Transport Policy 64 (2018) 61–75

Contents lists available at ScienceDirect

Transport Policy

journal homepage: www.elsevier.com/locate/tranpol

"Bidding the project" vs. "bidding the envelope" in public sector infrastructure procurements☆

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ARTICLE INFO

JEL codes:
H57
L98
L32

Keywords:
Public infrastructure procurement
Auction mechanisms
Prices vs quantities

ABSTRACT

We study the relative efficiency of two mechanisms actually employed in large-scale public procurements, often for transportation projects such as roads, bridges and rapid transit systems. In the more common "bidding the project" mechanism, the government specifies the size of the project (a quantity) and firms bid prices (the lowest bid winning). In the "bidding the envelope" mechanism the government specifies what it is willing to spend and firms bid quantities (the highest winning). With uncertainty about project costs and benefits, the much less frequently applied "bidding the envelope" mechanism can lead to higher value for money. Its advantage lies in its ability to allow quantity to adjust to high or low costs.

1. Introduction

Governments around the world are looking at innovative ways to procure large-scale public projects such as roads, bridges, rapid transit lines, hospitals, schools and prisons. The widely-recognized "infrastructure deficit" experienced in many countries, with both developed and developing economies, helps explain this interest. A recent report by the World Economic Forum suggested that an investment of the equivalent of US$2 trillion would need to be made each year for the next twenty years to bring the world's infrastructure to proper levels.[1]

There exists an extensive economics and management science literature on the use of auctions to procure public infrastructure, most of which focuses on the design and properties of optimal or near optimal mechanisms. In contrast, the purpose of this paper is to explore the cost and efficiency properties of two very basic mechanisms which are frequently employed in practice even though not optimal in any formal sense.

In the standard representation of a large-scale public procurement the government defines the project it would like delivered. It may leave a lot of discretion to bidders about how that project is to be delivered, but what we will call the "quantity" of services or "size of the project" to be delivered is precisely defined before bidding begins. Potential private partners will then bid competitively to provide that quantity at the lowest possible price to the government with the winner being the party with the lowest (quality controlled) price. We refer to this kind of procurement as "bidding the project" (or BTP). Competitive bidding will then lead to the provision of the defined quantity/project at a price close to the private sector provider's costs. The optimality of the mechanism then turns on the degree to which the government correctly specified the project before it asked for bids. If the government is uncertain about either the benefits of the project and/or the costs of delivery, it may not specify the optimal project size –determined by balancing the marginal benefits and costs of larger and smaller projects– before asking for bids. The final project, while delivered at close to cost, may not then be of the optimal size, resulting in some deadweight loss through this procurement.

A second method for procuring this project would involve the government determining how much money it was prepared to spend on the project (the "envelope") and then letting bidders compete through the quantity or size of the project they will provide for that amount of money. We refer to this as "bidding the envelope" (BTE). This approach has been

☆ The authors wish to express their gratitude for helpful discussions and comments to Larry Blain, Sarah Clark, Jean-Etienne de Bettignies, Ronald Fischer, Tae Oum, Alan Russell and two anonymous referees; to Jennifer Ng for excellent research assistance; and to the Phelps Centre for the Study of Government and Business at the Sauder School of Business and Chilean grants Conicyt FB0816 and Milenio P-05-004-F for financial support for this research.

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[1] World Economic Forum (2010). Deloitte (2008) provides a brief discussion about the deficits in various parts of the world. A McKinsey Global Institute Report in 2013 found that globally, $57 trillion in investment in infrastructure was needed by 2013. See McKinsey Global Institute (2013) at p. 10.

https://doi.org/10.1016/j.tranpol.2017.09.004
Received 4 March 2017; Received in revised form 17 June 2017; Accepted 6 September 2017
Available online 5 October 2017

used, for example by the Province of British Columbia, Canada in its procurement of large-scale improvements to the Sea-to-Sky Highway linking Vancouver with the mountain resort community of Whistler.[2]

An important variant of this approach, likely much more common, involves governments specifying a project quantity but then also a maximum amount they will pay (an "affordability ceiling" or "affordability cap"). If their specified project is not feasible given this affordability limit, the bidding becomes essentially a BTE competition to give the government a project as large as possible within that envelope.[3]

Whether the government was trying to maximize total welfare or "value for money" (defined below), with full information the government can achieve the first best without really needing to choose a mechanism: it can simply offer to buy a project of the optimal size at the known lowest cost of production or the implied envelope. The efficiency of the first-best is lost, however, when there is uncertainty/asymmetric information about costs and/or benefits. In these cases, the government will likely incorrectly (*ex post*) set the quantity or envelope. For example, after setting a project size based on their best estimation of benefits and costs (which will depend on the type of bidding mechanism used), a government finding that firms actually had lower costs than estimated would prefer a larger project. Under the BTE mechanism the firms will indeed bid greater quantities than previously expected; however, under the BTP mechanism the project size will not change.

The purpose of this paper, then, is to explore the conditions under which each of these mechanisms will be superior to the other in terms of minimizing the inefficiencies associated with second-best project sizes *ex post*. We will see that the relative advantages of the mechanisms will depend on a number of factors including the general level of benefits derived from the project and the expected size and distribution function of the marginal cost. It should also be clear that when we talk about the project "size" or "quantity" we could alternatively be talking about "quality" as long as, in this case, the quantity is fixed and quality is a measurable and contractible output.

The next section reviews the related literature including that on the regulation of prices versus quantities and scoring auctions. Section 3 then presents an overview of the model with the key results presented in Section 4. Section 5 offers our conclusions.

2. Related literature

As indicated, there is now a large literature on public procurement investigating the properties of various procurement methods for infrastructure and other goods and services. Previous research has explored many topics such as the design of optimal procurement auctions, scoring rules for multidimensional projects, second-sourcing, contract design for complex projects, collusion in bidding and transparency issues.[4] Our goal here, again, is to contribute to this literature by focusing on the efficiency properties of these two simple yet practical mechanisms.

The ideas here are clearly related to the pioneering work on the uses of prices versus quantity controls as regulatory mechanisms. In Weitzman's (1974) classic contribution, he asked whether it was better to control the behavior of a regulated private firm by setting the price it receives for its output and letting it choose profit-maximizing quantities, or by directly setting the quantity to be produced by the firm. As is true here, these mechanisms will trivially produce identical results when the regulator has full information. However, when there is uncertainty about the benefits and/or the costs of output, introduced much as we have here, the mechanisms are not equivalent and the superiority of one over the other will depend on the shapes of the benefit and cost functions.

Laffont (1977) clarified and extended Weitzman's results, distinguishing between "genuine randomness" –random elements of costs and benefit functions unknown to all players (regulator/planner, producers and consumers)– and random elements that, while unknown to the regulator/planner, are known to the consumer (in the case of benefits) and producer (in the case of costs). This second type of randomness contributes to the information gap that drives the differences between mechanisms. In a similar way, we show below that genuine randomness in project benefits will not affect the relative merits of the two procurement mechanisms we study.[5]

Despite these similarities, there are significant differences between the present paper and this prior literature. First, in Weitzman (1974) the regulator sets a quantity after balancing expected marginal benefits and costs, but Weitzman never discusses how the firm is compensated (problematic given that costs are uncertain). Our BTP mechanism, which also establishes a quantity, clarifies this: bidding will determine how much the winning firm is paid. Second, our BTE mechanism is quite different from the price mechanism in Weitzman (1974). This becomes most apparent when unit ("marginal") costs are constant: a firm responding to a fixed price per unit would either supply zero output (if the price was below its unit costs) or an infinite quantity (if the price was above), hence the Weitzman price mechanism cannot work here.

The most important difference here, however, derives from the fact that we are exploring a procurement model in which bidding modifies firms' behavior in an important way. In fact, it is largely the bidding that regulates firms in our model and, without it, neither of our mechanisms would produce satisfactory results.

Our focus here on two very simple mechanisms –both in current use and each one-dimensional– also sets this paper apart from the literature on scoring auctions. That literature, for example Che (1993), and Asker and Cantillon (2008, 2010), considers procurements in which the government invites prospective suppliers to quote on multiple dimensions of a project including price and possibly numerous aspects of project quality. In contrast, we consider simple mechanisms in which prospective suppliers quote just one number –either a price or a quantity.[6] And, importantly, one of these mechanisms –unlike virtually all of those discussed in the scoring literature – requires the government to specify an envelope and does not ask the bidders to quote a price as part of their bids. In other words, while in either a scoring auction or under the BTP mechanism the government will not know at the bidding stage what the final cost of the project will be, in a BTE mechanism it will know, something that governments may find desirable.[7]

In its focus on the efficiency properties of practical procurement

[2] This was a public-private partnership – an increasingly important procurement mode for large-scale infrastructure projects in many countries. On this project see: www.partnershipsbc.ca/files-4/project-seatosky.php. To be precise, the original call did specify baseline requirements that bidders must satisfy, but then let them offer up further improvements beyond that. The winning bidder provided many additional benefits beyond the baseline and within the envelope provided (e.g. more kilometers of passing lanes, better lighting and signage, and improved highway maintenance etc).

[3] British Columbia has used this approach as well (e.g. for a hospital: http://www.partnershipsbc.ca/files/documents/FSJH-RFP_Volume_1-Revision1.pdf). See also the description of the implementation of affordability caps in Ireland in OECD (2008 at p. 169). The concept is described in the Certified PPP Professional (CP^3P) certification program materials (created with support from the World Bank and other development banks): https://ppp-certification.com/ppp-certification-guide/164-tender-and-award.

[4] See, for example, the collection of essays in Piga and Thai (2007) and Dimitri et al. (2006). Important papers in the procurement auction literature also include Porter and Zona (1993), Compte et al. (2005), Compte and Jehiel (2002), Bajari and Tadelis (2001) and Anton and Yao (1987).

[5] Laffont (1977, p. 180) does recognize that if the different parties have different expectations about genuine randomness, the mechanisms will not be equivalent. This would be true in our model as well.

