

Null tests of the standard model using the linear model formalism

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We test both the Friedmann-Lemaître-Robertson-Walker geometry and Λ CDM cosmology in a model-independent way by reconstructing the Hubble function $H(z)$, the comoving distance $D(z)$, and the growth of structure $f\sigma_8(z)$ using the most recent data available. We use the linear model formalism in order to optimally reconstruct the above cosmological functions, together with their derivatives and integrals. We then evaluate four of the null tests available in the literature that probe both background and perturbation assumptions. For all the four tests, we find agreement, within the errors, with the standard cosmological model.

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I. INTRODUCTION

The late-time accelerated expansion of the Universe has led cosmologists to revise the theory according to which the cosmos evolves either by introducing a new form of matter called dark energy [1] or by modifying directly the laws of gravity [2]. Within the framework of Friedmann-Lemaître-Robertson-Walker (FLRW) cosmologies, a phase of accelerated expansion can be produced by a simple cosmological constant Λ . Although the above gives rise to severe coincidence and fine-tuning problems, observations seem in agreement with such an explanation [3–5].

In order to make progress and understand what is causing the Universe to accelerate, one has, generally speaking, two options. One can assume that the Λ CDM model is correct and work out its consequences. If inconsistencies with data are found, then physics beyond the standard model is necessary. The other complementary approach is to assume a specific model beyond Λ CDM and work out if it has advantages with respect to the standard paradigm. The second approach may be more powerful and may trigger new ideas and methodologies. The first approach is simpler so that it is possible to study and understand the model phenomenology in much greater details. The methodology of this paper belongs to the first approach. In particular, we will analyze null tests of the standard model.

Null or consistency test analyses do not aim at finding the parameters of the model in question. Rather, they aim at uncovering possible tensions in data, which could be due to unaccounted-for systematics or a failure of the model itself. These tests are model independent in the sense that they use directly the data and have the added advantage that, if violated, one knows which set of theoretical assumptions have to be reanalyzed. Equivalently, null tests have the ability to extract information that one may miss if restricting to

parameter estimation. It is worth stressing that it is imperative to corroborate the underlying cosmological model in such a way in view of future experiments that will collect an enormous amount of data, spanning over a wide range of redshift; see, for example, DES [5], eBOSS [6], J-PAS [7], DESI [8], LSST [9], Euclid [10], and SKA [11].

Here, we consider four null tests that have been proposed during the past ten years: the Om_1 diagnostic by Sahni *et al.* [12], the Om_2 diagnostic by Zunckel and Clarkson [13], the Ok diagnostic by Clarkson *et al.* [14], and the ns diagnostic by Nesseris and Sapone [15]. In order to evaluate these tests, we reconstruct the Hubble function $H(z)$, the comoving distance $D(z)$, and the growth of structure $f\sigma_8(z)$ using the most recent data available. We use the linear model formalism in order to optimally reconstruct the above cosmological functions. This method is simple and powerful as one can obtain an exact statistical description of the reconstructed functions, including their derivatives and integrals. Furthermore, it is an analytical approach which is suitable to be used with large data sets. Therefore, we propose this method as a possible alternative to methods previously used in the literature, to list a few, binning data and discrete derivatives, principal component analysis, genetic algorithms, the Padé approximation, Gaussian processes, nonparametric smoothing, and machine learning [16–21].

This paper is organized as follows. In Sec. II, we review basic equations and the notation adopted in this paper, and in Sec. III, we briefly present the four null tests considered in this work. We describe the data we use in Sec. IV and detail our methodology in Sec. V. The results of Sec. VI show that the standard cosmological model passes all the tests. The busy reader can jump to Fig. 4. We conclude in Sec. VII.

II. BASIC EQUATIONS

Here, we review the basic equations upon which all the null tests are built. The evolution of the dark-energy component can be expressed in terms of its present energy density parameter Ω_{de_0} and its equation-of-state parameter $w(z) = p/\rho$, with p and ρ being the pressure and energy density, respectively. The subscript 0 will denote the present-day value of the corresponding quantity. The Hubble parameter is then

$$\frac{H^2(z)}{H_0^2} = \Omega_{m_0}(1+z)^3 + \Omega_{de_0}(1+z)^{3(1+\hat{w})} + \Omega_{k_0}(1+z)^2, \quad (1)$$

where

$$\hat{w}(z) = \frac{1}{\log(1+z)} \int_0^z \frac{w(z')}{1+z'} dz', \quad (2)$$

and Ω_{m_0} and Ω_{k_0} are the matter and curvature density parameters, respectively. If we are dealing with the cosmological constant, then $w(z) = -1$ and $\hat{w}(z) = -1$ at all redshifts. Furthermore, the relation $\Omega_{m_0} + \Omega_{de_0} + \Omega_{k_0} = 1$ has to be satisfied.

