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On the effect of inventory policies on distribution network design with several demand classes

P. Escalona[a,*], V. Marianov[b], F. Ordóñez[c], R. Stegmaier[a]

[a] Department of Industrial Engineering, Universidad Técnica Federico Santa María, Avenida España 1680, Valparaíso, Chile
[b] Department of Electrical Engineering, Pontificia Universidad Católica de Chile, Avda. Vicuña Mackenna 4860, Santiago, Chile
[c] Department of Industrial Engineering, Universidad de Chile, República 701, Santiago, Chile

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ABSTRACT

This paper studies the effect of several inventory policies on the design of a distribution network for fast-moving items able to provide differentiated service levels in terms of product availability for several demand classes. We consider the distribution network design problem when the *global round-up*, *single class allocation*, *local separate stock*, *local round-up*, and *critical level* inventory policies are used. We show how to formulate these problems as conic quadratic mixed-integer problems and prove that the critical level policy provides the lowest cost distribution network design. Further results and a computational study show how these different models compare in practice.

1. Introduction

Several types of inventory policies can be implemented in a distribution network of fast-moving consumer goods (FMCG) to deal with different service requirements in terms of product availability. Escalona et al. (2015) classified these policies into two types when a distribution network observes demand from several classes of customers, where each class demand is a group of customers with the same preset service level. The first group imposes general service conditions over the entire network and the second group imposes conditions on the operation of the inventory system at each distribution center (DC). The first policy group includes the *global round-up policy* (GRU policy), which sets the service level of the entire distribution network based on the highest priority class, and the *single class allocation* (SCA policy), where each DC serves a single demand class. The second policy group includes: the *local separate stock policy* (LSS policy), according to which each DC serves the demand assigned to it from a common stockpile and uses separate safety stocks for each demand class; the *local round-up policy* (LRU policy), in which each DC serves all demand assigned to it from a common stockpile and sets the safety stock as the maximum among the sets of classes assigned to it; and the *local critical level policy* (LCL policy), in which each DC serves the demand assigned to it from a common stockpile and uses rationing to provide differentiated service levels.

To the best of our knowledge, only the critical level policy has been used to design a distribution network able to provide differentiated service levels to different demand classes. This policy is an efficient way of providing differentiated service levels that outperforms the round-up and separate stock policies in a FMCG single-echelon system Escalona et al. (2017b,a). However, we have no evidence that the critical level policy has the best performance when designing a distribution network that observes demand from several classes of customers with different service level requirements. In addition, it seems surprising that only the inventory policy that minimizes total cost in a single-echelon is used in designing a distribution network that can deal with different service level

* Corresponding author.
E-mail address: pablo.escalona@usm.cl (P. Escalona).

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requirements. Therefore, the SCA, LSS, or LRU policies should also be considered owing to customer configurations, security, the existence of contracts, image, or simplicity.

The objective of this paper is to expand the design alternatives of distribution networks that provide differentiated service levels to different demand classes. We do so by modeling and solving the SCA, LSS, and LRU policies. Furthermore, we compare these models, including GRU and LCL policies, to establish the demand configuration and spatial distribution of customer classes that make each design alternative attractive. For each policy, we formulate an integer non-linear model (INLP) proving various relations between them, e.g., the location-inventory model using GRU, LSS, or LRU policies are lower bounds of the SCA policy. We show how to formulate GRU, SCA, LSS, and LRU models as conic quadratic mixed-integer models (CQMIPs) that can be solved using standard optimization solvers. Finally, using different configurations of demand and spatial distribution of customer classes, we establish the most likely order relationship of these policies in terms of total cost.

The research questions we answer in this paper are: (i) Are there alternative ways to design a supply chain that provides different service levels in terms of product availability, which have not been previously analyzed? (ii) If so, is it possible to prove total cost order relations between them?; and (iii) what are these order relations, or the most likely ones, under different configurations of demand and spatial distribution of customer classes?

The rest of this paper is structured as follows. In Section 2, we discuss relevant results in the literature. In Section 3, we present the mathematical programming models for each policy. In Section 4, we show how to formulate the models as CQMIPs. In Section 5, we explore total cost order relations among the policies. In Section 6, we report computational results. Finally, in Section 7, we conclude with managerial insights and future extensions to this work.

2. Literature review

In the last decade, there has been a trend towards integration of inventory and location decisions, because when these decisions are addressed separately it often results in sub-optimal solutions (see Farahani et al., 2015). These integrated models determine at the same time which DCs will be opened, their location, and customer allocations to them, how much inventory to keep and the optimal parameters of the inventory policy in each DC, while minimizing the total cost of the system. A comprehensive review in location-inventory models can be found in Farahani et al. (2015) and a characterization can be found in Sadjadi et al. (2015).

There is a rich body of literature on location-inventory problems that consider the ability of the distribution network to guarantee a desired service level in terms of product availability. Daskin et al. (2002) incorporated safety stocks into their location-inventory model such that the probability of a stockout at each DC is equal to some preset service level. They considered the same preset service level for all the DCs. The model is formulated as a INLP and solved by Lagrangian relaxation. The model of Daskin et al. (2002) has been reformulated and extended in a number of ways by relaxing one or more of their underlying assumptions. The immediate generalizations are the capacitated versions studied by Miranda and Garrido (2004) and later by Ozsen et al. (2008), the multi-commodity version studied by Shen (2005), and the stochastic version studied by Snyder et al. (2007). Other extensions were given by Sourirajan et al. (2007) in which the assumption of identical replenishment lead time was relaxed, Shen and Qi (2007) who considered the shipment from a DC to its customers using a vehicle routing model instead the linear direct shipping of Daskin et al. (2002),Shahabi et al. (2014) who relaxed the assumption of customer demand independence. Some reformulations of the Daskin et al. (2002) model include the set-covering integer programming model of Shen et al. (2003) solved using column generation, the mixed integer non-linear problem (MINLP) of You and Grossmann (2008) solved using heuristic method and a Lagrangian relaxation algorithm, and the CQMIP of Atamtürk et al. (2012). Using a different approach, Miranda and Garrido (2009) proposed a two-stage heuristic approach to determine the distribution network optimal preset service level using a known unit penalty cost for unfulfilled demand. The first step optimizes the preset service level and the second step optimizes the location and inventory decisions. All the above authors considered the same preset service level for the distribution network, i.e., they considered only one demand class because all customers require the same service level.

