \in \mathbb{R}_+$ implies that $\Pi'(\theta) = [p^* - C'(Q(p^*, \theta))]Q_\theta(p^*, \theta)$, and thus there exists a *critical type* $\theta_c(p^*) \in \Theta$ such that $\Pi'(\theta) \leq 0$ iff $\theta \leq \theta_c(p^*)$.[13] Unlike the optimal quantity mechanism, the nature of the allocative inefficiency that the price mechanism induces is different for low and high enough realizations of the demand: for low demand $\theta \leq \theta_c(p^*)$, production is higher than efficient, whereas the opposite holds for high demand $\theta \geq \theta_c(p^*)$.

Observe that the type which is most reluctant to participate is the critical type $\theta_c(p^*)$, which is left with zero profits. It follows that transfers are $T(p^*) = C(Q(p^*, \theta_c(p^*))) - p^* Q(p^*, \theta_c(p^*))$. It is easy to see that any price strictly below $\underline{p} = C'(Q(\underline{p}, \bar{\theta}))$ is dominated by $\underline{p}$, and any price strictly above $\bar{p} = C'(Q(\bar{p}, \underline{\theta}))$ is dominated by $\bar{p}$. Hence, the regulator chooses $p^* \in [\underline{p}, \bar{p}]$ to maximize:

$$\mathbb{E}_\theta SW(p^*, T(p^*), \theta) = \mathbb{E}_\theta \left\{ \int_{p^*}^\infty Q(x, \theta)dx - T(p^*) + \alpha[p^* Q(p^*, \theta) - C(Q(p^*, \theta)) + T(p^*)] \right\}.$$

(6)

Further characterizing the optimal price p^* is not an easy task. For example, as opposed to quantity regulation, the price instrument does not have a monotone behavior as α changes, because the focus no longer lies in inducing truth–telling and, therefore, ensuring informational rents. A sharper characterization requires extra assumptions.

Proposition 2. Assume that demand shocks are additive (i.e., $Q_{p\theta} = 0$) and that marginal costs are decreasing $C'' < 0$. If marginal costs are convex (concave), then the critical type $\theta_c \leq (\geq)\mathbb{E}(\theta)$, the optimal price $p^* \geq (\leq)C'(Q(p^*, \mathbb{E}(\theta)))$, and it rises (falls) in α. If marginal costs are linear, then the critical type $\theta_c = \mathbb{E}(\theta)$, the optimal price $p^* = C'(Q(p^*, \mathbb{E}(\theta)))$, and it is independent of α.

Proposition 2 shows that, if marginal costs are decreasing and linear, not only the optimal price is independent of the weight α that the regulator assigns to the firm's profits, but also it requires the regulator to know only the first moment of the distribution $G(\cdot)$. All in all, the optimal quantity mechanism has the usual flavor of a fully separating menu, well known from the mechanism design literature, whereas the optimal price mechanism varies dramatically, including a case where it ends up being very simple (a single contract), at the cost of making no use of the firm's information. It is then natural to compare the performance of both mechanisms, which we pursue in the next section.

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[11] Differentiating (i) in Definition 2 yields $p'(\theta) = C''Q_\theta/(1 - C''Q_p)$, which is positive if $C'' \geq 0$, as $Q_\theta > 0 > Q_p$.
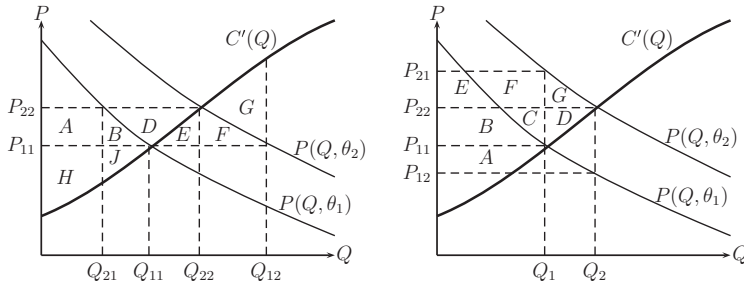
[12] This result is general in the sense that it holds for any market demand function $Q(p, \theta)$ with $Q_p < 0$.

[13] In particular, the critical type $\theta_c(p^*)$ could be either $\underline{\theta}$ or $\bar{\theta}$ if the price is too large or too small.

FIGURE 1

PRICE AND QUANTITY (FIRST-BEST) REGULATION WITH INCREASING MARGINAL COSTS

LEFT: The first-best price mechanism is incentive compatible. If type θ_1 reports $\theta_2 > \theta_1$, it gets a price P_{22} and gains $A + H$, but transfers $A + H + J + B + D$ to the regulator. If θ_2 reports θ_1, it gets a price P_{11} and gains $H + J - (E + F + G)$ and transfers $H + J$ to the regulator. [14]RIGHT: The first-best quantity mechanism is *not* incentive compatible. If θ_2 reports $\theta_1 < \theta_2$, then it produces Q_1 and must transfer A. However, at Q_1, the firms charges P_{21} and gains $A + B + C + E + F$.

3. Comparison of sophisticated mechanisms

■ When marginal costs are increasing, the price mechanism achieves the first-best whereas the quantity mechanism never does. Why is this so? Under price regulation, to obtain a higher price (and thus exercise market power), the firm must overstate the demand, but that increases the transfer to the regulator, making misreporting nonprofitable and the first-best feasible. Likewise, if a firm understates its demand, then transfers fall, but net profits fall much more. So misreporting is never profitable, as explained in Figure 1 and its caption. On the other hand, under quantity regulation, to restrict output (and thus exercise market power), the monopolist must understate its demand which, in addition, decreases the transfer paid to the regulator, making misreporting desirable and the first-best—as in Definition 2—unfeasible (see Figure 1). Altogether, price regulation dominates quantity regulation for $\alpha < 1$, for the former coincides with the first-best.

For decreasing marginal costs, where transfers are now subsidies to the firm, the incentives in the quantity mechanism are unchanged, but the first-best price mechanism ceases to be incentive compatible. Indeed, it is profitable for a firm to *understate* its demand to get a higher price and a lower subsidy, because all units sold below marginal cost are still subsidized, but the ones sold above marginal cost generate positive profits, as shown by Lewis and Sappington (1988). Thus, the optimal mechanism involves bunching and not a menu. The comparison of price and quantity regulation is less clear now, for we need to contrast an inefficient menu with an inefficient bunching. Yet, when $\alpha = 1$, quantity regulation unambiguously dominates, as it is allocative efficient by Proposition 1.

Proposition 3. The price mechanism strictly dominates the quantity mechanism if marginal costs are nondecreasing, except when $\alpha = 1$, in which case the two mechanisms generate the same level of welfare. If marginal costs are strictly decreasing, then there exists a weight $\alpha^* \in [0, 1)$ such that the quantity mechanism strictly dominates the price mechanism for all $\alpha \in (\alpha^*, 1]$.
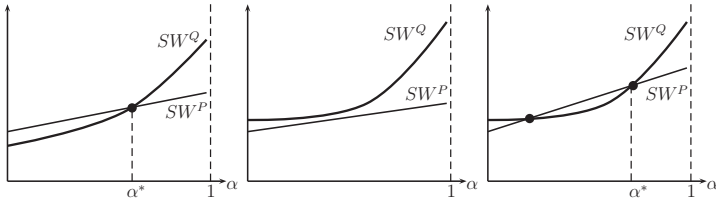
From Proposition 3, we know that quantity regulation is better, under very general conditions regarding demand shocks and cost technologies,[15] for values of α above some α^* that is strictly

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FIGURE 2

COMPARING PRICES AND QUANTITIES AS THE WEIGHT α CHANGES

All panels assume $C''' = 0 = Q_{p\theta}$. LEFT: When price regulation achieves a greater social welfare for α near 0, quantity and price regulation cross only once at an interior α^*. MIDDLE AND RIGHT: When quantity regulation achieves a greater welfare for α near 0, then, generically, no crossing or a double crossing may occur if, for example, the regulator's prior belief $G(\cdot)$ places substantial mass near the boundaries of the support.

below one. However, what happens when $\alpha < \alpha^*$? In order to make comparisons for all values of α, a preliminary step is to explore how the respective social welfares SW^Q and SW^P induced by the optimal quantity and price mechanism change with α.

Lemma 3. Assume decreasing marginal costs $C'' \leq 0$. The social welfare function for the quantity mechanism, SW^Q, is strictly increasing and strictly convex in α. On the other hand, the social welfare for the price mechanism, SW^P, is strictly increasing and convex in α; yet, if marginal costs are linear and demand shocks additive ($Q_{p\theta} = 0$), then the social welfare SW^P is linear in α.

