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Energy prices in the presence of plant indivisibilities

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Abstract

In several countries (Chile, Bolivia, Argentina and Peru, among others), power plants are dispatched according to merit order, i.e. based on the marginal operating costs of the plants. In this scheme, the operating plant with the highest marginal cost sets the spot price at which firms trade the energy required to fulfill their contracts. The underlying peak-load pricing model assumes that plants can operate at any level up to capacity, whereas real power plants have minimum operating levels. This implies that a low cost plant might have to reduce its supply in order to accommodate the minimum operating level of a more expensive power plant. This paper derives the welfare maximizing price rules in this case and shows that the standard peak-load pricing rules no longer apply.

JEL classifications: L51; L94

Keywords: Peak-load pricing; Indivisibility; Electricity

1. Introduction

Early deregulators of electric industries such as Chile, Bolivia and Peru adopted the dispatch and pricing system developed by *Electricité de France* (EDF).[1] In this approach demand is assumed to be unresponsive to price, hence the role of the system operator is to accommodate power supply to the fixed demand. Plants are dispatched according to the *merit order*, i.e. they are ranked according to their

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[1] See Joskow (1976) for a description of the EDF system.

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marginal operating costs and dispatched in ascending order until demand is satisfied. The pricing system has two components. First, plants are paid the price of energy for their production, which is given by the marginal cost of the most expensive plant in operation.[2] In addition, each plant receives a capacity payment equal to the power it delivers at peak demand times a capacity charge which equals the cost of the least expensive means of expanding capacity.[3] Assuming inelastic demand, divisible plants and no uncertainty, it can be shown that this system of rewards, known as peak-load pricing, satisfies the following three desirable properties (Turvey, 1968).

- Any plant that is dispatched by the system operator obeys willingly.
- Each plant pays for its operation and investment costs.
- The rules of dispatch minimize the long and short term cost of providing electricity.

These conditions imply that the system can be decentralized, i.e. given these rules of operation, the market will provide the optimal investment mix that replicates the planner's solution. There have been many extensions of peak-load pricing in order to adapt these results to the real world. Most researchers have explored the consequences of eliminating the assumption of no uncertainty.[4] Less work has been done on the effects of incorporating operating constraints such as minimum operating levels, minimum run time, spinning reserves and maximum ramp time. These constraints, which are known in the engineering–economics literature of electric power systems under the heading of 'unit commitment constraints' have a significant effect on how units are dispatched.

System operators are used to deal with the problem of determining a schedule of units that achieve the minimum operational cost while meeting the forecasted demand and operational constraints.[5] This is a mixed-integer programming problem, and is in the class of the NP-hard problems (Tseng et al., 2000). Hence, there is a vast literature dealing with heuristic methods to find an adequate solution. However, there have been no theoretical analysis of the pricing system that provides the welfare maximizing short and long-run signals to generating companies. A possible explanation is that in state owned monopolies the assignment of revenues to specific

[2] To be precise, the marginal cost of energy is normally used only for transactions between generating companies so that they can satisfy their energy contracts with clients.

[3] In the bidding approach (developed first in the UK, and used in many European countries, Colombia, and some states in the US) generating firms make bids on the amounts and the prices of electricity they are willing to provide the system. The system operator uses these bids to construct the energy supply function and sets the dispatch order.

[4] Under conditions of supply uncertainty, it is necessary to include an outage cost. For an extension with uncertainty in demand and supply, see Chao (1983) and for a survey of peak-load pricing literature readers are referred to the paper by Crew et al. (1995).

[5] Most of the literature, however, focuses on the economic optimization problem of a power utility that decides the day-ahead which units will commit given uncertain electricity prices, electricity demanded by its customers and operating constraints. This problem is commonly referred to as the UC problem. See for instance Allen and Ilic (1997) and Valenzuela and Mazumdar (2001). Other papers center on the valuation of generation assets with UC constraints under uncertain fuel and electricity prices (Skantze et al., 2000).

plants is not an issue, but the most probable explanation is the difficulty in finding an analytical solution.