Our work focuses on a location-inventory model able to provide differentiated service levels in terms of product availability for several demand classes. In this context, Escalona et al. (2015) analyzed a location-inventory model with differentiated service levels, in which the DCs observe demand from two classes of customers, high and low priority. To provide differentiated service levels, they assumed, at each DC, a continuous review (Q,r,C) inventory policy and that the service level provided by a DC is measured by the probability of satisfying the entire demand of each class assigned to the DC during a replenishment cycle from on-hand inventory. The location-inventory model with differentiated service levels is formulated as a MINLP with chance constraints and the authors propose a decomposition heuristic to solve it. Using a different approach, Berman et al. (2012) considered a location-inventory model where the DCs operate under a periodic review (R,S) policy, i.e., where a replenishment order is placed every R periods (review range) such that the inventory position reaches S. Berman et al. (2012) included differentiated shortage costs for each DC in their model. This allows the service level provided by different DCs to be different. The model is formulated as an INLP and solved with Lagrangian relaxation using the procedure proposed by Daskin et al. (2002). Liu et al. (2010) studied a capacitated location-inventory model that assigns online demands to regional warehouses currently serving in-store demands in a multi-channel supply chain. Each regional warehouse provided differentiated service levels using an order-up-to inventory policy with differentiated shortage costs. The model is formulated as an INLP and a Lagrangian relaxation-based procedure is proposed to solve it. Tsao et al. (2012) studied a location-inventory problem for designing a distribution network with several local DCs and retailers. Each local DC operate under a continuous review (Q,r) policy with type I service level where the preset service level is different for each local DC. They develop a continuous approximation approach, with the motivation of solving larger-scale problems.

In summary, only Escalona et al. (2015) considered a location-inventory model when a distribution network observes demand

Table 1
Sets, parameters and variables.

Index sets	
I	Set of retailers, indexed by i
J	Set of possible DC locations, indexed by j
K	Set of customer classes, indexed by k
$N_k = \{i \in I \mid i \text{ is class } k\}$	Set of retailers of class k, with $k \in K$
Parameters and notation	
μ_i	Mean demand per unit time of retailer i
σ_i^2	Variance of demand per unit time of retailer i
f_j	Fixed cost per unit time of locating a DC j
d_{ij}	Cost per unit to ship between retailer i and DC j
c_{ij}	Transport rate between retailer i and DC j
a_j	Fixed shipment cost from external supplier to DC j.
S_j	Ordering cost at DC site j
h_j	Inventory holding cost per unit and unit time at DC j
L_j	Fixed lead time in unit time from the supplier to DC j
α_k	Preset service level for class demand k, where $\alpha_1 > \alpha_2 > \ldots > \alpha_{\lvert K\rvert}$
z_{α_k}	Standard normal deviate such that $\mathbb{P}(z \leqslant z_{\alpha_k}) = \alpha_k$
Decision Variables	
X_j	1 if site j is selected as a DC and 0 otherwise
Y_{ij}	1 if retailer i is served by a DC located at j and 0 otherwise

from different classes of customers with differentiated service level requirements in terms of product availability. This paper builds on this previous work by considering other inventory control policies in the design of distribution networks capable of providing differentiated service levels when facing demand from several classes of customers with different service level requirements.

3. Model formulation

We consider the distribution network design for a three-stage single-product supply chain in which a single supplier ships product of high demand volume (FMCG) to a set of retailers (customers) via a set of DCs to locate. We assume that demands per time unit at each retailer are independent and normally distributed as an approximation of the non-negative demand; that the supplier and DCs have unlimited capacity; and that the location of the supplier, site candidates, and retailers are known.

In this distribution network, there are several categories of retailers or demand classes, where each demand class is a group of retailers with the same preset service level in terms of product availability. We order demand classes according to their preset service level, where the high-priority retailers (class 1) require the high service level and the low-priority retailers require the lower service level. A retailer can only be assigned to a single demand class, exactly to one DC, and we assume that the class of each retailer is known.

Each DC follows a continuous review (Q,r) policy with full-backorder and deterministic lead time, i.e., when the inventory position falls below a reorder level r a replenishment order for Q units is placed and arrives L time units later. The service level is measured by service level type I and to provide differentiated service levels, we consider the SCA, LSS, and LRU policies. The problem is to determine how many DCs should be opened, where to locate them, which DC should serve which retailer, how much inventory to keep at each DC to minimize total location, shipment, and inventory costs, while meeting the preset service level for each demand class. We use the notation described in Table 1 throughout the paper.

Using the allocation variable Y_{ij}, we characterize the demand at candidate DC j. Let $D_j(L_j) = \sum_{k\in K} D_{kj}(L_j)$ be the total demand of all classes during the lead time L_j at DC j, where $D_{kj}(L_j)$ be the total demand of class k at DC j during the lead time L_j. In this paper, we assume that each retailer is assigned exactly to one DC and its demand is independent and normally distributed, therefore $D_{kj}(L_j)$ is also normally distributed with mean $L_j\mu_{kj}$ and variance $L_j\sigma_{kj}^2$, where $\mu_{kj} = \sum_{i\in N_k} \mu_i Y_{ij} \geqslant 0$ and $\sigma_{kj}^2 = \sum_{i\in N_k} \sigma_i^2 Y_{ij} \geqslant 0$; and $D_j(L_j)$ is normally distributed with mean $L_j\mu_j$ and variance $L_j\sigma_j^2$, where $\mu_j = \sum_{k\in K} \mu_{kj} = \sum_{i\in I} \mu_i Y_{ij}$ and $\sigma_j^2 = \sum_{k\in K} \sigma_{kj}^2 = \sum_{i\in I} \sigma_i^2 Y_{ij}$.

In what follow, the general cost function is presented. Then, the inventory-location models assuming GRU, SCA, LSS, and LRU policies are presented. For the location-inventory model using LCL policy we use the formulation of Escalona et al. (2015).

3.1. Cost function

Under a continuous review (Q,r) inventory policy at each DC and linear transportation costs, the average cost per unit time at DC j is

$$AC_j(Q_j,r_j,X_j,Y_{ij}) = f_jX_j + S_j\frac{\sum_{i\in I}\mu_iY_{ij}}{Q_j} + a_j\sum_{i\in I}\mu_iY_{ij} + \sum_{i\in I}d_{ij}\mu_iY_{ij} + h_j\left(\frac{Q_j}{2} + r_j - L_j\sum_{i\in I}\mu_iY_{ij}\right), \tag{1}$$

where Q_j and r_j are the replenishment order and the reorder point at candidate DC j, respectively. The first term of Eq. (1) is the fixed cost per unit time, the second term is the ordering cost per unit time, the third and fourth term are the supply and distribution costs per unit time, respectively, and the fifth term is approximately the holding cost per unit time, because we assume negligible back-orders. Similar to Daskin et al. (2002) and Shen et al. (2003), we assume the replenishment order Q_j is determined using an economic order quantity model (EOQ), i.e.,

$$Q_j = \sqrt{\frac{2S_j}{h_j}\sum_{i\in I}\mu_i Y_{ij}}. \tag{2}$$

Then, replacing Eq. (2) into Eq. (1), the average cost per unit time at DC j is

$$AC_j(r_j,X_j,Y_{ij}) = f_jX_j + \sum_{i\in I}\hat{d}_{ij}Y_{ij} + \psi_j\sqrt{\sum_{i\in I}\mu_i Y_{ij}} + h_j\left(r_j - L_j\sum_{i\in I}\mu_i Y_{ij}\right), \tag{3}$$

where $\psi_j = \sqrt{2h_jS_j}$ and $\hat{d}_{ij} = (a_j + d_{ij})\mu_i$.