For both mechanisms, there is a *direct* effect of an increase in α on the weighted social welfare, namely, an increase proportional to the firm's profits. For quantity regulation, as shown in Proposition 1, the optimal quantity mechanism distorts less the optimal allocation as α rises, creating an *indirect* effect: the firm's profits increase as α rises. For price regulation, when marginal costs are not linear, the optimal (unique) price adjusts to variations in α, introducing again an indirect effect, as in the case of quantity regulation. Thus, the respective social welfares SW^P and SW^Q are increasing and convex in α. Although quantity regulation dominates for $\alpha > \alpha^*$, the welfares SW^P and SW^Q may intersect multiple times, creating alternating regions of dominance by one or the other mechanism. Alternatively, they may intersect only once, or none at all, meaning that quantity dominates for all α. To illustrate these cases, consider linear marginal costs and additive demand shocks so that SW^P is linear in α, by Lemma 3. Then, it may be the case, as shown in Figure 2, that SW^P and SW^Q cross once (left panel), that SW^P is always below SW^Q (middle panel), or that they cross twice (right panel), suggesting that price dominates for intermediate values of α.

As we show in the Appendix, if we consider a linear demand function, quadratic costs, and a uniform distribution for θ, there is a single crossing, that is, the weight α^* is unique and interior. However, as discussed before, if the information is sufficiently disperse, in the sense that the type distribution has heavy tails, then price regulation could be worse than quantity regulation when α is 0, and typically both a double crossing and no crossing may occur.

Proposition 3 shows that price regulation performs worse than quantity regulation when transfers are not too socially costly (i.e., α is near 1), but how much worse? Call DWL^P the deadweight loss induced by the price mechanism. In linear environments, we offer a precise answer to this question in the following proposition.

Proposition 4. Let $\alpha = 1$, and assume a linear demand with additive shocks. If marginal costs are linearly decreasing, then the relative deadweight loss induced by the optimal price mechanism is:

$$\frac{DWL^P}{SW^Q} = \frac{C''^2}{P_q^2} \cdot \frac{\int_{\underline{\theta}}^{\bar{\theta}} (\theta - \mathbb{E}(\theta))^2 \, g(\theta) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} (P(0) + \theta - C'(0))^2 g(\theta) d\theta}. \tag{7}$$

Observe that DWL^P/SW^Q falls as the marginal costs C' become less steep, in line with Proposition 3. Also, DWL^P/SW^Q rises as the type distribution becomes heavier on the tails. Intuitively, when extreme demand realizations are more likely, the price mechanism performs worse, as it is far off the mark more often. More precisely, notice that because $C''/P_q < 1$ (by Assumption 1) and $P(0) - C'(0) > 0$, our formula (7) obeys $DWL^P/SW^Q < \text{var}(\theta)/\mathbb{E}(\theta^2) = 1 - \mathbb{E}(\theta)^2/\mathbb{E}(\theta^2)$. So in the extreme case, when half of the mass is in $\underline{\theta}$ and the remaining half is in $\bar{\theta}$, we have $DWL^P/SW^Q < 1/2$, namely, the price mechanism leads to a relative deadweight loss below 50%.[16] For the uniform distribution, this loss is less than 25%, whereas if the asymmetric information vanishes, or when G is "close" to a Dirac delta at $\bar{\theta}/2$, the price mechanism is efficient.

4. Simple regulatory mechanisms

■ Now we characterize the optimal simple, or bunching, mechanisms in both quantities and prices. In a simple mechanism, the regulator does not delegate the choice of price or production to the firm, but instead uses its imperfect information to set one price or one quantity without attempting to induce information revelation. Although sophisticated mechanisms that offer a menu with many options may generate greater social gains, they have been criticized for being difficult to implement in practice (Crew and Kleindorfer, 2002; Vogelsang, 2002; Rogerson, 2003; Chu and Sappington, 2007). Indeed, there is little evidence of situations in which authorities have used sophisticated mechanisms. For example, the Federal Communications Commission in the United States tried out price regulation with menus for the Bell operating companies in the early 1990s, but the practice was abandoned after a few years (Vogelsang, 2006). In 2004, the Office of Gas and Electricity Markets in the United Kingdom introduced menus to deal with asymmetric information on capital expenditures. In 2005, the system operator in England and Wales was offered menus of up to three contracts (Joskow, 2005). Yet these are some exceptions; usually the firm is offered a single option.

Some possible reasons are that the regulator is not experienced enough to offer something more complex, that it fears that the monopoly is not sufficiently sophisticated to make the right choice, or that it can find a loophole to exploit. In general, simple mechanisms are easier to implement and understand, so it is important to study how the choice of instrument changes if the regulator decides to use such mechanisms.

Definition 3 (simple mechanisms). A simple regulatory mechanism consists of a unique assignment and transfer $(x^*, T^*) \in \mathbb{R}_+ \times \mathbb{R}$ (where x is either p or q), for all types $\theta \in \Theta$.

Let us start by analyzing the case of decreasing marginal costs. As discussed in Section 2, the optimal simple price p^* is the one that maximizes (6). Next, notice that for any quantity $q \geq 0$ and transfer T, the firm's profits are $\pi(q, \theta, T) = P(q, \theta)q - C(q) + T$. Because $P_\theta > 0$, transfers are optimally set to leave the lowest type $\underline{\theta}$ with no rents, $T(q) \equiv C(q) - P(q, \underline{\theta})q$. So, the regulator chooses $q \geq 0$ to maximize $\mathbb{E}_\theta SW(q, \theta, T(q))$ in (4). We now present our first comparison for simple mechanisms:

Proposition 5. Assume a linear demand with additive shocks (i.e., $Q_{p\theta} = 0$). If marginal costs are linearly decreasing, the optimal price mechanism dominates that of quantity for all weights $\alpha \in [0, 1]$.

For an intuition, suppose that $\alpha = 1$. Then, with linear demand and marginal costs, both the optimal simple price and quantity mechanisms make prescriptions based on the intersection of

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[16] This case is likely to emerge if the firm has private information about a binary outcome that affects the demand for the regulated good, for example, the production of a good that is complement/substitute to the regulated one. The regulator knows that this may shift the demand, but only the firm knows if this happens in the relevant time frame.

FIGURE 3

DEADWEIGHT LOSSES OF SIMPLE PRICE AND QUANTITY REGULATION
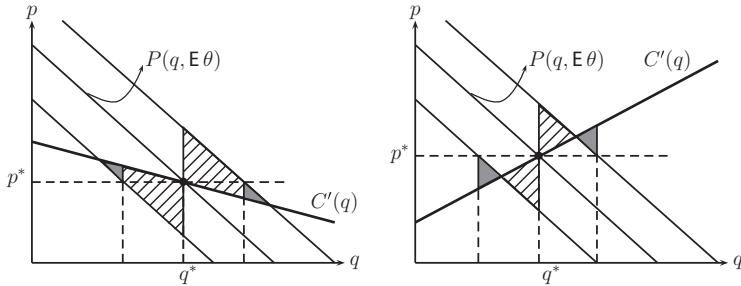
Both panels assume a linear demand, linear marginal costs, and costless transfers ($\alpha = 1$). The shaded areas are the deadweight losses of price regulation. At left, when marginal costs are decreasing, the sum of the dashed and shaded areas are the deadweight losses of quantity regulation. At right, when marginal costs are increasing, only the shaded areas are the deadweight losses of quantity regulation.

marginal costs and the demand of the expected type $\mathbb{E}(\theta)$; see Proposition 2 and the Appendix. Now suppose that the demand realization turns out to be larger than expected. Then, the expost efficient quantity is larger than the one prescribed by the intersection of marginal cost and expected demand, thus creating potential for ex post deadweight losses. Obviously, the quantity mechanism has no way to adjust ex post; yet, in the price mechanism, more units than expected would be sold, as the demand was larger than expected. However, as shown in Figure 3 (left panel), this adjustment will not fully reach the ex post efficient level if marginal costs are decreasing, that is, the quantity produced never "overshoots." Thus, price regulation dominates quantity regulation, for underproduction is milder in the former. The process is analogous if demand is smaller than expected.[17]

We now turn to the case in which marginal costs are strictly increasing $C'' > 0$. In Section 3 we showed that sophisticated price regulation dominates (Proposition 3). However, an unambiguous comparison of regulatory instruments is not possible when the regulator restricts to simple mechanisms. To show this point, it is enough to consider the case in which transfers are costless, $\alpha = 1$.

Proposition 6. Let $\alpha = 1$, and assume linear marginal costs and a linear demand with additive shocks. If $C'' > 0$, then the optimal simple price mechanism dominates that of quantity iff $P_q + C'' \leq 0$.

