Special values of Dirichlet series and zeta integrals

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For f and g polynomials in p variables, we relate the special value at a non-positive integer s = -N, obtained by analytic continuation of the Dirichlet series ?(s;f, g) = ? k1 = 0 ??? kp = 0 ?g(k 1,?,k p)f(k 1,?,k p) -s (Re(s) ? 0), to special values of zeta integrals Z(s;f,g) = ? x?[0, ?)p g(x)f(x) -s dx (Re(s) ? 0). We prove a simple relation between ?(-N;f,g) and Z(-N;f a, g a), where for a ? ? p, f a(x) is the shifted polynomial f a(x) = f(a + x). By direct calculation we prove the product rule for zeta integrals at s = 0, degree(fh)·Z(0;fh, g) = degree(f)·Z(0;f, g) + degree(h)·Z(0;h, g), and deduce the corresponding rule for Dirichlet series at s = 0, degree(fh)·?(0;fh, g) = degree(f)·?(0;f, g)+degree(h)·?(0;h, g). This last formula generalizes work of Shintani and Chen-Eie.