The reorder point r_j in Eq. (3) depends on the policy used to provide differentiated service levels. GRU, SCA, LSS, and LRU policies differ from each other in the reorder point formulation.

3.2. Inventory-location model under GRU policy

The GRU policy considers that each DC serves all demand assigned to it from a common stockpile and sets the service level of the entire distribution network based on the highest priority class. The reorder point of the GRU policy at DC j is obtained from $F_{D_j(L_j)}(r_j) = \alpha_1$. Under normally distributed demand

$$r_j = L_j\sum_{i\in I}\mu_i Y_{ij} + z_{\alpha_1}\sqrt{L_j\sum_{i\in I}\sigma_i^2 Y_{ij}}. \tag{4}$$

Replacing Eq. (4) in (3) and rearranging terms, the location-inventory model under the GRU policy is formulated as an INLP, denoted (**GRU**), as follows.

Model (**GRU**):

$$\min_{X,Y}\ \sum_{j\in J}\left\{f_jX_j + \sum_{i\in I}\hat{d}_{ij}Y_{ij} + \psi_j\sqrt{\sum_{i\in I}\mu_i Y_{ij}} + \hat{\theta}_j z_{\alpha_1}\sqrt{\sum_{i\in I}\sigma_i^2 Y_{ij}}\right\} \tag{5}$$

$$\text{s.t:}\quad \sum_{j\in J}Y_{ij} = 1 \qquad \forall\ i\in I \tag{6}$$

$$Y_{ij} \leqslant X_j \qquad \forall\ i\in I, j\in J \tag{7}$$

$$X_j,\ Y_{ij}\in\{0,1\} \qquad \forall\ i\in I, j\in J, \tag{8}$$

where $\hat{\theta}_j = h_j\sqrt{L_j}$. Constraints (6) establish that each customer is assigned exactly to one DC, constraints (7) ensure that one customer can be assigned to location j only if a DC is installed there, and constraints (8) define the domain of the decision variables.

3.3. Inventory-location model under SCA policy

The SCA policy considers that each DC serves a single demand class. To formulate this policy, we define a new variable:

$$V_{kj} = \begin{cases}1 & \text{if a DC installed at } j \text{ serves class } k,\\ 0 & \text{otherwise.}\end{cases} \tag{9}$$

Using Eq. (9), we ensure (i) that each installed DC serves only one class and (ii) the allocation of each customer to a DC that serves its class, respectively through the following constraints.

$$\sum_{k\in K}V_{kj} \leqslant X_j \qquad \forall\ j\in J \tag{10}$$

$$Y_{ij} \leqslant V_{kj} \qquad \forall\ i\in N_k, j\in J, k\in K \tag{11}$$

The reorder point of the SCA policy is obtained from $r_j = \sum_{k\in K} r_{kj}V_{kj}$ with r_{kj} obtained as the solution of $F_{D_{kj}(L_j)}(r_{kj}) = \alpha_k$, where $F_{D_{kj}(L_j)}(x)$ is the distribution function of $D_{kj}(L_j)$. Under normally distributed demand, $r_{kj} = \mu_{kj}L_j + z_{\alpha_k}\sigma_{kj}\sqrt{L_j} = L_j\sum_{i\in N_k}\mu_i Y_{ij} + z_{\alpha_k}\sqrt{L_j\sum_{i\in N_k}\sigma_i^2 Y_{ij}}$, and the reorder point of the SCA policy can be obtained using,

$$r_j = L_j\sum_{i\in I}\mu_i Y_{ij} + \sum_{k\in K} z_{\alpha_k}\sqrt{L_j\sum_{i\in N_k}\sigma_i^2 Y_{ij}}, \tag{12}$$

because constraints (10) and (11) ensure that each installed DC serves only one class and the allocation of each customer to a DC that serves its class.

Replacing Eq. (12) in (3) and rearranging terms, we formulate an integrated location-inventory model under SCA policy as an INLP, denoted (**SCA**), as follows.

Model (**SCA**):

$$\min_{X,Y,V} \sum_{j\in J}\left\{f_jX_j+\sum_{i\in I}\hat{d}_{ij}Y_{ij}+\psi_j\sqrt{\sum_{i\in I}\mu_iY_{ij}}+\hat{\theta}_j\sum_{k\in K}z_{\alpha_k}\sqrt{\sum_{i\in N_k}\sigma_i^2Y_{ij}}\right\} \quad (13)$$

s.t: (6),(7),(8),(10),(11)

$$V_{kj}\in\{0,1\} \qquad \forall\ j\in J,k\in K. \quad (14)$$

Constraints (14) define the domain of the new decision variables V_{kj}.

3.4. Inventory-location model under LSS policy

The LSS policy considers that each DC serves the demand assigned to it from a common stockpile and it uses separate safety stocks for each class. The reorder point of the LSS policy is obtained from $r_j=\sum_{k\in K}r_{kj}$. Under normally distributed demand:

$$\begin{aligned} r_j &= \mu_jL_j+\sum_{k\in K}z_{\alpha_k}\sigma_{kj}\sqrt{L_j}\\ &= L_j\sum_{i\in I}\mu_iY_{ij}+\sum_{k\in K}z_{\alpha_k}\sqrt{L_j\sum_{i\in N_k}\sigma_i^2Y_{ij}}. \end{aligned} \quad (12)$$

Replacing Eq. (12) in (3) and rearranging terms, we formulate an integrated location-inventory model under LSS policy as an INLP, denoted (**LSS**), as follows.

Model (**LSS**):

$$\min_{X,Y} \sum_{j\in J}\left\{f_jX_j+\sum_{i\in I}\hat{d}_{ij}Y_{ij}+\psi_j\sqrt{\sum_{i\in I}\mu_iY_{ij}}+\hat{\theta}_j\sum_{k\in K}z_{\alpha_k}\sqrt{\sum_{i\in N_k}\sigma_i^2Y_{ij}}\right\} \quad (13)$$

s.t: (6),(7),(8).