In a general FLRW model with curvature, the angular diameter distance can be written as

$$D_A(z) = \frac{c}{1+z} \frac{1}{H_0 \sqrt{-\Omega_{k_0}}} \sin \left(\sqrt{-\Omega_{k_0}} \int_0^z \frac{dz'}{H(z')} \right). \quad (3)$$

The angular diameter distance is related to the luminosity distance D_L and the dimensionless comoving distance D by the relations

$$D_L(z) = (1+z)^2 D_A(z), \quad (4)$$

$$D(z) = \frac{H_0}{c} (1+z) D_A(z). \quad (5)$$

In the Universe, matter clusters, forming perturbations $\delta\rho(t, x)$ to the underlying background energy density $\rho(t)$. The growth of matter perturbation in the Λ CDM model is given by (assuming homogeneity and isotropy)

$$\delta''(z) + \left(\frac{5}{1+z} - \frac{H'(z)}{H(z)} \right) \delta'(z) - \frac{3}{2} \frac{\Omega_m(z)}{(1+z)^2} \delta(z) = 0, \quad (6)$$

where a prime refers to the derivative with respect to z , and the time evolving matter energy density is

$$\Omega_m(z) = \frac{\Omega_{m_0}(1+z)^3}{H^2(z)/H_0^2}. \quad (7)$$

III. NULL TESTS OF THE STANDARD MODEL

In this section, we list the null tests that we consider in this paper; we refer to the corresponding literature for further details.

A. *Om* diagnostic

The *Om* diagnostic was introduced to test deviations with respect to the flat Λ CDM scenario. Setting $w = -1$ and $\Omega_{k_0} = 0$ in Eq. (1), one can solve for Ω_{m_0} and obtain the following diagnostic [12]:

$$Om_1(z) = \frac{H(z)^2/H_0^2 - 1}{(1+z)^3 - 1}. \quad (8)$$

Equivalently, one can solve for Ω_{m_0} in Eq. (3) and obtain the alternative diagnostic [13]:

$$Om_2(z) = \frac{1/D'(z)^2 - 1}{(1+z)^3 - 1}. \quad (9)$$

Clearly, within the flat Λ CDM model, one has

$$Om_1(z) = Om_2(z) = \Omega_{m_0}, \quad (10)$$

which has to be valid at any redshift. Any violation of the above relation will falsify the flat Λ CDM model. As discussed below, we will obtain the luminosity distance $D_L(z)$ (and so D) from supernova Ia data and the Hubble function $H(z)$ from the cosmic chronometer data.

B. *Ok* diagnostic

The following constant-curvature test can falsify not only the flat Λ CDM model but all the FLRW models at once. The *Ok* diagnostic is defined according to Ref. [14] (see also Ref. [22]):

$$Ok(z) = \frac{D'(z)^2 H(z)^2 / H_0^2 - 1}{D(z)^2}. \quad (11)$$

If the FLRW models are the correct background models, one has

$$Ok(z) = \Omega_{k_0}. \quad (12)$$

Any violation not caused by standard-model perturbations¹ would have profound implications as the FLRW model is at the basis of almost any cosmological model (except inhomogeneous [26] and backreaction models; see the Classical Quantum Gravity special issue [27] and also Ref. [28]).

¹Standard-model perturbations can produce an additional systematic error on H_0 [23] and on the dark-energy equation of state [24,25].

C. ns diagnostic

While the previous tests probe the background structure of the Universe, the test proposed in Ref. [15] (and deeply investigated in Ref. [29]) is sensitive to both background and perturbation observables. The ns diagnostic is able to test the validity of the growth of matter perturbations under the assumption of an FLRW metric with a cosmological constant (which has no perturbations). In brief, the ns diagnostic is obtained from Eq. (6): first, the corresponding Lagrangian is found, and, then, with the help of Noether's theorem, the associated conserved quantity is obtained. The conserved quantity is the ns diagnostic that is found to be

$$ns(z) = \frac{1}{1+z} \frac{H(z) f \sigma_8(z)}{H(0) f \sigma_8(0)} \times \exp \left[\frac{3 \times 10^4}{2} \omega_m \int_0^z (1+x)^2 \frac{\sigma_{8,0} - \int_0^x \frac{f \sigma_8(y)}{1+y} dy}{H(x)^2 f \sigma_8(x)} dx \right], \quad (13)$$

where $\sigma_{8,0}$ is the normalization of the power spectrum (present-day mass fluctuation on a scale of $8 h^{-1}$ Mpc; this refers to linear perturbation theory), and

$$f \sigma_8(z) = f(z) G(z) \sigma_{8,0}, \quad (14)$$

in which $G(z)$ is the growth function of matter perturbations normalized to unity today [$G(z) = \delta(z)/\delta(0)$, from (6)] and the growth factor is $f = d \ln G / d \ln(1+z) + 1$.

