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SPECULATION AND HEDGING IN ART MARKETS

MEMORIA PARA OPTAR AL TÍTULO DE
INGENIERO CIVIL INDUSTRIAL

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POR: VICTOR EDUARDO VALDENEGRO CABRERA
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Alternative Investments are considered a possible option for investors willing to diversify their portfolios. They refer collectively to the many asset classes falling outside the traditional definition of stocks and bonds. Including categories like hedge funds, private equity, real estate, commodities and tangible collectible assets such as fine wines, stamps, automobiles, antique furniture, and for the purpose of this work: Art.

The market for this type of collectible assets like art tend to be illiquid, and gains result only from the increase in the prices of these assets, but some investors are willing to take the trade-off for a higher return and/or the pleasure of owning such a piece.

This works begins with a theoretical model between two group of Agents trading in an Art Market. The main findings stay that the price of an Art piece is determined by it's fundamental value plus the option to re-sell it in a future period which depends on the difference of beliefs about the value of the piece between the agents, so their is a risk-sharing component included in the valuation.

Even though intuition makes as think while bigger the difference in opinion, bigger is the valuation of the asset. Our findings stay this true in most cases for a Risk-Aversed world, but there is a trade-off between the valuation from the possible resale option and the risk sharing component which also tends to increase with the increase of the beliefs disagreement between agents, so for a certain level of risk-aversion there is a certain interval which makes the increase of disagreement devaluate the art piece; contrary to the intuition introduced by Miller (1977).

The second part of this work is a look for determinants of the art indexes calculated by Renneboog and Spaenjers (2013), W. N. Goetzmann, Renneboog, and Spaenjers (2009) and Mei and Moses (2002). The methodology of this second part includes a Capital Asset Pricing Model adjusted by factors as determinant for art returns measures. The main findings of this second part are that uncertainty of new art-styles and crises tend to decrease art returns, while the search for luxury appetite, the need for hedge against possible inflations and the appearance of good sentiment in the market tends to increase art returns.

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Inversiones Alternativas son consideradas una opción viable para inversionistas que quieren diversificar sus carteras; se considera dentro de esta categoria aquellos que caen fuera de los activos tradicionales como acciones o bonos. Incluyendo categorias como Hedge Funds, Private Equity, Commodities, Inmobiliaria y bienes tangibles como vinos finos, estampillas, automoviles, mueblería antigua y para el proposito de este trabajo: Arte.

El mercado para este tipo de activos coleccionables como el arte tiende a ser abstante iliquido, por lo que las ganancias resultan solo en el incremento en precio de estos, sin embargo se ven inversionistas dispuestos a tomar este trade-off en cambio de un mayor retorno y/o el placer de ser el poseedor de dicha pieza.

Este trabajo comienza con un modelo teorico entre dos grupos de agentes tranzando en el mercado del arte. Los descubrimientos principales son que el precio d euna pieza de arte depende de su valor fundamental más el valor de la opción de reventa del activo en un futuro periodo, la cual depende de la diferencia en creencias sobre el valor entre los distintos grupos de agentes, lo que implica que hay un componente de risk-sharing incluido.

Aunque intuitivamente se crea que a mayor diferencia de opinion, mayor el precio del activo. En este modelo se descubre que aunque sea verdad en la mayor parte de los casos del mundo averso al riesgo, igual hay un trade-off entre la posible opcion de reventa y el componente de Risk-Sharing, crece monotonamente con la diferencia en creencias; por lo que a ciertos niveles de tolerancia al riesgohay un intervalo en el cual el incremento en el desacuerdo en el precio devalua la pieza de arte; contrario a la lo que se conoce intuitivamente de Miller (1977).

La segunda parte de este trabajo es una mirada en los determinantes de los precios de Indices de arte ya calculados por Renneboog and Spaenjers (2013), W. N. Goetzmann et al. (2009) y Mei and Moses (2002). La metodologia incluye el modelo de CAPM ajustado por factores como determinantes de las medidas de retorno de arte. Los mayores descubrimientos en esta segunda parte son que la incertidumbre de nuevos estilos de arte y las crisis tienden a disminuir los retornos, mientras que el aumento en apetito al lujo, la necesidad de cubrirse frente a una posible inflacion o el sentimiento positivo en el mercado tiende a incrementar los retornos.

An investment in knowledge pays the best interest.
Benjamin Franklin

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Finalmente decir que escribo este documento con nostalgia ya que representa un cierre de una bonita étapa durante mi vida, siendo Beauchef una Facultad donde entre con mucha energía y ganas de aprender en primer año, pero que al final termine "semi-odiando un poco". Pero es un lugar donde se trabaja, sufre pero se aprende y pasa bien; la formación obtenida me ha servido bastante durante el comienzo de mi vida laboral, así que termino esta étapa agradecido por haber tenido el privilegio de estudiar en una institución de excelencia como lo es la FCFM.

[1] De verdad ha sido harta la paciencia que me ha tenido para que termine de escribir.

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Chapter 1

Introduction

Art works have experienced scandalous peak prices over time, or at least from a middle-class person's perspective. The biggest quantities have been paid for creations of death artists, but living ones do not necessarily stay behind. Alive artists like Hirst have works which have been paid multi-millionary figures in USD or GBP. Today, the mystery of art prices and their patterns are undetermined, so we try to make a step in the answer in this paper.

Empirical literature in art returns is wide and the real art return studied in long periods seems to be pretty low. According to Ashenfelter and Graddy (2002) survey, real return estimations for art vary between 0.6% and 5.0% for paintings. Returns are heterogeneous between periods (and even in the same period) and construction methodologies, due to the difficulty of estimating expected changes in price for highly different and non-liquid goods.

Form an historical point of view, the art market has been consumed mostly from the art lover and not the investor. We could think from the art consumer point of view, distinguishing between price and art returns is merely a curiosity, but many collectors are nowadays looking for the value of their financial assets (Burton and Jacobsen (1999)), so in this work we also look for the distinction between art lovers and art investors translated as heterogeneous beliefs, which is exactly what we do in chapter 4 and 5.

In a first part we fit and extremely simple model of short sale constraints and heterogeneous beliefs between two agents to study the behavior of art, as it is itself an asset or luxury good which can not be short sold, and as mentioned before is traded from art lovers and also investors. What really caught our attention is that in Spaenjers, Goetzmann, and Mamonova (2015) they propose to price art separating the price into two parts: the first one corresponding to emotional dividends and the second one to resale revenues. Our theoretical model posseses this two components in the equilibrium prices, similarly to the model used in Hong, Scheinkman, and Xiong (2006), but with some of our own distinctions.

In the second part, we do an empirical analysis, searching for determinants of art index returns using different factors such as Protection Values, luxury pleasure, love for art, changes in art styles and crises. This empirical analysis is used in two diferent markets, The United States and The United Kingdom for the period of 1958-2007. Our empirical analysis uses the capital asset pricing model, similarly as Bryan (1985), but instead of adding a consumption component, we add factors related to the variables we consider as determinants, and in an annual basis, taking the indexes calculated by Renneboog and Spaenjers (2013), W. N. Goetzmann et al. (2009) and Mei and Moses (2002).

The rest of this work is organized as follows: chapter 2 presents the objectives of this work, chapter 3 summarizes the literature review we did for this study, chapter 4 presents our simple model pricins a single risky art asset, chapter 5 studies the different dynamics of this model and shows some intuitive plots about prices behavior when varying each variable and estates our hypotheses to be tested, chapter 6 presents an empirical analysis similar to the one used by Bryan (1985) connecting the indenpendent variables to the variables to the theoretical model and showing the results of our hypotheses and chapter 7 gives some concluding remarks.

Chapter 2

Objectives

2.1 General Objective

The general objective of this work is to study the behavior of art asset prices in a theoretical and empirical point of view. Making the match between the theoretical Model and the empirical analysis looking for determinants.

2.2 Specific Objectives

- Study the behavior of the asset price under the assumption of heterogeneous beliefs between agents about the pleasure dividend from earning the art asset.
- Study the behavior of the price by the movements in the risk-aversion coefficient, heterogeneous beliefs and uncertainty in the market.
- Estimate if variables such as Protection Values, Luxury Pleasure, Heterogeneous Beliefs and Crises are determinants of the Art prices in a linear estimation.

Chapter 3

Literature Review

The literature review made for this paper can be divided into two main parts: first, the Art index pricing literature and the literature for pricing assets with short-sale constraints and speculative markets with heterogeneous beliefs.

Even if there is not a vast literature. Several Researchers have estimated Art price Indexes, their price/return relation with other financial assets and possible determinants that capture the behavior of the art returns.

Bryan (1985) studies the characteristics of investments and consumptions on the art market between 1971-1984, using the Capital Asset Pricing Model. He establishes considering art owners, esthetic consumers and investors which possesses demand in the future consumption. Paintings belong to the Durable good class of commodities. They provide the actual consumption and the future's demand, so in this sense there is little difference with cars or real estate. As art work produces a service to the owner over time as contrary as the nominal income produced by financial assets. Propietaries of durable goods are in a certain way hedged against Unexpected inflations because the value of the service income increases with the level of prices in general. So, Art returns is hedged against inflation in a certain way that other assets such as Stocks or Bonds are not. This might cause the differentiation of the art market with other markets in inflation periods. The analysis concludes with art being an asset that does not fit in the consumers world, nor in the investors world.

W. N. Goetzmann (1993) uses the prices of paintings sold in the market at least two times during the period 1715-1986 to construct and index of art return by repeate sales regression. With the index he made a comparison between the fluctuations of art prices and stock market. He found that art work demand increases with the wealth of art collectors. This relationship between art prices and wealth implies art is an investment tool, even though it finds little evidence that art is an attractive asset for a risk-averse investor. Art in absence of esthetic dividends is just potentially attractive for agents who would pick a relatively volatil portfolio. For Goetzmann, this helps explain why the art market is correlated with the stock market. If wealth is the limitation for the appreciation of paintings, then an increase in the stock market would relax this restriction.

Pesando (1993) uses repeat sales regressions as Goetzmann from copies of modern art works in auctions to estimate a semi-annual index of the art prices for the period 1977-1992. He determines that copies are not fairly comparable with traditional financial assets. He found that modern art copies have lower returns than stocks and bonds during the studied period, but their volatility could be lower than stocks or bonds.

Mei and Moses (2002) construct an annual index of art prices using a repeat sales regression for the period 1975-2000 with a new database of art repeatedly sold based on the record of the New York Public Library and the Watson library from the Art Metropolitan Museum. Their work estimates the index to price art as investment. They find contrary to previos studies that art has been a more interested investment than fixed income, although it has a poor return when compared to the stock market. Their index is less volatile and has a lower correlation with other financial assets than other indexes. This different results are supposed due to a bigger sample which makes the index more diversified, but their results suggest that the art prices tend to fluctuate in the same direction the stock market does which is consistent with the wealth effect studied by W. N. Goetzmann (1993).

The study of luxury goods -including art- allows us to partially respond for the stock puzzle as presented by Ait-Sahalia, Parker, and Yogo (2004). The puzzle stays that the stock market risk measured by its co-movements with the aggregate consumption is not enough to justify its expected return overpasses the goverment debt in short term. This study also proposes a partial solution to the puzzle by distinguishing the consumption of basic goods form the luxury goods. Wealthy homes with bigger heritage are almost satiated in their basic goods consumption, so wealthy shocks are translated into consumption of luxury which is more sensitive to the stock market return than the basic goods. They finally show luxury consumption has a much significative correlation to stock amrket returns than basic goods consumption has with complementary levels of risk aversion. It also shows that prices of luxury goods with fix offer (as the case of art) reveal information about bonus shares.

Given different behaviors of art returns when compared to other financial assets. Some studies started to look for their determinants causing this effects. Mandel (2009) stays that art return determinants differ from those of stock market returns and other investments due to the charasteristic of consumption good that art has over the other financial assets. Art owners are pleased by the intrinsic value, and as a luxury good. The joy of having a wealthy signal for owning art also gives a pleasure dividend. From a theoretica point of view,art must be treated differently from stocks and other risky assets.

W. Goetzmann, Renneboog, and Spaenjers (2011) constructs a repeated sales regression index of art prices between 1830-2007. They use this index to analyse the impact of art prices over time. They also make a focus on the proxy on the wealth of collectors to purchase art which consist on the study of the evolution of the high incomes over time. Thus, they search empirically the relation between hign income distribution in one hand and art prices on the other. They conclude that art prices increase when there is a bigger inequality even when controlling by stock market tendencies. In the long term, wealthy incomes or at least the biggest ones seem to be a factor in the art price formation. This relationships support the Veblenian thought of art as an instrument of social competence between the rich. Veblen (1899) named the term ostentatious consumption to refer to the consumption iunrelated to the intrinsic value of the good.

Even the different methodologies and samples used for constructing the indexes. Several show fall prices during financial crises. W. N. Goetzmann (1993) found three low markets in art: 1830-1840, 1880-1900 and 1930-1940 which correspond to wide periods of economic recesions in the United States and the United Kingdom. The esgtimated index by Mei and Moses (2002) also indentifies a price fall during the oil crises in 1974-1975 and the Great Depression of 1929-1934. W. Goetzmann et al. (2011) get significant falls during the first World War, the Great Depression of the thirties and after the Oil Crise of 1973.

Renneboog and Spaenjers (2013) search for the determinants of art prices and the returns of art investments. They differ themselves from the others by constructing an index by applying an hedonic regression to a new sample of more than one million transactions from painting and paper work auctions. They find lower returns for the art market in comparison with W. N. Goetzmann (1993) and Mei and Moses (2002). They also show confidence of high-income consumers and the art amrket sentiment predict tendencies of art prices. Nevertheless, their results can not explain boom or falls in the art market which have happened in the last decades. Thus mightbe explain by the fundamental value of art, as already defined, hard to understand. Combining this with the impossibility of short-selling, this uncertainty implies a potential role to the art buyer sentiment, which is defined as unjistifiably optimistic (or pessimistic) over the future values of re-selling. They suggest also variation of optimism in time over the potential value of "art as investment" might explain part of the art market cycles. Finally their results show that even the demand for luxury consumption and the art amrket sentiment are determinants of art market cycles.

For the literature review about models of Heterogeneous Beliefs and short sale constraints which is large, we started by Miller (1977) which main conclusions are that uncertainty and risk imply divergence of opinion, so a market without short-selling for an aset will be held by the minority with the most optimistic expectations about its returns. It's also quite possible risky securities will have lower returns when facing divergence of opinion, since it tends to increase with risk. In a same static framework Chen, Hong, and Stein (2002) analyze the overvaluation generated by Heterogeneous Beliefs, their model estates heterogeneous beliefs only appear when the short-selling constriction is present, they conclude after an empirical study that the evidence shown by short-sales constraints impacts into stock prices and expected returns. Hong and Stein (2003) developp a model where the paired Heterogeneous beliefs-short sale contraints might lead to market crashes.