3.5. Inventory-location model under LRU policy

The LRU policy considers that each DC serves all demand assigned to it from a common stockpile and sets the safety stock as the maximum among the sets of classes assigned to it. The reorder point of the LRU policy at DC j is obtained from $F_{D_j(L_j)}(r_j)=\max_{k\in K}\{\alpha_kV_{kj}\}$. Under normally distributed demand

$$\begin{aligned} r_j &= \mu_jL_j+\max_{k\in K}\{z_{\alpha_k}V_{kj}\}\sigma_j\sqrt{L_j}\\ &= L_j\sum_{i\in I}\mu_iY_{ij}+\max_{k\in K}\{z_{\alpha_k}V_{kj}\}\sqrt{L_j\sum_{i\in I}\sigma_i^2Y_{ij}}. \end{aligned} \quad (15)$$

Let Z_j be the maximum of the inverse normal distribution for the service levels of the demand that DC j serves, i.e., $Z_j=\max_{k\in K}\{z_{\alpha_k}V_{kj}\}$. Then, rearranging terms in Eq. (15), the reorder point of the LRU policy at DC j is

$$r_j=L_j\sum_{i\in I}\mu_iY_{ij}+\sqrt{L_j\sum_{i\in I}\sigma_i^2W_{ij}^2}, \quad (16)$$

where $W_{ij}=Z_jY_{ij}$. Note that Y_{ij} is binary and $Y_{ij}=Y_{ij}^2$. Replacing Eq. (16) in (3), and rearranging terms, we formulate a location-inventory model under the LRU policy as an MINLP, denoted (**LRU**), as follows.

Model (**LRU**):

$$\min_{X,Y,V,W,Z} \sum_{j\in J}\left\{f_jX_j+\sum_{i\in I}\hat{d}_{ij}Y_{ij}+\psi_j\sqrt{\sum_{i\in I}\mu_iY_{ij}}+\hat{\theta}_j\sqrt{\sum_{i\in I}\sigma_i^2W_{ij}^2}\right\} \quad (17)$$

s.t: (6),(7),(8),(11),(14)

$$Z_j\geqslant z_{\alpha_k}V_{kj} \qquad \forall\ j\in J,k\in K \quad (18)$$

$$W_{ij}\geqslant Z_j+\hat{z}(Y_{ij}-1) \qquad \forall\ j\in J \quad (19)$$

$$W_{ij}\geqslant 0 \qquad \forall\ i\in I,j\in J \quad (20)$$

$$Z_j\geqslant 0 \qquad \forall\ j\in J. \quad (21)$$

Constraints (18) stipulate that Z_j is equal to the maximum of the inverse normal distribution of the service levels of the demand served by the DC j. Constraints (19) and (20) are a valid linearization of $W_{ij}=Z_jY_{ij}$ because (**LRU**) is a minimization model, where $\hat{z}$ is an upper bound for Z_j. We observe that the best value for $\hat{z}$ is z_{α_1}. Constraints (21) define thedomain of the decision variable Z_j. This variable is greater than or equal to zero, because we assume $\alpha_k \geqslant 0.5$ for any $k \in K$.

Note that models (**GRU**),(**SCA**), and (**LSS**) are INLP, and model (**LRU**) is a MINLP, where the non-linearity is in the objective function. In what follow, we show how to reformulate these models to eliminate the non-linear terms from the objective.

4. A CQMIP formulation

In this section, we show how to reformulate models (**GRU**),(**SCA**),(**LSS**), and (**LRU**) as CQMIPs using the procedure presented in Atamtürk et al. (2012),Zhang et al. (2014), and Escalona et al. (2015). The advantage of the CQMIP formulation is that it can be solved directly using standard optimization software packages such as CPLEX or Mosek.

The square root term in the objective function of models (**GRU**),(**SCA**),(**LSS**), and (**LRU**) can give rise to difficulties in the optimization procedure. When the DC j is not selected, the square root terms would take a value of 0, which leads to unbounded gradients in the INLP optimization and, hence, numerical difficulties. Thus, we reformulate the models (**GRU**),(**SCA**),(**LSS**), and (**LRU**) to eliminate the square root terms. We first note that $Y_{ij}^2 = Y_{ij}$. Then, we introduce four sets of non-negative continuous variables, $H1_j$,$H2_j$,$H3_{kj}$, and $H4_j^2$, to represent the square root terms in (13), (17), and (5):

$$H1_j^2 = \sum_{i\in I} \mu_i Y_{ij}^2\,, \quad \forall\, j \in J \tag{22}$$

$$H2_j^2 = \sum_{i\in I} (\sigma_i W_{ij})^2\,, \quad \forall\, j \in J \tag{23}$$

$$H3_{kj}^2 = \sum_{i\in N_k} (\sigma_i Y_{ij})^2\,, \quad \forall\, j \in J, k \in K \tag{24}$$

$$H4_j^2 = \sum_{i\in I} (\sigma_i Y_{ij})^2 \quad \forall\, j \in J \tag{25}$$

$$H1_j \geqslant 0\,, \quad \forall\, j \in J \tag{26}$$

$$H2_j \geqslant 0\,, \quad \forall\, j \in J \tag{27}$$

$$H3_{kj} \geqslant 0\,, \quad \forall\, j \in J, k \in K \tag{28}$$

$$H4_j \geqslant 0 \quad \forall\, j \in J. \tag{29}$$

Because the non-negative variables $H1_j$,$H2_j$,$H3_{kj}$, and $H4_j$ are introduced in the objective function of (**GRU**),(**SCA**),(**LSS**), and (**LRU**) with positive coefficients, and these are minimization models, Eqs. (22)–(25) can be further relaxed as the following inequalities:

$$H1_j^2 \geqslant \sum_{i\in I} \mu_i Y_{ij}^2\,, \quad \forall\, j \in J \tag{30}$$

$$H2_j^2 \geqslant \sum_{i\in I} (\sigma_i W_{ij})^2\,, \quad \forall\, j \in J \tag{31}$$

$$H3_{kj}^2 \geqslant \sum_{i\in N_k} (\sigma_i Y_{ij})^2\,, \quad \forall\, j \in J, k \in K \tag{32}$$

$$H4_j^2 \geqslant \sum_{i\in I} (\sigma_i Y_{ij})^2\,, \quad \forall\, j \in J. \tag{33}$$

Note that constraints (30)–(33), together with constraints (26)–(29) define second-order cone constraints. Thus, we can reformulate models (**GRU**),(**SCA**),(**LSS**), and (**LRU**) as the following MINLP with second-order cone constraints denoted as ($\mathbf{CQ_{GRU}}$),($\mathbf{CQ_{SCA}}$),($\mathbf{CQ_{LSS}}$), and ($\mathbf{CQ_{LRU}}$), respectively.