[6] As a result we do not need to score multiple attributes of a bid. Of course, it may be the case in the BTE mechanism that there are multiple dimensions of "quantity" that the government cares about, in which case it will have to create some scoring mechanism to determine which of a set of different bids provides the greatest aggregate quantity for the purposes of winning the competition.

[7] Cost overruns in the provision of public infrastructure would appear to be an important problem. This has been most comprehensively documented with respect to transportation infrastructure; see for example Cantarelli et al. (2010) and the studies cited therein. For example, these authors (at p.6) cite one study that found that 77% of highway projects in the United States experienced cost escalation, and another that found that the average cost overrun of infrastructure projects was over 50%.

mechanisms, the papers that come closest to the present paper are those by Engel et al. (1997, 2001) and Dasgupta and Spulber (1990). In the articles by Engel et al. the authors study the choice between two variants of what we are calling BTP procurements for new highways. In both cases, the project is specified by the government. In one mechanism bidders are asked to bid based on tolls to be charged or (if tolls are regulated) by franchise length, with the lowest tolls or shortest franchise lengths generally winning. In their second mechanism – newly proposed in this article but since then put into practice – bidders bid on the net present value of toll revenues they will accept, recognizing that their franchise will terminate when that amount has been collected.[8] In contrast, in this paper we compare the more common version of the BTP mechanism with a different mechanism in which the scope of the project is not defined by the public body –as assumed by Engel et al.– but rather is itself the dimension over which bidders compete.

Dasgupta and Spulber (1990) compare the efficiency of three types of procurement auctions, two of them for sole-sourcing as we consider here. One is essentially equivalent to our BTP mechanism with firms bidding for contracts that specify a fixed quantity. The other, however, involves the buyer specifying a schedule of payments as a function of quantities to be provided by the winning bidder. That is, their "envelope" is not a fixed number, as we have it here and as is often observed in practice, but is an amount that varies according to the quantity provided.

To be clear, mechanisms such as scoring rules and the Dasgupta-Spulber variable quantity payment schedule (and likely others) could outperform both the BTP and BTE mechanisms. These mechanisms do, however, require a level of information and sophistication on the part of governments and firms that may not be present. Indeed, we see the simpler BTP and BTE mechanisms regularly employed in practice and our goal here is to understand the implications of these choices.

3. Model

In our model, the government wishes to procure some assets, and possibly also services delivered using those assets. This could be, for example, a road with associated maintenance services. Collectively the assets are referred to here as the "project". A contract between the public and private parties will specify two key elements: the amount of services to be provided (q) and the price to be paid for those services by the public partner to the private partner. For some projects it will make sense to interpret q as the quantity of services provided, such as the number seats per hour travelling on an urban rail line procured using a public-private partnership. In other cases q may be better thought of as a measure of quality, for example the safety (measured in reduced vehicular accident costs) of a PPP road. It is assumed that this contract is fully enforceable on both parties.

The government will choose between two alternative bidding mechanisms, described below, for procuring this project. In making this choice we assume that the government seeks to maximize the "value for money" (VM) from this procurement, defined here to be the difference between the value or benefit of the project procured and the cost to the government to procure it. This objective – commonly adopted in the procurement literature – is somewhat different from the maximization of total social surplus, notably in its treatment of the profits of the winning firm. While these profits would count as part of social surplus, they do not contribute to value for money. We use VM as our objective largely because we believe it more closely reflects actual procurement agency objectives, but in the increasingly global world of large-scale procurements (e.g. in public-private partnerships) where the profits may flow to foreign-owned firms, VM may also more closely track national social surplus.[9] Of course, should highly competitive bidding reduce the winning firm's profits to near zero, there will be no difference between VM and total social surplus. In a later section of this paper we consider the implications for our analysis of adopting a total social surplus objective.

The key to our model is that the public partner will not have full information about costs and/or benefits. Uncertainty about benefits of a project of size q is represented by the function $B(q;\theta)$ where θ is random variable capturing the imprecision with which the government assesses the public's need for the services. This uncertainty is shared – θ is also unknown to all potential private partners. Conventionally, we assume that the first and second derivatives of the benefit function with respect to q obey: $B_q>0$ and $B_{qq}<0$. We also assume that higher levels of θ are associated, for a given project size, with higher levels of benefits, $B_\theta>0$, as well as higher levels of marginal benefits, $B_{q\theta}>0$. The common inability of governments to properly assess the value of large public projects is widely recognized – governments have been notoriously poor at predicting the demand for transportation projects, for example.[10]

Similarly, the government will not have precise information about the private sector's costs of delivering the public services. We represent these imperfectly known costs with the function $TC=C(\eta)q$. Notice that this functional form assumes that marginal costs, $C(\eta)$, are constant. We assume that higher levels of η correspond to higher levels of marginal costs: i.e. $C_\eta>0$. While the public partner knows the shape of the total cost function, it does not know the value of the random component, η_i, for any firm i out of the N firms that can potentially bid. The private firms do know their own costs when they bid, however, so this is not general uncertainty of the sort we saw with respect to the benefits function, it is a case of asymmetric information. Specifically, firm i will draw its cost parameter, η_i, from a common distribution function known by everyone.

Again, in choosing between the mechanisms, the government will look to maximize expected value for money given by $VM=B(q;\theta)–T(q;\theta)$, where $T(q;\theta)$ represents the price paid by the government to procure the project. [11]

In the first mechanism, which we call the "bid the project" (BTP) mechanism, the government specifies the project it wants delivered and firms bid by indicating the total price they would charge to provide that project. Here, the project is defined by the level of q the government wants provided, q^p, and the winning bidder will be the one that quotes the lowest price. This means that the project is delivered for an amount equal to $C(\tilde{\eta})q^p$ where $C(\tilde{\eta})$ is the second lowest marginal cost among all bidders (a second-order statistic), while the actual cost of production is $C(\widehat{\eta})q^p$, where $C(\widehat{\eta})$ is the lowest marginal cost among all bidders (a first-order statistic). This is more easily understood if the auction follows a second-price format. It is direct to observe that bidding its true cost is a dominant strategy for each firm and, therefore, the winning firm will receive the second lowest marginal cost as price per unit. The result, though, does not depend on using a second-price auction format; it is also obtained in a first-price auction if used, since in the unique symmetric equilibrium each firm bids the expectation of the second lowest marginal cost conditional on winning the auction. See the appendix for the proof.

In the second, "bid the envelope" (BTE) mechanism, the government determines the total amount it is prepared to pay for the project (the "envelope") and bidders compete by offering to provide the greatest quantity (or quality) for that total price. If we denote this envelope as T^e, we can see that a second price auction would result in a quantity defined by: $q^e=T^e/C(\tilde{\eta})$. The project would cost the government T^e, though the actual cost of production would be $C(\widehat{\eta})T^e/C(\tilde{\eta})$. The result is, again, the same if a first-price auction was used (see the appendix).

[8] This second mechanism can more efficiently allocate demand risk in their model in which bidders are risk averse. Beginning in 1998, this mechanism has been employed four times in Chile. The largest case was the tendering, in 1998, of the Santiago-Valparaíso route.

[9] For more on the distinction between value for money and total surplus as procurement objectives, see Ross and Yan (2015).

[10] See, e.g. Flyvbjerg et al. (2005) and Cantarelli et al. (2010).

[11] To the extent that there are implementation costs associated with these mechanisms, we assume that they are not different between the two mechanisms and that they are largely fixed costs that will not affect the optimal size of a project.

In the first best, when the government knows that, for example, $\theta = \theta^*$ and the lowest η_i is equal to η^*, the value for money and social welfare maximizing project will be known to come from setting the marginal benefits of q equal to the marginal cost of q, giving q^*:

$$B_q(q^*;\theta^*) = C(\eta^*) \tag{1}$$

With full information, the government can achieve this first best simply by offering to buy a project of the optimal size at the known lowest cost of production or the implied envelope.

With asymmetric information, however, the two mechanisms will in general perform differently. To build intuition, consider the case of known benefits but unknown costs, and suppose first that the government believes (or estimates) that the constant marginal cost will be C_0. It would then, under the BTP mechanism, choose a quantity that equates marginal benefits with C_0. If, however, actual costs come out lower than C_0, the actual first-best quantity will be higher, though under the BTP mechanism, the same quantity is provided. The fact that costs are lower does mean that the government will pay less than it expected for its project, but there will be a deadweight loss associated with the inefficient choice of quantity. This deadweight loss is illustrated in Fig. 1a. Here B_q represents the known (in this case) marginal benefits curve, C_0 the anticipated marginal costs curve and C_1 the realized, lower, marginal costs curve. Under a BTP mechanism the government will specify a project q_0 and the winning bidder will provide it at a price of $C_0 q_0$. The true first-best quantity will be q_1, however, and the resulting deadweight loss associated with the lower output is illustrated as area *DWLa* in Fig. 1a.

Under a BTE mechanism, there will be output adjustments in response to costs that are above or below expected levels. This is illustrated for this case in Fig. 1b. If the government specifies an envelope based on the expected costs of a project q_0 it chooses $T_0 = C_0/q_0$. Given that costs are lower than expected in this case, bidders will be able to provide a larger quantity – indeed, highly competitive bidding leads to a quantity close to that given by

$$q_2 = T_0/C_1 = \left(\frac{C_0}{C_1}\right) q_0$$

While, with lower costs some increase in quantity is efficient, there is no guarantee that the quantity produced by bidding in the BTE mechanism will not overshoot the true first best quantity (q_1). Fig. 1b illustrates a case where such overshooting has taken place, generating a deadweight loss represented by area *DWLb*.

These two figures illustrate the key tradeoffs studied in this paper – essentially the BTP mechanism is inflexible with respect to quantity in the face of changing costs while the BTE adapts imperfectly to changing costs. The next section presents our formal results, examining the tradeoffs illustrated here. It will also explore the implications of uncertainty about project benefits.