Johnson et al. (1997) examine the effects of competition and decentralized ownership on resource scheduling in a pool-based electric power system with unit commitment restrictions. They show that decentralized scheduling of multi-owned resources under imperfect information faces serious difficulties. In fact, they demonstrate resorting to an heuristic algorithm that variations in near-optimal schedules that have negligible effect on total system cost can have significant consequences on the total payments by customers to generators and on the distribution of profits among the generators. Thus a system operator charged with making efficient central unit commitments decisions is in the delicate position of making distributional decisions among customers and resource owners with no economic rationale to back its choices. These findings are not surprising: UC restrictions result in non-convexities, and decentralization does not work in non-convex economies.

In this paper, we focus on the effects of one type of UC restrictions: minimum operational levels (MOLs) below which some plants cannot operate. This raises the possibility that within certain demand ranges it might be necessary to reduce the supply of a low operating cost plant, which is replaced by the output from a higher operating cost plant pinned at its MOL, in order to adjust supply to demand in real time.[6] We analyze the properties of standard peak-load pricing in the presence of MOLs, as we are interested in finding the reward structure that will provide the welfare maximizing short and long term signals to generating firms. Hence, in our context, the rationale for making short term distributional decisions is long-run welfare maximization (we omit transitional effects).[7]

We model a simple case with two types of plants, in order to highlight our results. The first type of plant has a high investment cost but a low operating cost as compared to the second type of plant. We examine the case in which a high operating cost plant operating at its MOL displaces part of the production of a low operating cost plant. Hence, the marginal cost of the system corresponds to that of the displaced low operating cost plant, as this is the one that absorbs small demand fluctuations. The problem is that if we set price equal to this marginal cost, the high operating cost plant makes losses, violating the first optimal property of peak-load pricing. We show that keeping marginal cost pricing requires capacity charges above the cost of expansion, because MOLs increase the total cost of the system. Moreover, we show that plants with no MOL restrictions should receive higher

[6] Other cases in which plants with higher operating costs displace plants with lower operating costs occur when the former have long ramp up periods so it is not economical to make them run only at peak time or when the former are forced to operate in order to maintain the integrity of the system.

[7] Valenzuela and Mazumdar (2001) solve the UC problem for a power utility in a model where the electricity price is set by open competition. A stochastic model of the market takes into account the uncertainty on demand and the generating unit availabilities. However, their price model ignores the UC constraints and assumes that a strict predetermined merit order of loading prevails. Not surprisingly, the market-clearing price is shown to be the variable cost or bid of the last unit used to meet the aggregate load prevailing at a particular hour.

capacity payments. MOL constraints have a significant effect on how units are dispatched, and therefore on operating costs. 'By ignoring the MOL constraints, one is likely to undervalue plants with significant flexibility while overvaluing inflexible plants.'[8]

Note that with MOLs the simplicity of the peak-loading pricing rule is lost. First, on occasion the high operating cost plant must be compelled to operate, violating the condition that any *plant that is dispatched by the system operator obeys willingly*. Second, when capacity charges exceed the cost of expansion, there are incentives for an entrant to supply all demand at peak-load using a high operating cost plant, since it receives the cost of capacity plus the cost of power and an additional amount that allows it to earn rents. However, this means that there is no supply in low demand periods (or supply is provided only with high cost plants) since low operating cost plants would exit. Hence, implementation of peak-load pricing in the presence of MOLs requires that the system operator pay capacity payments only to those plants that generate whenever it is requested. In addition, a major problem with MOLs lies in the complexity of the solution, since the system operator must perform the computations needed to derive the optimal rewards, which lacks the transparency of the rewards in the idealized peak-load pricing model.

In the remainder of this paper we formalize these results in a simple model. In particular, we find the power payments that lead to the correct short and long term operation when there are indivisibilities.[9]

2. The model

In order to simplify the exposition we assume that there are two types of plants. Type 1 plants have lower operating costs but higher investment costs than type 2 plants. Hence, in general the former operate as base load plants while the second type operates at peak time. Operating costs are $c_1 < c_2$ and the unit costs of capacity are given by $f_1 > f_2$. The generating capacities of the two types of plants are given by $\bar{q}_1$ and $\bar{q}_2$. We assume the existence of a minimum size for type 2 plants, and that demand justifies the installation of one plant. This plant has a minimum operating level of $\alpha\bar{q}_2$, with $0 \leqslant \alpha \leqslant 1$.