Model ($\mathbf{CQ_{GRU}}$):

$$\min_{X,Y,H1,H4} \quad \sum_{j\in J} \left\{ f_j X_j + \sum_{i\in I} \hat{d}_{ij} Y_{ij} + \psi_j H1_j + \hat{\theta}_j z_{\alpha_1} H4_j \right\} \tag{34}$$

s.t: (6),(7),(8),(30),(26),(33),(29).

Model ($\mathbf{CQ_{SCA}}$):

$$\min_{X,Y,V,H1,H3} \quad \sum_{j\in J} \left\{ f_j X_j + \sum_{i\in I} \hat{d}_{ij} Y_{ij} + \psi_j H1_j + \hat{\theta}_j \sum_{k\in K} z_{\alpha_k} H3_{kj} \right\} \tag{35}$$

s.t: (6),(7),(8),(11),(10),(14),(30),(32),(26),(28).

Model (**CQ$_{\mathbf{LSS}}$**):

$$\min_{X,Y,H1,H3} \sum_{j\in J}\left\{f_jX_j + \sum_{i\in I}\hat{d}_{ij}Y_{ij} + \psi_j H1_j + \hat{\theta}_j\sum_{k\in K} z_{\alpha_k}H3_{kj}\right\} \tag{35}$$

s.t: (6),(7),(8),(30),(32),(26),(28).

Model (**CQ$_{\mathbf{LRU}}$**):

$$\min_{X,Y,V,Z,W,H1,H2} \sum_{j\in J}\left\{f_jX_j + \sum_{i\in I}\hat{d}_{ij}Y_{ij} + \psi_j H1_j + \hat{\theta}_j H2_j\right\} \tag{36}$$

s.t: (6),(7),(8),(11),(14),(18),(21),(19),(20),(30),(31),(26),(27).

Models (**CQ$_{\mathbf{GRU}}$**),(**CQ$_{\mathbf{SCA}}$**),(**CQ$_{\mathbf{LSS}}$**), and (**CQ$_{\mathbf{LRU}}$**) can be trivially shown to be equivalent to models (**GRU**),(**SCA**),(**LSS**), and (**LRU**), respectively. However, they have a linear objective function and second-order cone constraints. We can solve this models using CPLEX 12.4, which handles second-order cone constraints in an efficient way.

5. Model properties

In what follows, we describe a number of properties of the models that allow us to establish a possible ordering among the optimal objective functions of models associated with GRU, SCA, LSS, LRU, and LCL policies. For the joint location-inventory model using LCL policy to provide differentiated service levels, we use the formulation of Escalona et al. (2015), denoted (**LCL**).

Proposition 1. *$Z^*_{LSS} \leqslant Z^*_{SCA}$ and $Z^*_{LRU} \leqslant Z^*_{SCA}$, where Z^*_{SCA},Z^*_{LSS},Z^*_{LRU} are the optimal objective function of models* (**SCA**),(**LSS**), *and* (**LRU**), *respectively.*

Proof. The proof of Proposition 1 follows directly from the fact that models (**LSS**) and (**LRU**) are relaxations of model (**SCA**). In the case of model (**LSS**), it is easy to show that is a relaxation of model (**SCA**). Therefore, the optimal solution of model (**LSS**) is a lower bound of model (**SCA**), i.e., $Z^*_{LSS} \leqslant Z^*_{SCA}$.

Let us now consider the following reformulation of model (**SCA**):

$$\min_{X,Y,V,W,Z} \sum_{j\in J}\left\{f_jX_j + \sum_{i\in I}\hat{d}_{ij}Y_{ij} + \psi_j\sqrt{\sum\nolimits_{i\in I}\mu_iY_{ij}} + \hat{\theta}_j\sqrt{\sum\nolimits_{i\in I}\sigma_i^2W_{ij}^2}\right\} \tag{17}$$

s.t: (6),(7),(8),(11),(10),(14),(18),(19),(20),(21).

It is easy to show that model (**LRU**) is a relaxation of the reformulation of (**SCA**). Therefore, the optimal solution of the model (**LRU**) is a lower bound of model (**SCA**), i.e., $Z^*_{LRU} \leqslant Z^*_{SCA}$. □

Proposition 2. *$Z^*_{LRU} \leqslant Z^*_{GRU}$, where Z^*_{GRU} is the optimal objective function of model* (**GRU**).

Proof. Consider the following reformulation of model (**GRU**):

$$\min_{X,Y,V,W,Z} \sum_{j\in J}\left\{f_jX_j + \sum_{i\in I}\hat{d}_{ij}Y_{ij} + \psi_j\sqrt{\sum_{i\in I}\mu_iY_{ij}} + \hat{\theta}_j\sqrt{\sum_{i\in I}\sigma_i^2W_{ij}^2}\right\} \tag{17}$$

s.t: (6),(7),(8),(11),(10),(14),(18),(19),(20),(21)

$$Z_j \geqslant z_{\alpha_1} \qquad \forall\, j\in J \tag{37}$$

It is easy to show that model (**LRU**) is a relaxation of the reformulation of model (**GRU**). Therefore, the optimal solution of model (**LRU**) is a lower bound of model (**GRU**), i.e., $Z^*_{LRU} \leqslant Z^*_{GRU}$. □

We observe that the SCA policy can have a better cost performance than the GRU policy when the demand classes are spatially separated, i.e., when the classes are highly spatially segmented. To illustrate when this occurs, consider a small example with two retailers, $I = \{A,B\}$, shown in Fig. 1. Retailer A is class 1 and retailer B is class 2, i.e. $N_1 = \{A\}$ and $N_2 = \{B\}$. In this example, each retailer location is also a candidate DC location, i.e. $I = J$.

For simplicity, we assume $a_j = 0, h_j = h$, and $S_j = S$ for any $j = A,B$. Under the SCA policy it is easy to show that it is optimal to assign demands at retailer A to a DC at A and to assign demands at retailer B to a DC at B, i.e., $X^*_A = X^*_B = 1, Y^*_{AA} = Y^*_{BB} = 1$. On the

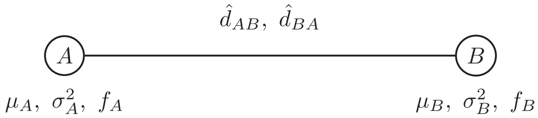

Fig. 1. Small segmented network.

other hand, using the GRU policy, it is easy to show that $X_A^* = X_B^* = 1, Y_{AA}^* = Y_{BB}^* = 1$ when $f_B \leqslant \hat{d}_{BA}$ and $f_A \leqslant \hat{d}_{AB}$. Therefore, under $f_B \leqslant \hat{d}_{BA}$ and $f_A \leqslant \hat{d}_{AB}$, the SCA policy has a better cost performance than the GRU policy, i.e., $Z_{GRU}^* > Z_{SCA}^*$, because $z_{\alpha_1} > z_{\alpha_2}$.