In order to be robust against systematics of a particular experiment, we need to reconstruct the $ns(z)$ test by using four independent observables: the Hubble parameter $H(z)$, the growth of structure $f \sigma_8(z)$, $\sigma_{8,0}$, and $\omega_m = \Omega_{m_0} h^2$. As discussed below, we will obtain $f \sigma_8$ from the Redshift-Space Distortion (RSD) measurements collected by different experiments. The Hubble data come from the cosmic

chronometers; instead, for $\sigma_{8,0}$, we will use alternatively the results from the SDSS-III BOSS [30] and KiDS [31] surveys, and for ω_m , the results come from the Planck satellite [4].

Within any (not necessarily flat) Λ CDM model, one has

$$ns(z) = 1. \quad (15)$$

Any violation of the above relation may imply a deviation from the FLRW model, nonzero dark-energy perturbations and/or a deviation from GR.

IV. DATA

The four tests above use three cosmological functions: $H(z)$, $D(z)$, and $f \sigma_8(z)$. We will estimate these functions using three distinct data sets. We will use the full covariance matrix when available.

A. Cosmic chronometers

The so-called cosmic chronometers are passively evolving galaxies from which it is possible to obtain in a model-independent way the Hubble parameter $H(z)$ at various redshifts [32]. Here, we use the 31 independent data points given in Table I. This is the most up-to-date collection of $H(z)$ data; see Fig. 1 for a plot. The linear-model best fit to the data (Sec. V) is also shown in the plot.

B. Supernovae Ia

We obtain the luminosity distance (in Mpc) from the distance modulus μ :

$$D_L(z) = 10^{\frac{\mu(z)-25}{5}}. \quad (16)$$

The distance modulus can be obtained from supernova Ia data. We will consider the compressed Joint Light-curve Analysis (JLA) compilation (Ref. [3], Appendix E) so that

TABLE I. The 31 cosmic chronometer data points used in this analysis.

z	$H(z)$	$\sigma_{H(z)}$	References	z	$H(z)$	$\sigma_{H(z)}$	References
0.07	69.0	19.6	[33]	0.4783	80.9	9.0	[34]
0.09	69.0	12.0	[35]	0.48	97.0	62.0	[36]
0.12	68.6	26.2	[33]	0.593	104.0	13.0	[37]
0.17	83.0	8.0	[35]	0.68	92.0	8.0	[37]
0.179	75.0	4.0	[37]	0.781	105.0	12.0	[37]
0.199	75.0	5.0	[37]	0.875	125.0	17.0	[37]
0.2	72.9	29.6	[33]	0.88	90.0	40.0	[36]
0.27	77.0	14.0	[35]	0.9	117.0	23.0	[35]
0.28	88.8	36.6	[33]	1.037	154.0	20.0	[37]
0.352	83.0	14.0	[37]	1.3	168.0	17.0	[35]
0.3802	83.0	13.5	[34]	1.363	160.0	33.6	[38]
0.4	95.0	17.0	[35]	1.43	177.0	18.0	[35]
0.4004	77.0	10.2	[34]	1.53	140.0	14.0	[35]
0.4247	87.1	11.2	[34]	1.75	202.0	40.0	[35]
0.4497	92.8	12.9	[34]	1.965	186.5	50.4	[38]
0.47	89.0	49.6	[39]				

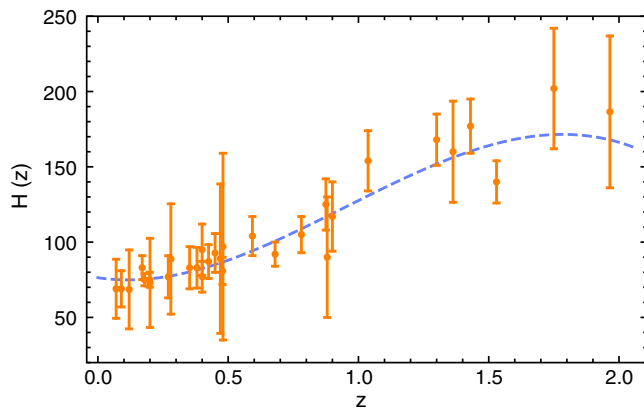


FIG. 1. The 31 cosmic chronometer data points and the corresponding linear model best fit (in light blue) that we use in this work. See Table I for the numerical values.