Consider the case in which the existence of the minimum operating level in type 2 plant alters the merit order of dispatch within some range of demand. For example, in order to satisfy an increase in demand, the system operator may be forced to

[8] A similar result is found in a bidding context by Skantze et al. (2000) when analyzing the market based valuation of generation assets. They state that operational constraints have a significant effect on how units are bid into, and dispatched by, a spot market operator, and therefore on the owners' cash flow. By ignoring the UC constraints, one is likely to undervalue plants with significant flexibility while overvaluing large inflexible fossil plants.

[9] The indivisibilities discussed in this paper are different from those studied in the early paper by Williamson (1966). In that paper, indivisibility meant that plants had a fixed minimum size, but it could produce at any power level. In the type of indivisibility considered in this paper, the plant cannot operate below a certain power level

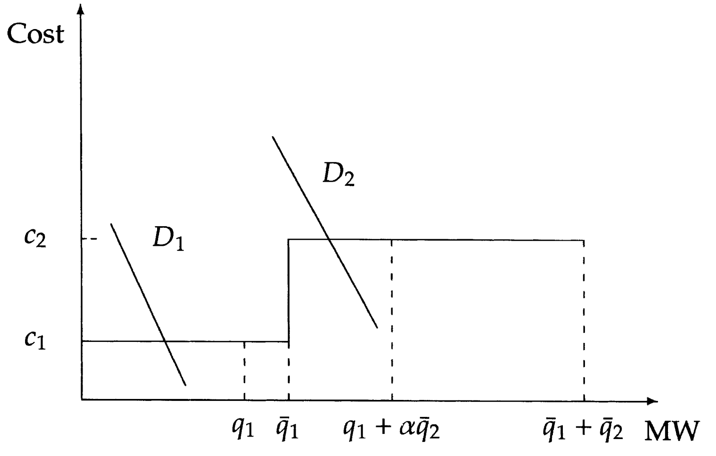

Fig. 1. The problem of minimum operating levels.

dispatch the type 2 plant. Because it has a MOL, its entry displaces part of the capacity supplied by a lower cost type 1 plant. Fig. 1 shows the change in the dispatch order when demand increases from D_1 to D_2. Suppose that at price c_2, demand is less than $\bar{q}_1 + \alpha\bar{q}_2$, but higher than $\bar{q}_1$. When demand is given by curve D_2, the type 2 plant must enter in order to supply the market, but the MOL displaces some of the capacity of the type 1 plant, which produces at a level $q_1 < \bar{q}_1$.

The *load duration curve* described in Fig. 2 orders the 8,760 hours of the year according to the demand for energy, which is assumed inelastic. Let D denote the maximum demand, $q(t)$ the demand in the t-th hour with highest demand, and $t(q)$ its inverse. For simplicity, we assume that function t is differentiable, hence $t'(q) \leqslant 0$. In what follows we use the following notation: $T = 8760$, $\bar{q}_0 = \bar{q}_1 + \alpha\bar{q}_2$, $T_0 = t(\bar{q}_0)$ and $T_1 = t(\bar{q}_1)$. In the figure, during the $(T - T_1)$ hours of low demand only the type 1 plants operate and spot price equals c_1. At T_1, these plants are operating at full capacity and the type 2 plant must begin to provide energy. Given that the type 2 plant is pinned by its MOL, the type 1 plants must cut back their supply. This inversion of the normal merit order occurs in the range $\overline{T_0 T_1}$. At T_0, demand is such that the type 1 plants are operating at full capacity and the type 2 plant needs to generate above its MOL, so the spot price is given by the cost of the type 2 plant.

Result 1 *A MOL leads to an inversion of the merit order of dispatch.*

2.1. The case with no MOLs

When there are no indivisibilities $\alpha = 0$, i.e. $T_0 = T_1$, then the total cost of plants (investment plus operational cost) are:[10]

[10] See Boiteux (1960) for the earliest analysis of the peak-load problem.