Proposition 3. *For two demand classes, $Z_{LCL}^* < Z_{LSS}^*$ and $Z_{LCL}^* < Z_{LRU}^*$, where Z_{LCL}^* is the optimal objective function of model* (**LCL**).

Proof. Consider the optimal distribution network for the joint location-inventory model using the LCL policy to provide differentiated service levels to two demand classes. This network will have a lower total cost than implementing the LSS or LRU policy, because, in a single echelon, Escalona et al. (2017b) showed that, under normally distributed demand and two demand classes, the optimal reorder point of the LCL policy is strictly less than the reorder point induced by the LRU and LSS policies. □

The main consequences of Propositions 1–3 is that, for a network with two demand classes:

- the LCL policy induces the lowest cost for a distribution network able to provide differentiated service levels in terms of product availability; and
- the LSS and LRU policies compete for being the best alternative to LCL policy, while GRU and SCA policies are the worst alternatives to LCL policy, because there are only five ways to order the optimal objective function:
  (i) $Z_{LCL}^* \leqslant Z_{LSS}^* \leqslant Z_{LRU}^* \leqslant Z_{GRU}^* \leqslant Z_{SCA}^*$;
  (ii) $Z_{LCL}^* \leqslant Z_{LSS}^* \leqslant Z_{LRU}^* \leqslant Z_{SCA}^* \leqslant Z_{GRU}^*$;
  (iii) $Z_{LCL}^* \leqslant Z_{LRU}^* \leqslant Z_{LSS}^* \leqslant Z_{GRU}^* \leqslant Z_{SCA}^*$;
  (iv) $Z_{LCL}^* \leqslant Z_{LRU}^* \leqslant Z_{LSS}^* \leqslant Z_{SCA}^* \leqslant Z_{GRU}^*$; and
  (v) $Z_{LCL}^* \leqslant Z_{LRU}^* \leqslant Z_{GRU}^* \leqslant Z_{LSS}^* \leqslant Z_{SCA}^*$.

In what follows we establish the most likely order relationship under different demand configurations and spatial distribution of customer classes.

6. Computational study

In this section, we present the numerical tests and their results. The main objectives of the computational study are to: (i) determine good alternative policies at the LCL policy to design distribution networks able to provide differentiated service levels under different demand configurations and spatial distribution of customer classes; and (ii) to quantify the relative cost of using different policies at the LCL policy to design distribution networks able to provide differentiated service levels. To illustrate the performance of the different policies, we carried out computational experiments for instances with 49 nodes from Daskin (2011). We generated several test problems with different demand configurations and spatial distribution of customer classes, and compared the LSS, SCA, GRU, LRU, and LCL inventory policies to determine the most likely order relationship among the policies in terms of total cost. In all cases, each retailer location is also a candidate DC location, i.e., there are as many candidate DC locations as retailer locations for each instance.

We analyze 10 different configurations in terms of demand and spatial distribution of customer classes. The first 5 configurations consider that the spatial distribution of customer classes is random and the last 5 configurations consider that the spatial distribution of customer classes is segmented, i.e., demand classes are spatially separated. For each configuration, we generate 1000 random instances. Table 2 shows the 10 configurations.

In each configuration, as shown in Table 2, what changes is the class that dominates on demand and/or nodes. To illustrate the concept of dominance in demand and/or nodes, consider configuration 2 where class 1 dominates on demand, i.e., $\sum_{i \in N_1} \mu_i > \sum_{i \in N_2} \mu_i$, and class 2 dominates in nodes, i.e., $|N_1| < |N_2|$.

Test problems used the following common criteria and parameters: service level requirements $\alpha_1 = U[0.95, 0.99]$ and $\alpha_2 = U[0.5, 0.9]$; cost per unit to ship between retailer i and candidate DC site j, d_{ij} equal to the distance between retailer i and

Table 2
Configurations.

Configuration	Spatial distribution	Network dominance: Demand	Network dominance: Nodes
1	Random	No class dominates	No class dominates
2	Random	Class 1	Class 2
3	Random	Class 1	Class 1
4	Random	Class 2	Class 1
5	Random	Class 2	Class 2
6	Segmented	No class dominates	No class dominates
7	Segmented	Class 1	Class 2
8	Segmented	Class 1	Class 1
9	Segmented	Class 2	Class 1
10	Segmented	Class 2	Class 2

Table 3
Order of the distribution network cost according to an inventory policy.

	Spatial random configurations					Segmented spatial configurations				
	1	2	3	4	5	6	7	8	9	10
$\widehat{Z}_{LCL} \leqslant Z^*_{LSS} \leqslant Z^*_{LRU} \leqslant Z^*_{GRU} \leqslant Z^*_{SCA}$	88%	32%	65%	54%	80%	27%	9%	53%	8%	18%
$\widehat{Z}_{LCL} \leqslant Z^*_{LSS} \leqslant Z^*_{LRU} \leqslant Z^*_{SCA} \leqslant Z^*_{GRU}$	0%	1%	1%	46%	17%	48%	12%	12%	89%	56%
$\widehat{Z}_{LCL} \leqslant Z^*_{LRU} \leqslant Z^*_{LSS} \leqslant Z^*_{GRU} \leqslant Z^*_{SCA}$	6%	52%	4%	0%	3%	11%	44%	13%	0%	8%
$\widehat{Z}_{LCL} \leqslant Z^*_{LRU} \leqslant Z^*_{LSS} \leqslant Z^*_{SCA} \leqslant Z^*_{GRU}$	0%	0%	1%	0%	0%	13%	34%	7%	2%	17%
$\widehat{Z}_{LCL} \leqslant Z^*_{LRU} \leqslant Z^*_{GRU} \leqslant Z^*_{LSS} \leqslant Z^*_{SCA}$	6%	16%	30%	0%	0%	0%	1%	15%	0%	0%

candidate DC j multiplied by a transport rate $c_{ij} = c$, $\forall\ i \in I, j \in J$, where $c = U\,[0.005,0.02]$; demand per unit time at each retailer is normally distributed with coefficient of variation $CV_i = U\,[0.1,0.5]$; fixed (per unit time) cost of locating a DC at candidate site $j, f_j = U\,[100,300]$; cost per unit to ship between external supplier and candidate DC site $j, a_j = a$, $\forall\ j \in J$, where $a = U\,[0.3,0.7]$; ordering cost from candidate DC site $j, S_j = S$, $\forall\ j \in J$, where $S = U\,[500,1000]$; and lead time, $L_j = \{2,3,4\}$ with discrete uniform distribution. In Appendix A, we show how each configuration was built.