4. Results

In this section we provide the formal analysis of these two mechanisms and demonstrate conditions under which either might be preferred to the other in terms of the government's objectives. Again, project benefits and costs are given by $B(q,\theta)$ and $C(\eta)q$ respectively with θ and η both randomly and independently distributed, each with mean 0.

To carry the analysis further at certain points, below we will add further structure to the problem in which the random components enter in a linear additive fashion and the marginal benefit curve is itself linear. Specifically:

$$B_q(q,\theta) = a - bq + \theta \quad \text{and} \quad C(\eta) = c + \eta$$

Therefore in this special case θ and η shift, respectively, the marginal benefit and marginal cost functions in a parallel fashion. Note that, once we have assumed that the error terms are additive, irrespective of whether the non-random part is linear or not, assuming that the random variables θ and η have zero mean comes at no further loss of generality. In what follows we will refer to this case as *the linear case.*

4.1. Bidding the project (BTP)

Knowing only the distributions of θ and η but not their realized values, a government choosing the BTP mechanism and using a second price auction, will specify a project size q^p to maximize the expected value for money given by $E_{\theta,\eta}[B(q^p;\theta) - C(\tilde{\eta})q^p]$, yielding first-order conditions:

$$E\left[B_q(q^p;\theta)\right] = E[C(\tilde{\eta})]$$

where $C(\tilde{\eta})$ was defined to be the second lowest marginal cost among all bidders. Recall that the actual cost of production is $C(\hat{\eta})q^p$, where $C(\hat{\eta})$ is the lowest marginal cost among all bidders; however, under the value for money objective what is of interest is the cost to the government.

If θ enters the marginal benefit function in an additive fashion, with $g(q)$ the non-random part, and, as expected, marginal benefit is downward sloping, then

$$E[g(q^p) + \theta] = E[C(\tilde{\eta})] \Rightarrow q^p = g^{-1}(E[C(\tilde{\eta})])$$

Which shows that the project size does not depend on the distribution of θ but does depend on the distribution of η. Moreover, expected value for money, $E[VM]$, and the expected costs of a project using the BTP mechanism, T^p, will also not depend on the distributions of θ :

$$E[VM^p] = E\left[\int_0^{q^p} B_q(q;\theta)dq - C(\tilde{\eta})q^p\right] = E\left[\int_0^{q^p} (g(q)+\theta)dq - C(\tilde{\eta})q^p\right]$$

$$E[VM^p] = E\left[\int_0^{q^p} g(q)dq + \theta q^p - C(\tilde{\eta})q^p\right] = \int_0^{q^p} g(q)dq - E[C(\tilde{\eta})]q^p$$

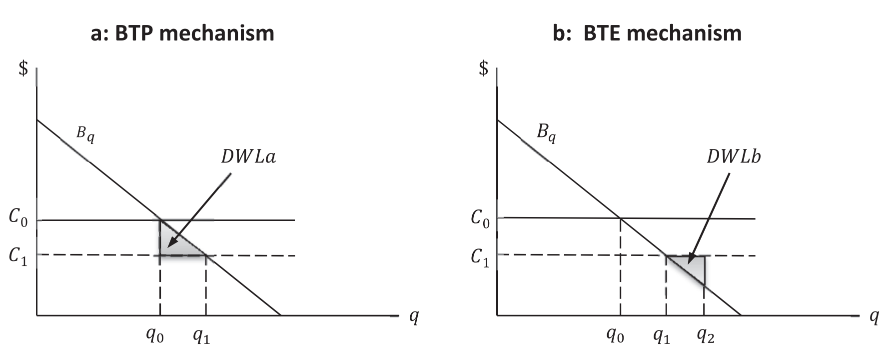

Fig. 1. a and b: Costs lower than expected.

and $T^p = E[C(\tilde{\eta})]\cdot q^p$

Closed form solutions can be obtained for the linear case:

$$q^p = \frac{a-c-E[\tilde{\eta}]}{b}, E[VM] = \frac{(a-c-E[\tilde{\eta}])^2}{2b}, E[T^p] = \frac{(c+E[\tilde{\eta}])(a-c-E[\tilde{\eta}])}{b}$$

4.2. Bidding the envelope (BTE)

A government choosing the BTE mechanism will select an envelope T^e knowing that the project size will then be determined (through the bidding process) to be given by $q = T^e/C(\tilde{\eta})$ (where, again, $C(\tilde{\eta})$ represents the second lowest realization of costs among bidders). The government's problem is to maximize expected value for money with respect to the envelope:

$$Max_{T^e} E[VM^e] = E\left[B\left(\frac{T^e}{C(\tilde{\eta})};\theta\right) - T^e\right]$$

yielding first-order conditions:

$$E\left[B_q\left(\frac{T^e}{C(\tilde{\eta})};\theta\right)\cdot\left(\frac{1}{C(\tilde{\eta})}\right)\right] = 1$$

Notice that if marginal benefits are linearly additive in θ, and given the independence of θ and η, this random component will again drop out of the first-order conditions. Therefore the randomness of the benefits function will not determine the choice of envelope size.

Indeed, under additively separable random component of the marginal benefit function, the first-order condition gives us

$$E\left[\frac{1}{C(\tilde{\eta})}\cdot g\left(\frac{T^e}{C(\tilde{\eta})}\right)\right] = 1$$

This condition then implicitly defines the optimal envelope for the BTE mechanism, T^e.

A closed form solution for T^e can be easily obtained in the linear case[12]:

$$T^e = \frac{aE\left[\frac{1}{c+\tilde{\eta}}\right] - 1}{bE\left[\left(\frac{1}{c+\tilde{\eta}}\right)^2\right]}$$

Expected value for money under BTE can then be calculated in a straightforward fashion:

$$E[VM^e] = \frac{\left(aE\left[\frac{1}{c+\tilde{\eta}}\right] - 1\right)^2}{2bE\left[\left(\frac{1}{c+\tilde{\eta}}\right)^2\right]}$$

which clearly will depend on the distribution of $\tilde{\eta}$, the second lowest value (second order statistic) of all random variables though not on the distribution of θ, as explained.

4.3. Comparing the two mechanisms

We are now in a position to compare the two mechanisms. We do this by looking at the expected levels of value for money each mechanism reaches, but also by looking at the expected expenditure and expected project size under each mechanism. In all cases, we use the results from the special linear case. We push the analytics as far as we can but this will prove insufficient at times; in order to analyze the effects of the distribution of η on the comparison, or the actual magnitude of differences we will need to resort to numerical techniques. For this here we use a uniform distribution in which η ranges within $[-d; d]$. In a later section we show that the results for a normal distribution would be qualitatively similar but with changes in magnitudes.

4.3.1. Value for money comparison

In order to compare the performance of the two mechanisms we construct the ratio of expected value for money under both and call this $EVM^{e/p}$:

$$EVM^{e/p} = \frac{E[VM^e]}{E[VM^p]} = \frac{\left(aE\left[\frac{1}{c+\tilde{\eta}}\right] - 1\right)^2}{(a-c-E[\tilde{\eta}])^2 E\left[\left(\frac{1}{c+\tilde{\eta}}\right)^2\right]}$$

Therefore, when $EVM^{e/p} > 1$, the BTE mechanism delivers greater expected value for money, while the BTP mechanism dominates when $EVM^{e/p} < 1$.

Notice, first, that this ratio does not depend on the slope of the marginal benefits function, b or, again, on the random component of the benefits function, θ. It will depend on the vertical intercept of the marginal benefits curve a (an indicator of the size and importance of the project) as well as the non-random and random components of the cost function.

We first establish that no mechanism will dominate the other under all conditions, i.e., that $EVM^{e/p}$ can be either greater than or less than one for some parameter values. We proceed in two steps, mapped out in the Appendix. First, we show that $EVM^{e/p}$ is negatively related to the marginal benefits intercept term, a. We then demonstrate that there is a value of a, which we label a', at which $EVM^{e/p} = 1$ and which also satisfies the condition $a' > c$ (so that the project is worth undertaking at some scale). Taken together we see that these results imply that for $a > a'$ the BTP mechanism will generate greater value for money, while for $a < a'$, the BTE mechanism will be superior.

Lemma 1. The expected value for money from the BTE mechanism, relative to that from the BTP mechanism will be negatively related to the level of a, i.e. $dEVM^{e/p}/da < 0$.

Proof: See appendix.

Lemma 2. There exists a value of a, a', such that: (i) $EVM^{e/p}$ evaluated at a' will equal one; and (ii) $a' > c$.

Proof: See appendix

This then gives us our first key comparative finding.

Proposition 1. For some values of the parameters a and c, the BTP mechanism will generate greater value for money, while for other values the BTE mechanism will generate greater value for money.

Proof: Follows directly from discussion above applying Lemmas 1 and 2.

Proposition 1, when combined with Lemma 1, tells us that for a given level of costs, c, the BTP mechanism will generate greater value for money than the BTE mechanism for values of a beyond some critical level, and that the BTP relative advantage grows as a increases further above that critical level. But, of course, it also confirms that the BTE mechanism will generate greater value for money for levels of a below that critical value and the further below that level a is, the greater will be the BTE mechanism's advantage.

Fig. 2 below illustrates ranges of parameter values over which each mechanism dominates the other for the case in which the demand

[12] Jensen's Inequality indicates that $E[1/(c+\tilde{\eta})] > 1/(c+E[\tilde{\eta}])$. Since the distribution of has expectation equal to zero, then the expectation of the 2nd order statistic will always be below zero, implying $E[\tilde{\eta}] < 0$ and therefore $E[1/(c+\tilde{\eta})] > 1/(c+E[\tilde{\eta}]) > 1/c$ or, written differently, $cE[1/(c+\tilde{\eta})] > c/(c+E[\tilde{\eta}]) > 1$. Finally, using the fact that $a > c$, it follows that $aE[1/(c+\tilde{\eta})] > 1$. This ensures that $T^e > 0$.