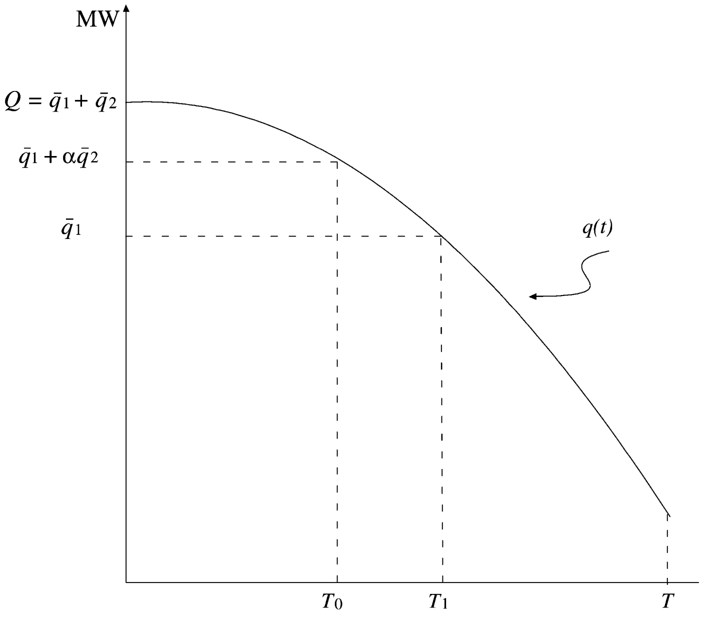

Fig. 2. A load duration curve with indivisibilities.

$$C_1 = c_1 \int_{T_1}^{T} q(t)\mathrm{d}t + c_1 \int_0^{T_1} \overline{q}_1 \mathrm{d}t + f_1 \overline{q}_1$$

$$C_2 = c_2 \int_0^{T_1} \{q(t) - \bar{q}_1\}\mathrm{d}t + f_2 \overline{q}_2$$

We consider a classical peak-load pricing system. Hence, plants receive a payment per unit of energy equal to the marginal cost of energy. They also receive a capacity payment equal to the marginal investment cost in type 2 plants f_2. The revenues accruing to each type (including the capacity charges) are:

$$R_1 = c_1 \int_{T_1}^{T} q(t)\mathrm{d}t + c_2 \int_0^{T_1} \overline{q}_1 \mathrm{d}t + f_2 \overline{q}_1$$

$$R_2 = c_2 \int_0^{T_1} \{q(t) - \bar{q}_1\}\mathrm{d}t + f_2 \overline{q}_2$$

Clearly, the type 2 plants are in a zero-profit equilibrium in all assignments. The type 1 plants are in equilibrium when $R_1 = C_1$, which implies that

$$T_1 = \frac{f_1 - f_2}{c_2 - c_1}$$

The assignment of capacity to type 1 plants is determined using the load duration curve. If $T_1 \leqslant T$, then $\bar{q}_1 = q(T_1)$, otherwise $\bar{q}_1 = 0$. Boiteux (1960) shows that this decentralized equilibrium is optimal as it minimizes the total cost of the system.

2.2. The general case with MOLs

When $\alpha \neq 0$, we have that $T_0 < T_1$. In the interval $\overline{T_0 T_1}$, type 1 plants do not operate at full capacity. Hence, assuming that $\bar{q}_1 \geqslant \alpha \bar{q}_2$ the total cost of each plant is:

$$C_1 = c_1 \int_0^{T_0} \bar{q}_1 \mathrm{d}t + c_1 \int_{T_0}^{T_1} \{q(t) - \alpha \bar{q}_2\} \mathrm{d}t + c_1 \int_{T_1}^{T} q(t) \mathrm{d}t + f_1 \bar{q}_1$$

$$C_2 = c_2 \int_0^{T_0} \{q(t) - \bar{q}_1\} \mathrm{d}t + c_2 \int_{T_0}^{T_1} \alpha \bar{q}_2 \mathrm{d}t + f_2 \bar{q}_2$$