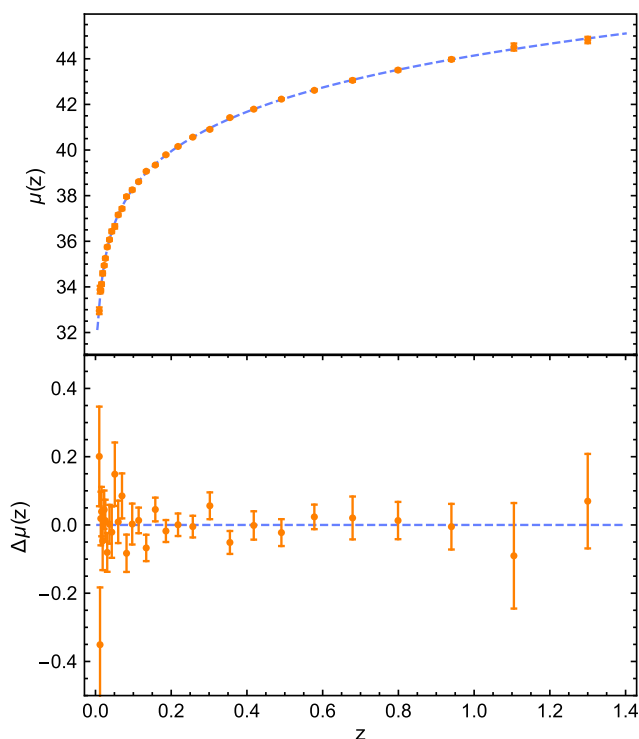


FIG. 2. The compressed JLA compilation (Ref. [3], Appendix E) and the corresponding linear model best fit (in light blue) that we use in this work. For this plot, we have adopted $M = 0$.

$$\mu(z) = \mu_b(z) - M, \quad (17)$$

where $\mu_b(z)$ is the binned distance modulus and M is an unknown offset (a nuisance parameter). Figure 2 shows the JLA data.

C. Redshift-space distortion data

In order to reconstruct $f\sigma_8(z)$, we will consider the robust and independent measurements given in the ‘‘Gold’’ RSD compilation from Ref. [40]; see Fig. 3.

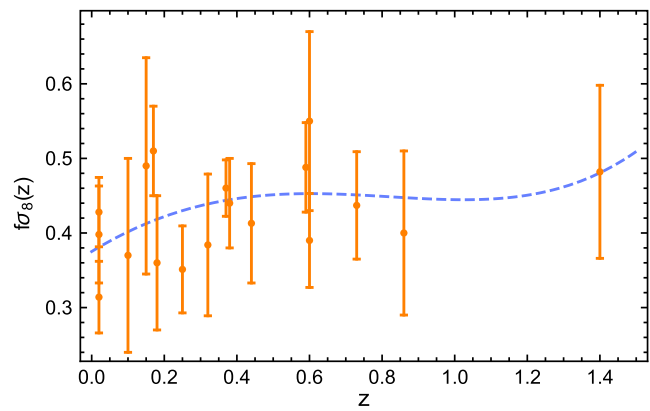


FIG. 3. The 18 $f\sigma_8$ data points [40] and the corresponding linear model best fit (in light blue) that we use in this work.

The three sets of data used in this paper come from several surveys that span different angular and redshift ranges. Therefore, it should be justified to assume that we are considering a fair sample of the Universe.

D. Addition parameters

For the parameter ω_m , necessary for the ns test, we use the results from Planck (Ref. [4], Table 4, TT,TE,EE+lowP):

$$\omega_m = 0.1427 \pm 0.0014. \quad (18)$$

For the parameter $\sigma_{8,0}$, also necessary for the ns test, we use three different values from two surveys. Indeed, the Planck cosmological constraints on $\sigma_{8,0}$ are in tension with those from Planck clusters [41] and from weak lensing measurements, and it is important to test how different values of $\sigma_{8,0}$ impact our results. We will consider results from the SDSS-III BOSS [30] and KiDS [31] surveys, which are independent from Planck. SDSS is a spectroscopic galaxy survey, while KiDS is a survey that uses cosmic shear, galaxy-galaxy lensing, and redshift-space distortion galaxy clustering measurements. The values we consider are

$$\begin{aligned} \sigma_{8,0} &= 0.804 \pm 0.051 \text{ SDSS-III BOSS,} \\ \sigma_{8,0} &= 0.832 \pm 0.080 \text{ KiDS,} \\ \sigma_{8,0} &= 0.747 \pm 0.109 \text{ KiDS (conservative cut).} \end{aligned} \quad (19)$$