Harrison and Kreps (1978) develop a model which determines the price of a dividend-yielding asset traded by two types of agents in a multi-period framework. They define *Speculative Behavior* as the phenomenon occured when investors are willing to pay more for an asset if they have the right to resell than they would pay if obliged to hold it forever. Their model features Heterogeneous Beliefs, incomplete markets and short sales constraints and the speculative phenomenon shows that a price is determined by its fundamental value and by its resale value which is the speculative component. Morris (1996), Scheinkman and Xiong (2003) also develop models of asset prices taking a speculative component into account, following the idea.

Our model follows the idea of the Hong et al. (2006) model making a resale option or bubble based on the recursive expectations of traders in their beliefs about the future value of the dividend, differently from rational bubbles, which are incapable of connecting bubbles with asset float (Hong et al. (2006)). But we take out the effect of signaling beliefs, so in our model investors are acknowledged about the others preferences, and we add a term called additional pleasure which makes a certain group of investors receive and additional payment which we explain in chapter 4.

Chapter 4

Theoretical Model

Consider an economy with three dates, $t = 0, 1, 2$, and a single traded asset, which will represent art. This asset cannot be sold short, it exists in a fixed supply equal to Q, and it pays D at $t = 2$, where D is normally distributed, $D \sim N\left(\bar{D}, \sigma_D\right)$. For simplicity the risk free rate is set to zero.

There are two groups of investors, denoted by A and B. Both groups find themselves earning a pleasure dividend from the art asset at $t = 2$; nevertheless investors from group A have an extra fixed incentive which denotes the belief disagreement between the groups. At dates 0 and 1 investors can trade the art asset and at date 2 they consume.

Let P_t denote the price of art at date t and x_t^{i} denote group's i position in it. We abstract from transaction costs. Investors's wealth at dates 1 and 2 can then be written as:

$$W_1^{\mathrm{i}} = (P_1 - P_0)\, x_0^{\mathrm{i}} \;\text{ and }\; W_2^{\mathrm{i}} = (D - P_1)\, x_1^{\mathrm{i}} \tag{4.1}$$

Investors maximize their subjective expected utility over wealth. This reduces to the usual mean-variance problem:

$$\mathbb{E}_t^{\mathrm{i}}\left[W_{t+1}^{\mathrm{i}}\right] - \frac{\theta}{2} Var_t^{\mathrm{i}}\left[W_{t+1}^{\mathrm{i}}\right] \tag{4.2}$$

where θ denotes the risk aversion of each group, which we consider to be the same for both groups of investors.

At date 0 the two groups' prior beliefs regarding D are normally distributed and equal, i.e. the mean $(\bar{D})$ and variance (σ_D^2) are the same for both groups. The beliefs of the two groups of investors at date 1 are also normally distributed, with equal variance (σ_D^2) and with means given by

$$\mathbb{E}_1^A[D] = \bar{D} + k + \varepsilon_A \;\text{ and }\; \mathbb{E}_1^B[D] = \bar{D} + \varepsilon_B \tag{4.3}$$

where ε is independent and normally distributed, $\varepsilon_{A,B} \sim N\left(0, \sigma_\varepsilon^2\right)$, ε_{i} with $\mathrm{i} = A, B$ represents the uncertainty investors perceives about the asset payment, and k represents an extra consumption income itself for investors of group A. To find our equilibriums, we solve per period prices by backward induction. Given these forecasts, we solve for the equilibrium holdings and price at date 1. Group A investors solve then

$$\max_{x_1^A} \mathbb{E}_1^A \left[(D - P_1)\, x_1^A \right] - \frac{\theta}{2} Var_1^A \left[(D - P_1)\, x_1^A \right] \tag{4.4}$$

and group insvestors from group B,

$$\max_{x_1^B} \mathbb{E}_1^B \left[(D - P_1)\, x_1^B \right] - \frac{\theta}{2} Var_1^B \left[(D - P_1)\, x_1^B \right] \tag{4.5}$$

where the first-order conditions are:

$$\left(\mathbb{E}_1^A[D] - P_1 \right) - \theta x_1^A \sigma_D^2 = 0 \quad \text{and} \quad \left(\mathbb{E}_1^B[D] - P_1 \right) - \theta x_1^B \sigma_D^2 = 0$$

With mean-variance preferences and short sale constraints, given the price P_1^a, investor demands $\left(x_1^A, x_1^B \right)$ for the asset are given by:

$$x_1^A = \max \left[\frac{\mathbb{E}_1^A[D] - P_1}{\theta \sigma_D^2}, 0 \right] \quad \text{and} \quad x_1^B = \max \left[\frac{\mathbb{E}_1^B[D] - P_1}{\theta \sigma_D^2}, 0 \right] \tag{4.6}$$

So we study 4 different cases:

4.1 A Model without Beliefs Disagreements and Symmetric Uncertainties

Lets assume there are no disagreements in the beliefs about the asset payment at $t = 2$, so $k = 0$ with equation 4.6 and market clearing, we obtain the equilibrium at $t = 1$ to be:

$$x_1^A = x_1^B = \frac{Q}{2}$$

$$P_1 = \bar{D} + \varepsilon - \frac{\theta Q \sigma_D^2}{2} \tag{4.7}$$

The term $D + \varepsilon$ representes the market value at $t = 1$ this would be a *Risk Sharing Portfolio* under the definiton of Simsek (2013), as the total risk from holding the asset between $t = 1$ to $t = 2$ $\theta Q \sigma_D^2$ is being captured by both group of investors which means they prefer to share the risk of holding the asset instead of taking sgressive positions in the entire market.

We next solve for equilibrium at date 0. Given investors' mean-variance preferences, their demands at date 0 are given by

$$x_0^A = \max\left[\frac{\mathbb{E}_0^A[P_1] - P_0}{\theta Var_0^A[P_1 - P_0]}, 0\right] \quad \text{and} \quad x_0^B = \max\left[\frac{\mathbb{E}_0^B[P_1] - P_0}{\theta Var_0^B[P_1 - P_0]}, 0\right] \tag{4.8}$$

As we impose no beliefs disagreements. There are also homogeneous beliefs at $t = 0$, so

If both group of investors have homogeneous beliefs at the initial period ($t = 0$) then their Expectations of P_1 are:

$$\mathbb{E}_0^A[P_1] = \mathbb{E}_0^B[P_1] = \bar{D} - \frac{\theta Q \sigma_D^2}{2}$$

and

$$Var_0^A[P_1 - P_0] = Var_0^B[P_1 - P_0] = \sigma_\varepsilon^2$$

So, imposing market-clearing and the above equations, we obtain the equilibrium

$$x_0^A = x_0^B = \frac{Q}{2}$$

$$P_0 = \bar{D} - \frac{\theta Q\left(\sigma_D^2 + \sigma_\varepsilon^2\right)}{2} \tag{4.9}$$

The intuition behind equation 4.9 is that both investors are sharing the risk from holding the asset between $t = 0$ to $t = 1$ which is $\theta Q\left(\sigma_\varepsilon^2\right)$ and also the risk from holding it between $t = 1$ to $t = 2$ which is$\theta Q \sigma_D^2$, so a model with no beliefs disagreements just captures the effects of the risk-sharings between investors making them share the market and its associated risks and the market value of the asset.

In the following sub-section we introduce the effect of a deterministic variable $k > 0$.

4.2 A Model with Beliefs Disagreements and Symmetric Uncertainties

So, let's introduce the effect of a deterministic variable $k > 0$ which represents an extra income for agent's of group A, as already defined in equation 4.3

Following the same intuition, imposing market-clearing in equation 4.6:

Proposition 1. When the beliefs disagreements are just given by $k > 0$ (symmetric uncertainties are holding) i.e. $\bar{D}_A - \bar{D}_B = k$ the position for A-agents in the market is strictly positive.

Imposing market clearing in equations 4.6:

$$x_1^A = \frac{Q}{2} + \frac{k}{2\theta\sigma_D^2}\,\mathbb{I}_{k \leq \theta Q \sigma_D^2} + \frac{Q}{2}\,\mathbb{I}_{k > \theta Q \sigma_D^2}$$

$$x_1^B = \frac{Q}{2} - \frac{k}{2\theta\sigma_D^2}\,\mathbb{I}_{k\leq\theta Q\sigma_D^2} - \frac{Q}{2}\,\mathbb{I}_{k>\theta Q\sigma_D^2}$$

$$P_1 = \bar{D} + \varepsilon + \frac{k}{2} - \frac{\theta Q\sigma_D^2}{2} + G(k, \theta Q\sigma_D^2) \tag{4.10}$$

The first effect we notice is in the market value of the asset, which is added in $\frac{k}{2}$ as investors of group A have an extra income which affects in the market's price if compared with equation 4.7. In the same line, there is also the risk-sharing component in the price from holding the asset between $t = 1$ to $t = 2$, and finally the term $G(k, \theta Q\sigma_D^2)$ is defined as the Resale Option, which represents the value from selling the asset to investors in the group with higher beliefs.

Definition 1. *The Resale function* Let k be the diference in beliefs about the dividends payment at $t = 1$. Due to our definition: $\mathbb{E}_1^A[D] - \mathbb{E}_1^B[D] = k$, so the resale option is:

$$G(k, \theta Q\sigma_D^2) = \frac{1}{2}max(k - \theta Q\sigma_D^2, 0) \tag{4.11}$$

Equation 4.11 represents the option value of reselling the art asset at $t = 1$ from the group with lower beliefs to the one with higher beliefs, it's clear from equations 4.3 that the difference of beliefs is just k, so the resale option's underlying is the extra-income perceived by investors of group A and the strike price is the total risk from holding the asset between $t = 1$ to $t = 2$.

In the work of Hong et al. (2006) the equilibrium price represents three cases the middle case where both investors are sharing the market asset, and two symmetric cases where investors of the most optimistic group take the entire market while the other group sits out of it, which is called an optimism effect. In this model the introduction of k makes impossible for investors of group B to take out the entire market, so the question to study is if k is big enough to make investors of group A take the total risk $\theta Q\sigma_D^2$ or if this extra income is not enough and they rather share the risky market with B-agents.

In this market set-up we find our agents in a shared market, so the question begins wether B-agents exercise the call option to re-sell to A-agents at $t = 1$. Which means if the extra income generated to A-agents is large enough to make A-agents willing to be long the entire market, or they rather share it with B-agents. So, the key of our model is the resale option which

We next solve for equilibrium at date 0. Given investors' mean-variance preferences, their demands at date 0 are given by equation 4.8

$$x_0^A = \max\left[\frac{\mathbb{E}_0^A[P_1] - P_0}{\theta Var_0^A[P_1 - P_0]}, 0\right] \quad \text{and} \quad x_0^B = \max\left[\frac{\mathbb{E}_0^B[P_1] - P_0}{\theta Var_0^B[P_1 - P_0]}, 0\right] \tag{4.12}$$

As we impose no beliefs-disagreements. There are also homogeneous beliefs at $t = 0$, so

If both group of investors have homogeneous beliefs at the initial period ($t = 0$) then their Expectations of P_1 are:

$$\mathbb{E}_0^A [P_1] = \mathbb{E}_0^B [P_1] = \bar{D} + \frac{k}{2} - \frac{\theta Q \sigma_D^2}{2} + G(k, \theta Q \sigma_D^2)$$

So

$$P_0 = \bar{D} + \frac{k}{2} - \frac{\theta Q \sigma_D^2}{2} + G(k, \theta Q \sigma_D^2) - \frac{\theta Q \sigma_\varepsilon^2}{2} \tag{4.13}$$

There are four terms in equation 4.13 the term $\bar{D} + \frac{k}{2}$ represents the market value of the asset, so there is a first optimistic effect given by A-agents biasing the asset up from its fundamental value, the term $\frac{\theta Q \sigma_D^2}{2}$ the risk required to hold the asset between $t = 1$ to $t = 2$ $G(k, \theta Q \sigma_D^2)$ represents the call option for B-agents to re-sell the asset to A-agents with strike price $\theta Q \sigma_D^2$ and underlying k and finally $\frac{\theta Q \sigma_\varepsilon^2}{2}$ the risk premium required from holding the asset between $t = 0$ to $t = 1$.

Intuition makes us think if agents are risk-aversed, they would rather share the market by taking *Risk Sharing Positions* on it instead of having and optimis effect and being long in the asset, but the introduction of k makes the speculative effect in the prices i.e. as k increases the resale option increases, so the price at $t = 0$ becomes higher just because B-agents are long at $t = 0$ with the purpose of re-selling the asset at $t = 1$ which is the *Speculative Behavior* defined by Harrison and Kreps (1978), so this extra-income perceived by A-agents who tend to have a preference for art; works as a source of speculation and affects the positions of both group of agents in the market making A-agents take a higher position as k increases, letting them take the entire market if k is big enough to make B-agents excersise the Resale Option, i.e if the underlying of the resale option k is bigger than the strike $\theta Q \sigma_D^2$ which is the risk of holding the art market.

For the next, we dismiss the belief disagreement effect k, but add assymetries in the uncertainties, i.e. $\varepsilon_A = \varepsilon$ and $\varepsilon_B = -\varepsilon$

4.3 A Model without Beliefs Disagreements and Asymmetric Uncertainties

Solving the equilibrium at $t = 1$ with $\varepsilon_A = \varepsilon$ and $\varepsilon_B = \varepsilon$, and imposing market clearing in equation 4.6, we obtain for the price the following lemma.

Lemma 1. *Let $l = \mathbb{E}_1^A[D] - \mathbb{E}_1^B[D]$ be the difference in opinions between the investors in groups A and B at date 1. The solution for the stock holdings and price on this date are given by the following three cases:*

- *Case 1: $l > \theta \sigma_D^2 Q$*

$$x_1^A = Q_a, \quad x_1^B = 0, \quad P_1 = \mathbb{E}_1^A[D] - \theta \sigma_D^2 Q$$

- *Case 2:* $\|l\| \leq \theta\sigma_D^2 Q$

$$x_1^A = \frac{l}{2\theta\sigma_D^2} + \frac{Q}{2}$$

$$x_1^B = -\frac{l}{2\theta\sigma_D^2} + \frac{Q}{2}$$

$$P_1 = \frac{\mathbb{E}_1^A[D] + \mathbb{E}_1^B[D]}{2} - \frac{\theta\sigma_D^2 Q}{2}$$

- *Case 3:* $l < -\theta\sigma_D^2 Q$

$$x_1^A = 0, \quad x_1^B = Q, \quad P_1 = \mathbb{E}_1^B[D] - \theta\sigma_D^2 Q$$

This first Lemma is a restatement of Miller (1977) and Chen et al. (2002), as similarly done by Hong et al. (2006). As seen there are three different cases. In case 2 where the price is determined by the average beliefs of the two groups and a risk-premium, this case represents a case where both groups will be long the asset which is and intermediary situation in which the difference in opinions does not make each group consume the entire market. The cases 1 and 3 are symmetric and represent wether one group or the other has a drammatically superior opinion over the other, so they consume the entire market and the price will be given by the group's opinion and the risk-premium $\frac{\theta\sigma_D^2 Q}{2}$.