The total cost of the system is:

$$C = C_1 + C_2 = c_1 \int_0^{T} q(t) \mathrm{d}t + (c_2 - c_1) \left[\int_0^{T_0} (q(t) - \bar{q}_1) \mathrm{d}t + \int_{T_0}^{T_1} \alpha \bar{q}_2 \mathrm{d}t \right] + f_1 \bar{q}_1 + f_2 \bar{q}_2 \qquad (1)$$

Next we derive the capacities $\bar{q}_1$ and $\bar{q}_2$ that minimize the total cost of the system and satisfy demand. Hence, we need to impose the restriction $\bar{q}_1 + \bar{q}_2 \geqslant D$, that is total capacity must exceed peak demand D. In the previous analysis we implicitly imposed the condition that the planner does not operate plant 2 unless necessary, but this was not introduced as a constraint in the maximization problem. We generalize the presentation and assume that the central planner also decides the time T_1, at which it begins operating the type 2 plant. Thus, we need to impose the constraint $q(T_1) \leqslant \bar{q}_1$, that is, at the time at which a type 1 plant has to reduce its load in order to accommodate the entry of plant 2, demand must be lower or equal to the capacity of the type 1 plants. Letting λ and μ be the Lagrange multipliers associated to the first and second restriction, respectively, the Lagrange function equals:

$$\begin{aligned} \mathscr{L} = {} & c_1 \int_0^{T} q(t) \mathrm{d}t + (c_2 - c_1) \left[\int_0^{T_0} (q(t) - \bar{q}_1) \mathrm{d}t + \int_{T_0}^{T_1} \alpha \bar{q}_2 \mathrm{d}t \right] + f_1 \bar{q}_1 + f_2 \bar{q}_2 \\ & + \lambda (D - \bar{q}_1 - \bar{q}_2) + \mu (q(T_1) - \bar{q}_1) \end{aligned} \qquad (2)$$

The resulting Kuhn–Tucker conditions are:

$$\frac{\partial \mathscr{L}}{\partial \bar{q}_1} = (c_2 - c_1)\left[-T_0 + (q(T_0) - \bar{q}_1 - \alpha \bar{q}_2)\frac{\mathrm{d}T_0}{\mathrm{d}\bar{q}_1}\right] + f_1 - \lambda - \mu \geqslant 0, \quad \bar{q}_1 \frac{\partial \mathscr{L}}{\partial \bar{q}_1} = 0 \tag{3}$$

$$\frac{\partial \mathscr{L}}{\partial \bar{q}_2} = (c_2 - c_1)\left[(q(T_0) - \bar{q}_1 - \alpha \bar{q}_2)\frac{\mathrm{d}T_0}{\mathrm{d}\bar{q}_2} + \alpha(T_1 - T_0)\right] + f_2 - \lambda \geqslant 0, \quad \bar{q}_2 \frac{\partial \mathscr{L}}{\partial \bar{q}_2} = 0 \tag{4}$$

$$\frac{\partial \mathscr{L}}{\partial T_1} = (c_2 - c_1)\left[\alpha \bar{q}_2 + (q(T_0) - \bar{q}_1 - \alpha \bar{q}_2)\frac{\mathrm{d}T_0}{\mathrm{d}T_1}\right] + \mu q'(T_1) \geqslant 0, \quad T_1 \frac{\partial \mathscr{L}}{\partial T_1} = 0 \tag{5}$$

$$\frac{\partial \mathscr{L}}{\partial \lambda} = D - \bar{q}_1 - \bar{q}_2 \leqslant 0, \quad \lambda \frac{\partial \mathscr{L}}{\partial \lambda} = 0 \tag{6}$$

$$\frac{\partial \mathscr{L}}{\partial \mu} = q(T_1) - \bar{q}_1 \leqslant 0, \quad \mu \frac{\partial \mathscr{L}}{\partial \mu} = 0 \tag{7}$$