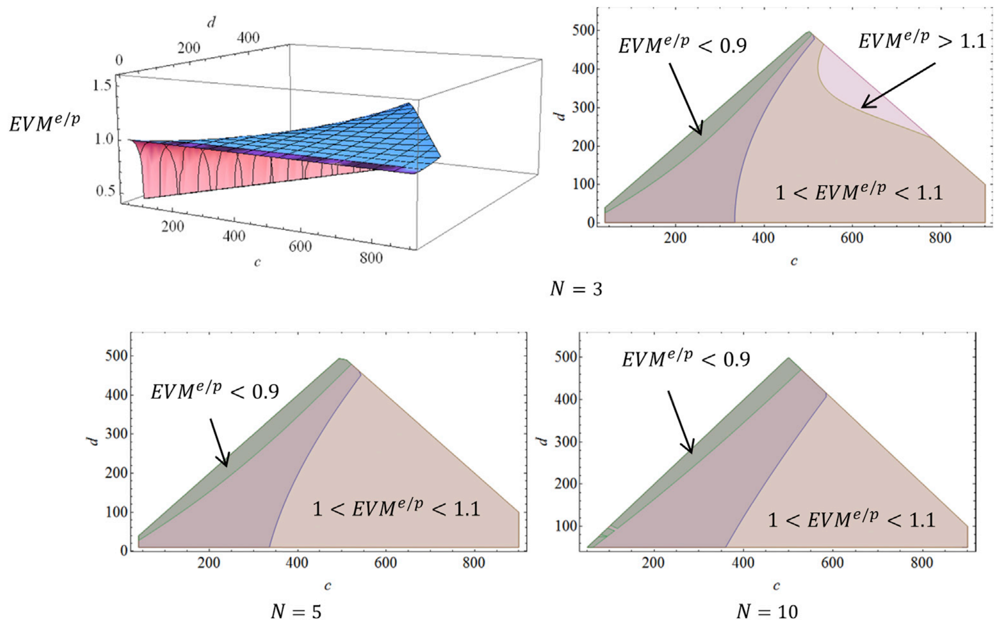

Fig. 2. Expected Value for Money comparisons (uniform distribution).

intercept parameter, *a,* is set at a value of $a = 1000$.

The graphs above are plotted only for values in which $c - d > 0$ and $c + d < a$ so that costs are never negative or above the maximum willingness to pay. The graphs show, for these cases, essentially, three things. First, in the largest portion of the feasible parameter space, BTE beats BTP as shown in the second panel of Fig. 2; this panel, together with the other panels, also shows that differences between mechanisms may be above 10% in parts of the parameter space. One can also see that, for a given level of c, more uncertainty about costs (i.e. a larger d) favours the BTP mechanism, while for a fixed level of d, larger values for c favour the BTE mechanism.

The second aspect that is clear is that as the bidding becomes more competitive (i.e. N increases) the range over which BTE wins shrinks. The intuition for this that as the number of bidders increase, the lowest marginal cost value becomes more certain for two reasons. The first one is purely statistical: as the number of firms drawing marginal costs from the common distribution increases, the expected lowest value approaches the lower bound with greater and greater probability. Increased competition also pushes bidders towards the lower end of the support, thus diminishing uncertainty. Recall that the main advantage of the BTE was, precisely, to allow for adjustments when the resulting marginal cost was different than expected. With less uncertainty, this advantage is diminished.

Summarizing the results above we have the following:

(a) The value for money generated using the BTE mechanism can be greater than or less than the value for money generated using the BTP mechanism, depending on the values of key parameters;
(b) A larger initial marginal value of the project (i.e. a), leading to larger projects, favors the choice of the BTP mechanism;

and for the special cases illustrated here,

(c) A larger value of expected unit costs of the project (i.e. c) favors the choice of the BTE mechanism;
(d) A wider distribution of possible costs, as captured by a larger value of d, favors the choice of the BTP mechanism; and
(e) More competitive bidding (as captured by larger N) favors the choice of the BTP mechanism.

4.3.2. *Project cost comparison*

Governments choosing between these mechanisms may also care about which will result in more costly projects in expectation. The expected costs of projects using the BTP and BTE mechanisms were made explicit above. We define the relative expected costs of the project under the two mechanisms to be $T^{e/p}$, which will then be given by:

$$T^{e/p} = \frac{T^e}{E[T^p]} = \frac{aE\left[\frac{1}{c+\tilde{\eta}}\right] - 1}{(a - c - E[\tilde{\eta}])(c + E[\tilde{\eta}])E\left[\left(\frac{1}{c+\tilde{\eta}}\right)^2\right]}$$

With this definition, whenever $T^{e/p}$ is larger than one, the BTE mechanism leads to higher expected costs for the project and vice versa. A similar approach to that applied for social welfare allows us to establish the following two lemmas:

Lemma 3. as the value of a increases, the expenditures under the BTE mechanism fall relative to those under the BTP mechanism (i.e. $dT^{e/p}/da < 0$).

Proof: See appendix.

Lemma 4. there is a value of a, called here a'' such that $a'' > c + E(\tilde{\eta})$ and at which $T^{e/p} = 1$.

Proof: See appendix.

Together these results imply Proposition 2.

Proposition 2. The expected costs of projects under BTE mechanism can be higher or lower than the expected costs under the BTP mechanism.

Proof: It follows from the discussion above (and the associated proofs in the appendix) that $T^{e/p}$ will be greater than one – implying that the expected costs of a BTE project are higher than those for a BTP project for $a < a''$ and that $T^{e/p}$ will be less than one – implying lower BTE project costs – for $a > a''$.

We can now ask when the BTE mechanism is more costly –in expectation– than the BTP mechanism and by how much? In order to assess this, we again assume that η is uniformly distributed in $[-d; d]$ in order to graph $T^{e/p}$. Fig. 3 shows the result for $a = 1,000$. The overall picture shows that, in expectation, the BTE mechanism is less costly in the largest portion of the parameter space and is never more than 10% more expensive than the BTP mechanism.

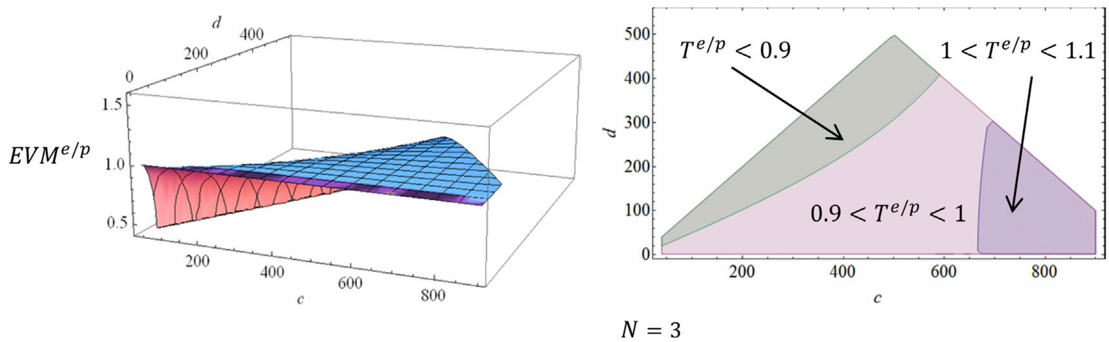

Fig. 3. Expected total costs comparison (uniform distribution).

Importantly, since the area in which the BTE mechanism is more efficient (in a value for money sense) than the BTP mechanism contains the area in which the BTE mechanism is more expensive, we can conclude that, in some cases the better performance will come from lower cost projects and other cases from more expensive projects. When the latter happens, it must be true that the extra (expected) cost is worthwhile. As we show now, whenever the BTE mechanism is more expensive, it delivers a larger project.

4.3.3. *Project size comparison*

Governments may also care about the size of the project they will end up procuring. For example, they might not want to fall short of transit seats, port capacity or textbooks for schools. In that sense, the BTP mechanism gives them an amount that is certain, but the BTE mechanism does not. Which mechanism generates a larger project in expectation and under what conditions? Recall that in the BTP mechanism the quantity is certain and given by

$$q^p = \frac{a - c - E[\tilde{\eta}]}{b}$$

The BTE mechanism quantity is not known in advance, we can only calculate an expectation:

$$E[q^e] = \frac{aE\left(\frac{1}{c+\tilde{\eta}}\right) - 1}{bE\left[\left(\frac{1}{c+\tilde{\eta}}\right)^2\right]} E\left(\frac{1}{c+\tilde{\eta}}\right)$$

Therefore, the ratio of (expected) project size is given by

$$q^{e/p} = \frac{aE\left(\frac{1}{c+\tilde{\eta}}\right) - 1}{(a - c - E[\tilde{\eta}])E\left[\left(\frac{1}{c+\tilde{\eta}}\right)^2\right]} E\left(\frac{1}{c+\tilde{\eta}}\right)$$

such that, whenever $q^{e/p}$ is larger than one, the BTE mechanism delivers a larger expected project size than the BTP mechanism.

Following the same procedure as above, we can show that $q^{e/p}$ may be larger or smaller than one, depending on parameter values.[13] But, again, the most important question is when and by how much the BTE mechanism project size is larger. Reorganizing terms it is easy to see that:

$$q^{e/p} = T^{e/p} cE\left(\frac{1}{c+\tilde{\eta}}\right)$$

Therefore, $q^{e/p}$ can be written as a constant times $T^{e/p}$, where that constant is strictly larger than one since $cE\left(\frac{1}{c+\tilde{\eta}}\right) > 1$ (see footnote 12), implying that $T^{e/p} < q^{e/p}$. It follows that the graphs we obtain for the expected project size (shown in the Appendix) look, essentially, like the ones we presented above for $T^{e/p}$ (Fig. 3). More importantly, it means that, at times, the BTE mechanism may be less expensive in expectation while delivering more output in expectation than the BTP mechanism; this because there will be parameter values that induce $T^{e/p} < 1 < q^{e/p}$.

Also, the parameter space over which the BTE mechanism delivers more output is completely contained by the parameter space over which the BTE mechanism is more efficient. This means that whenever the BTE delivers more output in expectation, it will be more efficient. But it can also happen that it delivers less output in expectation and still be more efficient; this is a reflection of the fact that the quantity adaptation of the BTE mechanism can go either way.