Re-writting the expression for P_1:

$$P_1 = \bar{D} - \frac{\theta Q\sigma_D^2}{2} + G(l, \theta Q\sigma_D^2)$$

Where the resale function now is written as:

$$G(l, \theta Q\sigma_D^2) = \frac{1}{2}max(l - \theta Q\sigma_D^2, 0) - \frac{1}{2}min(l + \theta Q\sigma_D^2, 0)$$

In comparison with the case of symmetric uncertainties in equation 4.11, now we obtain an extra term $-\frac{1}{2}min(l + \theta Q\sigma_D^2, 0)$ which represents the symmetry ob both group of investors, i.e. know the uncertainty between both groups is symmetric and the resale option could be exercise by either group, letting the other hold the entire market in case the difference $l = \mathbb{E}_1^A[D] - \mathbb{E}_1^B[D]$ is big or small enough.

Now, let's calculate at $t = 0$, assuming Homogeneous initial beliefs

We next solve for equilibrium at date 0. Given investors' mean-variance preferences, their demands at date 0 of equation 4.8 and imposing the market clearing condition $x_0^A + x_0^B = Q$.

Lemma 2. *The solution for the stock holdings and price on $t = 0$ are given by the following three cases:*

- *Case 1:* $\mathbb{E}_0^A[P_1] - \mathbb{E}_0^B[P_1] > \theta Var_0^A[P_1 - P_0]Q$

$$x_0^A = Q, \quad x_0^B = 0, \quad P_0 = \mathbb{E}_0^A[P_1] - \theta Var_0^A[P_1 - P_0]Q$$

- *Case 2:* $-\theta Var_0^B[P_1 - P_0]\,Q < \mathbb{E}_0^A[P_1] - \mathbb{E}_0^B[P_1] \leq \theta Var_0^A[P_1 - P_0]\,Q$

$$x_0^A = \frac{\mathbb{E}_0^A[P_1] - \mathbb{E}_0^B[P_1]}{\theta\left(Var_0^A[P_1 - P_0] + Var_0^B[P_1 - P_0]\right)} + \frac{Var_0^B[P_1 - P_0]\,Q_a}{Var_0^A[P_1 - P_0] + Var_0^B[P_1 - P_0]}$$

$$x_0^B = -\frac{\mathbb{E}_0^A[P_1] - \mathbb{E}_0^B[P_1]}{\theta\left(Var_0^A[P_1 - P_0] + Var_0^B[P_1 - P_0]\right)} + \frac{Var_0^A[P_1 - P_0]\,Q}{Var_0^A[P_1 - P_0] + Var_0^B[P_1 - P_0]}$$

$$P_0 = \frac{Var_0^B[P_1 - P_0]}{Var_0^A[P_1 - P_0] + Var_0^B[P_1 - P_0]}\mathbb{E}_0^A[P_1] + \frac{Var_0^A[P_1 - P_0]}{Var_0^A[P_1 - P_0] + Var_0^B[P_1 - P_0]}\mathbb{E}_0^B[P_1] - \frac{\theta Var_0^A[P_1 - P_0]\,Var_0^B[P_1 - P_0]\,Q}{Var_0^A[P_1 - P_0] + Var_0^B[P_1 - P_0]}$$

- *Case 3:* $\mathbb{E}_0^A[P_1] - \mathbb{E}_0^B[P_1] < -\theta Var_0^B[P_1 - P_0]\,Q$

$$x_0^A = 0, \quad x_0^B = Q, \quad P_0 = \mathbb{E}_0^B[P_1] - \theta Var_0^B[P_1 - P_0]\,Q$$

The intuition behind Lemma 2 is similar to the previous one. The equilibrium price at $t = 0$ is biased because of short-sale constraints as the most optimistic group has a bigger influence due to optimistic beliefs (cases 1 and 3) and the second case shows a case when the two different groups are trading the asset, so both beliefs have an influence in its price. In the following we provide the intuition when the beliefs about $\bar{D}_A$ and $\bar{D}_B$ are identical, so there is no optimism effect due to the equality of $\mathbb{E}_0^A[P_1]$ and $\mathbb{E}_0^B[P_1]$.

Proposition 2. If both group of investors have homogeneous beliefs at the initial period ($t = 0$) then their Expectations of P_1 are:

$$\mathbb{E}_0^A[P_1] = \mathbb{E}_0^B[P_1] = \mathbb{E}_0[P_1]$$

and

$$Var_0^A[P_1 - P_0] = Var_0^B[P_1 - P_0] = Var_0[P_1 - P_0]$$

It then follows that the equilibrium price at $t = 0$ is:

$$P_0 = \bar{D} - \frac{\theta Q \sigma_D^2}{2} - \frac{1}{2}\theta Q Var_0[P_1 - P_0] + \mathbb{E}_0\left[G(l, \theta Q \sigma_D^2)\right] \tag{4.14}$$

There are again 4 terms in the the price. The first $\bar{D}$ represents the pleasure dividend from owning the art piece. The second $\frac{\theta Q \sigma_D^2}{2}$ represents the risk premium for holding the asset between $t = 1$ and $t = 2$. The third $\frac{1}{2}\theta Q var_0 [P_1 - P_0]$ represents the risk premium for holding it between $t = 0$ and $t = 1$ and the last term $\mathbb{E}_0 \left[G(l, \theta Q\sigma_D^2)\right]$ represents the option value from selling the art piece to investors in the optimistic group when they have higher beliefs.

The Expected value of the Resale function, i.e. the resale option at $t = 0$ is calculated as:

$$B(g, \sigma_l) = \mathbb{E}\left(G(l, g)\right) = \frac{\sigma_l}{\sqrt{2\pi}} \mathrm{e}^{-\frac{g^2}{2\sigma_l^2}} - gN\left(-\frac{g}{\sigma_l}\right) \tag{4.15}$$

Proposition 3. For a null level of additional pleasure, the resale option increases with the Uncertainty degree $\sigma_l = \sqrt{2}\sigma_\varepsilon$ and decreases with its strike $g = \theta Q\sigma_D^2$

Proposition 3 means while more risk-aversed agents are (bigger g), their *Speculative Behavior* tends to vanish [1] as investors are less willing to resell the art asset when there is an increase in risk aversion. And, in the other hand, while bigger uncertainty degree agents are more willing to re-sell the asset at $t = 1$ so there is a trade of between uncertainty and risk-aversion which tells whether agents re-sell or hold the asset.

And the variance of this Resale Option is given by:

$$V(g, \sigma_l) = Var_0 \left[G(l, g)\right] = \frac{1}{2}\left[\left(g^2 + \sigma_l^2\right) N\left(-g/\sigma_l\right) - \frac{g\sigma_l}{\sqrt{2\pi}} \mathrm{e}^{-\frac{g^2}{2\sigma_l^2}}\right] - \mathbb{E}\left(G(l, g)\right)^2 \tag{4.16}$$

Proposition 4. It exist a value $\bar{\sigma_l}$ for which P_0 decreases for all $\sigma_l > \bar{\sigma_l}$

The intuition behind proposition 4 stays that when the dispersion of uncertainty is taking into account, the risk-sharing term obtained in equation 4.14 grows at a stronger pace than the Resale Option, so the price tends to decrease for a bis dispersion of the disagreement distribution.

In the last section of this chapter we combine the effect of having $k > 0$ and also the assymetries in uncertainties $\varepsilon_A = -\varepsilon_B = \varepsilon$

4.4 A Model with Beliefs Disagreements and Asymmetric Uncertainties

With the introduction of Belief Disagreement or extra-income in agent A's expectations for the pleasure dividend, the difference in opinion $l_k = \mathbb{E}_1^A[D] - \mathbb{E}_1^B[D]$ is written:

$$l_k = k + (\varepsilon_A - \varepsilon_B) = k + l \tag{4.17}$$

Thus, l_k follows a Gaussian distibution with mean k and a variance σ_l^2:

$$\sigma_l^2 = 2\sigma_\varepsilon^2 \tag{4.18}$$

[1] We use the definition of *Speculative Behavior* given by Harrison and Kreps (1978) which tells if investors are willing to pay more for an asset if they have the opportunity to re-sell later, they show speculative behavior

From lemmas 1, 2 and market clearing. The equilibrium prices in this cases are similar to the previous one:

$$P_1 = \bar{D} + \frac{k}{2} - \frac{\theta Q \sigma_D^2}{2} + G(l_k, \theta Q \sigma_D^2)$$
$$P_0 = \bar{D} + \frac{k}{2} - \frac{\theta Q \sigma_D^2}{2} + \mathbb{E}\left(G(l_k, \theta Q \sigma_D^2)\right) - \frac{\theta Q}{2} Var\left[G(l_k, \theta Q \sigma_D^2)\right]$$

Proposition 5. Defining $B(l, g)$ the expected value of the resale option at t=0 with strike g and differences in opinion l, for l_k this expected value is defined as follows:

$$\mathbb{E}\left(G\left(l_k, g\right)\right) = \frac{1}{2}B(g+k, \sigma_l) + \frac{1}{2}B(g-k, \sigma_l)$$

Also defining $V(g, \sigma_l)$ the Variance of this resale-option at 0. The introduction of l_k results into:

$$Var\left[G(l_k, g)\right] = \frac{1}{2}V(g+k, \sigma_l) + \frac{1}{2}V(g-k, \sigma_l) + \frac{1}{4}\left(B(g+k, \sigma_l) - B(g-k, \sigma_l)\right)^2$$

This results are sumarized and have implicancies in the proposition 6:

Proposition 6. For a belief disagreement k and assymetric uncertainties:

1. The Resale option $\mathbb{E}\left(G\left(l_k, g\right)\right)$ increases strictly with k and is convex.
2. It exists a value k^* that for all $k > k^*$ the price at time 0 is strictly increasing with respect to k.

Proposition 6 stays the introduction of a certain agent with a constant biased preference for the art asset k makes the resale option more expensive, so agents tend to have a more speculative behavior due to the effect of increasing the value of re-selling the art asset at $t = 1$ which is logic; as an agent receives more pleasure for owning art, he/she will be more reluctant to the idea of selling later.

Chapter 5

Model Dynamics

During this chapter we make some numerical calculations and formulate hypotheses about the assets sensitivity with respect to the model variables, which are k, θ and σ_ε. For the exercises made we set the variables as $\bar{D} = 10$; $Q = 1$; $\sigma_D = 1$ and we check for different levels of risk aversion θ.

5.1 Beliefs Disagreement sensitivity

First, we simulate how the equilibriums change by varying the value of k, so at first we set $\sigma_\varepsilon = 1$, that is, the uncertainty at $t = 1$ follows a white noise. Setting also at $t = 1$ $\varepsilon = 1$, our simulations for P_1 in figure 5.1 show the price increases with respect to k [1], and for different levels of risk aversion θ, the curve tends to move down the vertical axis, so prices decrease. Also as θ increases, there is a wider range of k's for which the B-agents are still holding there positions after $t = 1$, leaving the slope of P_1 to value 1/2 with respect to k, but as k exceeds θ or the risk from holding the asset between $t = 1$ and $t = 2$, then its enough for A-agents to be willing to take the entire market and let the slope to be one which represents the case where B-agents exercize the resale-option making A-agents take the entire market at $t = 1$.

Figure 5.3 represents the effect in the resale option at the 4th case which tends to increase when the additional pleasure increases, the value of re-selling the art asset at $t = 1$ increases which is logic; as an agent receives more pleasure for owning art, he/she will be more reluctant to the idea of selling later.

[1] See the appendix for the theoretical proof

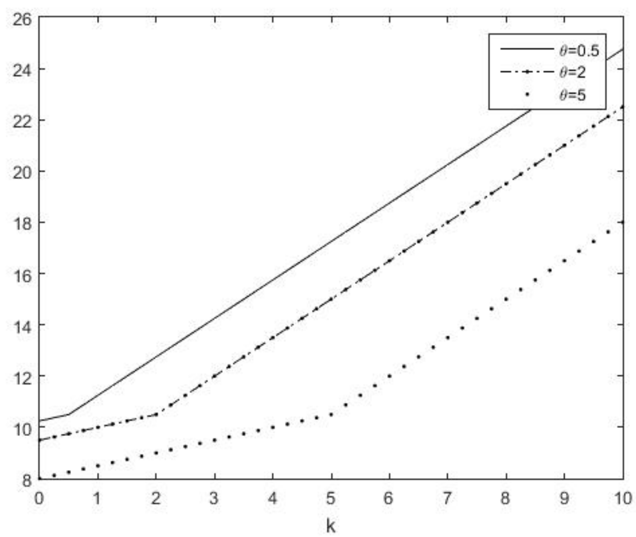


Figure 5.1: Prices at time 1 under variation of k case 2

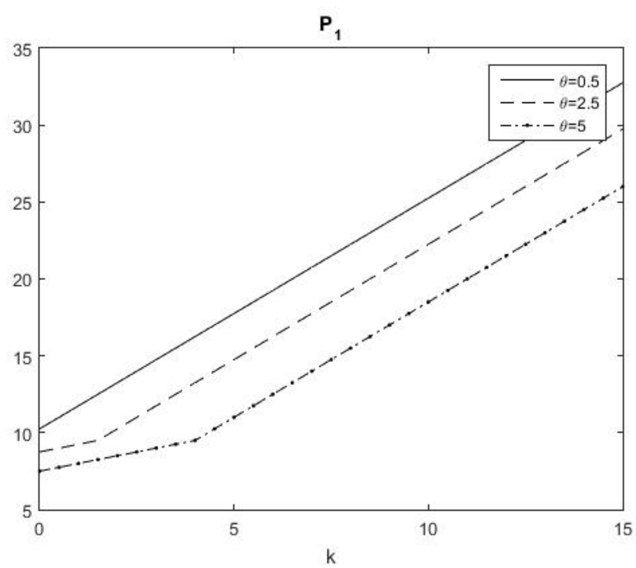


Figure 5.2: Prices at time 1 under variation of k case 4

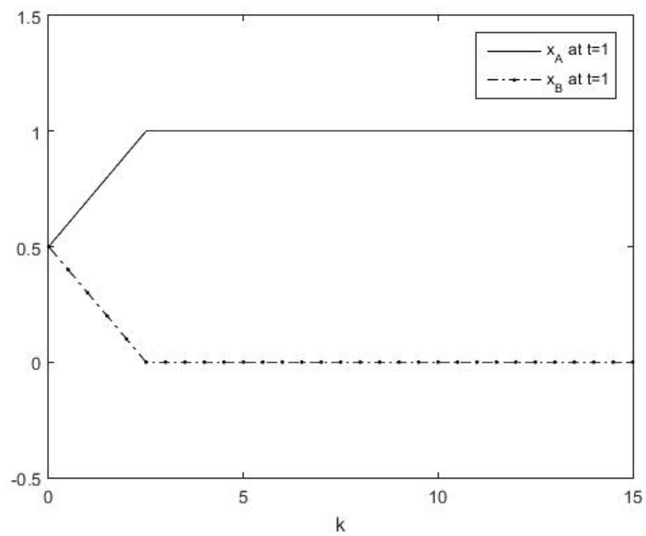


Figure 5.4: Positions at time 1 under variation of k case 2

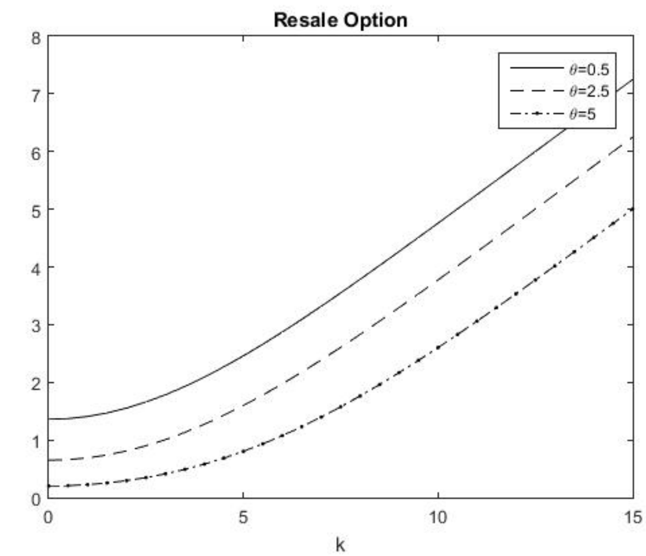


Figure 5.3: Resale Option under variations of k

Figure 5.4 shows how the positions of both groups vary with respect to k. In this case $\theta Q\sigma_D^2 = 2$ so at first $k = 0$ we obtain the *Risk-sharing positions* where there is no speculation, then after k starts to increase the more optimistic investors begin to linearly increase their position with respect to the less optimistics, and when k is large enough to make the optimistic group want to take the entire market, B-agents sit out of the market exercising the resale option to re-sell to A-agents at $t = 1$.