Models $(\mathbf{CQ_{GRU}})$,$(\mathbf{CQ_{SCA}})$,$(\mathbf{CQ_{LSS}})$, and $(\mathbf{CQ_{LRU}})$ were solved using CPLEX 12.4. The location-inventory model using the LCL policy was also solved with CPLEX 12.4 using the procedure described in Escalona et al. (2015), from which an upper bound is obtained for model (**LCL**). Let $\widehat{Z}_{LCL}$ be the objective function of model (**LCL**) using the procedure described in Escalona et al. (2015). For all instances and inventory policies tested, we used a termination criterion of a 10^{-5} optimality gap. All tests were carried on a PC with Intel Core i7 2.3 GHz processor and 16 GB RAM.

6.1. Experimental results for the test problems

For all configurations, we observed that the solution obtained using the procedure described in Escalona et al. (2015) induces the lowest cost for a distribution network able to provide differentiated service levels. Therefore, for the instances we test, the consequences of Propositions 1–3 are still valid when using $\widehat{Z}_{LCL}$ instead of Z^*_{LCL}. Table 3 shows, for each configuration, the percentage of instances associated with each ordering of the objective functions.

As Table 3 shows, for most of the tested instances of the configurations 1, 3, 4, 5, 6, 8, 9, and 10, LSS is the best alternative to the LCL policy. However, in most tested instances of configurations 2 and 7, LRU is the best alternative to LCL, i.e., when the high-priority class dominates on demand and the low-priority class dominates in nodes, the best alternative to LCL is the LRU policy in most of the tested instances.

We also observed that the spatial distribution of customer classes determines the worst performing policy when designing a distribution network able to provide differentiated service levels. From Table 3, we observe that for a random spatial distribution of customer classes, the SCA policy performed worst in most of the tested instances. For example, when spatial distribution of customer classes is random and no class dominates in demand or nodes (configuration 1) it is very likely that the SCA policy will be the worst-performing policy, because in this configuration, in 100% of the tested instances, it was the policy with the worst performance. On the other hand, from Table 3, we observe that in most of the instances tested in configurations 6, 9, and 10, the worst-performing policy is the GRU policy. Then, as we expected, when the high-priority class does not dominate on demand in a spatially segmented network, the GRU policy has the worst performance in most of the tested instances.

To quantify the benefit of designing a distribution network able to provide differentiated service levels using the LCL policy, we compute the benefit for each instance tested as $Benefit\,(\%) = 100 \times (\widehat{Z}_{(\cdot)} - Z^*_{LCL})/\widehat{Z}_{(\cdot)}$. Table 4 shows the average and maximum relative benefit of the LCL policy versus LSS, LRU, GRU, and SCA policies for random and segmented spatial distribution.

From Table 4, we observe that the benefit induced by the LCL policy with respect to the other policies tested is higher in a random network than in a segmented network. Furthermore, when the spatial distribution of demand classes is random, the cost of a distribution network able to provide differentiated service levels using the SCA policy is on average 8% more expensive than the cost of using LCL policy and in the worst case 18.3% when no class dominates in demand or nodes (configuration 1). When the low-priority class dominates in demand, high-priority class dominates in nodes, and the spatial distribution of customer classes is segmented, the cost of a distribution network able to provide differentiated service levels using the GRU policy is on average 6.6% more expensive than using LCL policy, and in the worst case is 18.9% more expensive.

We compute the CPU time of each location-inventory model for the test set described above. Table 5 shows the average and maximum CPU times of the GRU, SCA, LSS, LRU, and LCL policies for random and segmented spatial distribution.

From Table 5, we observe that for configurations 1, 3, 4, 5, 8, 9, and 10, LRU has the highest average CPU time, and for the spatial random configuration, LSS policy has the lowest average CPU time. We observe that problems generally become more difficult to solve in terms of CPU time when the holding cost per unit and unit time is high and the transportation rate is low. This is because, higher values of $\widehat{\theta}_j$ and ψ_j assign more weight on the nonlinear terms of the objective function of (**GRU**),(**SCA**),(**LSS**),(**LRU**), and (**LCL**) models, and low values of $\widehat{d}_{ij}$ assign little weight on the linear terms of these objective functions.

Table 4
Benefit of LCL versus GRU, SCA, LSS, and LRU policies for random and segmented spatial distribution.

	Spatial random configuration									
	Configuration 1		Configuration 2		Configuration 3		Configuration 4		Configuration 5	
Benefit (%)	Average	Max	Average	Max	Average	Max	Average	Max	Average	Max
LCL vs GRU	2.0%	6.1%	2.7%	6.9%	0.6%	2.4%	7.6%	19.3%	3.5%	9.7%
LCL vs SCA	11.9%	18.3%	8.9%	17.7%	7.0%	14.6%	7.7%	14.3%	7.3%	17.3%
LCL vs LSS	0.9%	2.4%	1.9%	4.8%	0.5%	1.6%	0.2%	1.9%	0.6%	2.4%
LCL vs LRU	1.6%	4.5%	1.7%	4.7%	0.6%	3.5%	4.1%	9.4%	1.4%	5.6%

	Segmented spatial configuration									
	Configuration 6		Configuration 7		Configuration 8		Configuration 9		Configuration 10	
Benefit (%)	Average	Max	Average	Max	Average	Max	Average	Max	Average	Max
LCL vs GRU	2.2%	6.0%	2.3%	6.8%	0.6%	2.5%	6.6%	18.9%	3.0%	9.5%
LCL vs SCA	2.1%	13.8%	2.6%	12.8%	1.9%	9.6%	2.2%	10.0%	1.9%	10.6%
LCL vs LSS	0.3%	2.2%	0.7%	3.7%	0.3%	1.1%	0.2%	1.0%	0.2%	1.4%
LCL vs LRU	0.4%	3.0%	0.6%	3.5%	0.3%	1.4%	1.4%	6.8%	0.4%	2.4%

Table 5
Configurations CPU time.

	Spatial random configuration									
	Configuration 1		Configuration 2		Configuration 3		Configuration 4		Configuration 5	
Time (s)	Average	Max	Average	Max	Average	Max	Average	Max	Average	Max
GRU	69	10552	20	7246	85	10410	20	8299	76	8940
SCA	201	9345	30	8987	103	10245	28	1150	104	8319
LSS	34	8598	6	39	84	10689	8	485	34	7640
LRU	220	9726	22	304	172	9729	113	9755	327	10471
LCL	73	10801	10	2388	105	10801	10	2102	55	7014

	Segmented spatial configuration									
	Configuration 6		Configuration 7		Configuration 8		Configuration 9		Configuration 10	
Time (s)	Average	Max	Average	Max	Average	Max	Average	Max	Average	Max
GRU	136	9774	13	3976	15	229	10	2648	26	8898
SCA	53	9423	32	6688	16	389	21	1091	12	194
LSS	130	10797	66	10447	14	276	12	277	11	166
LRU	139	4695	33	3414	198	6343	134	8489	109	3940
LCL	154	10579	25	1862	88	10030	32	4323	85	7722

7. Conclusions

We have studied the effect of using GRU, SCA, LSS, LRU, and LCL policies on the design of a distribution network providing differentiated service levels. For each policy, we have formulated an INLP and we have shown how to reformulate these models as CQMIPs that can be solved using standard optimization solvers. Various relations among the models have been proved, from which we have obtained the following managerial insights for a network with two demand classes.