The Hubble constant H_0 can be determined using the reconstructed $H(z)$ function at $z = 0$. However, it may also be interesting to consider an independent prior on H_0 . This could be useful to test for systematics. We will consider results from local measurements [42] and from the cosmic microwave background (CMB) (Ref. [4], Table 4, TT,TE,EE+lowP), respectively:

$$H_{0,\text{loc}} = 73.24 \pm 1.74 \frac{\text{km/s}}{\text{Mpc}}, \quad (20)$$

$$H_{0,\text{cmb}} = 67.27 \pm 0.66 \frac{\text{km/s}}{\text{Mpc}}. \quad (21)$$

The parameter M enters in the determination of the dimensionless comoving distance D that is used in the tests Om_2 and Ok . As M is degenerate with $-5 \log_{10} H_0$, we leave M free.

V. METHOD

In order to reconstruct the cosmological functions, and also their derivatives and integrals, we will use the linear model formalism; see, for instance, Ref. [43].

A. Linear model analysis

Let us choose a set of base functions $g_\alpha(z)$ of which the linear combination will constitute the template function $t(z, c_\alpha)$,

$$t(z, c_\alpha) = \sum_{\alpha=0}^{\alpha_{\max}} c_\alpha g_\alpha(z), \quad (22)$$

where α is an integer. The assumption is that $t(z, c_\alpha)$ can describe the actual functions that we want to reconstruct: $H(z)$, $\mu(z)$ or $f\sigma_8(z)$. Clearly, this is conditional to an appropriate choice of $g_\alpha(z)$ and α_{\max} , for each of the functions $H(z)$, $\mu(z)$, and $f\sigma_8(z)$. We call α_{\max} the ‘‘order’’ of the template, which will then have $\alpha_{\max} + 1$ coefficients.

Let us then assume that the data are given by

$$d_i = t_i + e_i, \quad (23)$$

where $t_i = t(z_i, c_\alpha)$ and e_i are Gaussian errors with covariance matrix C_{ij} .

Next, we fit the template t to the data and use the linear model formalism to calculate the Fisher matrix relative to the parameters c_α . This gives an exact description of the likelihood as the template is linear in its parameters. The Fisher matrix is

$$F_{\alpha\beta} = g_{\beta i} C_{ij}^{-1} g_{\alpha j}, \quad (24)$$

where $g_{\alpha i} = g_\alpha(z_i)$, and the best-fit values of c_α are

$$c_{\alpha,\text{bf}} = F_{\alpha\beta}^{-1} B_\beta, \quad (25)$$

where $B_\alpha = d_i C_{ij}^{-1} g_{\alpha j}$.

Summarizing, we have propagated the covariance matrix C_{ij} into the covariance matrix $F_{\alpha\beta}^{-1}$ on the parameters.

B. Error propagation

Let us denote with $\phi(z, \theta_\alpha)$ either Om_1 , Om_2 , Ok , or ns . $\phi(z, \theta_\alpha)$ will be a nonlinear function of the various templates $t(z, c_\alpha)$ [one for each $H(z)$, $\mu(z)$, and $f\sigma_8(z)$] and their derivatives and integrals. The parameter vector $\{\theta_\alpha\}$ comprises the template parameters of Sec. VA and the

additional parameters of Sec. IV D that enter $\phi(z, \theta_\alpha)$. The corresponding covariance matrix $\Sigma_{\alpha\beta}$ is obtained by forming an appropriate block diagonal matrix using the covariance matrices of the corresponding parameters. We have chosen independent data (see Sec. IV) so that correlations among different sets of data are not expected to be important.

In order to compute the error on $\phi(z, \theta_\alpha)$ due to the uncertainty encoded in the covariance matrix $\Sigma_{\alpha\beta}$, a straightforward approach is to apply a change of variable from $\{\theta_\alpha\}$ to ϕ . At the first order, the error is then given by

$$\sigma_\phi^2 = J_\alpha \Sigma_{\alpha\beta} J_\beta, \quad (26)$$

where

$$J_\alpha = \left. \frac{\partial \phi(z, \theta_\alpha)}{\partial \theta_\alpha} \right|_{\theta_{\alpha,\text{bf}}}. \quad (27)$$

Equation (26) is exact if $\phi(z, \theta_\alpha)$ is $t(z, c_\alpha)$, its derivative or integral. Indeed, in this case, it will depend linearly on the parameters $\{\theta_\alpha\}$.

