Contents

1 Introduction **1**

1.1 Measures on a Set of Boxes 2
1.2 Matching Points with Boxes 4
1.3 Removing Redundancies in a Set of Boxes 6
1.4 Thesis Structure and Contributions 7

2 Foundations **10**

2.1 Three Fundamental Problems 10
2.2 Matchings and Maximum Independent Sets 12
2.2.1 Finding Matchings via Independent Sets 12
2.2.2 Approximation Algorithms for Maximum Independent Set 13
2.2.3 Variants and Special Cases of Matchings with Rectangles 15
2.3 Maximum Clique and the Klee's Measure 17
2.3.1 A dynamic data structure for the Klee's Measure 18
2.3.2 A divide and conquer algorithm for the Klee's Measure 20
2.3.3 Special cases and lower bounds 22
2.4 From Covering Sets to Orthogonal Polygons 24
2.4.1 Coverage Kernels: between Box and Polygon Coverings 24
2.4.2 Approximation algorithms for the Box Cover problem 26
2.5 Final Remarks 29

3 Depth Distribution of a Set of Boxes **30**

3.1 Introduction 30
3.2 Algorithms Computing the Depth Distribution 31
3.2.1 Worst-case Analysis 32
3.2.2 Adaptive Analysis 36
3.3 Lower Bounds for the Depth Distribution problem 38
3.3.1 Conditional lower bound for the case of sorted slabs 39
3.3.2 Conditional lower bound for sets of planar boxes 43
3.4 Discussion 45

4 Matching Colored Points with Rectangles **48**
4.1 Introduction 48
4.2 Hardness Results 49
4.2.1 Hardness of the MonoMRM problem 50
4.2.2 Extension to BicMRM, and special cases of MonoMRM 53
4.3 Approximation algorithms 58
4.3.1 A special case solvable in polynomial time 58
4.3.2 Approximation algorithms 61
4.4 Discussion 63

5 Computing Minimum Coverage Kernels **65**
5.1 Introduction 65
5.2 Hardness under Restricted Settings 66
5.2.1 Hardness of Minimum Coverage Kernel 66
5.2.2 Extension to Box Cover 73
5.3 Efficient approximation of Minimum Coverage Kernels 74
5.3.1 An Efficient Weight Index for a Set of Boxes 75
5.3.2 Approximation algorithms for Minimum Coverage Kernel. 79
5.4 Discussion 82

6 Conclusions and Further Research **83**

Bibliography **87**