The first three inequalities can be written as:

$$\frac{\partial \mathscr{L}}{\partial \bar{q}_1} = -(c_2 - c_1)T_0 + f_1 - \lambda - \mu \geq 0, \quad \bar{q}_1 \frac{\partial \mathscr{L}}{\partial \bar{q}_1} = 0 \tag{8}$$

$$\frac{\partial \mathscr{L}}{\partial \bar{q}_2} = (c_2 - c_1)\alpha(T_1 - T_0) + f_2 - \lambda \geq 0, \quad \bar{q}_2 \frac{\partial \mathscr{L}}{\partial \bar{q}_2} = 0 \tag{9}$$

$$\frac{\partial \mathscr{L}}{\partial T_1} = (c_2 - c_1)\alpha \bar{q}_2 + \mu q'(T_1) \geqslant 0, \quad T_1 \frac{\partial \mathscr{L}}{\partial T_1} = 0 \tag{10}$$

Assuming $\bar{q}_1, \bar{q}_2 > 0$, the first two inequalities become equalities. From Eq. (9) follows that $\lambda > 0$ and from Eq. (6), we have that $D = \bar{q}_1 + \bar{q}_2$ and therefore $D = q(0) > \bar{q}_1$. Therefore, from Eq. (7) we have $T_1 > 0$. Thus, Eq. (10) implies $\mu > 0$, from which Eq. (7) is an equality and $T_1 = t(\bar{q}_1)$. Finally, Eq. (10) can be rewritten:[11]

$$t'(\bar{q}_1)\frac{\partial \mathscr{L}}{\partial T_1} = (c_2 - c_1)\alpha \bar{q}_2 t'(\bar{q}_1) + \mu = 0 \tag{11}$$

[11] Since $\lambda > 0$, Eq. (6) is also an equality, i.e. it is optimal not to have more capacity than required at peak demand. Eq. (7) states that it is optimal not to start operations in plant 2 unless demand exceeds the capacity of plant 1.

Using equalities Eqs. (8), (9) and (11), we obtain the optimality condition:

$$\frac{d\mathscr{L}}{d\bar{q}_1}=\frac{\partial\mathscr{L}}{\partial\bar{q}_1}-\frac{\partial\mathscr{L}}{\partial\bar{q}_2}=f_1-f_2-(c_2-c_1)\left[T_0+\alpha(T_1-T_0)-\alpha\bar{q}_2t'(\bar{q}_1)\right]=0. \tag{12}$$

As a final result, consider the effects of an increase in the MOL of type 2 plants (an increase in α) on the optimal mix of investment in the two types of plants. Total differentiation of Eq. (12) with respect to α leads to:

$$\begin{aligned}&(1-\alpha)\bar{q}_2t'(\bar{q}_0)-\bar{q}_2t'(\bar{q}_1)+(T_1-T_0)\\&+\left[(1-\alpha)^2t'(\bar{q}_0)+2\alpha t'(\bar{q}_1)-\alpha\bar{q}_2t''(\bar{q}_1)\right]\frac{d\bar{q}_1}{d\alpha}=0\end{aligned} \tag{13}$$

Note that unless function $t(q)$ is strongly concave, an increase in α, the minimum operating level of type 2 plant, leads to an increase in the optimal investment in type 1 plants.[12] The intuition is quite simple. An increase in the MOL implies that the system will be operating farther away from the optimal equilibrium without the MOL, and therefore type 2 plants have a higher cost, so that type 1 plants become relatively more attractive.

2.3. Peak-load pricing

2.3.1. A theoretical solution

Next we extend peak-load pricing to the situation in which MOLs are binding. We begin by setting the price of energy equal to the marginal cost, which is c_2 in the interval $0T_0$ and c_1 in the interval T_0T. Next we determine the capacity charge. Since the λ multiplier represents the cost of a marginal increase in peak demand, from Eq. (9) it follows that the capacity charge should be set equal to:

$$f_2^*=f_2+(c_2-c_1)\alpha(T_1-T_0) \tag{14}$$

The MOL raises the price of the system by more than the price of additional capacity. The revenues of the two types of plants can be written as:

$$R_1=c_2\int_0^{T_0}\bar{q}_1dt+c_1\int_{T_0}^{T_1}\{q(t)-\alpha\bar{q}_2\}dt+c_1\int_{T_1}^{T}q(t)dt+f_2^*\bar{q}_1$$

$$R_2=c_2\int_0^{T_0}\{q(t)-\bar{q}_1\}dt+c_1\int_{T_0}^{T_1}\alpha\bar{q}_2dt+f_2^*\bar{q}_2$$

which is sufficient for the type 2 firm to break even and recover the losses it makes by operating between T_0 and T_1, while being paid c_1 per unit of energy delivered.