The analysis for P_0 in figure 5.5 is similar to the one made for P_1, but now the extra-effect of uncertainty risk-premium from holding the asset between $t = 0$ to $t = 1$ makes the curves to be more separated and move down the Y-axis in a higher way, so the effect of θ makes prices decrease, and the speculative effect of k makes prices increase with a slope 1/2 when

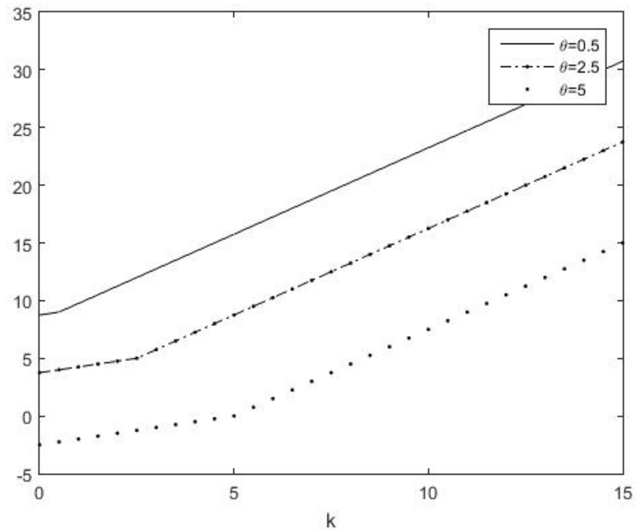


Figure 5.5: Prices at time 0 under variation of k case 2

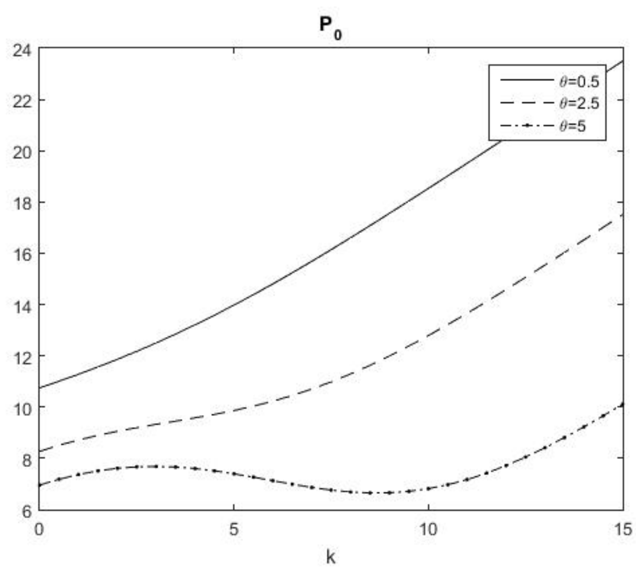


Figure 5.6: Prices at time 0 under variation of k case 4

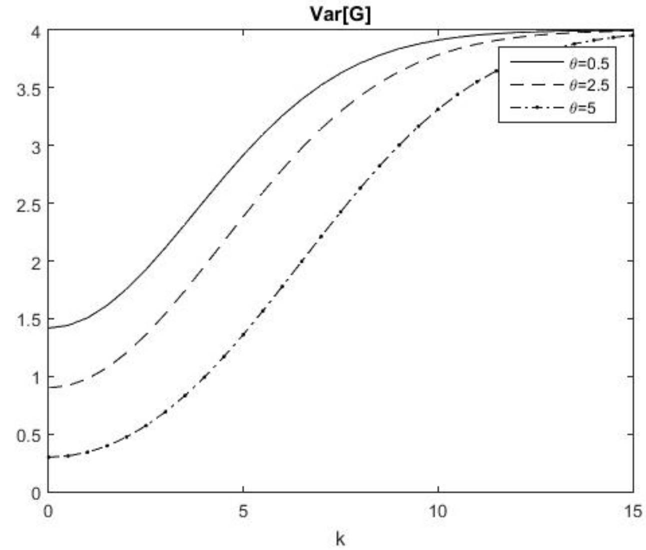


Figure 5.7: Variance of $G(l_k, g)$

both groups are sharing the market, and with a slope of 1 when the optimistic group takes the entire market in their positions. By constrast in case 4 or figure 5.6 we observe a change in convexity due to the effect of the Risk-Sharing component from holding the asset between $t = 0$ and $t = 1$ in figure 5.7 which has an initial negative effect on the Price for small values of k, but then converges to a constant value, so the constant growing effect of the resale option over the price prevails. [2], but the intuition behind this reveals that even an increase in the disagreement between both group of agents implies a decrease in the price at 0 under certain conditions, rejecting the Miller (1977) hypotheses due to the inclusion of the Risk-Sharing component in the Price.

Hyphoteses 1: The Additional Pleasure in art makes the prices increase, specifically P_0, so we attribute this to three different scenarios: Fist, investors buy art as protection from possible inflations. Second, Investors buy art for luxury purposes. Third and final, Investors buy art for the love or pleasure they receive as collectors or art owners.

5.2 Uncertainty Degree's sensitivity

So, our interest in this sub-section relies on the behavior of price at $t = 0$ which decreases with respect to σ_ε quadratically, as expected from equation 3 and it also decreases with respect to θ, so as the risk premium from holding the asset between $t = 0$ to $t = 1$ increases their is a drop in prices.

Hyphoteses 2: Under the influence of σ_ε the price P_0 decreases when there is an augmentation in the Uncertainty Degree σ_ε, so we attribute this as a new art-style appearance. When a new art styles appears, people starts comparing it with other styles and they tend to have diferent opinions of how good it is i.e. their expectancies about the style will be diferent

[2] See appendix for a proper proof of this statement

which translates in the uncertainty.

Hyphoteses 3: As the Risk Aversion of consumers/investors θ increases. There is a negative consequence in the equilibrium prices at time 0 P_0, and also agents tend to vanish their *Speculative Behavior* (i.e. The strike of the Resale Option increases), meaning they prefer to hold the asset for hedging purposes, instead of speculating in later sales, so risk aversion is related to financial events such as crisis due to agents's preference for hedging.

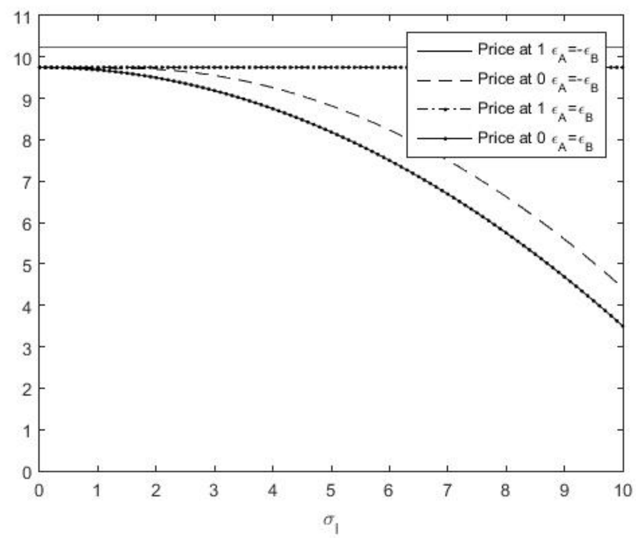


Figure 5.8: Price and Benchmarks under variations of σ_l

Figure 5.8 shows the interaction of prices of cases 3 and the Benchmark which is case 1 at both dates, as we see for the first case ($\varepsilon_A = \varepsilon_B$) at $t = 1$ is a straight horizontal line representing the case both consumers are trading in the market while the third case ($\varepsilon_A = \varepsilon_B$)
.

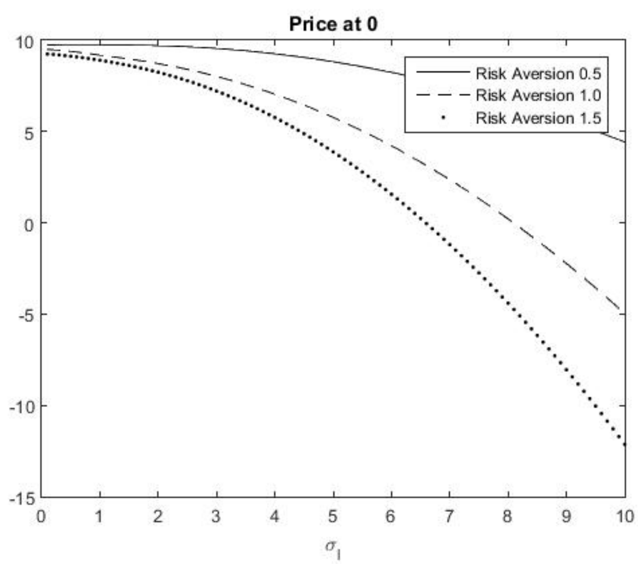


Figure 5.9: Prices at time 0 under variation of σ_l

Figure 5.9 represents the behavior of P_0, as already done with P_1, we focus on σ_l[3], so as it increases, there is a decreasing effect in the price at 0. Also as increasing the risk aversion coefficient. P_0 curve tends to move down the vertical axis, meaning that while more risk-aversed are the agents, smaller the price will be and less willing to pay for art.

[3]There is a price symmetry here in case A-investors or B-investors are consuming the entire market.

Chapter 6

Linear Estimation Analysis

Here we study the determinants of the already calculated indexes for art calculates by Renneboog and Spaenjers (2013), W. N. Goetzmann et al. (2009) and Mei and Moses (2002). See 8.2 for a better detail of each index.

In this chapter we first run the CAPM for the three different indexes. We use the ordinary least squares estimator and the Newy West standard errors. This estimator is consistent when there is possibly heteroskedasticity and serial autocorrelation. After running the CAPM we estimate the model with an extrafactor to study if it corresponds to a determinant for the art returns. The factors studied in this paper are Inflation, Luxury Appetite, Heterogeneous Beliefs-Change in art movements and Crises.

We call R_a^{RS} the Renneboog and Spaenjers (2013) index, R_a^G the W. N. Goetzmann et al. (2009) and R_a^{MM} the index calculated by Mei and Moses (2002). The first two are run for the period 1958-2007 and the last one for 1958-1999.

We first begin our empirical estimation by studying the Capital Asset Pricing Model for art returns in the United States and the United Kingdom as art indexes are calculated globally, we use this two economies. We run the linear regression $R_{a,t} - R_{f,t} = \alpha + \beta_0(R_{m,t} - R_{f,t}) + \varepsilon_t$ for the three mentioned indexes and both countries in table 6.1. We use the Ordinary Least Squares estimator and the Newey West errors for reporting the p-values.

For the case of the united States, the market return[1] used corresponds to the return market constructed by Fama and French (1993) which corresponds to the weighted return of all enterprises in the *Center for Research in Security Prices*(CRSP), *American Stock Exchange*(AMEX), or *National Association of Securities Dealers Automated Quotation*(NASDAQ). The market return for the U.K. corresponds to the index return of the *Financial Times Stock Exchange* All Share (FTSE All Share) and was obtained from *Datas-*

[1]Data obtained from`http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`, even they published at 1993, authors have calculating their factors progresively, so information is updated until June 2017.

tream [2]
The risk free rate chosen for the United States is the interest rate of the 1-month Treasury bonds, extracted from Fama and French data, which is taken by *Ibbotson Associates*. For the United Kingdom, the Central Bank (*Bank of England*) constructed a data base with the most important macroeconomic variables of the country from which was taken the R_f

Our first results in table 6.1 mainly show the art systemic risk is non significative in any index or country, and its returns are mainly explained by abnormal returns coefficient α, in a 99% confidence for the indexes of W. N. Goetzmann et al. (2009) and Mei and Moses (2002) in the United States. In a 95% confidence for Mei and Moses (2002) in the United Kingdom and 90% confidence for W. N. Goetzmann et al. (2009) in the same country. So, this first analysis lets us conclude art returns are explained by other factors instead of their investment condition.

Table 6.1: CAPM & Art

The table reports the OLS coefficient, its respective Newey-West standard error where ***, **, and * denote significance at 1%, 5%, and 10%, respectively and the Adjusted R^2 for each regression. Each column represents a regression where the dependent variable are the real returns of the Art Indexes which are independent of the country for Renneboog and Spaenjers (2013) R_a^{RS} and R_a^G W. N. Goetzmann et al. (2009) and the one from Mei and Moses (2002) R_a^{MM} was transformed with each country's CPI. The independent variables are separated for each country. The R_m of the U.S. corresponds to Market return factor built by Fama and French (1993), for the U.K. is the FTSE All Share. The risk free rates are the 1 month T-Bill extracted from Fama-French data for the U.S.