Let us explain why this ambiguity arises when marginal costs are increasing. Suppose that the demand θ turns out to be larger than $\mathbb{E}(\theta)$. Then, the optimal price regulation would raise the production again, but, this time, more than the efficient level, inducing social losses due to overproduction; with increasing marginal cost, the price mechanism overshoots its quantity adjustment. On the other hand and as before, the quantity mechanism induces losses caused by underproduction, as seen in the right panel of Figure 3. Analogously, if demand is lower than expected, price regulation induces social losses caused by underproduction, whereas quantity regulation induces losses due to overproduction (Figure 3). So, which mechanism is better and for how much depend on primitives.

Now, note that without asymmetric information (i.e., var(θ) = 0), the advantage of prices over quantities vanishes, as expected. However, when var(θ) > 0, the choice of instrument depends

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[17] If the demand or marginal cost is nonlinear, the optimal mechanisms are not characterized by the intersection of the expected demand and marginal cost, even if $\alpha = 1$. In Figure 3, there would be not only one tuple (p^*, q^*), but two (one per mechanism), making the comparison ambiguous. Still, with "enough" linearity, our result holds.

on the slope of marginal costs (C'') and demand (P_q).[18] If the magnitude of the slope of demand is greater (lower) than that of marginal costs, then price regulation is better (worse) than quantity regulation, for the ex post excess production (the "overshooting") is small (large); see Figure 3, right panel.

5. Further results

☐ **The benefits of full sophistication.** As shown in Section 3, when marginal costs are increasing and $\alpha < 1$, sophisticated price regulation dominates quantity, whereas both mechanisms coincide when $\alpha = 1$. However, if the regulator is restricted to choose simple mechanisms, then the welfare loss *within* prices can be larger than that of quantities, implying that either mechanism may be preferable depending on primitives (e.g., on the difference between the slope of the demand and marginal costs). This shows that it is important to understand how good or bad simple mechanisms perform conditional on the regulatory instrument; that is, what the benefits of sophistication are.

We measure the welfare loss *within* price- and quantity-based regulation schemes. Consider a linear demand and marginal costs and costless transfers (i.e., $\alpha = 1$). Observe first that in a price-based regulation, there is no welfare loss when marginal costs are nonincreasing, as the sophisticated and simple price mechanism are equivalent. Now, suppose that marginal costs are increasing. In this setting, the simple price mechanism is the one that is efficient for the average type, by Lemma 1. So our estimate for welfare losses in Section 3 (Proposition 4) applies to this case.

Corollary 1. Let $\alpha = 1$, and assume a linear demand with additive shocks. If marginal costs are linearly increasing, then the welfare loss induced by the simple price mechanism relative to the sophisticated one is given by equation (7).

As we argued in Section 3, these losses rise as the type distribution puts more mass at the extremes of the support. Now, consider quantity regulation, and call SW_{soph}^Q and SW_{sim}^Q the respective expected social welfare induced by the optimal sophisticated and simple quantity mechanism.

Proposition 7. Let $\alpha = 1$, and assume a linear demand with additive shocks and linear marginal costs. The welfare induced by the simple quantity mechanism relative to the sophisticated one is:

$$\frac{SW_{sim}^Q}{SW_{soph}^Q} = \frac{\left[\int_{\underline{\theta}}^{\bar{\theta}} (P(0) + \theta - C'(0))g(\theta)d\theta \right]^2}{\int_{\underline{\theta}}^{\bar{\theta}} (P(0) + \theta - C'(0))^2 g(\theta)d\theta}. \tag{8}$$

Let $d \equiv P(0) - C'(0) > 0$, then $SW_{sim}^Q / SW_{soph}^Q = [\mathbb{E}(\theta + d)]^2 / \mathbb{E}(\theta + d)^2 = 1/(1 + \mathrm{var}(\theta)/[\mathbb{E}(\theta + d)]^2)$. Thus, this ratio falls as the variance of demand realizations rises, or as the information becomes more dispersed. Intuitively, a mechanism that does not discriminate across types becomes much worse, for allocative inefficiencies arise more often. If θ is, for example, uniformly distributed, the regulator induces at most a 25% welfare loss when compared to the optimal sophisticated mechanism. Also, when G puts equal weights on $\{\underline{\theta}, \bar{\theta}\}$, the regulator can secure at least 50% of the maximal welfare.

☐ **Partial sophistication: semi simple mechanisms.** We have considered settings in which the regulator offers the firm a choice among an unlimited number of contracts, and settings in which the regulator offers no choice to the firm. An intermediate case is one in which the regulator offers the firm a choice between, say, two contracts. This is important to analyze,

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[18] These insights first appear in Weitzman (1974) and Laffont (1977) but in a context of price-taking firms.

because it provides a way to slightly sophisticate a simple regulation, without running into the implementation problems of large menus discussed in Section 4. In this case, how much welfare can be secured?

To this end, we define a *semi simple quantity mechanism* as a mechanism $\{q(\cdot), T(\cdot)\}$ in which $(q(\theta), T(\theta)) = (q_1, T_1) \in \mathbb{R}_+ \times \mathbb{R}$ for all $\theta < \hat{\theta}$, and $(q(\theta), T(\theta)) = (q_2, T_2) \in \mathbb{R}_+ \times \mathbb{R}$ for all $\theta \geq \hat{\theta}$, for some $\hat{\theta} \in \Theta$. Next, we characterize the best mechanism in this class.

Proposition 8. The optimal semi simple quantity mechanism $(q_1, q_2, T_1, T_2, \hat{\theta})$ obeys:

$$\mathbb{E}[P(q_1, \theta)|\theta \leq \hat{\theta}] = C'(q_1) + \mathbb{E}\left[z_\alpha(\theta)P_\theta(q_1, \theta)\left(1 + q_1 \frac{P_{\theta q}(q_1, \theta)}{P_\theta(q_1, \theta)}\right)\bigg| \theta \leq \hat{\theta}\right] \quad (9a)$$

$$\mathbb{E}[P(q_2, \theta)|\theta \geq \hat{\theta}] = C'(q_2) + \mathbb{E}\left[z_\alpha(\theta)P_\theta(q_2, \theta)\left(1 + q_2 \frac{P_{\theta q}(q_2, \theta)}{P_\theta(q_2, \theta)}\right)\bigg| \theta \geq \hat{\theta}\right] \quad (9b)$$

$$V(q_1, \hat{\theta}) - C(q_1) - z_\alpha(\hat{\theta})P_\theta(q_1, \hat{\theta})q_1 = V(q_2, \hat{\theta}) - C(q_2) - z_\alpha(\hat{\theta})P_\theta(q_2, \hat{\theta})q_2. \quad (9c)$$

Transfers T_1 and T_2 adjust so that $\Pi(\underline{\theta}) = 0$ and $\Pi(\hat{\theta}) = \int_{\hat{\theta}}^{\bar{\theta}} P_\theta(q_2, \theta')q_2 d\theta'$.

Given $\theta \leq (\geq)\hat{\theta}$, the regulator fixes a quantity $q_1(q_2)$ so that the expected price equals the expected *total* marginal costs (i.e., production plus informational costs), whereas the cutoff type $\hat{\theta}$ is chosen so that the quantities (q_1, q_2) achieve the same social welfare when realized demand is $\hat{\theta}$.

If the demand is linear and the cost function is quadratic, Proposition 9 below provides a precise estimate of the welfare losses of moving from a sophisticated mechanism to a semi simple one. Recall that SW^Q_{soph} is the expected social welfare induced by the optimal quantity mechanism. Call SW^Q_{ssim} the expected social welfare induced by the optimal semi simple mechanism.

Proposition 9. Fix $\hat{\theta} \in \Theta$, and assume linear marginal costs and a linear demand with additive shocks. Then, the relative welfare loss induced by the semi simple mechanism is given by:

$$\frac{SW^Q_{soph} - SW^Q_{ssim}}{SW^Q_{soph}} \leq \frac{\text{var}(\theta - z_\alpha(\theta)|\theta \leq \hat{\theta})G(\hat{\theta}) + \text{var}(\theta - z_\alpha(\theta)|\theta \geq \hat{\theta})(1 - G(\hat{\theta}))}{\mathbb{E}[(P(0) - C'(0) + \theta - z_\alpha(\theta))^2]}. \quad (10)$$

Proposition 9 shows us how much of the social welfare achievable by an optimal sophisticated mechanism can be captured by a semi simple mechanism. For an illustration, let $\alpha = 1$. Now, if θ is uniformly distributed on $[0, \bar{\theta}]$, and if $\hat{\theta} = \bar{\theta}/2$, the regulator secures at least 88% of the social welfare that she can achieve using a sophisticated mechanism. Because the optimal semi simple mechanism elects $\hat{\theta}$, the regulator can secure strictly more than 88%. So by our previous analysis, moving from simple regulation to semi simple yields an extra benefit about 13% of the maximal welfare.