C. Choice of base functions

We will adopt the following base functions:

$$\begin{aligned} H(z) &\rightarrow g_\alpha(z) = z^\alpha \quad \text{with } 0 \leq \alpha \leq \alpha_{\max}, \\ \mu(z) &\rightarrow g_\beta(z) = (\ln z)^\beta \quad \text{with } 0 \leq \beta \leq \beta_{\max}, \\ f\sigma_8(z) &\rightarrow g_\gamma(z) = (1+z)^\gamma \quad \text{with } 0 \leq \gamma \leq \gamma_{\max}. \end{aligned}$$

We have chosen the latter as they can reproduce the fiducial Λ CDM functions with not too high orders α_{\max} , β_{\max} , and γ_{\max} , that is, with not too many parameters. Furthermore, the base functions chosen for $f\sigma_8(z)$ allow us to compute analytically the inner integral in Eq. (13).

In order to choose the orders α_{\max} , β_{\max} , and γ_{\max} on which the template functions $t(z, c_\alpha)$ depend, see (22), we use the following strategy:

- (1) We pick values for $\{\alpha_{\max}, \beta_{\max}, \gamma_{\max}\}$. Each order has to be within 0 and $N - 1$, where N is the number of data of the corresponding catalog.²
- (2) We reconstruct $H(z)$, $\mu(z)$, and $f\sigma_8(z)$ using the method of Sec. VA with mock catalogs created using a fiducial Λ CDM cosmology. These mocks share the same redshift values and covariance matrix of the real data but do not have fluctuations. In other words, the corresponding $H(z)$, $\mu(z)$, and $f\sigma_8(z)$ values are exactly the fiducial ones. Mock catalogs without fluctuations are often used when one wants to consider the average behavior of a large number of ‘‘real life’’ mocks with fluctuations.

²With N points, one can fit up to N coefficients so that, for example, the order α_{\max} cannot exceed $N - 1$.

- (3) We evaluate the four null tests of Sec. III.
- (4) We obtain the sigma bands of the null test functions using the results of Sec. VB.
- (5) We calculate the following vector,

$$\Delta_\phi = \left\{ \frac{|\phi(z_i, \theta_{\alpha,\text{bf}}) - \phi_{\text{fid}}(z_i)|}{\sigma_\phi(z_i, \theta_{\alpha,\text{bf}})} \right\}, \quad (28)$$

with $i = 1, \dots, N$.

- (6) We set a reconstruction accuracy Q , and we repeat the steps (1)–(5) until we find the smallest values $\{\alpha_{\text{max}}, \beta_{\text{max}}, \gamma_{\text{max}}\}$ so that the following condition is satisfied:

$$\max\{\Delta_{Om_1}, \Delta_{Om_2}, \Delta_{Ok}, \Delta_{ns}\} < Q. \quad (29)$$

By setting $Q = 0.1$, we find

$$\{\alpha_{\text{max}}, \beta_{\text{max}}, \gamma_{\text{max}}\} = \{3, 6, 4\}. \quad (30)$$