	$R_a^{RS} - R_f$		$R_a^G - R_f$		$R_a^{MM} - R_f$	
α	0.04	0.02	0.05	0.04	0.08	0.06
	(0.02)	(0.02)	(0.02)***	(0.02)*	(0.03)***	(0.03)**
$R_m^{US} - R_f$	-0.02		-0.05		0.14	
	(0.09)		(0.07)		(0.16)	
$R_m^{UK} - R_f$		0.05		0.04		-0.04
		(0.08)		(0.06)		(0.11)
Adj. R^2	-0.02	-0.01	-0.01	-0.01	-0.01	-0.02

For the following sections the equations to estimate are similar to those estimated by Bryan (1985), but ours in the period already mentioned and in an annual basis. Also, instead of using a consumption component, we add variables as the factors we consider determinant for our hypotheses.

$$R_{a,t} - R_{f,t} = \alpha + \beta_0(R_{m,t} - R_{f,t}) + \beta_1 F_t + \varepsilon_t \tag{6.1}$$

Where F_t represents an extra factor taken into account for estimating Art returns, in this study this factors represent Inflation Proxies, Luxury Appetite, Love for art proxies, Heterogeneous beliefs proxies and crises, so

[2]Datastream is the original distribuitor of *FTSE All Share*

6.1 Beliefs Disagreement Hypotheses

As said in the first Hypotheses of the previous chapter, we now test the effect of Inflation, Luxury Appetite and Love for art in the determination of Art Indexes returns.

6.1.1 Protection Values

When considering painting owners as aesthetics consumers and also investors with a demand in the future conumption. Painting are classified under the class of durable goods. As studied by Bryan (1985), art works produce a service flow to the propietary in contrast with the general financial assets. The durable goods propietaries are in a certain hedged against unexpected inflation as the value of the service incresases along with the general prices value.

Expected Inflation refers to the one by which expectations and general public behaviour expect it before happenning. In other words, it is the type of inflation which people and corporations are already prepared. Unexpecte inflation is the one which takes public by surprise, causing distortions in the economic system. We study if art allows us to hedged against unexpected inflation.

In this work we take as measure of protection against inflation, two commodities: the real returns of gold R_{Gold} and the real returns of the real estate $R_{RealEstate}$. We also consider as effective measure of inflation the risk free rate R_f. Thus the unexpected inflation proxies for us are the substraction of both values: $R_{Gold} - R_f$ and $R_{RealEstate} - R_f$.

To make this analysis, we take the real gold prices from *Macrotrends* whichs is a web site containing several macroeconomical time series, sourced by the *London Bullion Market Association, Bureau of Labor Statistics*. We consider it as the world price of gold in this work, so we use it for both countries. For the Real Estate variable in the United States, we use the real price index of Case-Shiller from *Standard & Poor*, extrated from Robert Shiller's website [3]. For the case of the United Kingdom, this study uses the real price index provided by the Nationwide Bank[4], calculated by the *Nationwide Building Society* and considered valid as index by the *Bank of England*.

As we see in table 6.2 for the gold we see significance in both countries for the period 1958-2007 which are the indexes of Renneboog and Spaenjers (2013) and W. N. Goetzmann et al. (2009) and even though there is no signinficance for the index calculated by Mei and Moses (2002). The OLS estimates are all positive in both countries for the three indexes, so there is a first confirmation about our hypotheses. Prices will increase when investors are buying art to Protect against inflation.

In the case of the real estate, there is no significance in the case of the United States, but for the United Kingdom we have 99% significancy in the Renneboog and Spaenjers (2013) and Mei and Moses (2002) indexes and 95% in the W. N. Goetzmann et al. (2009) index,

[3] http://www.econ.yale.edu/~shiller/data.htm

[4] http://www.nationwide.co.uk/about/house-price-index/headlines

Table 6.2: Art & Protection Values

The table reports the OLS coefficient, its respective Newey-West standard error where ***, **, and * denote significance at 1%, 5%, and 10%, respectively and the Adjusted R^2 for each regression. Each column represents a regression where the dependent variable are the real returns of the Art Indexes which are independent of the country for Renneboog and Spaenjers (2013) R_a^{RS} and R_a^{G} W. N. Goetzmann et al. (2009) and the one from Mei and Moses (2002) R_a^{MM} was transformed with each country's CPI. The independent variables are separated for each country. The R_m and R_f at the beginning of this chapter, the R_{Gold} from *Macrotrends*, the $R_{RealEstate}$ for the United States from Robert Shiller's data library and for the United Kingdom from the Nation Wide Bank.

	$R_a^{RS} - R_f$				$R_a^{G} - R_f$				$R_a^{MM} - R_f$			
α	0.03	0.02	0.04	0.01	0.05	0.03	0.05	0.03	0.07	0.06	0.07	0.06
	(0.02)	(0.02)	(0.02)*	(0.02)	(0.02)**	(0.02)	(0.02)***	(0.02)	(0.03)***	(0.03)**	(0.03)***	(0.02)**
$R_m^{US} - R_f$	0.03		-0.02		-0.03		-0.06		0.18		0.17	
	(0.09)		(0.09)		(0.07)		(0.08)		(0.16)		(0.17)	
$R_m^{UK} - R_f$		0.11		0.05		0.07		0.04		-0.02		-0.06
		(0.07)		(0.09)		(0.06)		(0.07)		(0.13)		(0.11)
$R_{Gold} - R_f$	0.21	0.25			0.06	0.10			0.14	0.08		
	(0.08)***	(0.09)			(0.03)**	(0.05)**			(0.10)	(0.12)		
$R_{RealEstate}^{US} - R_f$			0.05				0.19				-0.38	
			(0.45)				(0.26)				(0.82)	
$R_{RealEstate}^{UK} - R_f$				0.77				0.32				0.68
				(0.15)***				(0.15)**				(0.24)***
Adj. R^2	0.07	0.11	-0.04	0.23	-0.02	0.02	-0.03	0.07	-0.02	-0.04	-0.04	0.05

all with positive OLS estimators which also confirms our first hypotheses.

6.1.2 Luxury Appetite

Propietaries of art are pleased by its intrinsic value, between this are the aesthetic value of owning an art work and the measure of how this work is classified as a luxury good, deryving the additional pleasures of the richness signal that owning a piece of art means. From a theoretic point of view, this is similar to the Protection subsection which told us that art should be treated differently from stocks or other risky assets. In contrast with a "pure" financial asset. This is for the mix of pecunary and non-pecunary benefits caused by the ownership of an art work. Thus art returns could be explained in one part by the aesthetic dividends generated.

Due the difficulty of measuring "the pleasure" which makes people buy art, we use different proxies. First, we study the relationship with the variation in income of people which the highest income in each country $R_{HighIncome}$ with the art returns. As the highest income goes for the richest people, the have saturated their basic goods consumption and consume luxury goods due to the pleasure given. We use Atkinson and Piketty (2010) data which is based in gubernamental records to construct a consisten time-serie with the share of the Total Income earned by the highest 0.1% of all salaried in the United States and the United Kingdom. The incomes of 0.1% of each country are available in *The World Wealth and Income Database* and is called as $R_{HighIncome}$ which represents the yearly change of Income earned by the 0.1 % with highest income.

Art is the last instance as a luxury durable good consumption, so if we consider vairables such as consumption of other luxury goods, we see them as determinants for art returns. To do so, we just have U.S.A. data. We use the Personal Consumption Expenditures, provided by the *National Income and Product Accounts* (NIPA). We took them from the *Federal Reserve Bank of St. Louis*[5]. We chose two categories, Jewelry and Watches, and Boats and Aircrafts. As we are currently estimating art returns, we used the change in expenditure from one year to the next one in this two variables.

Table 6.3 shows us the results for the linear regression estimations using the three indexes in each Panel for both countries. In panel A we estimate the Renneboog and Spaenjers (2013) index and we found for the case of the United States 95% significances in the $R_{HighIncome}$ and the $R_{Boats\&Aircrafts}$ with positive signe in their estimators, so this is a first hand of showing that art and luxury appetite are related and their relationship is positively correlated, as estated in our hypotheses in the previous chapter. We also find a positive estimator and a 90% significance in the NIPA index for the W. N. Goetzmann et al. (2009) art return index, and for the Mei and Moses (2002) we find a 95% significance in the $R_{HighIncome}$ for the United States, so we can still verify fot the case of the United States that our hypotheses is well stated and luxury appetite is a determinant of art returns positively related.

[5]https://research.stlouisfed.org/

Table 6.3: Art & Luxury Appetite

The table reports the OLS coefficient, its respective Newey-West standard error where ***, **, and * denote significance at 1%, 5%, and 10%, respectively and the Adjusted R^2 for each regression. Each column represents a regression where the dependent variable are the real returns of the Art Indexes which are independent of the country for Renneboog and Spaenjers (2013) R_a^{RS} and W. N. Goetzmann et al. (2009) R_a^G and the one from Mei and Moses (2002) R_a^{MM} was transformed with each country's CPI. The R_m and R_f are mentioned in the beginning of this chapter, The $R_{HighIncome}$ from the *World Wealth Income Database*, the $R_{Boats\&Aircrafts}$ and $R_{Jewelry\&Watches}$ for the US are the PCE returns provided by NIPA. US GDP from chapter 26 of Shiller (2015) and for the UK from the *Bank of England*.

	$R_a^{RS} - R_f$						$R_a^G - R_f$						$R_a^{MM} - R_f$				
α	0.03	0.02	0.03	0.02	-0.07	-0.08	0.04	0.03	0.05	0.04	0.02	-0.01	0.06	0.07	0.06	0.06	-0.06
	(0.03)	(0.03)	(0.02)	(0.02)	(0.03)**	(0.03)**	(0.03)	(0.02)	(0.02)**	(0.02)	(0.03)	(0.03)	(0.05)	(0.03)**	(0.03)*	(0.03)**	(0.05)
$R_m^{US} - R_f$	-0.01	-0.01	-0.01		0.03		-0.04	-0.05	-0.05		-0.04		0.16	0.15	0.18		0.23
	(0.09)	(0.09)	(0.08)		(0.08)		(0.07)	(0.07)	(0.07)		(0.07)		(0.15)	(0.16)	(0.16)		(0.18)
$R_m^{UK} - R_f$				0.05		0.09				0.04		0.06				-0.03	
				(0.08)		(0.07)				(0.07)		(0.06)				(0.11)	
$R_{Jewelry\&Watches}$	0.17						0.17						0.25				
	(0.28)						(0.21)						(0.47)				
$R_{Boats\&Aircrafts}$		0.34						0.37						0.19			
		(0.16)**						(0.21)*						(0.25)			
$R_{HighIncome}^{US}$			0.33						0.07						0.70		
			(0.16)**						(0.16)						(0.34)**		
$R_{HighIncome}^{UK}$				0.02						-0.26						0.39	
				(0.28)						(0.20)						(0.44)	
R_{GDP}^{US}					3.23						0.87						3.83
					(0.88)***						(0.64)						(1.36)***
R_{GDP}^{UK}						4.17						1.67					
						(1.05)***						(0.97)*					
Adj. R^2	-0.04	0.02	0.00	-0.04	0.18	0.23	-0.02	0.13	-0.03	0.00	0.00	0.06	-0.03	-0.03	0.05	-0.03	0.11

6.1.3 Love for Art

Table 6.4: General Art related terms

The table reports the OLS coefficient, its respective p-value where ***, **, and * denote significance at 1%, 5%, and 10%, respectively and the Adjusted R^2 for each regression. Each row represents the linear regression in equation 6.1 where the dependent variables are the real returns of the Art Indexes which are independent of the country for Renneboog and Spaenjers (2013) and W. N. Goetzmann et al. (2009) and the one from Mei and Moses (2002) was transformed with each country's CPI. The R_m and R_f and the words data are taken from *Chronicle*. For simplicity,we only report the OLS estimators, p-value and Adjusted R^2 to study the effect of each word in the regression and compare them between them, and not the complete regression with the market premia and α.

Panel A: US

	$R_{a,Shed}$			$R_{a,Grep}$			R_{aMrep}		
NYT	β	p-value	Adj. R^2	β	p-value	Adj. R^2	β	p-value	Adj. R^2
ArtHistorians	0.070	0.088	0.001	0.000	0.998	-0.036	0.061	0.410	-0.024
HighArt	0.066	0.093	0.001	0.004	0.846	-0.035	0.012	0.861	-0.039
Artstyle	0.018	0.039	-0.018	0.003	0.686	-0.034	0.029	0.237	-0.009
GenrePainting	0.030	0.083	0.005	0.027	0.117	0.051	0.046	0.051	0.017
NewArtist	0.039	0.223	0.010	0.010	0.493	-0.028	0.013	0.667	-0.037
ArtCollection	0.117	0.042	0.049	0.006	0.842	-0.035	0.118	0.172	0.005
NewPaint	0.010	0.728	-0.040	0.050	0.009	0.058	-0.025	0.637	-0.035
ArtWork	0.073	0.001	-0.004	-0.010	0.543	-0.034	0.056	0.453	-0.028
ArtShow	0.087	0.135	-0.006	0.002	0.965	-0.036	0.111	0.276	-0.014
ArtAuction	0.051	0.175	-0.010	0.001	0.985	-0.036	0.039	0.594	-0.031
AdvanceGuard	0.004	0.770	-0.041	0.004	0.642	-0.034	0.048	0.071	0.020
AvantGarde	0.277	0.014	0.077	0.085	0.151	-0.011	0.320	0.054	0.037
Connoisseur	0.090	0.062	-0.014	-0.052	0.220	-0.015	0.088	0.298	-0.026
ArtCriticism	0.086	0.002	0.060	0.022	0.110	-0.020	0.040	0.335	-0.030

Panel B: UK

	$R_{a,Shed}$			$R_{a,Grep}$			R_{aMrep}		
NYT	β	p-value	Adj. R^2	β	p-value	Adj. R^2	β	p-value	Adj. R^2
ArtHistorians	0.083	0.040	0.026	0.018	0.356	-0.027	0.042	0.528	-0.040
HighArt	0.072	0.072	0.016	0.011	0.555	-0.030	-0.001	0.989	-0.049
Artstyle	0.024	0.028	0.004	0.009	0.335	-0.022	0.023	0.322	-0.031
GenrePainting	0.029	0.121	0.006	0.026	0.150	0.041	0.050	0.029	0.019
NewArtist	0.044	0.174	0.023	0.012	0.353	-0.023	0.024	0.414	-0.040
ArtCollection	0.121	0.040	0.061	0.010	0.732	-0.031	0.099	0.194	-0.016
NewPaint	0.009	0.749	-0.035	0.048	0.015	0.052	-0.028	0.579	-0.042
ArtWork	0.072	0.000	0.001	-0.012	0.229	-0.031	0.060	0.446	-0.034
ArtShow	0.108	0.066	0.019	0.023	0.618	-0.028	0.085	0.333	-0.034
ArtAuction	0.069	0.037	0.024	0.022	0.414	-0.019	0.021	0.743	-0.046
AdvanceGuard	0.002	0.846	-0.036	0.002	0.801	-0.032	0.052	0.037	0.026
AvantGarde	0.303	0.003	0.107	0.116	0.064	0.012	0.275	0.083	0.013
Connoisseur	0.113	0.025	0.010	-0.022	0.547	-0.029	0.055	0.566	-0.043
ArtCriticism	0.092	0.002	0.080	0.029	0.058	-0.008	0.026	0.441	-0.045

We now use as proxy for art love a series of words related to the art world which we took from several museums glossaries. Renneboog and Spaenjers (2013) studied the use of a variable as the art market sentiment as the number of sales of art works and quantity of appearances

of certain words in the *Economist* magazine, words such as *art markets, art prices* and *art auctions*. In our case we look for words reflecting love for art, or vanguardism. So, we take the number of articles wich metion an art related word per year in the *New York Times* using the *Chronicle* tool, and as we are studying art returns, we took the yearly change in this quantity.
Table 6.4 is divided into two panels which represent both countries (US and UK). and each row reports the OLS coefficient, Newey-West p-value estimator and Adjusted R^2 for each index of equation 6.1. In general we see positive estimators for the three indexes and several significancies for both countries. One of the words used is *Avant-Garde* which is a French term related to vanguardism in art, so when a new art work or style appears this is a typical word used by art lovers which is significant for the Mei and Moses (2002) in the UK and Renneboog and Spaenjers (2013) for both countries, at a 99% in the case of the United Kingdom, we see similar results for other words listed in the table which tells us that a proxy of art love increases the prices as estated in our first hypotheses. Other examples similars to *Avant-Garde* are *Connoisseur* which is a specific term to denote an acknowledgeable person in the art world, *Art Collection*, *Art Auction* which represent a general term related to art or as our proxy, love for art and confifm our hypotheses.