An important related article is Rogerson (2003). He considers the Laffont and Tirole (1986) model of cost-based procurement and regulation and focuses on a class of regulatory mechanism that he calls Fix Price Cost Reimbursement (FPCR). FPCR specifies a threshold type, fixes the price for delivering the good for low-type firms, and reimburses exactly the realized production cost of high-type firms. So in our terminology, a FPCR is a semi simple mechanism. Rogerson finds that, in linear environments in which information is uniformly distributed, the regulator using a FPCR can capture 75% of the gains achievable by a sophisticated mechanism.[19]

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[19] Chu and Sappington (2007) consider the same environment as Rogerson (2003) and study whether his conclusions extend to more general settings and whether alternative simple contracts can perform better than Rogerson's FPCR.

19   The case of unknown costs.   Suppose now that the firm is privately informed only about its production cost $C(q, \theta)$, where θ denotes a cost realization. Baron and Myerson (1982) showed that the optimal price mechanism, say $(p^*(\theta), T^*(\theta))$, exhibits the usual properties. Thus, it is easy to see that when the demand $Q(p)$ is common knowledge, the optimal quantity mechanism is $q^*(\theta) \equiv Q(p^*(\theta))$. As a result, price and quantity regulation lead to the same level of welfare.

Proposition 10.   When there is private information on the costs, but not on the demand, both regulatory instruments—price and quantity—lead to the same results in terms of final price, production, transfers, consumer surplus, and firm profits.

Readers familiar with Weitzman (1974) may find Proposition 10 puzzling, for his main result is that when costs are private information to the firms, the relative advantage of one mechanism over the other depends on the slopes of demand and cost functions and the variance of the cost function. As we briefly argue in Section 1, Weitzman's analysis does not extend to monopoly regulation. In his model, a simple quantity mechanism obeys $q^* = \arg\max_q \mathbb{E}_\theta[V(q, \theta) - C(q, \theta)]$, whereas a simple price mechanism solves $p^* = \arg\max_p \mathbb{E}_\theta[V(h(p, \theta), \theta) - C(h(p, \theta), \theta)]$, with $h(p, \theta) = \arg\max_q pq - C(q, \theta)$. So Weitzman assumes that once a regulated price is given, the firm (or firms) will act as price taker(s) and choose a quantity of production according to its (their) cost function, ignoring the demand. However, in monopoly regulation, the firm faces the whole demand, and thus the quantity produced for a regulated price p will not be given by profit maximization but simply by the market demand $Q(p)$.[20]

6.   Conclusions

■   This article examines the optimal design of monopoly regulation in the presence of asymmetric information. We focus on the case in which the demand is private information to the firm, and the regulator—aside from setting lump-sum transfers—specifies either the quantity that the firm must produce or the price that it must set. We consider sophisticated regulation, when the regulator is allowed to offer an unlimited number of options to the firm, and also simple regulation, when the regulator is restricted to offer a single option. The ranking of instruments depends critically on the firm's cost technology and on whether the regulator employs sophisticated or simple mechanisms.

Although in almost all real-world situations price has been the regulation instrument of choice, this article suggests that in many settings, quantity regulation is a better regulatory instrument. In particular, under decreasing marginal cost and with the possibility of offering menus, a regulator would significantly improve welfare regulating through quantity. As long as the regulator can enforce the actual sale of the product, which is the case, for example, when airport slots are assigned or the delivery of a basic and measurable service is involved, such as water or electricity, quantity seems an attractive regulatory instrument.

It is important to stress that there are significant gains from offering (semi-) sophisticated mechanisms. As long as the regulator is willing and able to offer some choice to the firm, benefits can be realized from this flexibility by inducing a more efficient allocation and limiting rents to the firm. Indeed, offering a semi simple menu with only two options would capture an important share of maximal welfare gains. Also, relevant for decision making is that the optimal choice of instrument may depend on the level of sophistication that the regulator is able to implement. Thus, the gains from introducing menus in real-world situations are underestimated if one considers the instrument as exogenously fixed, which we believe may have misled regulators in the past.

With respect to where to drive this research next, we believe that there are several avenues to extend the comparison of instruments. To name a few, it would be interesting to

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[20] Laffont (1977) studied the case in which demand is private information but costs are uncertain to both parties. He assumes that for any fixed price, the total quantity produced is given by the market demand, making his model applicable to market power, as opposed to Weitzman (1974), who assumed competitive firms.

(i) study the robustness of each mechanism to renegotiations, (ii) explore the incentives for costly investments—affecting the costs or demand—in each case, (iii) study settings in which lump-sum transfers are not feasible, and (iv) examine environments in which the regulator is willing to shut down low types.

Appendix

This appendix contains all omitted proofs and the analysis of examples cited in Section 3.

Proof of Lemma 1. Prices. The single crossing property holds iff $\pi_p(p, T, \theta)/\pi_T(p, T, \theta)$ is increasing in θ. Because the profit function is quasilinear in T, the SCP holds iff $\pi_{p\theta} \geq 0$. The profit function obeys $\pi(p, T, \theta) = pQ(p, \theta) - C(Q(p, \theta)) + T$, and thus $\pi_{p\theta} = Q_{p\theta}(p - C') + Q_\theta(1 - C''Q_p)$.

Quantities. The SCP holds iff $\pi_q(q, T, \theta)/\pi_T(q, T, \theta)$ rises in θ. However, because π is quasilinear in T, the SCP holds iff $\pi_{q\theta} \geq 0$. Finally, $\pi(q, T, \theta) = P(q, \theta)q - C(q) + T$, and so $\pi_{p\theta} = qP_{q\theta} + P_\theta$. ☐

Proof of Lemma 2. Prices. By the Envelope Theorem, $\Pi'(\theta) = \pi_{\hat\theta}(\hat\theta, \theta)|_{\hat\theta=\theta} = [p(\theta) - C'(Q(p(\theta), \theta))]Q_\theta(p(\theta), \theta)$. Next, $\pi_{\hat\theta}(\theta, \theta) = 0$, by incentive compatibility (IC). Totally differentiating in θ: $\pi_{\hat\theta\theta}(\theta, \theta) + \pi_{\hat\theta\hat\theta}(\theta, \theta) = 0$. So,

$$\pi_{\hat\theta\theta}(\theta, \theta) = p'(\theta)[Q_{p\theta}(p - C') + Q_\theta(1 - C''Q_p)] \geq 0,$$

because $\pi_{\hat\theta\hat\theta}(\theta, \theta) \leq 0$, by IC. Thus, $p' > 0$ iff $Q_\theta(1 - C''Q_p) > -Q_{p\theta}(p - C')$. The proof of the converse is standard (see, e.g., Baron and Myerson, 1982).

Quantities. First, by the Envelope Theorem, $\Pi'(\theta) = \pi_{\hat\theta}(\hat\theta, \theta)|_{\hat\theta=\theta} = P_\theta(q(\theta), \theta)q(\theta)$. Next, $\pi_{\hat\theta}(\theta, \theta) = 0$, given (IC). Totally differentiating in θ: $\pi_{\hat\theta\theta}(\theta, \theta) + \pi_{\hat\theta\hat\theta}(\theta, \theta) = 0$. By IC, $\pi_{\hat\theta\hat\theta}(\theta, \theta) \leq 0$, and thus,

$$\pi_{\hat\theta\theta}(\theta, \theta) = q'(\theta)[P_{q\theta}(q(\theta), \theta)q(\theta) + P_\theta(q(\theta), \theta)] = q'P_\theta[qP_{q\theta}/P_\theta + 1] \geq 0.$$

So, $q' > 0$ iff $qP_{q\theta}/P_\theta > -1$, for $P_\theta > 0$. The converse is standard (Baron and Myerson, 1982). ☐