Fig. 4 summarizes the areas in which each situation can occur. In area A the BTP mechanism generates greater expected value for money than the BTE mechanism. In areas B, C, and D the BTE mechanism dominates but, in area B, it does so by providing both a smaller project size and lower project cost, in area C it provides larger size but lower cost and in area D it provides both larger size and higher cost.

5. Extensions

5.1. *Costly public funds*

Recognizing that large scale procurements involve large commitments of public funds and that raising those funds from taxpayers comes at a social cost – the deadweight loss of taxation – we can ask how changes in the marginal cost of raising tax revenues to pay for the procurement will affect the relative merits of the two mechanisms. To examine this let $\lambda > 1$ represent the cost to the economy of withdrawing a dollar by taxation, something known as the *marginal cost of public* funds *(MCPF)*.[14] In this case, value for money will be given by:

$$VM = B(q, \theta) - \lambda T$$

for a project leading to an output of q and a government payment to the provider of T. It is straightforward then to show that under the BTP mechanism the government will specify a project quantity:

$$q^p = \frac{a - \lambda(c + E[\tilde{\eta}])}{b}$$

and expected value for money under the BTP mechanism will be:

[13] Because the proof is rather similar to the analogous proofs for $EVM^{e/p}$ and $T^{e/p}$ we omit it here and in the appendix. It is available upon request from the authors.

[14] See Auerbach and Hines Jr (2002) for a theoretical discussion. The actual value of λ has been estimated by a number of authors. Its estimated value depends on the years, the tax being considered, and the country studied, among other things. For example Harrison et al. (2002) find a MCPF for Chile that is between 1.08 and 1.18 depending on the tax considered. Ballard et al. (1985) estimate a range of 1.17–1.33 for the U.S., while Auriol and Warlters (2012) find an average MCPF for 38 African countries of 1.2.

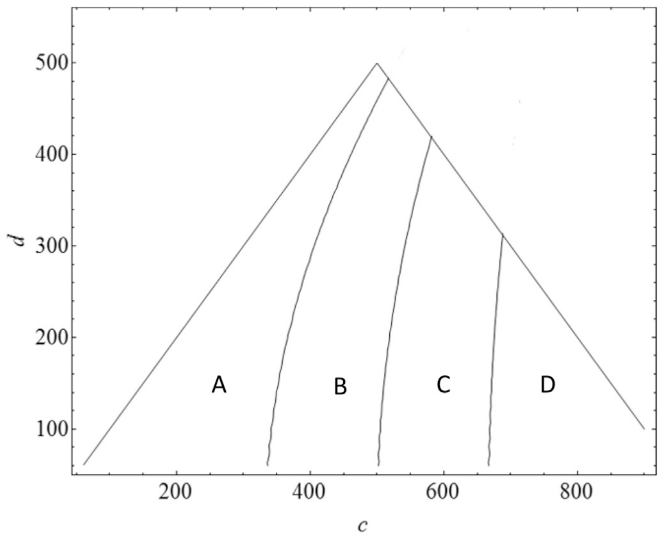

Fig. 4. BTE dominance (illustrated for the case $a = 1,000$ $N = 3$).

$$E[VM^p] = \frac{(a - \lambda(c + E[\tilde{\eta}]))^2}{2b}$$

We note that a MCPF larger than 1 diminishes both the project size and expected value for money.

For the BTE mechanism it is easy to show that the new envelope will be given by:

$$T^e = \frac{aE\left[\frac{1}{c+\tilde{\eta}}\right] - \lambda}{bE\left[\left(\frac{1}{c+\tilde{\eta}}\right)^2\right]}$$

And the expected value for money under the BTE mechanism will be:

$$E(VM^e) = \frac{\left[aE\left(\frac{1}{c+\tilde{\eta}}\right) - \lambda\right]^2}{2bE\left[\left(\frac{1}{c+\tilde{\eta}}\right)^2\right]}$$

We note that a MCPF larger than 1 diminishes both the size of the envelope and expected value for money.

Using these revised measures of expected value for money, we can now express $EVM^{e/p} = E[VM^e]/E[VM^p]$ as a function including the shadow price of public funds, λ:

$$EVM^{e/p}(\lambda) = \frac{\left[aE\left(\frac{1}{c+\tilde{\eta}}\right) - \lambda\right]^2}{[a - \lambda(c + E[\tilde{\eta}])]^2 E\left[\left(\frac{1}{c+\tilde{\eta}}\right)^2\right]}$$

From here we are then able to establish how increases in the deadweight loss from taxation affect the relative merits of the two mechanisms.

Proposition 3. Increases in the marginal cost of taxation will increase the expected value for money of the BTE mechanism relative to that for the BTP mechanism, that is $dEVM^{e/p}/d\lambda > 0$.

Proof: see appendix

We first note that this property does rely on the linearity assumptions but not on any specific distribution for η. In this case both mechanisms have now to consider that the MCPF is larger than one and that, therefore, transfers are more costly than before. But providing exact control over transfers is something the BTE mechanism does while the BTP mechanism only delivers the transfer value *ex post*, and therefore, cannot control expenditures as well. The effect may be very important and is certainly highly non-linear. For example, if we take a central point in the parameter space we have been considering, namely $a = 1,000$, $c = 450$, $d = 200$ *and* $N = 3$, the social surplus arising from the BTE mechanism is larger than that arising from the BTP mechanism by the following amounts:

λ	1	1.15	1.5	2
BTE is better by	2.2%	4.3%	12.8%	80.3%

5.2. Different shapes of marginal benefit functions

We can make a few simple observations about how the shape of the marginal benefits function will affect the relative value for money outcomes of the two mechanisms. In the linear special case developed here, the slope of the marginal benefits curve, while affecting the expected value for money under either mechanism, does not affect their ratio. This is due to the fact that, in this model, changing the slope of the marginal benefits curve without changing the intercept term amounts to a rotation of the marginal benefit curve around the intercept term *a*. So the optimal scale of the project changes along with the sensitivity of marginal benefits to project size.[15] Alternatively, if we consider two different linear marginal benefits curves that intersect the expected marginal cost at the same point, the flatter curve will also correspond to a smaller intercept term. By Proposition 2 this will favor the BTE mechanism, which takes advantage of the greater sensitivity of the optimal project size with respect to changes in costs when the marginal benefits curve is flatter.

Referring back to Fig. 1a, it is easy to see that the flatter the marginal benefit curve, the greater the increase in optimal project size ($q_1 - q_0$) when costs are lower than expected. When the optimal project size is so different from that under the estimated minimum cost, the BTE mechanism will dominate the fixed-size project under the BTP mechanism.

However, if the marginal benefit curve took the shape of a 0–1 demand curve (or even a kinked curve that was vertical below the kink but negatively sloped above), with all the possible cost levels crossing in the vertical portion, there would clearly be no value to larger projects and smaller projects would lose great value, with the result that only a project of a very specific size would be efficient. The BTP mechanism will secure that project at a minimum cost, preserving the full-information solution.

5.3. Other distribution functions

Consider that each bidder draws its marginal cost from a normal rather than a uniform distribution. Because the normal distribution has a probability weight much more centered around the mean, the overall uncertainty (measured in terms of the variance) is smaller. And with less uncertainty the two mechanisms look more similar. The following graph shows the value of $EVM^{e/p}$ for two distribution functions: the uniform and a normal around the mean. We fixed $a = 1000$, and consider $d = 200$ for the uniform distribution. In the case of the normal distribution one cannot really put actual boundaries to the values of *c* but, since we consider a standard deviation equal to 50, values of c smaller than 200 or larger than 800 are quite improbable.

The graph shows that the parameter space in which each mechanism dominates remain unchanged. Yet, the magnitudes by which this domination occurs do change because, as explained, less general uncertainty about c makes the mechanisms look more like each other. As the variance in the normal distribution grows, the curve for the normal distribution in Fig. 5 becomes steeper.

[15] This implies that the elasticity of project size with respect to marginal benefits will be constant with respect to changes in the slope for a constant level of marginal benefits.

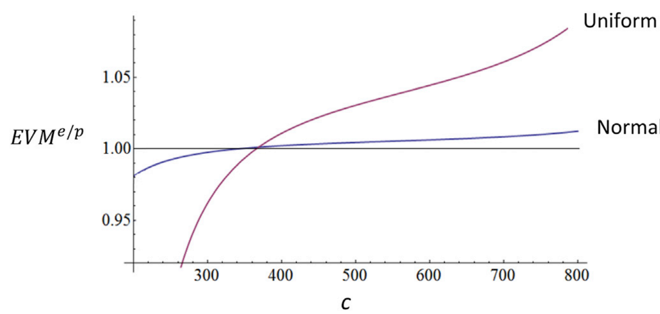

Fig. 5. Expected Value for Money comparisons: Normal vs Uniform distribution.

5.4. Social welfare

We consider here briefly the choice between BTP and BTE mechanisms for a government that seeks to maximize social welfare (defined as the difference between the benefits of the project and the actual costs of production).

