

## Study of a variable mass Atwood's machine using a smartphone

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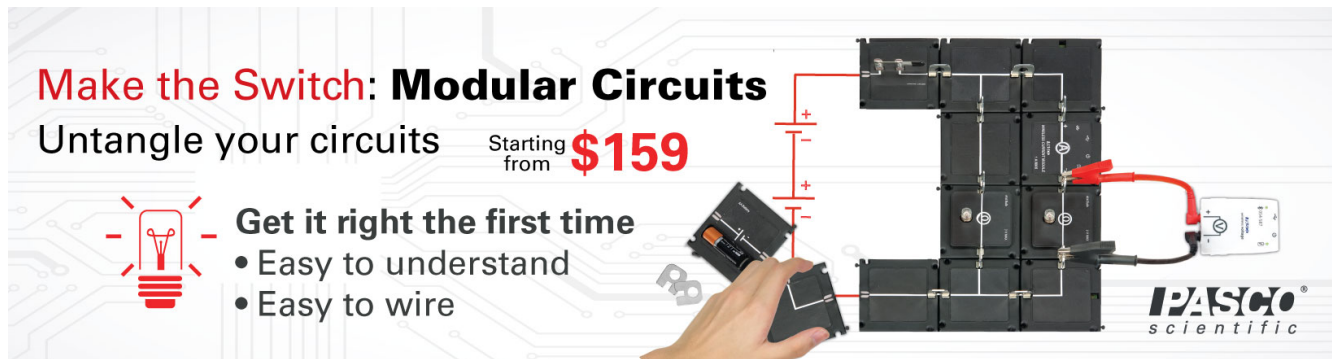
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## Study of a variable mass Atwood's machine using a smartphone

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The Atwood machine was invented in 1784 by George Atwood<sup>1</sup> and this system has been widely studied both theoretically<sup>2-3</sup> and experimentally<sup>4-5</sup> over the years. Nowadays, it is commonplace that many experimental physics courses include both Atwood's machine and variable mass to introduce more complex concepts in physics. To study the dynamics of the masses that compose the variable Atwood's machine, laboratories typically use a smart pulley. Now, the first work that introduced a smartphone as data acquisition equipment to study the acceleration in the Atwood's machine was the one by M. Monteiro et al.<sup>5</sup> Since then, there has been no further information available on the usage of smartphones in variable mass systems. This prompted us to do a study of this kind of system by means of data obtained with a smartphone and to show the practicality of using smartphones in complex experimental situations.

### Theory

The variable mass Atwood's machine consists of two variable masses  $m_a(t)$  and  $m_b(t)$ . Without loss of generality, we considered that only  $m_a(t)$  changes in time. Both masses are connected by an inextensible massless string over an ideal massless pulley. The acceleration equation of this system has been studied in depth by José Flores et al.<sup>6</sup> Based on the results obtained there, the common acceleration is given by

$$a = \left( \frac{m_a(t) - m_b}{m_a(t) + m_b} \right) g, \quad (1)$$

under the influence of an effective gravitational field  $g$ . As we can see, the integrability of Eq. (1) is largely dependent on  $m_a(t)$ . After some considerations that are well explained in Ref. 6 for  $m_a(t) = c_0(t) + m_a(t=0)$ , the common speed is given by:

$$v(t) = v(t=0) + \frac{gt}{\lambda} + \left( \frac{M_0 - \lambda m_{ab,0}}{\lambda^2 C_0} \right) g \ln \left( 1 - \frac{c_0 \lambda t}{M_0} \right), \quad (2)$$

where

$$\lambda = 1 - \left( \frac{m_b}{M_0 - (1/2) m_a(t=0)} \right), \quad (3)$$

and  $c_0$  is the flow rate of sand when the system is not accelerating,  $M_0(t) = m_b + m_a(t=0)$  and  $m_{ab,0} = m_a(t=0) - m_b$ .

Equation (2) can be compared directly with the results of our smartphone measurements.

### The experiment

#### Assembling

The experimental setup is shown to the left of Fig. 1. Two pulleys were located at the top. A negligible mass string was attached to it. A syringe (filled with sand) was added on the side of the string (position A in Fig. 1 left), which mimics our variable mass. On the other side of the string, a smartphone (iPhone 5) was attached (position B in Fig. 1 left). The "Vernier data acquisition"<sup>7</sup> program was used to collect the data over a certain period of time.



Fig. 1. Left: Experimental setup of the variable mass Atwood's machine. Right: Discharging flow of sand experimental setup. A PASCO Force Sensor was positioned at the top to collect the data.

#### Experimental procedure

The experimental procedure is similar to the one used in the recent work by M. Monteiro et al.<sup>5</sup> However, it was necessary to check that the flow rate of sand had a constant decrease<sup>6</sup> over time. To do so, PASCO Force Sensor software was used (in Fig. 1 right, the experimental assembly is shown). This can be checked in the inset of Fig. 3, in which the flow of sand is clearly constant. Once we had considered all the factors mentioned above, the data could be collected. Initially the smartphone was placed at position B (the mass of the smartphone and support is  $m_b = 0.15135 \pm 0.00001$  kg) while the syringe was placed at position A, filled with sand (the initial mass of the syringe and sand is  $m_a [t=0] = 0.16170 \pm 0.00001$  kg).