Proof of Proposition 1. First, by standard methods in mechanism design (Baron and Myerson, 1982), Lemma 1, and integration by parts allow us to rewrite the regulator's objective function as

$$\int_{\underline\theta}^{\bar\theta} \{V(q(\theta), \theta) - C(q(\theta)) - z_\alpha(\theta)P_\theta(q(\theta), \theta)q(\theta)\}g(\theta)d\theta - (1 - \alpha)\Pi(\underline\theta), \tag{A1}$$

where $V(q, \theta) \equiv \int_0^q P(q', \theta)dq'$. In any optimal mechanism, the inefficient type gets no rents, or $\Pi(\underline\theta) = 0$. Next, observe that for each θ, the integrand of the objective function (A1) is concave in $q(\theta)$, by Assumptions 1 and 2. Thus, the first-order condition (FOC) of the relaxed problem (A1) for each θ yields the allocation rule $q^o(\theta)$ in (5a). Finally, totally differentiating (5a) yields:

$$\frac{dq^o(\theta)}{d\theta} = \frac{-P_\theta + z'_\alpha(\theta)P_\theta + z_\alpha(\theta)\varphi_\theta(q^o, \theta)}{P_q - C'' - z_\alpha(\theta)\varphi_q(q^o, \theta)} > 0,$$

where $\varphi(q, \theta) \equiv P_\theta(q, \theta) + qP_{\theta q}(q, \theta) \geq 0$ by the SCP, and it satisfies $\varphi_q(q, \theta) \geq 0$ by Assumption 2, and $\varphi_\theta(q, \theta) \leq 0$ by Assumption 3. The inequality then follows because, in addition, $z'_\alpha(\theta) < 0$ and $P_q < C''$ by Assumption 1. Thus, q^o is increasing in θ and solves the regulator's problem. Transfers $T^o(\theta)$ are then found by integrating $\Pi'(\theta) = P_\theta(q^o(\theta), \theta)q^o(\theta)$ in Lemma 2-(iv). Finally, fix θ and implicitly differentiate equation (5a) in the weight α to get:

$$\frac{dq^o(\theta)}{d\alpha} = \frac{-\varphi(q^o, \theta)(1 - G(\theta))/g(\theta)}{P_q - C'' - z_\alpha(\theta)\varphi_q(q^o, \theta)} > 0,$$

where the inequality follows by analogous arguments. ☐

Proof of Proposition 2. The first-order condition to maximize (6) in p^* yields:

$$(1 - \alpha)T'(p^*) = \mathbb{E}_\theta[-(1 - \alpha)Q(p^*, \theta) + \alpha Q_p(p^*, \theta)(p^* - C'(Q(p^*, \theta)))].$$

Next, as $T(p^*) = C(Q(p^*, \theta_c(p^*))) - p^*Q(p^*, \theta_c(p^*))$ and $p^* = C'(Q(p^*, \theta_c(p^*)))$, we have:

$$T'(p^*) = C'(Q_p + Q_\theta\theta'_c) - p^*(Q_p + Q_\theta\theta'_c) - Q = -Q. \tag{A2}$$

Also, as $Q_p(p^*, \theta) \equiv Q_p(p^*)$ by additive demand shocks, we obtain:

$$-(1 - \alpha)Q(p^*, \theta_c(p^*)) = \mathbb{E}_\theta[-(1 - \alpha)Q(p^*, \theta) + \alpha Q_p(p^*)(p^* - C'(Q(p^*, \theta)))], \tag{A3}$$

which is equivalent to

$$\mathbb{E}_\theta\{(1 - \alpha)[Q(p^*, \theta_c) - Q(p^*, \theta)] + \alpha Q_p[p^* - C'(Q(p^*, \theta))]\} = 0. \tag{A4}$$

Next, observe that if marginal costs are convex $C''' \geq 0$, then

$$C'(Q(p^*, \theta)) \geq C'(Q(p^*, \theta_c)) + C''(Q(p^*, \theta_c))[Q(p^*, \theta) - Q(p^*, \theta_c)]. \quad (A5)$$

Thus, as $p^* = C'(Q(p^*, \theta_c))$ and $Q_p < 0$, inequalities (A4) and (A5) imply that for all $\alpha \in [0, 1]$:

$$\mathbb{E}_\theta[Q(p^*, \theta_c) - Q(p^*, \theta)][1 - \alpha + \alpha Q_p C''(Q(p^*, \theta_c))] \leq 0. \quad (A6)$$

Finally, because $Q_p C'' > 0$, inequality (A6) implies $Q(p^*, \theta_c) \leq \mathbb{E}_\theta Q(p^*, \theta) = Q(p^*, \mathbb{E}\theta)$, or $\theta_c \leq \mathbb{E}\theta$. If marginal costs are concave $C''' \leq 0$, then the inequality sign in (A5) reverses, and so by analogous arguments, we have that the critical type is greater than the expected type $\theta_c \geq \mathbb{E}\theta$. As a corollary, if marginal costs are linear, then the critical type θ_c is the expected type $\mathbb{E}(\theta)$.

Now, lets show how the optimal price p^* changes with α. First, rewrite (6) as $CS(p) + \alpha PS(p)$, where $CS(p) \equiv \mathbb{E}_\theta \int_p^\infty Q(x, \theta)dx - T(p)$ is the *net consumer surplus*, and $PS(p) \equiv \mathbb{E}_\theta[pQ(p, \theta) - C(Q(p, \theta)) + T(p)]$ is the *net producer surplus*. Next, by FOC in p, we have that at the optimum: $CS'(p^*) + \alpha PS'(p^*) = 0$. Thus, implicitly differentiating this last equation in α yields:

$$CS''(p^*)\frac{dp^*}{d\alpha} + \alpha PS''(p^*)\frac{dp^*}{d\alpha} + PS'(p^*) = 0 \implies \frac{dp^*}{d\alpha} = -\frac{PS'}{CS'' + \alpha PS''}. \quad (A7)$$

By the FOC, $PS'(p^*) \geq 0$ iff $CS'(p^*) = Q(p^*, \theta_c) - \mathbb{E}_\theta Q(p^*, \theta) \leq 0$, because $T'(p^*) = -Q(p^*, \theta_c)$ (see (A2)) and $CS'' + \alpha PS'' < 0$ by the second-order condition (SOC). Finally, because $Q_{p\theta} = 0$, we have $CS'(p^*) = \theta_c - \mathbb{E}\theta \leq 0$ iff $\theta_c \leq \mathbb{E}\theta$, where the last condition holds if $C'' \geq 0$ by our previous analysis. If $C'' \leq 0$, then $\theta_c \geq \mathbb{E}\theta$, and so by analogous arguments, $dp/d\alpha \leq 0$. $\qquad\square$

Proof of Proposition 3. The case in which marginal costs are nondecreasing is obvious, as the price mechanism achieves the first-best. So, suppose that marginal costs are strictly decreasing. First, it is easy to see that the social welfares in price and quantity regulation, say $SW^P(\alpha)$ and $SW^Q(\alpha)$, are continuous in α. Next, when $\alpha = 1$, the quantity mechanism achieves the first-best, whereas the price mechanism induces allocative inefficiencies; thus, $\varphi(\alpha) \equiv SW^Q(\alpha) - SW^P(\alpha) > 0$ at $\alpha = 1$. Finally, take $\epsilon > 0$ with $\varphi(1) - \epsilon > 0$, by continuity, there exists $\delta > 0$ such that for all $\alpha > 1 - \delta \equiv \alpha^*$, we have $\varphi(\alpha) = SW^Q(\alpha) - SW^P(\alpha) > \varphi(1) - \epsilon > 0$, that is, $SW^Q(\alpha) > SW^P(\alpha)$. $\qquad\square$

Proof of Lemma 3. Given (A1), for the quantity mechanism, we have:

$$SW^Q \equiv \int_{\underline{\theta}}^{\bar{\theta}} \left\{ V(q^o(\theta), \theta) - C(q^o(\theta)) - (1 - \alpha)\frac{1 - G(\theta)}{g(\theta)}P_2(q^o(\theta), \theta)q^o(\theta) \right\} g(\theta)d\theta,$$

where $V(q, \theta) \equiv \int_0^q P(q', \theta)dq'$ is the gross consumer welfare. Next, by the Envelope Theorem:

$$\frac{\partial SW^Q}{\partial \alpha} = \mathbb{E}_\theta\left[\frac{1 - G(\theta)}{g(\theta)}P_\theta(q^o(\theta), \theta)q^o(\theta)\right] > 0,$$

where the inequality follows, as $P_\theta(q, \theta)q$ rises in q by the SCP, and q^o rises in α, by Proposition 1.

Now we turn to price regulation. First, as in the proof of Proposition 2, write (6) as $SW^P(\alpha) = \max_p CS(p) + \alpha PS(p)$, where $CS(p) \equiv \mathbb{E}_\theta \int_p^\infty Q(x, \theta)dx - T(p)$ is the net consumer surplus, and $PS(p) \equiv \mathbb{E}_\theta[pQ(p, \theta) - C(Q(p, \theta)) + T(p)]$ is the net producer surplus. Next, by the Envelope Theorem, $\partial SW^P/\partial \alpha = PS(p^*) > 0$, for it is an expectation of a random variable that is strictly positive almost everywhere, and so SW^P rises in α. Differentiating SW^P once again yields:

$$\frac{\partial^2 SW^P}{\partial \alpha^2} = PS'(p^*)\frac{dp^*}{d\alpha} = \frac{-PS'(p^*)^2}{CS'' + \alpha PS''} \geq 0,$$

as $dp^*/d\alpha = -PS'/(CS'' + \alpha PS'')$ by (A7), and $CS'' + \alpha PS'' < 0$ by the SOC.