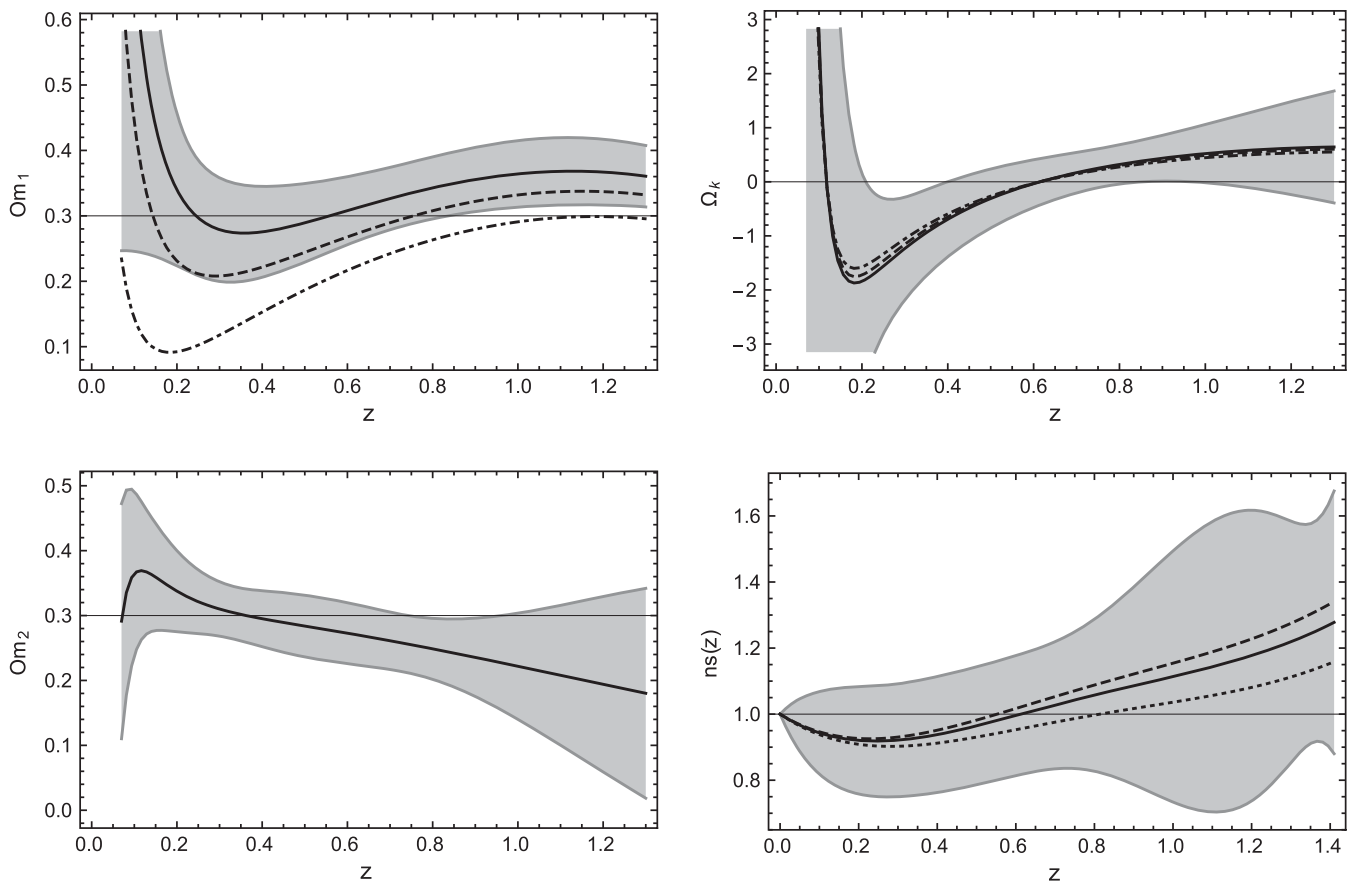


FIG. 4. The four null tests considered in this work with 1σ bands. The flat Λ CDM model is falsified if the Om diagnostics of Sec. III A do not give a constant value; a reference value of $\Omega_{m_0} = 0.3$ is shown (solid line) to guide the eye. All the FLRW models are ruled out if the Ok diagnostic of Sec. III B is not compatible with a constant value (the solid line shows the reference value $\Omega_{k_0} = 0$). Finally, the (possibly curved) Λ CDM model is falsified if the ns diagnostic of Sec. III C is incompatible with a constant value of unity. As these plots show, the latest cosmological data pass all the standard-model null tests. For the Om_1 and Ok tests, we adopted the following values of the Hubble constant: the $H_{0,\text{cmb}}$ value of Eq. (21) (solid line and gray bands), $H_0 = 70 \text{ Km s}^{-1} \text{ Mpc}^{-1}$ (dashed line), and the $H_{0,\text{loc}}$ value of Eq. (20) (dot-dashed line). For the ns test, we adopt the values given in Eq. (19): $\sigma_{8,0} = 0.804$ (black solid line), $\sigma_{8,0} = 0.832$ (black dashed line), and $\sigma_{8,0} = 0.747$ (black dotted line). See Sec. VI for more details.

Figures 1–3 show the reconstruction of the cosmological functions that we obtained.

The above strategy guarantees that the template functions can replicate the fiducial Λ CDM functions without inserting modeling biases in the analysis, while, at the same time, keeping the template order as low as possible. It is worth stressing that within our methodology a unnecessarily high order would lead to higher uncertainties in the Fisher matrix of Eq. (24), thus degrading the constraining power of the null tests.