When using the BTP mechanism, the government will now care about the actual cost of production and no longer about what it has to pay, which are now simply transfers. Since the second price auction ensures that the lowest marginal cost among all bidders wins, the problem the government now solves is max $E_{\theta,\eta}[B(q^p;\theta) - C(\widehat{\eta})q^p]$, where the only difference is that now the first-order statistic $\widehat{\eta}$, and not the second order-statistic $\tilde{\eta}$, is considered. Therefore, if the government seeks to maximize social welfare we would observe expressions that are analogous to the ones for the value for money case:

$$q^p = \frac{a-c-E[\widehat{\eta}]}{b}, E[SW^p] = \frac{(a-c-E[\widehat{\eta}])^2}{2b}, T^p = \frac{(c+E[\widehat{\eta}])(a-c-E[\widehat{\eta}])}{b}$$

Things are a little bit more complex with the BTE. A government choosing this mechanism will select an envelope T^e knowing that the project size will then be determined – because of second price bidding – as $q = T^e/C(\tilde{\eta})$ *where* $\tilde{\eta}$ refers to the *second lowest* draw of η. . But since the government's problem is now to maximize expected social welfare with respect to the envelope, it cares about the actual cost at which the winning project will be produced:

$$Max_{T^e} E[SW^e] = E\left[B\left(\frac{T^e}{C(\tilde{\eta})};\theta\right) - C(\widehat{\eta})\frac{T^e}{C(\tilde{\eta})}\right]$$

yielding first-order conditions:

$$E\left[B_q\left(\frac{T^e}{C(\tilde{\eta})};\theta\right)\cdot\left(\frac{1}{C(\tilde{\eta})}\right)\right] = E\left[\frac{C(\widehat{\eta})}{C(\tilde{\eta})}\right]$$

Under additively separable random components and the linear case marginal benefit function, a closed form solution for T^e can be easily obtained[16]:

$$T^e = \frac{aE\left[\frac{1}{c+\tilde{\eta}}\right] - E\left[\frac{c+\widehat{\eta}}{c+\tilde{\eta}}\right]}{bE\left[\left(\frac{1}{c+\tilde{\eta}}\right)^2\right]}$$

Expected social welfare under BTE can then be calculated in a straightforward fashion:

$$E[SW^e] = \frac{\left(aE\left[\frac{1}{c+\tilde{\eta}}\right] - E\left[\frac{c+\widehat{\eta}}{c+\tilde{\eta}}\right]\right)^2}{2bE\left[\left(\frac{1}{c+\tilde{\eta}}\right)^2\right]}$$

which clearly will depend on the distribution of $\tilde{\eta}$, the second lowest value (a second order statistic) and $\widehat{\eta}$, the lowest value (a first order statistic). And this is the main difference: while the BTP changes from expressions depending on the second-order statistic to expressions depending on the first-order statistic, the BTE mechanism mixes both.

The ratio of these expected social welfare values can then be expressed as:

$$ESW^{e/p} = \frac{E[SW^e]}{E[SW^p]} = \frac{\left(aE\left[\frac{1}{c+\tilde{\eta}}\right] - E\left[\frac{c+\widehat{\eta}}{c+\tilde{\eta}}\right]\right)^2}{(a-c-E[\widehat{\eta}])^2 E\left[\left(\frac{1}{c+\tilde{\eta}}\right)^2\right]}$$

Analyses analogous to those above show that, again, whether one mechanism dominates the other depends on parameter values. In this case, however, the parameter space over which BTE dominates and the magnitude by which it dominates are greatly diminished, to the point of being almost imperceptible. This is due to the fact that the first order statistic has less uncertainty than the second order statistic. This hurts the BTE mechanism since, with respect to social welfare, it depends on both order statistics while the BTP mechanism depends only on the first.

6. Discussion and conclusions

In this paper we have described two mechanisms, both in actual use by governments, for the procurement of large public projects including many large transportation projects. The more common BTP mechanism involves the government specifying the size of the project it wishes delivered and competitive bidders competing to provide that project by offering bids close to cost levels. In the second mechanism, BTE, the government specifies a budget or envelope that represents the total amount it is willing to spend on the project and bidders compete by offering greater quantities until the quantity they provide results in costs equal to the size of the envelope.

Clearly, with either costs or benefits uncertainty, the project put out to bid may not be of the optimal size *ex post*. In addition, uncertainty on the part of government about costs will lead to differences in the efficiency of the two mechanisms. In contrast, uncertainty about benefits, if shared by all parties, does not affect the relative efficiency of the mechanisms.

Our first key result is that either mechanism can dominate the other in terms of the value for money generated, depending on the values taken by various parameters of the model. We have shown that which mechanism dominates the other will depend on a number of factors. First, larger project benefits (as captured by the intercept of the marginal benefits curve), favor the BTP mechanism. With respect to the shapes of the benefit and cost curves, the key determinant is the degree to which efficiency requires significant changes in project size when costs change. The BTE mechanism does allow project size to adjust in the initial direction of *ex post* optimality, though there is no guarantee that it will not overshoot. The BTP mechanism allows for no change in project size.

Our results indicate that the degree of the uncertainty in costs – influenced by the distribution of marginal costs and the competitiveness of the bidding (as captured by N) – are important in determining the differences between the mechanisms. Specifically, the less uncertainty there is, the less unequal the mechanisms will be.

What may be surprising about these results to many is the fact that the less conventional BTE mechanism dominates its BTP counterpart in a large fraction of the parameter space we explore, suggesting that its superiority does not only arise with extreme or unlikely parameter

[16] Jensen's Inequality indicates that $E[1/(c+\eta)] > 1/(c+E[\eta]) = 1/c$. And using the fact that $a > c$, it follows that $aE[1/(c+\eta)] > a/c > 1$. This ensures that $T^e > 0$.

configurations. We have also seen that the performance difference between the BTE and BTE mechanisms can be substantial and that as the marginal cost of public funds increases above 1, the BTE mechanism improves further relative to the BTP mechanism, a property that may be very important in some cases.

Further work on this model could elaborate on the conditions under which one mechanism dominates the other, and we offer a few conjectures here. First, our model above assumes that there is no correlation between the random elements of the benefit and cost functions. Given the analysis above, the introduction of non-zero correlation would have reasonably predictable qualitative effects. If benefits and costs are negatively correlated (e.g. higher benefits tend to occur with lower costs) the quantity adjustments coming from the BTE mechanism will be even more valuable at least initially – so this favours BTE procurement, other things equal. On the other hand, if benefits and costs are positively correlated the quantity adjustments from the BTE mechanism will be less valuable – and this favours the BTP mechanism.

Second, the implications of non-constant (with respect to quantity) marginal costs are also quite intuitive. If marginal costs are rising with quantity, the output adjustments that come from the BTE mechanism are less valuable. To be specific, the optimal adjustments to changing rising or falling marginal costs are smaller with a rising marginal cost curve and the deadweight loss associated with any given inefficiency in output is less as well. If marginal costs are falling (but more slowly than marginal benefits) the desired output adjustments with changed costs are larger and any changes are more valuable, suggesting a greater relative advantage of the BTE mechanism.

It is easy to see the implications should the uncertainty with respect to costs be with fixed rather than marginal costs. If it is only the firms' fixed costs that are uncertain the government, (and, for simplicity, assuming there is no uncertainty about benefits) changes in costs from expected levels will have no implications for optimal project size.[17] In such a case the BTP mechanism will continue to provide the optimal project size. The BTE mechanism, however, will adjust project size away from the optimal with fixed costs that are higher or lower than expected.

Third, it might be argued that governments care about the quality of a project and not just its size and cost. This would be handled in the above analysis if we can interpret the quantity or size variable as being in fact a measure of quality with other measures of quantity pre-specified (for example, to one unit). Or if the government knows what quality it wants and can observe when that quality is delivered (or not) it can simply make it a contractual obligation and, again, the analysis above applies with respect to quantity.

However, if the government does not know the costs of providing quality or quantity and these costs vary across the population of firms, it will not be clear *ex ante* what the optimal level of quality will be. In such a case the government will have to establish a scoring rule to assess multidimensional bids.[18]

Our analysis here has focused on purely economic trade-offs, but there is no denying that political considerations often weigh heavily in decision-making about large infrastructure projects and this could be an interesting avenue for future work. Some political considerations suggest themselves immediately. Governments that want certainty about their expenditures on a project might be attracted to the BTE mechanism whereas those that want more certainty about the scale of the project they are getting would get that from the BTP mechanism. It may also be the case that no government will be absolutely pure in its application of either mechanism, for example if bids under a BTP mechanism diverge dramatically from what the government expected, it may adjust the scale of the project and re-tender. In a similar way, if the bids received under a BTE envelope are for quantities far from what the government anticipated and wanted, it may go back to the drawing board. Of course, rational bidders anticipating these possibilities will adjust their bids in ways that would need to be analyzed as well.

Taken together, our results suggest that governments may wish to take a closer look at BTE mechanisms for public procurement. Of course there may be other challenges associated with the BTE approach that we have not considered here – for example, if "output" or "project size" is not such a simple one-dimensional variable as modelled here, some sort of scoring rule will be needed to compare the size of competing bids that differ on multiple dimensions. However, we think the results here are strong enough in support of the BTE mechanism that it merits serious consideration under the right conditions.

APPENDIX

First-price auction mechanism

There are N firms, and for each firm η are i.i.d. according to F and the associated density function f; the lowest value of the support of the distribution is η_0. The distribution is common knowledge for the firms.

Bidding the project

We denote $b_i(\eta_i)$ the bid of player *i*. A bid is a per unit price. Given the *ex ante* symmetry, we look for a symmetric Nash equilibrium in bidding strategies, that is $b_j(\eta_j) = \hat{b}(\eta_j)$ for all *j*. Moreover, we restrict our attention to strictly increasing bidding functions $\hat{b}'(\cdot) > 0$, since it is natural that higher values of η will lead to higher bid prices. As is usual in deriving equilibrium for first-price auctions, we will solve the problem faced by bidder i, when it uses strategy $\hat{b}(\cdot)$ and everybody else is also using it, i.e. all other players bid according to $\hat{b}(\eta_j)$ for all $j \neq i$.

For the bidding strategy to be optimal, bidder *i* has to maximize its expected profit when using $\hat{b}(\cdot)$ evaluated at its own value η_i. What we do is calculate bidder i's expected payoff if it uses the bidding strategy with some arbitrary value *r*, and then ensure that its payoff is maximal for $r = \eta_i$. First, bidder *i* wins the auction if $\hat{b}(r) < \hat{b}(\eta_j)\ \ \forall j \neq i$, i.e. if it offers the smallest per unit price. Second, since $\hat{b}(\cdot)$ is strictly increasing, this happens when *r* is smaller than the other N-1, values of η. The probability that this happens is $(1 - F(r))^{N-1}$.