Now, assume that $C''' = 0$ and that demand shocks are additive (i.e., $Q_{p\theta} = 0$). Proposition 2 shows that p^* is independent of α. Thus, $\partial SW^P/\partial \alpha = PS(p^*)$ is independent of α. $\qquad\square$

Proof of Proposition 4. Consider an inverse demand function $P(q, \theta) = P(0) + \theta + P_q q$ and marginal costs $C'(q) = C'(0) + C''q$, with $P_q < C'' < 0$ to satisfy Assumption 1. By Proposition 2, the optimal price mechanism is efficient for the expected type $\mathbb{E}(\theta)$. Thus, there are no welfare losses for this type; but, for low types $\theta < \mathbb{E}(\theta)$ and high types $\theta > \mathbb{E}(\theta)$, the optimal price mechanism mandates them to produce more and less than what is efficient, respectively. In particular, for the lowest type $\underline{\theta}$, the welfare loss is the area $\bar{\ell} = \bar{\gamma}\bar{\delta}/2$, depicted in Figure A1. Next, consider a type $\theta \in (\underline{\theta}, \bar{\theta}]$. The welfare loss for this type is $\gamma(\theta)\delta(\theta)/2$, as illustrated in Figure A1. The functions $\gamma(\cdot)$ and $\delta(\cdot)$ capture two sources of inefficiencies. Indeed, when the demand realization is θ, the function $\delta(\theta)$ is the absolute difference of the regulated price and the realized marginal cost, whereas $\gamma(\theta)$ is the absolute difference of the first-best production level and the one induced by the regulated price. Now, observe that $\gamma(\theta)$ and $\delta(\theta)$ are linear in θ, and so we can write them

FIGURE A1

PROPOSITION 4 DEPICTED [Color figure can be viewed at wileyonlinelibrary.com]
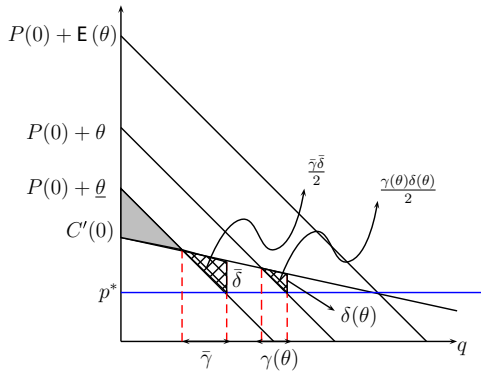
When demand and marginal costs are linear, the deadweight loss for each type $\theta < \mathbb{E}(\theta)$ induced by the optimal price regulation is proportional to that of $\underline{\theta}$.

as $\gamma(\theta) = \bar{\gamma} + \gamma'(\theta - \underline{\theta})$ and $\delta(\theta) = \bar{\delta} + \delta'(\theta - \underline{\theta})$. Because $\gamma(\mathbb{E}(\theta)) = \delta(\mathbb{E}(\theta)) = 0$ (as the optimal price p^* is efficient for the expected type), it follows that $\gamma(\theta) = \bar{\gamma}(\mathbb{E}(\theta) - \theta)/(\mathbb{E}(\theta) - \underline{\theta})$ and $\delta(\theta) = \bar{\delta}(\mathbb{E}(\theta) - \theta)/(\mathbb{E}(\theta) - \underline{\theta})$. Next, using the functional forms for the demand and marginal costs, it is easy to find that:

$$\bar{\ell} = \frac{\bar{\delta}\bar{\gamma}}{2} = \frac{C''(\mathbb{E}(\theta) - \underline{\theta})^2}{P_q^2(C'' - P_q)}.$$

Hence, the total deadweight loss induced by the price mechanism is given by:

$$DWL^P = \bar{\ell}\int_{\underline{\theta}}^{\bar{\theta}}\left(\frac{\mathbb{E}(\theta) - \theta}{\mathbb{E}(\theta) - \underline{\theta}}\right)^2 g(\theta)d\theta = \frac{C''}{2P_q^2(C'' - P_q)}\int_{\underline{\theta}}^{\bar{\theta}}(\mathbb{E}(\theta) - \theta)^2 g(\theta)d\theta. \tag{A8}$$

Now we turn to the optimal quantity mechanism $q^o(\theta)$, which is efficient when $\alpha = 1$, by Proposition 1. Thus, intersecting the demand and the marginal costs for each type yields a quantity rule: $q^o(\theta) = (P(0) + \theta - C'(0))/(C'' - P_q)$. Next, by the linearity of the demand and marginal costs, the social welfare $SW^Q(\theta)$ for a type θ is a triangular area with base $P(0) + \theta - C'(0)$ and height $q^o(\theta)$. For instance, the shaded region in Figure A1 is the social welfare for the lowest type $\underline{\theta}$. All told,

$$SW^Q(\theta) = \frac{(P(0) + \theta - C'(0))^2}{2(C'' - P_q)} \quad \Rightarrow \quad SW^Q = \mathbb{E}SW^Q(\theta) = \int_{\underline{\theta}}^{\bar{\theta}}\frac{(P(0) + \theta - C'(0))^2}{2(C'' - P_q)}g(\theta)d\theta. \tag{A9}$$

Finally, divide DWL^P in (A8) and SW^Q in (A9) to get the desired expression (7). □

Examples cited in Section 3. Consider an inverse demand function $P(q, \theta) = A + \theta - bq$ with $A, b \geq 0$, and a cost function $C(q) = c_0 q - kq^2/2$, with $k \geq 0$. Suppose that θ is uniformly distributed in $[0, \bar{\theta}]$, and thus $z_\alpha(\theta) = (1 - \alpha)(\bar{\theta} - \theta)$. Using Proposition 1, the optimal quantity mechanism is:

$$q^o(\theta) = \frac{A + \theta - z_\alpha(\theta) - c_0}{b - k} \quad \text{and} \quad T^o(\theta) = c_0 q^o(\theta) - \frac{1}{2}kq^o(\theta)^2 + \int_0^{\bar{\theta}} q^o(x)dx. \tag{A10}$$

By (6), the optimal price mechanism is:

$$p^* = \frac{bc_0 - k(A + \bar{\theta}/2)}{b - k} \quad \text{and} \quad T^* = \frac{k(A - c_0 + \bar{\theta}/2)^2}{2(b - k)^2}. \tag{A11}$$

Let $\Delta SW \equiv SW^Q - SW^P$. Computing the difference of social welfares, we get:

$$\Delta SW = \bar{\theta}\frac{-12Ab^2(1 - \alpha) + bk(1 - \alpha)\bar{\theta} + k^2\alpha\bar{\theta} + 4b^2(1 - \alpha)(3c_0 - \alpha\bar{\theta})}{24b^2(b - k)}, \tag{A12}$$

which implies that $\Delta SW|_{\alpha=0} < 0$, and thus $\alpha^* \in (0, 1)$. However, this does not hold if we consider distributions that put most of the mass in $\{0, \bar{\theta}\}$. Consider a sequence of distributions that converge to the one that only puts mass in $\{0, \bar{\theta}\}$, with $G(0) = 1/2$ and $G(\bar{\theta}) = 1$. Also, fix $A = 2 + c_0$, $b = 1$, $k = 1/2$, and $\bar{\theta} = \sqrt{5} - 1 + \epsilon$ with small $\epsilon > 0$. Then,

a double crossing emerges: $SW^Q = SW^P$ at $\alpha^* \sim 0.004$ and at $\alpha^* \sim 0.01$. Also, if $\bar\theta > \sqrt{5} - 1$ and $k < b < 1$, then $SW^Q > SW^P$ for all α.

Proof of Proposition 5. We start by characterizing the optimal simple quantity mechanism. As discussed in Section 4, for any regulated quantity q, transfers are set to leave the lowest type $\underline\theta$ with zero profits, that is, $T(q) \equiv C(q) - P(q, \underline\theta)q$. Next, the regulator chooses q to maximize $\mathbb{E}_\theta SW(q, \theta, T(q))$ in (4), that is,

$$\max_{q \geq 0} \mathbb{E}[V(q, \theta) - C(q) + P(q, \underline\theta)q + \alpha(P(q, \theta)q - P(q, \underline\theta)q)].$$

The FOC yields:

$$\alpha \mathbb{E} P(q, \theta) + (1 - \alpha)P(q, \underline\theta) = C'(q). \tag{A13}$$

If $P(q, \theta)$ is linear in θ, (A13) turns to $P(q, \alpha\mathbb{E}(\theta) + (1 - \alpha)\underline\theta) = C'(q)$. Thus, the optimal quantity mechanism is the one that is (allocative) efficient for a type weakly below the average.