6.2 Uncertainty & Risk Aversion Hypotheses

In a first time studied diferent styles of art that were famous on time, or appeared as a new style, so in this section we try to confimr our hypotheses that estates the effect of the Heterogeneity degree between groups of investors in the formation of art prices. We study the effect of the appearance of a new style as a proxy of heterogeneity degree, i.e. when a new style appears, people tend to compare it with the ones already existing and start to give their opinions, so the heterogeneity of different beliefs about the formation of a new or vanguardist art style painting would be affected by the speculative behavior between consumers.

Table 6.5 shows the result of the estimation of equation 6.1 with a new variable we created $D_{MovementChanges}$. To create this variable we looked art history and see yearly which movements or art styles were notorious at the time, and we looked for years were one new famous style appeared or when there was a notorious art style on top, but replaced for a new vanguardist one, so we created a dummy containing this effects. Each panel represents the United States and the United Kingdom respectively, and each column represents the linear estimation for the indexes we are working on. In the case of Mei and Moses (2002) the OLS estimator do not represent any important significancy, but the have the negative sign which lets us believe the art returns tend to decrease with the effect of a style change. For the cases of Renneboog and Spaenjers (2013) and W. N. Goetzmann et al. (2009) we find significancies of 95% in the case of the United States and 90% in the case of the United Kingdom using the Newey-West error estimations, also with a negative OLS estimator which confirms our hypotheses estating that the uncertainty created by a new art style tends to decrease art prices.

Table 6.5: Art and Change in Movements

The table reports the OLS coefficient, its respective Newey-West standard error where ***, **, and * denote significance at 1%, 5%, and 10%, respectively and the Adjusted R^2 for each regression. Each column represents a regression where the dependent variable are the real returns of the Art Indexes which are independent of the country for Renneboog and Spaenjers (2013) R_a^{RS} and W. N. Goetzmann et al. (2009) R_a^G and the one from Mei and Moses (2002) R_a^{MM} was transformed with each country's CPI. The R_m and R_f are mentioned at the beginning of this chapter, The dummie $D_{MovementChanges}$ is a variable we created by studying and comparing the diferent movements of art styles over time and observing when a new style appears.

	$R_a^{RS} - R_f$		$R_a^G - R_f$		$R_a^{MM} - R_f$	
α	0.06	0.04	0.06	0.05	0.08	0.07
	(0.03)*	(0.03)	(0.02)***	(0.02)**	(0.03)**	(0.03)*
$R_m^{US} - R_f$	0.03		-0.02		0.15	
	(0.09)		(0.08)		(0.17)	
$R_m^{UK} - R_f$		0.06		0.05		-0.04
		(0.08)		(0.06)		(0.11)
$D_{ChangeofStyle}$	-0.09	-0.08	-0.06	-0.05	-0.02	-0.02
	(0.04)**	(0.04)*	(0.03)*	(0.03)*	(0.07)	(0.06)
Adj. R^2	0.03	0.01	0.04	0.01	-0.04	-0.05

In this study we attribute changes in risk aversion when there is a crises coming or going on, so investors tend to be more risk averse in general. In other hand, many researchers have found that art prices have dropped severally during certain economic recession periods. This can be found in W. N. Goetzmann (1993), Mei and Moses (2002) W. Goetzmann et al. (2011).

Reinhart (2010) created an historic database containing information about financial crises: Banking Crises, Monetary Crises, Domestic Crises and Inflation Crises. In the studied period 1958-2007 there are not domestic crises in any the US nor the UK. For our estimations we used dummies containing 1 at year t if there was a crise or 0 if not. The variable D_{Crises} considers all kinds of them, meanwhile $D_{MonetaryCrises}$, $D_{InflationCrises}$, $D_{StockMarketCrush}$ and $D_{BankingCrises}$ correspond to individually created dummies for Monetary, Inflation, Stock Market Crush and Banking crises respectively.

Table 6.6 is divided in three panels, containing the estimations for Renneboog and Spaenjers (2013) index, W. N. Goetzmann et al. (2009) index and Mei and Moses (2002) index respectively. Each column represents and estimation reporting the OLS estimator and the Newey-West p-values.
In the case of the United States, we just find significancy for the R_{GDP} in Mei and Moses (2002) and Renneboog and Spaenjers (2013) indexes which gives us a positives estimators, and for the stock market crush crisis in the Renneboog and Spaenjers (2013) index with a 90% significancy and a negative estimator which is what we were looking for in our Hypotheses. On the other hand, we have several significancies for the United Kingdom estimations. First, the R_{GDP} has 99% significancy in the R_a^{RS}, 95% in R_a^{MM} index and 90% significancy in the R_a^G index, even though the estimators are positive which contradicts our hypotheses, but different dummies containing crises confirm it. For the R_a^{RS} we find 99% significacies in the dummy containing all kinds of crises ocurred during the period, for the monetary crises

and the Inflation crises. All with a negative estimator which confirms our Hypotheses. In the case of R_a^G we find a 95% significancy with a negative estimator in the Inflation and Banking Crises and for the R_a^{MM} index we find 99% significancy in Inflation crises and 95% significancies in the dummy containing all crises and the Monetary crises. So, we conclude that crises are a determinant of the return of art prices, and they influence negatively as expected for oue hypothese.

Table 6.6: Art & Crises

The table reports the OLS coefficient, its respective Newey-West standard error where ***, **, and * denote significance at 1%, 5%, and 10%, respectively and the Adjusted R^2 for each regression. Each column represents a regression where the dependent variable are the real returns of the Art Indexes which are independent of the country for Renneboog and Spaenjers (2013) R_a^{RS} and W. N. Goetzmann et al. (2009) R_a^G and the one from Mei and Moses (2002) R_a^{MM} was transformed with each country's CPI. The independent variables are separated for each country. The R_m and R_f are mentionedat the beginning of this chapter. The Crises Dummies were taken from Reinhart (2010).

					$R_a^{SP} - R_f$				
			US				UK		
α	0.04	0.04	0.07	0.02	0.07	0.05	0.03	0.04	0.03
	(0.02)	(0.03)	(0.03)***	(0.02)	(0.02)***	(0.02)*	(0.03)	(0.02)	(0.03)
$R_m - R_f$	-0.02	-0.02	-0.09	-0.02	0.01	0.12	0.01	0.06	0.15
	(0.09)	(0.09)	(0.09)	(0.09)	(0.06)	(0.07)*	(0.09)	(0.08)	(0.08)*
D_{Crisis}	0.00				-0.13				
	(0.04)				(0.03)***				
$D_{MonetaryCrisis}$		0.00				-0.17			
		(0.05)				(0.03)***			
$D_{StockMarketCrush}$			-0.09				-0.05		
			(0.04)				(0.05)		
$D_{BankingCrisis}$				0.08				-0.11	
				(0.07)				(0.07)	
$D_{Inflation}$									-0.28
									(0.09)***
Adj. R^2	-0.04	-0.04	0.03	0.01	0.15	0.12	-0.03	0.03	0.01
					$R_a^G - R_f$				
			US				UK		
α	0.05	0.05	0.07	0.05	0.03	0.04	0.02	0.05	0.04
	(0.03)*	(0.02)***	(0.03)**	(0.02)*	(0.02)*	(0.02)	(0.02)	(0.02)**	(0.02)*
$R_m - R_f$	-0.04	-0.05	-0.08	-0.05	0.04	0.05	0.12	0.05	0.11
	(0.08)	(0.07)	(0.10)	(0.07)	(0.07)	(0.06)	(0.11)	(0.06)	(0.07)
D_{Crisis}	0.01				0.00				
	(0.03)				(0.02)				
$D_{MonetaryCrisis}$		0.02				-0.01			
		(0.05)				(0.02)			
$D_{StockMarketCrush}$			-0.04				0.10		
			(0.04)				(0.06)		
$D_{BankingCrisis}$				-0.01				-0.08	
				(0.05)				(0.03)**	
$D_{Inflation}$									-0.18
									(0.07)***
Adj. R^2	-0.03	-0.03	0.00	-0.03	-0.03	-0.03	0.06	0.04	0.01
					$R_a^{MM} - R_f$				
			US				UK		
α	0.06	0.07	0.10	0.07	0.11	0.08	0.07	0.08	0.07
	(0.03)**	(0.03)***	(0.03)***	(0.02)	(0.03)***	(0.03)***	(0.03)**	(0.03)***	(0.03)**
$R_m - R_f$	0.16	0.14	0.10	0.14	-0.05	0.02	-0.09	-0.02	0.13
	(0.17)	(0.16)	(0.18)	(0.16)	(0.09)	(0.12)	(0.15)	(0.10)	(0.10)
D_{Crisis}	0.03				-0.14				
	(0.07)				(0.05)**				
$D_{MonetaryCrisis}$		0.06				-0.14			
		(0.18)				(0.06)**			
$D_{StockMarketCrush}$			-0.07				-0.09		
			(0.06)				(0.09)		
$D_{BankingCrisis}$				0.04				-0.14	
				(0.09)				(0.11)	
$D_{Inflation}$									-0.44
									(0.10)***
Adj. R^2	-0.03	-0.03	-0.02	-0.03	0.05	0.01	-0.04	0.00	0.02

Chapter 7

Concluding Remarks

In this paper, we developped a discrete time period model to understand the impact of how two diferent groups of agents interact in a specific market which has a short sale constraint, for our case, the art market in concrete. The idea is to see the impact of diferent variables as the heterogeneity degree between groups of agents, the effect of having one group with additional pleasure over the asset, the unceraintу present in the market for both groups and the risk aversion interact between them making the Price dynamics.

Our conclusion stays that under certain conditions, the additional pleasure makes prices increase, for the extra value added pleasure dividend, and also makes the resale option increase as art lovers or investors who get the extra benefit tend to be more reluctant to the idea of selling art. This tend to be confirmed in our empirical analysis where we also find some determinants of art returns. Our obtained results for non-expected inflation proxies shows that art as part of the durable goods of commodities. It behaves like it. Being art returns determines not only by for the income generated, but also by a service flow provided to the investor, so art returns allow from an investing point of view to hedge against unexpected inflations.
In the case of luxury pleasure proxies. Our conclusion is art returns are determined in the United States by the esthetic pleasure given for owning art. Specifically, it proves the existing relationship between art returns ant the change in the income of the people with a High Income. The fact this people has already fill their need for basic consumption, an increase in their incomes implies an augmentation in their luxury goods consumption -in this case, art. We do not find the same evidence for the United Kingdom, as our variables were mainly constructed for luxury pleasure proxies in the United States, and we do not account for variables of luxury personal consumption in the United Kingdom, so it remains for further research.
In the case of art love proxies, or general related terms for art we also find the evidence we expected, as several words are significant and tend to determine the art returns. The additional pleasure given to art collectors/owners represented as love for art determines art prices and gives as a proxy estating that love for art gives art returns an increase.

For the Heterogeneity degree, under certain conditions it tends to make art prices drop. For the empirical analysis, we used a change in art styles or art movements as a proxy due to the comparison of styles when a new one comes out. The estimations used by the the number of articles that counted that word in the *New York Times* proves change in styles are relevant for return of art indexes in a negative sense.

For the case of the risk aversion, we also find tendencie of declining prices while investor are more risk-aversed in our model, which it gets confirmed when using the crisis proxy. We assumed crises represent an increase in risk aversion as investors tend to be more reisk aversed when a crisis is presented. Our empirical analysis supports this as making crises significant, and also the fact a crise of any type presents itself makes a decrease in art returns.

Chapter 8

Appendix

8.1 *Technical Proofs & Calculations*

As already estated in chapter 4, at date 0 the two groups' prior beliefs regarding D are normally distributed and equal, i.e., the mean $(\bar{D}_0)$ and variance (σ_D^2) are the same for both groups. The beliefs of the two groups of investors at date 1 are also normally distributed, with equal variance (σ_D^2) and with means given by

$$\bar{D}_A = \bar{D} + k + \varepsilon \text{ and } \bar{D}_B = \bar{D} + varepsilon \tag{8.1}$$

where ε is independent and normally distributed, $\varepsilon \sim N\left(0, \sigma_\varepsilon^2\right)$ and k represents the pleasure from art consumption itself due to inflation hedging or luxury of owning art for investors of group A. Given these forecasts, we solve for the equilibrium holdings and price at date 1. Group A investors solve then

$$\max_{x_1^A} \mathbb{E}_1^A\left[\left(D - P_1\right) x_1^A\right] - \frac{\theta}{2} Var_1^A\left[\left(D - P_1\right) x_1^A\right] \tag{8.2}$$

and group B investors,

$$\max_{x_1^B} \mathbb{E}_1^B\left[\left(D - P_1\right) x_1^B\right] - \frac{\theta}{2} Var_1^B\left[\left(D - P_1\right) x_1^B\right] \tag{8.3}$$

which is equivalent

$$\max_{x_1^A} \left[\left(\mathbb{E}_1^A[D] - P_1\right) x_1^A - \frac{\theta\left(x_1^A\right)^2 \sigma_D^2}{2}\right]$$

and

$$\max_{x_1^B} \left[\left(\mathbb{E}_1^B[D] - P_1\right) x_1^B - \frac{\theta\left(x_1^B\right)^2 \sigma_D^2}{2}\right]$$

where the first-order conditions are:

$$\left(\mathbb{E}_1^A[D] - P_1\right) - \theta x_1^A \sigma_D^2 = 0 \text{ and } \left(\mathbb{E}_1^B[D] - P_1\right) - \theta x_1^B \sigma_D^2 = 0 \tag{8.4}$$

Markets clear, therefore $x_1^A + x_1^B = Q$.

Lets first study the case without beliefs disagreements.