- *The LCL policy induces the lowest cost for a distribution network providing differentiated service levels in terms of product availability.*
- *The LSS and LRU policies compete for being the best alternative to the LCL policy, while GRU and SCA policies are the worst alternatives to the LCL policy.*
- *Policies that impose general service conditions over the entire distribution network perform worse than those policies that impose conditions on the operation of the inventory system at each DC.*

We have conducted several test problems under different demand configurations and spatial distribution of customer classes, from which we have observed the following managerial insights.

- *The benefit induced by the LCL policy with respect to the other policies tested is higher in a random network than in a segmented network.*

- *When the high-priority class dominates on demand and the low-priority class dominates in nodes, the best alternative to the LCL policy is the LRU policy in most of the instances we tested. For any other demand configuration and spatial distribution of demand classes, LSS policy is the best alternative to the LCL policy in most of the instances we tested.*
- *For a random spatial distribution of customer classes, the SCA policy performed worst, becoming on average 8% more expensive than using LCL policy and in the worst case* 18.3% *when no class dominates in demand or nodes.*
- *When the low-priority class dominates on demand in a spatially segmented network, the GRU policy performed worst, becoming on average* 6.6% *more expensive than using the LCL policy, and* 18.9% *more expensive in the worst case when high-priority class dominates in nodes.*

The main issue left for future research is to formulate and solve the joint location inventory model using the LCL policy for more than two demand classes and determine whether our results and observations remain valid for more than two demand classes.

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Appendix A. Tested configurations

- Configuration 1. In this configuration, the spatial distribution of customer classes is random and no class dominates in demand or nodes. The instances, for this configuration, were generated with the following parameters: class of the retailer $i, r_i = \{1,2\}$ with discrete uniform distribution; and demand per unit time at each retailer is normally distributed with mean $\mu_i = U[10,50]$;
- Configuration 2. In this configuration, the high-priority class dominates in demand, i.e., $\sum_{i\in N_1}\mu_i > \sum_{i\in N_2}\mu_i$; the low-priority class dominates in nodes; and the spatial distribution of customer classes is random. The instances, for this configuration, were generated with the following parameters: $|N_1| = \{1,...,\lceil 0.25|I|\rceil\}$ with discrete uniform distribution; $|N_2| = |I|-|N_1|$; $\mu_i = U[100,500]$ for any $i \in N_1$; and $\mu_i = U[10,50]$ for any $i \in N_2$.
- Configuration 3. In this configuration, the high-priority class dominates in nodes and demand, and the spatial distribution of customer classes is random. The instances for this configuration were generated with the following parameters: $|N_2| = \{1,...,\lceil 0.25|I|\rceil\}$ with discrete uniform distribution; $|N_1| = |I|-|N_2|$; $\mu_i = U[10,50]$ for any $i = 1,2$.
- Configuration 4. Low-priority class dominates in demand, i.e., $\sum_{i\in N_2}\mu_i > \sum_{i\in N_1}\mu_i$; the high-priority class dominates in nodes; and the spatial distribution of customer classes is random. The instances, for this configuration, were generated with the following parameters: $|N_2| = \{1,...,\lceil 0.25|I|\rceil\}$ with discrete uniform distribution; $|N_1| = |I|-|N_2|$; $\mu_i = U[100,500]$ for any $i \in N_2$; and $\mu_i = U[10,50]$ for any $i \in N_1$.
- Configuration 5. Low-priority class dominates in nodes and demand, and the spatial distribution of customer classes is random. The instances, for this configuration, were generated with the following parameters: $|N_1| = \{1,...,\lceil 0.25|I|\rceil\}$ with discrete uniform distribution; $|N_2| = |I|-|N_1|$; $\mu_i = U[10,50]$ for any $i = 1,2$.
- Configuration 6. In this configuration, the spatial distribution of customer classes is segmented and no class dominates in demand or nodes. We select a node at random and then determine its nearest $|I|/2$ nodes. We assigned class 1 to this set of nodes and we assigned class 2 to the rest. Demand per unit time at each retailer is normally distributed with mean $\mu_i = U[10,50]$.
- Configuration 7. In this configuration, the spatial distribution of customer classes is segmented, the high-priority class dominates in demand, and the low-priority class dominates in nodes. We select a high-priority class node at random and then determine its nearest nodes. The instances, for this configuration, were generated with the following parameters: $|N_1| = \{1,...,\lceil 0.25|I|\rceil\}$ with discrete uniform distribution; $|N_2| = |I|-|N_1|$; $\mu_i = U[100,500]$ for any $i \in N_1$; and $\mu_i = U[10,50]$ for any $i \in N_2$.
- Configuration 8. In this configuration, the high-priority class dominates in nodes and demand, and the spatial distribution of customer classes is segmented. We select a low-priority class node at random and then determine its nearest nodes. The instances, for this configuration, were generated with the following parameters: $|N_2| = \{1,...,\lceil 0.25|I|\rceil\}$ with discrete uniform distribution; $|N_1| = |I|-|N_2|$; $\mu_i = U[10,50]$ for any $i = 1,2$.
- Configuration 9. In this configuration, the low-priority class dominates in demand, the high-priority class dominates in nodes, and the spatial distribution of customer classes is segmented. We select a high-priority class node at random and then determine its nearest nodes. The instances for this configuration were generated with the following parameters: $|N_2| = \{1,...,\lceil 0.25|I|\rceil\}$ with discrete uniform distribution; $|N_1| = |I|-|N_2|$; $\mu_i = U[10,50]$ for any $i \in N_1$; and $\mu_i = U[100,500]$ for any $i \in N_2$.
- Configuration 10. The low-priority class dominates in nodes and demand, and the spatial distribution of customer classes is segmented. We select a low-priority class node at random and then determine its nearest nodes. The instances, for this configuration, were generated with the following parameters: $|N_1| = \{1,...,\lceil 0.25|I|\rceil\}$ with discrete uniform distribution; $|N_2| = |I|-|N_1|$; $\mu_i = U[10,50]$ for any $i = 1,2$.

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