Now, we proceed with the proof of Proposition 5.

Step 1. $\alpha = 0$. First, recall that the optimal price mechanism is independent of α and solves $p^* = C'(Q(p^*, \mathbb{E}\theta))$, by Lemma 2. On the other hand, by (A13), the optimal simple quantity mechanism obeys $P(q^*, \underline\theta) = C'(q^*)$ and $T(q^*) = C(q^*) - P(q^*, \underline\theta)q^* = C(q^*) - C'(q^*)q^*$. The associated social welfare is

$$SW^Q = \mathbb{E}_\theta \left\{ \int_0^{q^*} [P(x, \theta) - P(q^*, \theta)]dx \right\} - T(q^*) = -P_q \frac{q^{*2}}{2} - T(q^*).$$

Now, let $\hat q(\theta) \equiv Q(p^*, \theta)$ be the quantity allocation induced by p^*, which is linear in θ. Then,

$$SW^P = \mathbb{E}_\theta \left\{ \int_0^{\hat q(\theta)} [P(x, \theta) - P(\hat q(\theta), \theta)]dx \right\} - T(\hat q(\mathbb{E}\theta)) = -\mathbb{E}_\theta \left[P_q \frac{\hat q(\theta)^2}{2} \right] - T(\hat q(\mathbb{E}\theta)).$$

By the linearity of marginal costs, and using that $T(q) = C(q) - C'(q)q$:

$$T(\hat q(\mathbb{E}\theta)) - T(q^*) = -C''q^*(\hat q(\mathbb{E}\theta) - q^*) - \frac{C''}{2}(\hat q(\mathbb{E}\theta) - q^*)^2 = -\frac{C''}{2}(\hat q(\mathbb{E}\theta)^2 - q^{2*}).$$

Finally, note that $\mathbb{E}(\hat q(\theta)^2) > \hat q(\mathbb{E}\theta)^2$, by Jensen's Inequality, and so $SW^P > SW^Q$ because:

$$SW^P - SW^Q + T(\hat q(\mathbb{E}\theta)) - T(q^*) > -\frac{P_q}{2}(\hat q(\mathbb{E}\theta)^2 - q^{2*}) > -\frac{C''}{2}(\hat q(\mathbb{E}\theta)^2 - q^{2*}) = T(\hat q(\mathbb{E}\theta)) - T(q^*).$$

Step 2. $\alpha = 1$. Again by (A13), the optimal quantity q^* solves $P(q^*, \mathbb{E}\theta) = C'(q^*)$, inducing a social welfare:

$$SW^Q = \mathbb{E}_\theta \left[\int_0^{q^*} P(x, \theta)dx - C(q^*) \right] = \int_0^{q^*} P(x, \mathbb{E}\theta)dx - C(q^*),$$

where the last equality holds by the linearity of the demand. Next, the social welfare for the price mechanism is

$$SW^P = \mathbb{E}_\theta \left[\int_0^{\hat q(\theta)} P(x, \theta)dx - C(\hat q(\theta)) \right] \equiv \mathbb{E}_\theta \psi(\theta).$$

Note that $\psi'' > 0$, as $\hat q'(\theta) > 0 = \hat q''(\theta)$, and thus $SW^P \geq \psi(\mathbb{E}\theta) = SW^Q$, by Jensen's Inequality.

Step 3. By Lemma 3, SW^Q is increasing and convex in α, whereas SW^P is increasing and linear in α. Thus, because $SW^P > SW^Q$ for $\alpha \in \{0, 1\}$, if follows that $SW^P > SW^Q$ for all $\alpha \in [0, 1]$. □

Proof of Proposition 6. First, we start by characterizing the optimal simple price mechanism for strictly increasing marginal costs. In this case, the maximized profits Π are concave in θ by Lemma 1, and thus the critical type θ_c lies in the corners, that is, $\theta_c \in \{\underline\theta, \bar\theta\}$.

Lemma 1. If $\theta_c = \underline\theta$, then the optimal price p^* satisfies $p^* \leq C'(Q(p^*, \alpha\mathbb{E}(\theta) + (1 - \alpha)\underline\theta))$. Conversely, if $\theta_c = \bar\theta$, then $p^* \geq C'(Q(p^*, \alpha\mathbb{E}(\theta) + (1 - \alpha)\bar\theta))$. Thus, if $\alpha = 1$, then $p^* = C'(Q(p^*, \mathbb{E}(\theta)))$.

Proof. Given $\theta_c \in \{\underline\theta, \bar\theta\}$, the regulator solves:

$$\max_{p \geq 0} \mathbb{E}_\theta \left[\int_p^\infty Q(x, \theta)dx - T(p) + \alpha(pQ(p, \theta) - C(Q(p, \theta)) - (1 - \alpha)T(p)) \right],$$

where $T(p) = C(Q(p, \theta_c)) - pQ(p, \theta_c)$. The first-order condition yields:

$$\mathbb{E}_\theta\{(1 - \alpha)[Q(p, \theta_c) - Q(p, \theta)] + Q_p[p - \alpha C'(Q(p, \theta)) - (1 - \alpha)C'(Q(p, \theta_c))]\} = 0.$$

Because Q is linear in θ and C' is linear in Q, the optimality condition reduces to:

$$(1 - \alpha)[Q(p, \theta_c) - Q(p, \mathbb{E}(\theta))] + Q_p[p - C'(Q(p, \alpha\mathbb{E}(\theta) + (1 - \alpha)\theta_c))] = 0. \tag{A14}$$

If $\theta_c = \underline{\theta}$, then $p \leq C'(Q(p, \alpha \mathbb{E}(\theta) + (1 - \alpha)\underline{\theta}))$ to satisfy (A14), as $Q_p < 0 < Q_\theta$. The case $\theta_c = \bar{\theta}$ is analogous. So if $\alpha = 1$, then $C'(Q(p^*, \mathbb{E}(\theta))) \leq p^* \leq C'(Q(p^*, \mathbb{E}(\theta)))$, and so $p^* = C'(Q(p^*, \mathbb{E}(\theta)))$. □

Now, we prove Proposition 6. First, let $\alpha = 1$ and call p^* and $\hat{q}$ the optimal simple price and quantity. Now define $q^*(\theta) = Q(p^*, \theta)$, and write the social welfare function as:

$$V(q, \theta) - C(q) = \int_0^{\hat{q}} P(q', \theta)dq' - C(\hat{q}) + [P(\hat{q}, \theta) - C'(\hat{q})](q - \hat{q}) + \frac{1}{2}[P_q(\hat{q}, \theta) - C''(\hat{q})](q - \hat{q})^2.$$

Now evaluate at $q = q^*(\theta)$, and take expectations in each side. Let $\Delta SW \equiv SW^P - SW^Q$, then:

$$\Delta SW = \mathbb{E}\left\{[P(\hat{q}, \theta) - C'(\hat{q})](q^*(\theta) - \hat{q}) + \frac{1}{2}[P_q(\hat{q}, \theta) - C''(\hat{q})](q^*(\theta) - \hat{q})^2\right\}.$$

Notice that $P(\hat{q}, \theta) - C'(\hat{q}) = P(\hat{q}, \theta) - P(\hat{q}, \mathbb{E}(\theta)) = P_\theta(\theta - \mathbb{E}(\theta))$, because $\hat{q}$ is optimally chosen and P is linear in θ. Likewise, $q^*(\theta) - \hat{q} = Q(p^*, \theta) - Q(p^*, \mathbb{E}(\theta)) = Q_\theta(\theta - \mathbb{E}(\theta))$, for Q is linear in θ. Finally, $Q_\theta = -P_\theta/P_q$, because $P(Q(p, \theta), \theta) \equiv p$. Thus,

$$\Delta SW = \mathbb{E}\left\{-\frac{P_\theta^2}{P_q}(\theta - \mathbb{E}(\theta))^2 + \frac{P_\theta^2}{2P_q^2}[P_q - C''](\theta - \mathbb{E}(\theta))^2\right\} = -\frac{P_\theta^2 \, \text{var}(\theta)}{2P_q^2}(P_q + C''),$$

and so $\Delta SW \geq 0$ if and only if $P_q + C'' \leq 0$. □

Proof of Proposition 7. First, by (A9), the social welfare induced by the optimal sophisticated quantity mechanism is:

$$SW_{soph}^Q = \int_{\underline{\theta}}^{\bar{\theta}} \frac{(P(0) + \theta - C'(0))^2}{2(C'' - P_q)} g(\theta)d\theta. \tag{A15}$$