Once the system is released, it is expected that each mass changes its velocity direction. In other words, as soon as the smartphone and syringe velocities go to zero due to the flow of sand, the smartphone velocity is expected to go from upwards to downwards. It's important to avoid hitting both the pulley against the syringe and the smartphone against the floor after observing the velocity change direction. Just before any of these happened, the system was stopped manually and the app was paused. The acceleration trend can be observed directly from the Vernier visualizer. An example is shown in Fig. 2. As expected for these variable mass systems, we observed a nonconstant acceleration over time (the area of interest is between  $\sim 5$  s to  $\sim 11$  s approximately in Fig. 2).

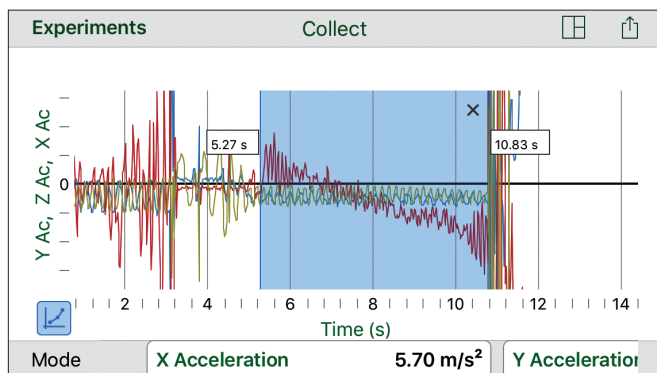


Fig. 2. Snapshot of the Vernier app. We observe three acceleration curves collected by the acceleration sensor. The yellow, blue, and red lines represent the  $x$ -,  $y$ -, and  $z$ -components, respectively. The only relevant component is the  $z$ . The interested area is highlighted in blue.

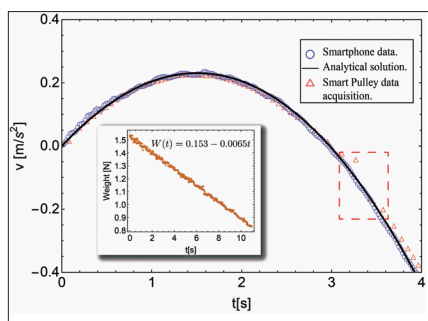


Fig. 3. Experimental result of the variable mass Atwood's machine. The circles represent the measured values of the velocity collected by a smartphone. The continuous line represents the analytical solution given by Eq. (2). The triangular dots represent the measured values of the velocity collected directly by using the Smart Pulley. The inset shows the linear flow rate of sand equal to  $0.0065 \text{ N/s}$ . The parameters of the system were  $m_a (t = 0) = 0.16170 \pm 0.00001 \text{ kg}$  and  $m_b = 0.15135 \pm 0.00001 \text{ kg}$ ,  $g = 9.78 \text{ m/s}^2$ .

the Vernier app (shown in the highlighted area in Fig. 2) were numerically integrated with respect to time by using the standard discrete integration formula.<sup>8</sup> After some minor adjustment of scale, the experimental velocity trend (collected using a smartphone) is shown in blue circles in Fig. 3 (this corresponds to one experimental realization only). Comparing this result with the analytical solution given by Eq. (2) (continuous line in Fig. 3), we observed a good agreement.

Finally, the data from the PASCO Smart Pulley software (triangular dots) are also plotted in Fig. 3. The results were qualitatively similar with respect to those obtained from the smartphone. Nevertheless, it is important to point out the fact that as soon as the masses changed their velocity directions, the Smart Pulley needed to change its rotational directions quickly, making it impossible to collect any data in that period of time. This inconvenience is highlighted in the dashed square area in Fig. 3. However, it is clear that a smartphone does not present any problem acquiring data when the system

changes its velocity direction abruptly. This might be quite useful to laboratory courses in order to study more complex variable mass systems.

## Conclusion

From Fig. 3 we can conclude that it is possible to study a variable mass system by using a smartphone. Indeed, a really good agreement between the experiment and theoretical prediction was observed. Moreover, using smartphones as data acquisition equipment instead of traditional laboratory equipment (such as a smart pulley) might be more adequate when systems present more complex dynamics, such as those faced when carrying out this experiment.

Despite the fact that this type of system is usually considered quite advanced to introduce in physics laboratory courses, it could be useful for students to get a deeper understanding about the physics behind variable mass systems. Moreover, introducing a smartphone as data acquisition equipment might lead students to think more carefully about the experimental data treatment and to question the meaning of the experimental results. Nonetheless, introducing both, variable mass Atwood's machine and smartphone, at the beginning might not be very useful and perhaps it is better to start with a more basic experiment, for instance those in Refs. 5, 6, and 9 to mention a few, before introducing this one.

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7. Vernier Graphical Analysis app, <https://itunes.apple.com/cl/app/verniergraphical-analysis/id522996341?mt=8>.
8. In order to integrate the experimental acceleration data numerically, we computed a discrete integration:
 
$$v_{n+1} = v_n + \left( \frac{a_{n+1} + a_n}{2} \right) (t_{n+1} - t_n), v_0 = 0.$$
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