VI. RESULTS

A. Om diagnostic

Figure 4 (left panels) shows the reconstruction of the diagnostics Om_1 and Om_2 . The 1σ gray bands are obtained using Eq. (26). The Om_1 test is, within its error, consistent with the reference value of about 0.3. Figure 4 was obtained adopting the $H_{0,\text{cmb}}$ value of Eq. (21) (solid line and gray

bands), $H_0 = 70 \text{ Km s}^{-1} \text{ Mpc}^{-1}$ (dashed line), and the $H_{0,\text{loc}}$ value of Eq. (20) (dot-dashed line).

In order to evaluate the Om_2 test, we need to specify the value of the nuisance parameter M , on which D depends. As discussed in Sec. IV D, M is degenerate with $-5 \log_{10} H_0$. Therefore, we can fix H_0 to an arbitrary value such as $H_0 = 70 \text{ Km s}^{-1} \text{ Mpc}^{-1}$ and adopt the value of M for which Om_2 is closest to a constant. If such a value of M does not exist, then the flat Λ CDM model is falsified. The lower left panel of Fig. 4 shows the Om_2 diagnostic for the value $M = 0$. Also, this test is passed. It is worth stressing that this test (and also the Ok test discussed in the next section) will be more constraining when the comoving distance D will be reconstructed using data which do not need unconstrained nuisance parameters such as M . Future Baryon Acoustic Oscillations data could be useful in this respect.

The Om_1 test (and also the Ok test) diverges at $z = 0$ as it involves a $0/0$ limit. Because of the noise in the data, this limit is not well behaved, and it does not follow the theoretical behavior. Nonetheless, this is not a problem as also the errors diverge so that this singular behavior is correctly taken into account and does not bias the result.

B. Ok diagnostic

In Fig. 4 (top right panel), we show the results of the reconstruction of the Ok diagnostic of Eq. (11). As with the Om_2 test of the previous section, one has to specify the value of the nuisance parameter M . Moreover, in this case, the Hubble constant H_0 appears directly in Eq. (11). In the plot, we show the Ok diagnostic that is obtained when adopting the $H_{0,\text{cmb}}$ value of Eq. (21) (solid line and gray bands), $H_0 = 70 \text{ Km s}^{-1} \text{ Mpc}^{-1}$ (dashed line), and the $H_{0,\text{loc}}$ value of Eq. (20) (dot-dashed line). It is clear that the reconstruction is consistent with $\Omega_{k_0} = 0$. The results found are consistent with those presented in Ref. [16].

C. ns diagnostic

In Fig. 4 (lower right panel), we show the results relative to the ns null test of Eq. (13). Present-day data again confirm within 1σ errors that the Λ CDM model is viable. In this case, for consistency [15], $H(0)$ is $H(z = 0)$, where the latter is the Hubble function reconstructed from cosmic

chronometer data. We could also use the H_0 values used for the other tests; however, the effect is an overall shift of the entire curve as $H(0)$ enters as a multiplicative factor. The $\sigma_{8,0}$ adopted (black solid line) is from the SDSS-III BOSS survey, but we also plotted the ns test for the $\sigma_{8,0}$ values from KiDS; see Sec. IV D.

Next-generation surveys are expected to be able to accurately reconstruct this test; for example, $H(z)$ data will be obtained via galaxy clustering, and $f\sigma_8(z)$ data will be obtained via independent weak lensing observations.

VII. CONCLUSIONS

In this paper, using $H(z)$, supernova Ia, and $f\sigma_8$ data, we have reconstructed four null tests, that can be used to probe deviations from either Λ CDM or the assumption of homogeneity and isotropy in the Universe. The reconstruction has been performed by fitting the data with the linear model formalism which provides an exact statistical description of the reconstructed functions together with their derivatives and integrals. We find that all four tests are in agreement with the standard cosmological model, and no interesting deviations were found. This also implies that we have not found any tension on the data. However, special attention goes to the value of H_0 used. As shown, the Om_1 test is the most sensitive to H_0 due to its direct dependence; from Fig. 4, we understand that lower values of H_0 favor a larger value of Om_1 . The other tests are less affected by the value of H_0 .

Current data give results that are far from the cosmic variance limit, and future data will revolutionize the usefulness of the null tests. The availability of hundreds of thousands of supernovae Ia, hundreds of millions of galaxy spectra, billions of galaxy shape measurements and 21-cm data to high redshifts will enable us to accurately use the null tests to assess the viability of the standard model of cosmology. In order to be ready for these large data sets, it is important to develop and explore alternative methodologies to reconstruct the relevant cosmological functions.

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