Model without Beliefs Disagreements and Symmetric Uncertainties

with the fact there are no disagreements about the asset's payment, $E_1^A[D] = E_1^B[D] = \bar{D}+\varepsilon$, $x_1^A = x_1^B$, and imposing market clearing $x_1^{A,B} = \frac{Q}{2}$, so from equation 8.4 we obtain

$$P_1 = \bar{D} + \varepsilon - \frac{\theta Q \sigma_D^2}{2}$$

Now let solve the equilibrium at $t = 0$. Given investors' mean-variance preferences, their demands at date 0 are given by

$$x_0^A = \max\left[\frac{\mathbb{E}_0^A[P_1] - P_0}{\theta Var_0^A[P_1 - P_0]}, 0\right] \quad \text{and} \quad x_0^B = \max\left[\frac{\mathbb{E}_0^B[P_1] - P_0}{\theta Var_0^B[P_1 - P_0]}, 0\right] \tag{8.5}$$

As we set the model to have homogeneous intial beliefs, $\mathbb{E}_0^A[P_1] = \mathbb{E}_0^B[P_1] = \bar{D} - \frac{\theta Q \sigma_D^2}{2}$, and there is the same effect with the variance $Var_0^A[P_1 - P_0] = Var_0^B[P_1 - P_0] = Var_0[P_1 - P_0]$, so assuming both groups share the market equally, and imposing market-clearing at $t = 0$.

$$P_0 = \bar{D} - \frac{\theta Q\left(\sigma_D^2 + Var_0[P_1 - P_0]\right)}{2}$$

Model with Beliefs Disagreements and Symmetric Uncertainties

Under the same logic, from equation 8.4 we obtain the demand functions:

$$x_1^A = \max\left[\frac{\mathbb{E}[D]_1^A - P_1}{\theta \sigma_D^2}, 0\right] \quad \text{and} \quad x_1^B = \max\left[\frac{\mathbb{E}[D]_1^B - P_1}{\theta \sigma_D^2}, 0\right] \tag{8.6}$$

which translate into three cases

- Case 1: $x_1^A = Q_a, \quad x_1^B = 0$

  $$P_1 = \mathbb{E}_1^A[D] - \theta Q \sigma_D^2$$

  Which occurs only if $\mathbb{E}_1^B[D] - P_1 < 0$, which is equivalent of $k > \theta Q \sigma_D^2$
- Case 2: $x_1^A > 0 \quad x_1^B > 0$ Imposing market-clearing

  $$x_1^A = \frac{k}{2\theta\sigma_D^2} + \frac{Q}{2}$$

$$x_1^B = -\frac{k}{2\theta\sigma_D^2} + \frac{Q}{2}$$

$$P_1 = \frac{\mathbb{E}_1^A[D] + \mathbb{E}_1^B[D]}{2} - \frac{\theta\sigma_D^2 Q}{2}$$

from 8.6 and the price in this case, we obtain that it holds only if $\|k\| \leq \theta Q\sigma_D^2$

- Case 3: $x_1^B = Q$ and $x_1^A = 0$

$$P_1 = \mathbb{E}_1^B[D] - \theta\sigma_D^2 Q$$

Combining the price calculated in this case with the demand function, it translates that $k < -\theta Q\sigma_D^2$ which may never hold as $k > 0$

Re-adjusting terms, we obtain:

$$P_1 = \frac{\mathbb{E}_1^A[D] + \mathbb{E}_1^B[D]}{2} - \frac{\theta Q\sigma_D^2}{2} + \frac{1}{2}max\left(\mathbb{E}_1^A[D] - \mathbb{E}_1^B[D] - \theta Q\sigma_D^2, 0\right) \tag{8.7}$$

Which results in equation 8.7, we next solve for equilibrium at date 0. Given investors' mean-variance preferences, their demands at date 0 are given by

$$x_0^A = \max\left[\frac{\mathbb{E}_0^A\left[P_1\right] - P_0}{\theta Var_0^A\left[P_1 - P_0\right]}, 0\right] \quad \text{and} \quad x_0^B = \max\left[\frac{\mathbb{E}_0^B\left[P_1\right] - P_0}{\theta Var_0^B\left[P_1 - P_0\right]}, 0\right] \tag{8.8}$$

Assuming Homogeneous initial beliefs $\mathbb{E}_0^A\left[P_1\right] = \mathbb{E}_0^B\left[P_1\right]$, and the same for the variances $Var_0^B\left[P_1 - P_0\right] = Var_0^A\left[P_1 - P_0\right]$ Imposing the Market-clearing codition in $t = 0$, i.e. $x_0^A + x_0^B = Q$.
So, we obtain $x_0^{A,B} = \frac{Q}{2}$ and directly from equation 8.8 we obtain:

$$P_0 = \mathbb{E}_0\left[P_1\right] - \frac{\theta Q}{2}Var_0\left[P_1 - P_0\right]$$

Proof 1 (Proof of Proposition 1)**.**

$$\bar{D}_A - P_1 < 0 <=> \bar{D} + k + \phi\frac{\varepsilon}{2} - \left(\bar{D} + \frac{k}{2} + \phi\frac{\varepsilon}{2} - \frac{\theta Q\sigma_D^2}{2}\right) \tag{8.9}$$

So, $\frac{k}{2} + \frac{\theta Q\sigma_D^2}{2} < 0$ which is impossible for risk-averse agents with positive extra-income, so the case where B-agents adquire the entire market is impossible with the extra income for A-agents.

Disagreement Sensitivity

Direct diferentiation with respect to k yields:

$$\frac{\partial P_0}{\partial k} = \frac{1}{2} + \frac{\partial G(k, \theta Q\sigma_D^2)}{\partial k}$$

Where $\frac{\partial G(k,\theta Q\sigma_D^2)}{\partial k}$ is null if $k \leq \theta Q\sigma_D^2$ and $\frac{1}{2}$ if $k > \theta Q\sigma_D^2$, so

$$\frac{\partial P_0}{\partial k} = \frac{1}{2} + \frac{1}{2}\mathbb{I}_{k>\theta Q\sigma_D^2}$$

So, the price at time $t = 0$ increases with respect to k.

Uncertainty Sensitivity

Direct differentiation of P_0 with respect to σ_ε yields:

$$\frac{\partial P_0}{\partial \phi} = -\theta Q\sigma_\varepsilon < 0$$

So, the price at time $t = 0$ decreases with respect to σ_ε.

Risk Aversion Sensitivity

$$\frac{\partial P_0}{\partial \theta} = -\frac{Q\sigma_D^2}{2} - \frac{Q\left(\phi\sigma_\varepsilon\right)^2}{2} + \frac{\partial G(k,\theta Q\sigma_D^2)}{\partial \theta}$$

Where

$$\frac{\partial G(k,\theta Q\sigma_D^2)}{\partial \theta} = -\frac{Q\sigma_D^2}{2}\mathbb{I}_{k>\theta Q\sigma_D^2}$$

So, the price at time $t = 0$ decreases with respect to θ.

Model without Beliefs Disagreements and Asymmetric Uncertainties

Proof 2 (Proof of Lemmas 1 and 2)**.** It follows from maximizing the mean-variance utilities (4.2) of each agent obtaining the demands function given in equations (4.6) and imposing the market clearing condition at $t = 1$ and $t = 0$.

Proof 3 (Proof of Proposition 2)**.** When investors from groups A and B have identical beliefs at $t = 0$, it follows from lemma 2 that the equilibrium price at this period is:

$$P_0 = \mathbb{E}_0\left[P_1\right] - \frac{\theta Q}{2}Var_0\left[P_1 - P_0\right] \tag{8.10}$$

Here the important thing is to understand the expecion of P_1 at $t = 0$, so re-writting the equilibrium price from lemma 1

$$P_1 = \frac{\mathbb{E}_1^A[D] + \mathbb{E}_1^B[D]}{2} - \frac{\theta\sigma_D^2 Q}{2} + \begin{cases} -\frac{l}{2} - \frac{\theta\sigma_D^2 Q}{2} & \text{if } l < -\theta\sigma^2 Q \\ 0 & \text{if } \|l\| \leq \theta\sigma_D^2 Q \\ \frac{l}{2} - \frac{\theta\sigma_D^2 Q}{2} & \text{if } l > \theta\sigma_D^2 Q \end{cases} \tag{8.11}$$

where $l = \mathbb{E}_1^A[D] - \mathbb{E}_1^B[D]$ represents the difference in expectations between the group of investors. For the expectation of B-investors at the initial period there are two uncertain terms in equation 8.10 which are $\bar{D}_B$ and the three pieces function already defined as the Resale function $G(l,g)$.
The expectation for $\bar{D}_B$ is $\bar{D}$ which is the value when investors are not able to sell their shares at $t = 1$ and the three pieces function represents the value of re-selling at $t = 1$ and it is calculated computionally as the integrating area weighted by the probability density of l (In this work l follows a Gaussian distribution).

Calculating the Resale Option

Direct Integration of G yields, the distribution of l is centered in 0, as $l = \varepsilon_A - \varepsilon_B$, so the expression of the resale option is:

$$\mathbb{E}\left[G\left(l, \theta Q\sigma_D^2\right)\right] = -\mathbb{E}\left[\frac{l + \theta Q\sigma_D^2}{2} I_{\{l<-\theta Q\sigma_D^2\}}\right] + \mathbb{E}\left[\left(l - \frac{\theta Q\sigma_D^2}{2}\right) I_{\{l>\theta Q\sigma_D^2\}}\right] \tag{8.12}$$

The fact that l has a symmetric distribution with zero-mean:

$\mathbb{E}\left[\frac{\theta Q\sigma_D^2}{2} I_{\{l<-\theta Q\sigma_D^2\}}\right] = \mathbb{E}\left[\frac{\theta Q\sigma_D^2}{2} I_{\{l>\theta Q\sigma_D^2\}}\right]$

So, the result is:

$$\mathbb{E}\left[G\left(l, \theta Q\sigma_D^2\right)\right] = \mathbb{E}\left[\left(l - \theta Q\sigma_D^2\right) I_{\{l>\theta Q\sigma_D^2\}}\right] \tag{8.13}$$

Direct Integration yields:

$$\mathbb{E}\left[G\left(l, \theta Q\sigma_D^2\right)\right] = \frac{\sigma_l}{\sqrt{2\pi}} \mathrm{e}^{-\frac{(\theta Q\sigma_D^2)^2}{2\sigma_l^2}} - \theta Q\sigma_D^2 N\left(-\frac{\theta Q\sigma_D^2}{\sigma_l}\right)^{1} \tag{8.14}$$

Proof 4 (Proof of proposition 3)**.** We directly derive the resale option with respect to σ_l. The density function for the distribution of l is $f(x) = \frac{1}{\sqrt{2\pi}\sigma_l}\mathrm{e}^{-\frac{x^2}{2\sigma_l^2}}$ and its derivative with respect to σ_l:

$$\frac{\partial f(x)}{\partial \sigma_l} = \frac{x^2}{\sqrt{2\pi}\sigma_l^4}\mathrm{e}^{-\frac{x^2}{2\sigma_l^2}} - \frac{1}{\sqrt{2\pi}\sigma_l^2}\mathrm{e}^{-\frac{x^2}{2\sigma_l^2}}$$

So, diferentiating the resale option and establishing $g = \theta Q\sigma_D^2$:

$$\partial_{\sigma_l}\mathbb{E}\left[G\left(l, g\right)\right] = \int_g^{\infty} (x - g)\frac{\partial f(x)}{\partial \sigma_l}\,\mathrm{d}x$$

[1] N() represents the cumulative distribution function of a standard normal distribution

And, integrating we obtain $\partial_{\sigma_l}\mathbb{E}\left[G\left(l,g\right)\right] = \frac{\mathrm{e}^{-\frac{g^2}{2\sigma_l^2}}}{\sqrt{2\pi}} > 0$ So, we conclude the resale option increases with σ_l

Now let's with respect to g, direct differentiation of $\mathbb{E}\left[G\left(l,g\right)\right]$ with respect to g yields: $\frac{\partial\mathbb{E}\left[G(l,g)\right]}{\partial g} = -N\left(-\frac{g}{\sigma_l}\right) < 0$

Calculating the variance of $P_1 - P_0$

From lemmas 1 and 2 is easy to see:

$$Var_0\left[P_1 - P_0\right] = Var_0\left[\bar{D} + \frac{k}{2} + G(l, \theta Q\sigma_D^2)\right] \tag{8.15}$$

Which results into:

$$Var_0\left[P_1 - P_0\right] = Var\left[G(l, \theta Q\sigma_D^2)\right] \tag{8.16}$$

By direct Integration, we obtain:

$$Var\left[G(l,g)\right] = \frac{1}{2}\left[\left(g^2 + \sigma_l^2\right)N\left(-g/\sigma_l\right) - \frac{g\sigma_l}{\sqrt{2\pi}\mathrm{e}^{-g^2/2\sigma_l^2}}\right] - \left(\mathbb{E}\left[G\left(l,g\right)\right]\right)^2 \tag{8.17}$$

Where $g = \theta Q\sigma_D^2$

Proof 5 (Proof of proposition 4)**.** First, lets differentiate the Variance terms in equation 8.17:

$$\frac{\partial\mathbb{E}\left[G^2\right]}{\partial\sigma_l} = \sigma_l - \sigma_l Erf\left(\frac{g}{\sqrt{2\pi}\sigma_l}\right) + \frac{g^3}{2\sqrt{2\pi}\sigma_l^2}\mathrm{e}^{-\frac{g^2}{2\sigma_l^2}} + \frac{2g}{\sqrt{2\pi}}\mathrm{e}^{-\frac{g^2}{2\sigma_l^2}2}$$

The derivative of the second term of this variance:

$$\frac{\partial\mathbb{E}\left[G\right]^2}{\partial\sigma_l} = 2\mathbb{E}\left[G\right]\frac{\partial\mathbb{E}\left[G\right]}{\partial\sigma_l} = \frac{\sigma_l}{2\pi}\mathrm{e}^{-\frac{g^2}{\sigma_l^2}} - g\frac{\mathrm{e}^{-\frac{g^2}{2\sigma_l^2}}}{\sqrt{2\pi}}N\left(\frac{g}{\sigma_l}\right)$$

Substracting both terms above, and taking the limit $\sigma_l \to \infty$ we obtain: $\frac{\partial Var(P_1 - P_0)}{\partial\sigma_l} \xrightarrow[\sigma_l\to\infty]{} \infty$

While $\frac{\partial\mathbb{E}(G)}{\partial\sigma_l} \xrightarrow[\sigma_l\to\infty]{} 1/\sqrt{2\pi}$ So, this means that the derivative of P_0 with respect to σ_l diverges to $-\infty$, and by the definition of the limit of a divergent function; for each value M there exist a $\bar{\sigma}_l$ where the value of this derivative follows: $\frac{\partial P_0}{\partial\sigma_l} < M$ for all $\sigma_l > \bar{\sigma}_l$, in particular there exists a value of $\bar{\sigma}_l$ for which this derivative is always negative, so the price will start to decrease continuously as estated in the proposition.

