

# NONLOCAL $s$ -MINIMAL SURFACES AND LAWSON CONES

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## Abstract

The nonlocal  $s$ -fractional minimal surface equation for  $\Sigma = \partial E$  where  $E$  is an open set in  $\mathbb{R}^N$  is given by

$$H_\Sigma(s)(p) := \int_{\mathbb{R}^N} \chi_E(x) - \chi_{E-c}(x) / |x - p|^{N+s} dx = 0$$
 for all  $p$  is an element of  $\Sigma$

Here  $0 < s < 1$ ,  $\chi$  designates characteristic function, and the integral is understood in the principal value sense. The classical notion of minimal surface is recovered by letting  $s \rightarrow 1$ . In this paper we exhibit the first concrete examples (beyond the plane) of nonlocal  $s$  minimal surfaces. When  $s$  is close to 1, we first construct a connected embedded  $s$ -minimal surface of revolution in  $\mathbb{R}^3$ , the nonlocal catenoid, an analog of the standard catenoid  $|x|^3 = \log(r + \sqrt{r^2 - 1})$ . Rather than eventual logarithmic growth, this surface becomes asymptotic to the cone  $|x|^3 = r \sqrt{1 - s}$ . We also find a two-sheet embedded  $s$ -minimal surface asymptotic to the same cone, an analog to the simple union of two parallel planes.

On the other hand, for any  $0 < s < 1$ ,  $n, m \geq 1$ ,  $s$ -minimal Lawson cones  $|v| = \alpha |u|$ ,  $(u, v)$ , is an element of  $\mathbb{R}^n \times \mathbb{R}^m$ , are found to exist. In sharp contrast with the classical case, we prove their stability for small  $s$  and  $n + m = 7$ , which suggests that unlike the classical theory (or the case  $s$  close to 1), the regularity of  $s$ -area minimizing surfaces may not hold true in dimension 7.

## Palabras clave

**KeyWords Plus:** [LEVEL SET APPROACH](#); [INEQUALITY](#); [REGULARITY](#); [CONSTANT](#)

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