Next, using (A13), the social welfare induced by the optimal simple quantity mechanism is:

$$SW_{sim}^Q = \frac{(P(0) + \alpha\mathbb{E}(\theta) - C'(0))^2}{2(C'' - P_q)}. \tag{A16}$$

Finally, letting $\alpha = 1$ and taking the ratio between (A16) and (A15) yields (8). □

Proof of Proposition 8. First, because any semi simple mechanism *is* a sophisticated mechanism, the regulator's objective function obeys (see the proof of Proposition 1 in the Appendix):

$$\int_{\underline{\theta}}^{\bar{\theta}} \{V(q(\theta), \theta) - C(q(\theta)) - z_\alpha(\theta)P_\theta(q(\theta), \theta)q(\theta)\}g(\theta)d\theta - (1 - \alpha)\Pi(\underline{\theta}),$$

where $q(\theta) = q_1$ if $\theta < \hat{\theta}$ and $q(\theta) = q_2$ otherwise. Hence, at the optimum, $\Pi(\underline{\theta}) = 0$. Next, the regulator chooses $(q_1, q_2, \hat{\theta})$ to maximize:

$$\int_{\underline{\theta}}^{\hat{\theta}} \{V(q_1, \theta) - C(q_1) - z_\alpha(\theta)P_\theta(q_1, \theta)q_1\}g(\theta)d\theta + \int_{\hat{\theta}}^{\bar{\theta}} \{V(q_2, \theta) - C(q_2) - z_\alpha(\theta)P_\theta(q_2, \theta)q_2\}g(\theta)d\theta.$$

The first-order conditions in $(q_1, q_2, \hat{\theta})$ yield the expressions (9a)–(9c).

Finally, let's check that the allocation is monotone, that is, $q_1 < q_2$. First, observe that for any q,

$$\mathbb{E}\left[z_\alpha(\theta)P_\theta(q, \theta)\left(1 + q\frac{P_{\theta q}(q, \theta)}{P_\theta(q, \theta)}\right)\Big| \theta \leq \hat{\theta}\right] > \mathbb{E}\left[z_\alpha(\theta)P_\theta(q, \theta)\left(1 + q\frac{P_{\theta q}(q, \theta)}{P_\theta(q, \theta)}\right)\Big| \theta \geq \hat{\theta}\right].$$

Because $P_\theta(q, \theta) + qP_{\theta q}(q, \theta)$ falls in θ, by Assumption 3, and $z'_\alpha(\theta) < 0$. Also, because $P_\theta > 0$,

$$\mathbb{E}[P(q, \theta)|\theta \leq \hat{\theta}] - C'(q) < \mathbb{E}[P(q, \theta)|\theta \geq \hat{\theta}] - C'(q).$$

Thus, because $P(q, \theta) - C'(q)$ falls in q, by Assumption 1, and $qP_{\theta q} + P_\theta$ rises in q, by Assumption 3; if $q_2 < q_1$, then that would imply that q_1 is not optimal, which is a contradiction. □

Proof of Proposition 9. Fix $\hat{\theta} \in \Theta$. Because the demand is linear and costs are quadratic, for any q, q_1 and θ we have:

$$V(q, \theta) - C(q) - z_\alpha(\theta)q = V(q_1, \theta) - C(q_1) - z_\alpha(\theta)q_1 + [P(q_1, \theta) - C'(q_1) - z_\alpha(\theta)](q - q_1) + [P_q - C'']\frac{(q - q_1)^2}{2},$$

where $V_q(q, \theta) = P(q, \theta)$ and $V_{qq}(q, \theta) = P_q$. Next, evaluate the above expression at $q = q^o(\theta)$, where q^o is optimal quantity (Proposition 1), and multiply both sides by $g(\theta)d\theta$ and integrate from $\underline{\theta}$ to $\hat{\theta}$:

$$\int_{\underline{\theta}}^{\hat{\theta}} [V(q^o(\theta), \theta) - C(q^o(\theta)) - z_\alpha(\theta)q^o(\theta)]g(\theta)d\theta = \int_{\underline{\theta}}^{\hat{\theta}} [V(q_1, \theta) - C(q_1) - z_\alpha(\theta)q_1]g(\theta)d\theta$$

$$+ \int_{\underline{\theta}}^{\hat{\theta}} [P(q_1, \theta) - C'(q_1) - z_\alpha(\theta)](q^o(\theta) - q_1)g(\theta)d\theta$$

$$+ \int_{\underline{\theta}}^{\hat{\theta}} [P_q - C''] \frac{(q^o(\theta) - q_1)^2}{2} g(\theta)d\theta. \tag{A17}$$

Next, by Proposition 1 and Proposition 8, one can see that

$$q^o(\theta) - q_1 = \frac{\theta - z_\alpha(\theta) - \mathbb{E}(\theta - z_\alpha(\theta)|\theta \le \hat{\theta})}{C'' - P_q}. \tag{A18}$$

Next, add and subtract $P(q_1, \mathbb{E}(\theta|\theta \le \hat{\theta})) - \mathbb{E}(z_\alpha(\theta)|\theta \le \hat{\theta})$ in $P(q_1, \theta) - C'(q_1) - z_\alpha(\theta)$, and then use Proposition 8 and that $P(q, \theta) = P(q) + \theta$ to get

$$P'(q_1, \theta) - C'(q_1) - z_\alpha(\theta) = P'(q_1, \theta) - P'(q_1, \mathbb{E}(\theta|\theta \le \hat{\theta})) + \mathbb{E}(z_\alpha(\theta)|\theta \le \theta) - z_\alpha(\theta)$$

$$= \theta - z_\alpha(\theta) - \mathbb{E}(\theta - z_\alpha(\theta)|\theta \le \hat{\theta}). \tag{A19}$$

Altogether, equations (A17)–(A19) imply:

$$\int_{\underline{\theta}}^{\hat{\theta}} [V(q^o(\theta), \theta) - C(q^o(\theta)) - z_\alpha(\theta)q(\theta)]g(\theta)d\theta = \int_{\underline{\theta}}^{\hat{\theta}} [V(q_1, \theta) - C(q_1) - z_\alpha(\theta)q_1]g(\theta)d\theta$$

$$+ \int_{\underline{\theta}}^{\hat{\theta}} \frac{[\theta - z_\alpha(\theta) - \mathbb{E}(\theta - z_\alpha(\theta)|\theta \le \hat{\theta})]^2}{2(C'' - P_q)} g(\theta)d\theta. \tag{A20}$$

Following the same logic, we can obtain an analogous expression for $\theta \ge \hat{\theta}$, namely:

$$\int_{\hat{\theta}}^{\bar{\theta}} [V(q^o(\theta), \theta) - C(q^o(\theta)) - z_\alpha(\theta)q(\theta)]g(\theta)d\theta = \int_{\hat{\theta}}^{\bar{\theta}} [V(q_2, \theta) - C(q_2) - z_\alpha(\theta)q_2]g(\theta)d\theta$$

$$+ \int_{\hat{\theta}}^{\bar{\theta}} \frac{[\theta - z_\alpha(\theta) - \mathbb{E}(\theta - z_\alpha(\theta)|\theta \ge \hat{\theta})]^2}{2(C'' - P_q)} g(\theta)d\theta. \tag{A21}$$

Then, adding (A20) and (A21) yield:

$$SW^Q - \overline{SW}^Q = \frac{1}{2(C'' - P_q)} \int_{\underline{\theta}}^{\hat{\theta}} \left[\theta - z_\alpha(\theta) - \mathbb{E}\left(\theta - z_\alpha(\theta)\middle|\theta \le \hat{\theta}\right)\right]^2 g(\theta)d\theta$$

$$+ \frac{1}{2(C'' - P_q)} \int_{\underline{\theta}}^{\hat{\theta}} \left[\theta - z_\alpha(\theta) - \mathbb{E}\left(\theta - z_\alpha(\theta)\middle|\theta \ge \hat{\theta}\right)\right]^2 g(\theta)d\theta$$

$$= \frac{\text{var}\left(\theta - z_\alpha(\theta)\middle|\theta \le \hat{\theta}\right) G(\hat{\theta}) + \text{var}\left(\theta - z_\alpha(\theta)\middle|\theta \ge \hat{\theta}\right)(1 - G(\hat{\theta}))}{2(C'' - P_q)}.$$

Finally, dividing the above expression by (A15) yields (10). The inequality follows, as $\hat{\theta}$ is fixed. □

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