[2]Erf represents the Gaussian error function

Proof 6 (Proof of proposition 5)**.** it follows from direct Integration of the expected value and the variance of $G(l_k, g)$ where $l_k = k + \Delta\varepsilon$ and $g = \theta Q\sigma_D^2$.

Lets begin by differentiating the Resale Option: $\frac{\partial \mathbb{E}\left[G(l_k,g)\right]}{\partial k} = \frac{1}{2}\frac{\partial}{\partial k}B(g+k,\sigma_l) + \frac{1}{2}\frac{\partial}{\partial k}B(g-k,\sigma_l)$

Which turns into:

$\frac{\partial \mathbb{E}\left[G(l_k,g)\right]}{\partial k} = \frac{1}{2}N\left(\frac{g+k}{\sigma_l}\right) + \frac{1}{2}N\left(\frac{g-k}{\sigma_l}\right) > 0$

As k and g are always positive, this difference turns to be positive, so the resale option increases with respect to k and as k diverges, the resale option increases with slope 1/2. Then for convexity, looking at the second derivative:

$$\frac{\partial^2 \mathbb{E}\left[G\left(l,g\right)\right]}{\partial k^2} = \frac{1}{2\sqrt{2\pi}\sigma_l}\left(\mathrm{e}^{-\frac{(k-g)^2}{2\sigma_l^2}} + \mathrm{e}^{-\frac{(k+g)^2}{2\sigma_l^2}}\right)$$

which is positive, so it proves the curve's convexity and the first part of the proposition. Then, lets move to the Risk-Sharing effect, but first, let's get back to the case with no belief disagreements:

$\frac{\partial Var[G(l,g)]}{\partial g} = -\left[\frac{\sigma_l}{\sqrt{2\pi}}\mathrm{e}^{-g^2/2\sigma_l^2} - gN\left(-g/\sigma_l\right)\right]\left[1 - 2N\left(-g/\sigma_l\right)\right] < 0$ This term is negative, so the Risk-Sharing decreases as g increases, but the derivative tends to zero as g diverges, making the variance to decrease until a certain level.

So, looking at the Risk-Sharing with the introducton of k.

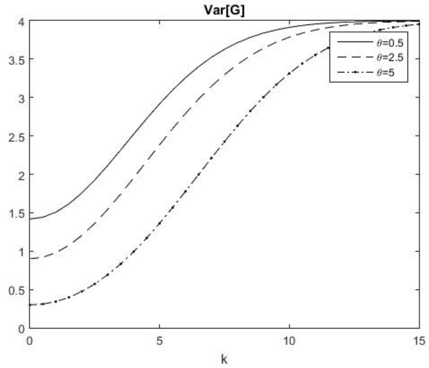
$$\partial_k Var\left[G(l_k,g)\right] = \frac{1}{2}\frac{\partial}{\partial k}V(g+k,\sigma_l) + \frac{1}{2}\frac{\partial}{\partial k}V(g-k,\sigma_l) + \frac{1}{2}\left(B(g+k,\sigma_l) - B(g-k,\sigma_l)\right)\left(N\left(\frac{g+k}{\sigma_l}\right) - N\left(\frac{g-k}{\sigma_l}\right)\right).$$

And looking at each termn separately:

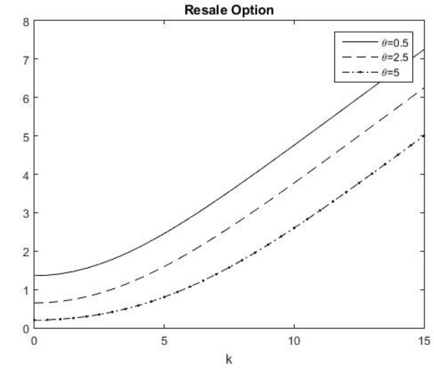
$$\frac{\partial}{\partial k}V(g+k,\sigma_l) = -\left[\frac{\sigma_l}{\sqrt{2\pi}}\mathrm{e}^{-\frac{-(g+k)^2}{2\sigma_l^2}} - (g+k)N(-\tfrac{g+k}{\sigma_l})\right]\left[1 - 2N(-\tfrac{g+k}{\sigma_l})\right]$$

$$\frac{\partial}{\partial k}V(g-k,\sigma_l) = \left[\frac{\sigma_l}{\sqrt{2\pi}}\mathrm{e}^{-\frac{-(g-k)^2}{2\sigma_l^2}} - (g-k)N(-\tfrac{g-k}{\sigma_l})\right]\left[1 - 2N(-\tfrac{g-k}{\sigma_l})\right]$$

Taking the limit when k diverges and knowing that $xN(-x/a) \xrightarrow[x\to\infty]{} 0$ of every term, we obtain after soem algebraic simplifications that the Variance of $G(l_k, g)$ converges to a constant value with the increase of k



(a) Variance of $G(l_k, g)$



(b) Resale Option

Figure 8.1: Resale Option vs Risk-Sharing Component

So, taking the values calculated in the equation :$\frac{\partial P_0}{\partial k} = \frac{\partial \mathbb{E}\left[G(l_k,g)\right]}{\partial k} - \frac{\theta Q}{2}\frac{\partial Var\left[G(l_k,g)\right]}{\partial k}$ and adding them, we can notice that this expression converges to 1/2 when $k \to \infty$. Which we can numerically seen in figure 8.1a and 8.1b by comparing the Variance of $G(l_k, g)$ and the Resale Option one derivative converges to 0 while the other converges to 1/2 which by definition of limit means that it exists a value k^* that for all $k > k^*$ the price at 0 increases with respect to k. The intuition behind this is the Risk-Sharing starts to increase with the adoption of A-agents into the market, while they still share the market with B-agents, but when k is too big, just A-agents are trading which means the risk premium for holding the asset between $t = 0$ and $t = 1$ stabilizes as just A-agents hold the market.

8.2 *Empirical Analysis*

Art Market Indexes

We deliver a little review about how researchers calculated their art market indexes which are used in this paper.

Rennebog Spaenjers Hedonic Index

Index calculated for prices in the period 1957-2007 with data from oil paintings and paper work which represent 85% of the turnover (Renneboog and Spaenjers (2013)). Authors determine a list of 10.442 artisit and chose data over thei sales from the Art Sales Index data base. they make the match between the artists name with all the sales in the data-base. This base contains data from auctions of several types of art. Their prices are hammer prices, abstracting from transactions costs and as historically Art Sales Index does not include *Buy-ins.* [3]

[3] *Buy-in* refers to art-works which did not reach the minimum bid-price in an auction.

Authors construct an hedonic index for pricing art. Their model relates natural logarithm of Prices (in US Dollars) yearly Dummy variables and control for several factors relating to the characteristic of the artist or the painting.

$$ln(P_{kt}) = \alpha + \sum_{m=1}^{M} \beta_m X_{mkt} + \sum_{t=1}^{T} \gamma_t D_{kt} + \varepsilon_{kt} \tag{8.18}$$

Where P_{kt} represents the price of the art object k at t, X_{mkt} is the value of the characteristic m at t and D_{kt} a time dummy which values 1 if the object is traded at t (0 if not). β_m reflects the attribution of a virtual price with respect to everty m characteristic, and the anti-logarithm of γ_t controls for the variation in time, so the hedonic Index at t is:

$$\pi_t = 100\mathrm{e}xp(\hat{\gamma}_t) \tag{8.19}$$

But, due to the logarithmic transformation, this index follows a geometric mean -not arithmetic- of prices over time, so they correct this bias by using the *tripplet2004* (n.d.) correction:

$$\pi_t^* = 100\mathrm{e}xp\left[\hat{\gamma}_t + \frac{1}{2}(\hat{\sigma}_t^2 - \hat{\sigma}_0^2)\right] \tag{8.20}$$

Where $\hat{\sigma}_t^2$ and $\hat{\sigma}_0^2$ represent the residual variances at times t and 0

Goetzmann et al. Index

Long-term index for the period 1830-2007. They identify every repeated Sale in Reitlinger book, which gives a data-set of 1.096 until 1961, then they take the data-set used by Renneboog and Spaenjers (2013) which contains more than a million sales from auctions until 2009. And identify the repeated sales of the same art-work at the U.K.

To estimate their index, they use a bayesian regression for repeated sales, which adds aditional constraints. The Bayesian formulation avoids spurios negative auto-correlation, which in returns. They assume continuosly compounded returns of an asset i $r_{\mathrm{i},t}$ is represented by μ_t compounded return of art index and an error term

$$r_{\mathrm{i},t} = \mu_t + \eta_{\mathrm{i},t} \tag{8.21}$$

Their estimator is computed as:

$$\hat{\mu}_{Bayes} = \left[\left(X'\Omega X + \kappa(I - \frac{1}{T}J)\right)\right]^{-1} X'\Omega^{-1}r \tag{8.22}$$

Where X represents an NxT matrix which rows are dummies for every asset in the sample and columns the retention time. Ω is a weight matrix which is determined by the time between each sale. J a matrix of 1's and κ a constant $\kappa = \frac{\sigma^2}{\sigma_\mu^2}$

Mei and Moses Index

Authors create a database for the American Art Market. For the second half of XX century, they construct a new data set of repeated sales of paintings and estimates an annual index of art prices for the period 1875-2000. They find that art outperforms fixed income securities as an investment, though it significantly under-performs the stock market. Art assets also have lower volatility and lower correlation with other assets, making it attractive for portfolio diversificaton. There is strong evidence of underperformance of masterpieces, meaning expensive paintings tend to under-perform the art market index. The evidence is mixed on whether the law of one price holds in the New York auction market.

Their Index follows a repeated sales regression as in the previous case. Now Ω is a matrix of weights are based in a three step error estimation used by Case and Shiller (1987) also known as a delta method in econometrics.

Data Construction

In the present paper in chapter 6 the dependent variable used is the art returns. The independent variables of interest are different measures of markets of each country along with the characterizations of determinants to study: Inflation proxies, luxury appetite, change in art movements and financial crises. The art returns as the other variables are treated in its real value. In case of counting with nominal prices, this are tranformed to its real value with the price of 2007 usin the consumer price index of the United States and the United Kingdom as an inflation measure. The equation to transform this nominal prices into their real value at 2007 is:

$$P_{\mathrm{i},real} = \frac{P_{\mathrm{i},nom} CPI_{2007}}{CPI_{\mathrm{i}}} \tag{8.23}$$

Where $P_{\mathrm{i},nom}$ corresponds to the nominal price at the year i, CPI_{i} the consumer price index at the year i and $P_{\mathrm{i},real}$ the real value in the year i.

We also work with rates. For the interest rate transformation to real values in 2007 we used the Fischer equation:

$$R_{\mathrm{i}} = \left((1 + r_{\mathrm{i}}) \frac{CPI_{\mathrm{i}-1}}{CPI_{\mathrm{i}}} \right) - 1 \tag{8.24}$$

Where r_{i} is the nominal value of the interest rate and R_{i} its real value.

The consumer price index of the United States is taken from the data showed in chapter 26 of the book Shiller (2015). For the United Kingdom the Bank of England built on 2014 a data base with the most importan macroeconomical variables of the country, here we extracted the CPI for the U.K.

Descriptive Statistics of the Data

During this paper the indexes used are

Table 8.1: Art Returns descriptive statistics

It is important to notice that $R_{a,Shed}$ and $R_{a,Grep}$ have the same statistics for the United States and the United Kingdom. This is due to the construction of the indexes which was made in real returns, so in this work we assume they the art amrket in general. In the case of $R_{a,Meirep}$ authors calculated a nominale price which is transformed with the Consumer Price Index of each country.

Variable	Period	Mean	Median	Standard Deviation	Skewness	Kurtosis	Min	Max
				US				
$R_{a,Shed}$	1958-2007	0.05	0.03	0.15	0.08	0.63	-0.37	0.41
$R_{a,Grep}$	1958-2007	0.06	0.06	0.10	0.46	1.58	-0.18	0.38
R_{aMrep}	1958-1999	0.10	0.11	0.22	0.02	-0.24	-0.40	0.57
				UK				
$R_{a,Shed}$	1958-2007	0.05	0.03	0.15	0.08	0.63	-0.37	0.41
$R_{a,Grep}$	1958-2007	0.06	0.06	0.10	0.46	1.58	-0.18	0.38
$R_{a,Mrep}$	1958-1999	0.09	0.10	0.22	-0.05	-0.05	-0.41	0.57

Table 8.2: Independent Variables descriptive statistics 1958-2007

Variable	Mean	Median	Standard Deviation	Skewness	Kurtosis	Min	Max
			US				
R_m	0.08	0.11	0.16	-0.46	-0.29	-0.34	0.40
R_f	0.01	0.02	0.02	-0.70	1.43	-0.05	0.05
R_{CPI}	0.04	0.03	0.03	1.61	2.49	0.01	0.14
R_{Gold}	0.04	-0.01	0.22	1.69	5.04	-0.37	0.94
$R_{RealEstate}$	0.01	0.00	0.04	0.30	0.21	-0.09	0.10
$R_{HighIncome}$	0.03	0.01	0.09	1.77	5.32	-0.12	0.40
$R_{Jewelry\&Watches}$	0.06	0.06	0.06	0.54	0.96	-0.08	0.24
$R_{Boats\&Aircrafts}$	0.07	0.07	0.10	-0.14	0.90	-0.23	0.31
$NYT_{AvantGarde}$	0.06	0.02	0.18	1.46	3.75	-0.22	0.76
R_{GDP}	0.03	0.04	0.02	-0.50	-0.01	-0.02	0.07
			UK				
R_m	0.06	0.08	0.23	0.47	3.96	-0.61	0.90
R_f	0.02	0.03	0.03	-1.53	3.95	-0.11	0.08
R_{CPI}	0.05	0.04	0.05	1.80	3.25	0.01	0.24
R_{Gold}	0.04	-0.01	0.22	1.69	5.04	-0.37	0.94
$R_{RealEstate}$	0.04	0.04	0.10	0.41	0.83	-0.17	0.33
$R_{HighIncome}$	0.02	0.02	0.07	0.63	2.31	-0.11	0.27
$NYT_{AvantGarde}$	0.06	0.02	0.18	1.46	3.75	-0.22	0.76
R_{GDP}	0.02	0.02	0.02	-0.18	1.16	-0.02	0.08

Table 8.3: Art Styles

Year	Art Style
1958	Hard-edge painting
1959	
1960	
1961	
1962	
1963	
1964	
1965	Op art, hard edge formalism, kinetic, minimalism and dada
1966	
1967	
1968	
1969	
1970	
1971	
1972	
1973	
1974	Post-modern art
1975	arte povera, photorealism, performance bodyart
1976	Raw Art/Rough Art
1977	Earth art,
1978	
1979	
1980	
1981	
1982	
1983	
1984	
1985	Neo-Geo/Appropiation
1986	
1987	
1988	
1989	
1990	
1991	
1992	Massurrealism, toyism, Transgressive art
1993	
1994	Lowbrow
1995	Tactical Media
1996	
1997	
1998	Pop surrealism
1999	
2000	
2001	
2002	
2003	
2004	
2005	
2006	
2007	

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