Bidder *i* receives a payoff, if it wins, given by $(\hat{b}(r) - c(\eta))q^p$, where we have dropped the subindex *i*, and q^p is the project specified by the

[17] This assumes that the fixed costs do not rise so high as to make the project inefficient at any size. Part of the fixed costs could also represent the opportunity cost for firms associated with not pursuing other projects (if they do not have the capacity to take any number of projects).

[18] As noted earlier, scoring rules in auctions are considered by Che (1993) and Asker and Cantillon (2008 and 2010).

government. Therefore, the expected payoff u from using the bidding strategy evaluated at r when its true value is η is given by

$$u(r,\eta) = (1 - F(r))^{N-1}\left(\widehat{b}(r) - c(\eta)\right)q^p$$

As explained, since $\widehat{b}(\cdot)$ is an equilibrium, $u(r,\eta)$ must be maximized when $r = \eta$. Therefore, we must derive $u(r,\eta)$ with respect to r, evaluate the derivative at η, and equate to zero. First we calculate:

$$\frac{du(r,\eta)}{dr} = (N-1)(1-F(r))^{N-2}(-f(r))\left(\widehat{b}(r) - c(\eta)\right)q^p + (1-F(r))^{N-1}\widehat{b}'(r)q^p$$

Evaluating the derivative at $r = \eta$, equating to zero and reorganizing we obtain

$$(N-1)(1-F(\eta))^{N-2}(-f(\eta))\widehat{b}(\eta) + (1-F(\eta))^{N-1}\widehat{b}^{'}(\eta) = -(N-1)(1-F(\eta))^{N-2}f(\eta)c(\eta)$$

Note that the left hand side may be rewritten as a derivative:

$$\frac{d}{d\eta}\left[(1-F(\eta))^{N-1}\widehat{b}(\eta)\right] = -(N-1)(1-F(\eta))^{N-2}f(\eta)c(\eta)$$

And then:

$$(1-F(\eta))^{N-1}\widehat{b}(\eta) = -(N-1)\int_{\eta_0}^{\eta} c(\eta)f(\eta)(1-F(\eta))^{N-2}d\eta$$

From where it follows that:

$$\widehat{b}(\eta) = -\frac{(N-1)}{(1-F(\eta))^{N-1}}\int_{\eta_0}^{\eta} c(\eta)f(\eta)(1-F(\eta))^{N-2}d\eta$$

$$\widehat{b}(\eta) = \frac{1}{(1-F(\eta))^{N-1}}\int_{\eta_0}^{\eta} c(\eta)d(1-F(\eta))^{N-1}$$

And because $(1-F(\eta))^{N-1}$ is the distribution function of the lowest value of η among all bidder's N-1 competitors, the bidding strategy in the symmetric equilibrium (which is also unique) is such that each firm bids the expectation of the second lowest marginal cost conditional on winning the auction. This means that, a firm with a high cost will make a large bid, because the probability of winning is low, while the winning firm –the one with the smallest marginal cost—will bid, precisely, the second lowest marginal cost. This ensures that, as in the second price auction, the project is delivered for an amount equal to $C(\tilde{\eta})q^p$ where $C(\tilde{\eta})$ is the second lowest marginal cost among all bidders (a second-order statistic), while the actual cost of production is $C(\widehat{\eta})q^p$, where $C(\widehat{\eta})$ is the lowest marginal cost among all bidders (a first-order statistic).

Bidding the envelope

We denote $B_i(\eta_i)$ the bid of player i. This bid is a quantity. Given the *ex ante* symmetry, we look for a symmetric Nash equilibrium in bidding strategies, that is $B_j(\eta_j) = \widehat{B}(\eta_j)$ for all j. Moreover, we restrict our attention to strictly decreasing bidding functions $\widehat{B}^{'}(\cdot) < 0$, since it is natural that smaller values of η will lead to higher bid quantities. As before, we will solve the problem faced by bidder i, when it uses strategy $\widehat{B}(\cdot)$ and everybody else is also using it, i.e. all other players bid according to $\widehat{B}(\eta_j)$ for all $j \neq i$.

For the bidding strategy to be optimal, bidder i has to maximize its expected profit when using $\widehat{B}(\cdot)$ evaluated at its own value η_i. What we do is calculate bidder i's expected payoff if it uses the bidding strategy with some arbitrary value r, and then ensure that its payoff is maximal for $r = \eta_i$. First, bidder i wins the auction if $\widehat{B}(r) > \widehat{B}(\eta_j)\ \forall j \neq i$, i.e. if it offers the largest project. Second, since $\widehat{B}(\cdot)$ is strictly decreasing, this happens when r is smaller than the other N-1, values of η. The probability that this happens is $(1-F(r))^{N-1}$.

Bidder i receives a payoff, if it wins, given by $T^e - c(\eta)\widehat{B}(r)$, where we have dropped the subindex i, and T^e is the envelope specified by the government. Therefore, the expected payoff U from using the bidding strategy evaluated at r when its true value is η is given by

$$U(r,\eta) = (1-F(r))^{N-1}\left(T^e - c(\eta)\widehat{B}(r)\right)$$

As explained, since $\widehat{B}(\cdot)$ is an equilibrium, $U(r,\eta)$ must be maximized when $r = \eta$. Therefore, we must derive $U(r,\eta)$ with respect to r, evaluate the derivative at η, and equate to zero:

$$\frac{dU(r,\eta)}{dr} = (N-1)(1-F(r))^{N-2}(-f(r))\left(T^e - c(\eta)\widehat{B}(r)\right) - (1-F(r))^{N-1}c(\eta)\widehat{B}'(r)$$

Evaluating the derivative at $r = \eta$, equating to zero and reorganizing we obtain

$$(N-1)(1-F(\eta))^{N-2}(-f(\eta))\widehat{B}(\eta) + (1-F(\eta))^{N-1}\widehat{B}^{'}(\eta) = -(N-1)(1-F(\eta))^{N-2}f(\eta)\frac{T^e}{c(\eta)}$$

Note that the left hand side may be rewritten as a derivative:

$$\frac{d}{d\eta}\left[(1-F(\eta))^{N-1}\widehat{B}(\eta)\right] = -(N-1)(1-F(\eta))^{N-2}f(\eta)\frac{T^e}{c(\eta)}$$

And then:
$(1-F(\eta))^{N-1}\widehat{B}(\eta)=-(N-1)\int_{\eta_0}^{\eta}\frac{T^e}{c(\eta)}f(\eta)(1-F(\eta))^{N-2}d\eta$

From where it follows that:

$$\widehat{B}(\eta)=-\frac{(N-1)}{(1-F(\eta))^{N-1}}\int_{\eta_0}^{\eta}\frac{T^e}{c(\eta)}f(\eta)(1-F(\eta))^{N-2}d\eta$$

$$\widehat{B}(\eta)=\frac{1}{(1-F(\eta))^{N-1}}\int_{\eta_0}^{\eta}\frac{T^e}{c(\eta)}d(1-F(\eta))^{N-1}$$

And because $(1-F(\eta))^{N-1}$ is the distribution function of the lowest value of η among all bidder's N-1 competitors, the bidding strategy in the symmetric equilibrium (which is also unique) is such that each firm bids a quantity according to the expectation of the second lowest marginal cost conditional on winning the auction. This means that, a firm with a high cost will make a small bid (a small project), because the probability of winning is low, while the winning firm –the one with the smallest marginal cost—will propose a project, precisely, according the second lowest marginal cost. This ensures that the first-price auction will result in a quantity defined by: $q^e=T^e/C(\tilde{\eta})$, where $C(\tilde{\eta})$ is the second lowest marginal cost among all bidders (a second-order statistic, just as in the second price auction. The project will cost the government T^e, though the actual cost of production will be $C(\widehat{\eta})T^e/C(\tilde{\eta})$, where $C(\widehat{\eta})$ is the lowest marginal cost among all bidders (a first-order statistic).

Proofs

We define $\psi=\frac{1}{c+\tilde{\eta}}$. We already establish in the text that

$$E(\psi)>\frac{1}{c+E(\tilde{\eta})}>\frac{1}{c} \quad (a.1)$$

And we will also use

$$(c+E(\tilde{\eta}))E(\psi^2)-E(\psi)>0 \quad (a.2)$$

Which holds since

$$\left(c+E(\tilde{\eta}))E(\psi^2)-E(\psi)=(c+E(\tilde{\eta}))(Var(\psi)+E(\psi)^2)-E(\psi)=(c+E(\tilde{\eta}))Var(\psi)+E(\psi)\underbrace{[E(\psi)\cdot(c+E(\tilde{\eta}))-1]}_{+}>0\right.$$

We now move on to prove lemmas and propositions.

Lemma 1. The expected value for money from the BTE mechanism, relative to that from the BTP mechanism will be negatively related to the level of a, i.e. $\partial EVM^{e/p}/\partial a<0$.

Proof: $EVM^{e/p}=\frac{[aE(\psi)-1]^2}{[a-c-E(\tilde{\eta})]^2E(\psi^2)}$. Differentiating with respect to a:

$$\frac{\partial EVM^{e/p}}{\partial a}=\frac{2(aE(\psi)-1)E(\psi)(a-c-E(\tilde{\eta}))^2E(\psi^2)-2(a-c-E(\tilde{\eta}))E(\psi^2)(aE(\psi)-1)^2}{([a-c-E(\tilde{\eta})]^2E(\psi^2))^2}$$

$$\frac{\partial EVM^{e/p}}{\partial a}=\frac{2(a-c-E(\tilde{\eta}))E(\psi^2)(aE(\psi)-1)}{([a-c-E(\tilde{\eta})]^2E(\psi^2))^2}(E(\psi)(a-c-E(\tilde{\eta}))-(aE(\psi)-1))$$

$$\frac{\partial EVM^{e/p}}{\partial a}=\left(\frac{2(a-c-E(\tilde{\eta}))E(\psi^2)(aE(\psi)-1)}{([a-c-E(\tilde{\eta})]^2E(\psi^2))^2}\right)\underbrace{(1-(c+E(\tilde{\eta}))E(\psi))}_{(-)}$$

where the first term in parentheses is positive since $aE(\psi)-1>cE(\psi)-1>0$, by (a.1). The second term is negative as well because of (a.1). It thus follows that

$$\frac{\partial EVM^{e/p}}{\partial a}<0$$

Lemma 2. There exists a value of a, a', such that: (i) $EVM^{e/p}$ evaluated at a' will equal one; and (ii) a' > c.