[12] Alternatively, the condition can be based on the concavity of $q(t)$.

In long-run equilibrium, type 1 plants need to break even. Hence,

$$c_1\int_0^{T_0}\bar{q}_1\mathrm{d}t+f_1\bar{q}_1=c_2\int_0^{T_0}\bar{q}_1\mathrm{d}t+f_2^*\bar{q}_1$$

Rearranging terms, we can rewrite this equality as

$$f_1-f_2-(c_2-c_1)\left[T_0+\alpha(T_1-T_0)\right]=0 \tag{15}$$

However, this solution is not optimal, as it differs from the optimality condition Eq. (12). The reason is quite intuitive. Observe that at any time different from T_1 and 0 (peak demand), the only effect of a marginal increase in demand is to increase operational costs. At T_1, however, a marginal increase in demand lengthens the period in which plant 2 operates at its technical minimum by $t'(\bar{q}_1)$, and this leads to an increase in on total costs of $-(c_2-c_1)\alpha\bar{q}_2t'(\bar{q}_1)>0$. Formally, since the μ multiplier represents the cost of a marginal increase in demand at T_1, from Eq. (12) it follows it is necessary to impose a capacity charge equal to:

$$\mu=-(c_2-c_1)\alpha\bar{q}_2t'(\bar{q}_1), \tag{16}$$

which should be paid only to the type 1 plants as these are the only plants operating at T_1. As we can see by comparing Eqs. (12) and (15) this amount is sufficient to provide the right investment signals for type 1 plants.

Result 2 *Peak-load pricing with a MOL requires a capacity charge, that is, higher than without a MOL and an additional capacity charge paid to low cost firms.*

Observe that when there is no indivisibility ($\alpha=0$), this second capacity charge (Eq. (16)) disappears and the capacity charge (Eq. (14)) becomes the standard capacity charge.

2.3.2. Implementing peak-load pricing

In this section, we show that peak-load pricing is incompatible with a decentralized system when there are MOLs. To see this point, recall that in the interval $\overline{T_0T_1}$ plant 2 operates at a loss. In the previous section, we showed that it was possible to compensate this plant via a capacity charge higher than the marginal cost of capacity. Note however, that this requires that plant 2 be compelled to operate, violating the condition that *any plant that is dispatched by the system operator obeys willingly*, i.e. this is not a decentralized solution. There is an additional problem: at the time of peak demand, users are required to pay the operational cost of the high cost plants, plus the cost of capacity (peak demand × investment cost of the type 2 plant), plus a surcharge to compensate the plant for receiving only c_1 while operating in $\overline{T_0T_1}$. Hence, there is an incentive for an entrant to supply all demand at peak-load with type two plants, receive the cost of capacity plus the cost of power, plus the additional surcharge (which would give them rents). The problem, of course, implies that low cost plants make losses and would exit.

Result 3 *Optimal peak-load pricing with a MOL cannot be decentralized.*

However, peak-load pricing can be implemented by introducing the rule that capacity will be paid only to plants that generate whenever they are requested to do so. Hence, explicit payment rules would solve the problem, but violate the condition that plants are always willing to generate power when requested. An additional problem lies in the complexity of the solution, which requires the explicit computation of capacity charges by the system operator, negating one of the advantages of the peak-load pricing rule in the absence of MOLs.