Proof: $EVM^{e/p}(a)=\frac{[aE(\psi)-1]^2}{[a-c-E(\tilde{\eta})]^2E(\psi^2)}$ is a continuous and differentiable function of a over $(c+E(\tilde{\eta}),+\infty)$. Lemma 1 established that $\frac{\partial EVM^{e/p}}{\partial a}<0$. It is also easy to see that:

$$\lim_{a\to c+E(\tilde{\eta})}EVM^{e/p}=+\infty,$$

since the numerator is finite and positive and the denominator goes to 0^+. Also, applying L'Hôpital's rule twice, we get that

$$\lim_{a\to+\infty} EVM^{e/p} = \frac{E(\psi)}{E(\psi^2)} < c,$$

following (a.2). It follows from the intermediate value theorem that there exists $a' > c + E(\tilde{\eta})$ such that $EVM^{e/p}(a') = 1$.

Proposition 1. For some values of the parameters a and c, the BTP mechanism will generate greater expected social welfare and for other values the BTE mechanism will generate greater expected social welfare.

Proof: none required, see text.

Lemma 3. as the value of a increases, the expenditures under the BTE mechanism fall relative to those under the BTP mechanism (i.e. $\partial T^{e/p}/\partial a < 0$).

Proof: As obtained in the paper, $T^{e/p} = \frac{T^e}{T^p} = \frac{aE[\psi]-1}{(a-c-E(\tilde{\eta}))(c+E(\tilde{\eta}))E[(\psi)^2]}$

$$\frac{\partial T^{e/p}}{\partial a} = \frac{E(\psi)(a-c-E(\tilde{\eta}))(c+E(\tilde{\eta}))E(\psi^2) - (c+E(\tilde{\eta}))E(\psi^2)(aE(\psi)-1)}{[(a-c-E(\tilde{\eta}))(c+E(\tilde{\eta}))E(\psi^2)]^2}$$

$$\frac{\partial T^{e/p}}{\partial a} = \frac{(c+E(\tilde{\eta})E(\psi^2)}{\left[(a-c)cE(\psi^2)\right]^2}[E(\psi)(a-c-E(\tilde{\eta}) - aE(\psi)+1]$$

$$= \left(\frac{(c+E(\tilde{\eta})E(\psi^2)}{\left[(a-c)cE(\psi^2)\right]^2}\right)\underbrace{(1-(c+E(\tilde{\eta})E(\psi))}_{(-)}$$

where the first term is positive and the second negative by (a.1), leaving us with:

$$\left(\partial T^{e/p}\right)/\partial a < 0$$

Lemma 4. there is a value of a, called here a'' such that $a'' > c$ and at which $T^{e/p} = 1$.

Proof: We show that the a'' that makes $T^{e/p} = 1$ is such that $a'' > c$ and therefore that $T^{e/p}$ will be above 1 for $c < a < a''$ and it will be below 1 for $a > a''$.

Consider a'' such that $T^{e/p} = 1$

$$a''E(\psi) - 1 = (a'' - c - E(\tilde{\eta}))\cdot(c+E(\tilde{\eta}))\cdot E\left(\psi^2\right)$$

$$\Rightarrow (c+E(\tilde{\eta}))^2 E\left(\psi^2\right) - 1 = a''((c+E(\tilde{\eta}))E\left(\psi^2\right) - E(\psi))$$

$$\Rightarrow a'' = \frac{(c+E(\tilde{\eta}))^2 E(\psi^2) - 1}{(c+E(\tilde{\eta}))E(\psi^2) - E(\psi)} = (c+E(\tilde{\eta}))\frac{(c+E(\tilde{\eta}))E(\psi^2) - 1}{cE(\psi^2) - E(\psi)}$$

Next, (a.2) ensures that that denominator is positive. And

$$(c+E(\tilde{\eta}))^2 E\left(\psi^2\right) > (c+E(\tilde{\eta}))E(\psi) > 1$$

where the first inequality follows from (a.2) and the second from (a.1). Therefore, $a'' > c$

Proposition 2. The expected costs of projects under BTE mechanism can be higher or lower than the expected costs under the BTP mechanism.

Proof: none required – see text.

Proposition 3. Increases in the marginal cost of taxation will increase the expected social welfare of the BTE mechanism relative to that for the BTP mechanism, that is $dESW^{e/p}/d\lambda > 0$.

Proof: We showed in the text that

$$EVM^{e/p}(\lambda) = \frac{[aE(\psi) - \lambda]^2}{[a - \lambda(c+E(\tilde{\eta})]^2 E(\psi^2)}$$

And differentiating:

$$\frac{\partial EVM^{e/p}}{\partial \lambda} = \frac{2(aE(\psi)-\lambda)(-1)(a-\lambda(c+E(\tilde{\eta}))^2E(\psi^2) - 2(a-\lambda(c+E(\tilde{\eta}))(-(c+E(\tilde{\eta}))E(\psi^2)(aE(\psi)-\lambda)^2}{\left[[a-\lambda(c+E(\tilde{\eta})]^2E(\psi^2)\right]^2}$$

$$\frac{\partial EVM^{e/p}}{\partial \lambda} = \frac{2[a-\lambda(c+E(\tilde{\eta})]\ E(\psi^2)(aE(\psi)-\lambda)}{\left[[a-\lambda(c+E(\tilde{\eta})]^2 E(\psi^2)\right]^2}[(c+E(\tilde{\eta})(aE(\psi)-\lambda)-a+\lambda(c+E(\tilde{\eta})]$$

$$\frac{\partial EVM^{e/p}}{\partial \lambda} = \left(\frac{2(a-\lambda(c+E(\tilde{\eta})E(\psi^2)(aE(\psi)-\lambda)}{[[a-\lambda(c+E(\tilde{\eta}]^2E(\psi^2)]^2}\right)[(c+E(\tilde{\eta})(aE(\tilde{\psi})-\lambda+\lambda)-a] = \left(\frac{2(a-\lambda(c+E(\tilde{\eta})E(\psi^2)(aE(\psi)-\lambda)}{[[a-\lambda(c+E(\tilde{\eta}]^2E(\psi^2)]^2}\right)a[(c+E(\tilde{\eta}))E(\psi)-1]$$

Where the last term is positive by (a.1). Therefore we have:

$dESW^{e/p}/d\lambda > 0$

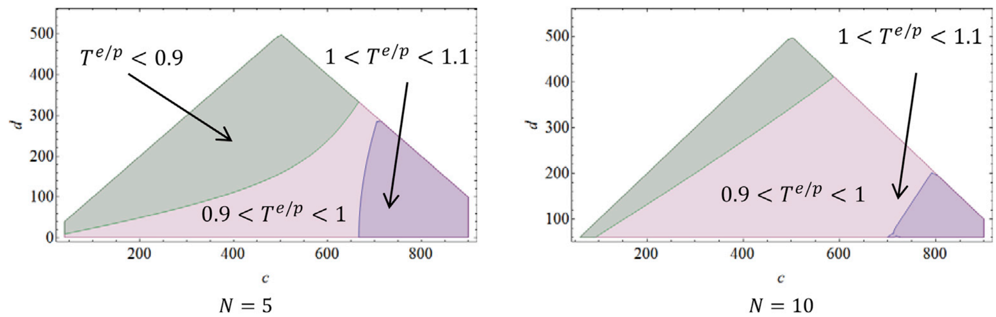

Fig. A.1. Expected total costs comparison.

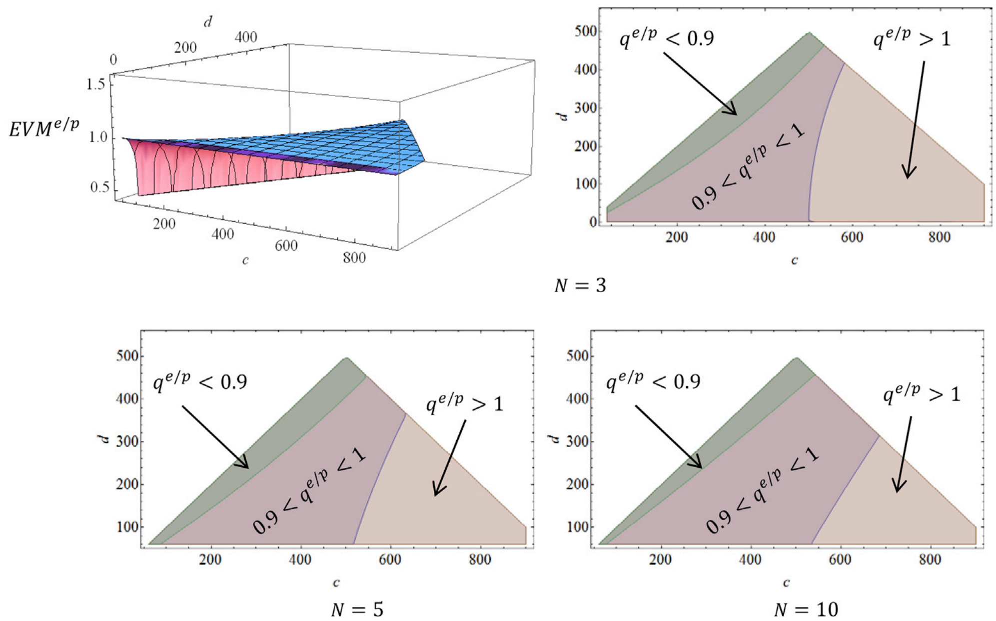

Fig. A.2. Expected project size comparison.

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