Up to now we have imposed that the price of energy be set equal to the marginal operational cost. We impose this requirement because if demand were elastic, this would be the requirement of an efficient pricing scheme. In our model, however, the load duration curve is independent of price, as is usual in this type of analysis, so the use of marginal cost pricing is not really necessary and is external to the model. Therefore we might as well have incorporated the additional cost caused by the MOL into the price of energy and kept the original capacity charge as the marginal cost of capacity. For instance, we can consider an energy charge equal to c_2 in $\overline{T_0T_1}$. In this case an energy charge above c_1 is required in $\overline{T_1T_2}$ in order to achieve the optimal solution. The required energy charge is:

$$w = c_1 + (c_2 - c_1)\frac{\displaystyle\int_{T_0}^{T_1}[\alpha D - q(t)]\mathrm{d}t - \alpha\bar{q}_2\bar{q}_1 t'(\bar{q}_1)}{\displaystyle\int_{\mathrm{T}_1}^{T} q(t)\mathrm{d}t}. \tag{17}$$

However, this solution also faces implementation problems. Note that in the period $\overline{T_0T}$ type 1 plants receive more than their marginal cost (since unless $t(q)$ is highly concave, $w > c_1$). This implies that all these plants would like to generate at full capacity, and hence supply would exceed demand. Hence, the effective demand faced by these plants would have to be assigned proportionally to the capacity of each plant.

3. Conclusions

This paper has shown that when plants have minimum operating levels, the standard peak-load pricing system must be modified in order to achieve the (long-run) optimal investment mix between different plants. It has also shown that the solution cannot be implemented via a decentralized mechanism.

MOLs are important in deregulated marginal cost dispatch systems, where the use of the standard peak-load pricing formulas can lead to inefficiencies. The fact that the legislation in the countries that use peak-load pricing does not cover these and other imperfections means that firms must use informal methods of settling these problems. However, this also implies that a new entrant faces unwritten rules, which might be one of the reasons for the lack of entry into the electric markets in these countries.

There are other problems that involve similar issues and which could be analyzed by analogous methods: for example, the long ramp-up times of some plants mean that high cost plants are sometimes required to operate as base-load plants, creating inversions in the merit order. We have made a strong simplification in our analysis, as we have assumed that demand is constant and does not respond to the existence of an additional power charge. Removing this restriction is another topic for future research.[13]

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References

Allen E., Ilic M., 1997. Stochastic Unit Commitment for Power Producers, IEEE Transactions on Energy Conversion (to be published).

Boiteux, M., 1960. Peak load pricing. J. Bus. 33, 157–179.

Chao, H.-P., 1983. Peak load pricing and capacity planning with demand and supply uncertainty. Bell J. Econ. 14 (1), 179–190.

Crew, M., Chitru, F., Kleindorfer, P., 1995. The Theory of Peak-load Pricing: A Survey. J. Regul. Econ. 8, 215–248.

Johnson, R., Oren, S., Svoboda, A., 1997. Equity and Efficiency of Unit Commitment in Competitive Electricity Markets. Utilities Policy 6, 9–19.

Joskow, P.L., 1976. Contributions to the theory of marginal cost pricing. Bell J. Econ. 7 (1), 197–206.

Oren, S., Smith, S., Wilson, R., 1985. Capacity pricing. Econometrica 53 (3), 545–566.

Skantze P., Visudhiphan P., Ilic M., 2000. Valuation of Generation Assets with Unit Commitment Constraints under uncertain fuel prices, The Laboratory for Energy and the Environment, MIT, November.

Tseng, C.L., Li, C.A., Oren, S.S., 2000. Solving Unit Commitment by a Unit Decommitment Method. J. Optimization Theory Appl. 105, 707–730.

Turvey, R., 1968. Optimal Pricing and Investment in Electricity Supply. MIT Press, Cambridge, MA.

Valenzuela, J., Mazumdar, M., 2001. Probabilistic Unit Commitment under a Deregulated Market. In: Hobbs, B., Rothkopf, M., O'Neill, R., Chao, H. (Eds.), The Next Generation of Unit Commitment Models. Kluwer Academic Publisher, Boston, pp. 139–152.

Williamson, O.E., 1966. Peak-load pricing and optimal capacity constraints under indivisibility constraints. Am. Econ. Rev. 56 (4), 810–827.

[13] As in Oren et al. (1985).