Evaluating crusher system location in an open pit mine using Markov chains *

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Abstract

We present a methodology for addressing the problem of deciding the location of a third ore crusher to be installed at Chile's Chuquicamata mine, the largest open pit copper mine in the world. This approach evaluates a complex trade-off between minimizing operational cost and assuring production goal when the probability of equipment failure is considered. The heart of the methodology is a Markov chain model that incorporates failure as a randomness factor in determining the productivity of the crusher systems. We evaluate two alternative location configurations using stationary probabilities of a Markov chain model and the results were validated with a discrete-time simulation model. Goodness-of-fit indicators demonstrate the model's suitability for replacing the simulation model for calculating configuration productivity levels. In addition to successfully solving the decision problem, the Markov model generates insights into the relationships between the relevant variables which the discrete-time simulation is unable to provide, and does so without the latter's greater costs and complexities of modeling, solving and calibration. The methodology was applied at Chuquicamata in 2010 to choose the optimal location configuration based on the then-existing crusher system configuration and company data.

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1 Introduction

The Chuquicamata copper mine, a division of Chile's state-owned CODELCO mining company, is an open-pit operation located in the far north of the country that has been worked continuously since 1915. The natural growth of extraction operations at the mine has driven the evolution of a mixed system of material transport that currently consists of 15 loading units (power shovels and front loaders), 96 high-payload haul trucks, 2 primary (gyratory) crushers and 4 conveyor belt systems.

The material extracted at the property is separated into waste rock, which has no economic value and is hauled away to waste dumps, and ore, which contains the valuable minerals and is sent to the first in a series of processing plants whose final output is copper metal. In the initial process the ore is crushed and the output is transported to the concentrator plant on conveyor belts.

When the problem considered in this study arose, the configuration of the mine included two crusher units of equal capacity, one installed inside the pit and the other outside of it (see Figure 1). Both of them fed their output to the concentrator plant through 2 conveyor belts. Since crushers and belts, like any other equipment, are subject to failure, it was resolved that a third crusher should be installed to improve reliability.

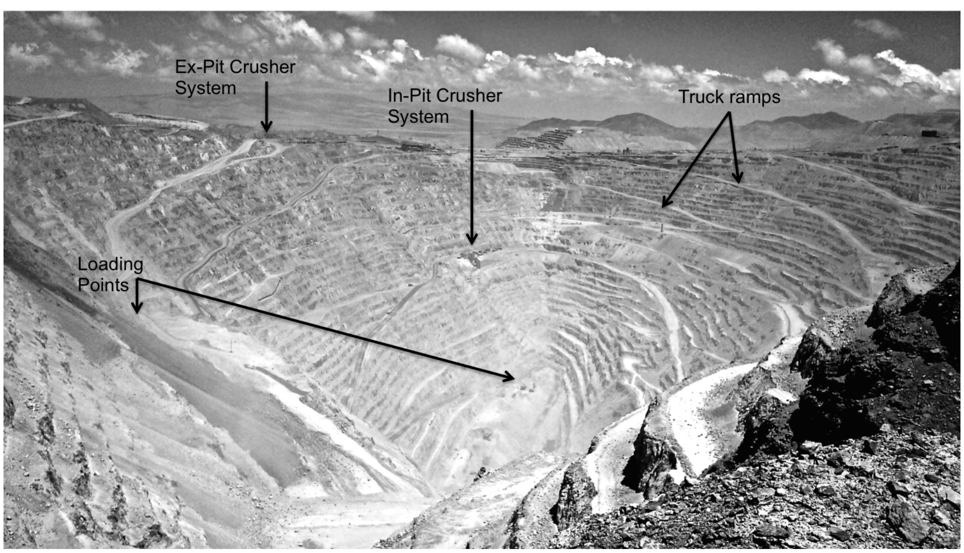

Figure 1: Current Chuquicamata operating configuration showing relative locations of equipment

For reasons beyond the scope of our enquiry, the new unit would have to be built alongside one of the existing crushers where the 2 units would share the same 2 conveyor belts. This posed a major decision problem defined by a complex interplay of factors including the failure rates of the belts and crushers, the operating costs of truck haulage versus conveyor belt transport, the capital costs of truck acquisition and the much greater expense involved in an in-pit (inside) crusher installation compared to an ex-pit (outside) one.

In broad terms, the decision problem was fundamentally one of evaluating the trade-off between cost and reliability, a decidedly non-trivial exercise that required a methodology which could take proper account of the element of randomness in the belt and crusher failures. The most commonly used method for studying the productivity of mine systems incorporating randomness is simulation. The first applications of the technique in this context date back to the 1960's in cases mainly involving underground mines (Rist [1962], Falkie and Mitchell [1963], Harvey [1964], Elbrond [1964] and Redmon [1964]). In the 1980's, with the growing use of dispatching systems in open-pit operations, simulation was employed to investigate the benefits of different computational algorithms (Chatterjee and Brake [1981], Wilke and Heck [1982], Tu and Hucka [1985] y Sturgul and Harrison [1987]). Subsequent advances in computer technology enabled the design of the first simulation model with graphical animations, which was used in the study of stockpiling strategies for minimizing costs at an iron mine [Baunach et al., 1989]. More recently, simulation has been applied to analyses of open-pit mine productivity under operating constraints [Awuah-Offei et al., 2003], new truck dispatching algorithms in real time [Wang et al., 2006] and autonomous vehicle dispatch algorithms [Saayman et al., 2006].

Yet despite its widespread application to stochastic mining problems, simulation is costly in time and resources spent on modeling, solving and calibration. Furthermore, it provides no insights into the problems it is used to study or the implications of the factors and parameters involved in them. Finally, practice, conclusions based on simulation are in practice based on the numerical results of multiple realizations. Solving mine operating problems analytically, on the other hand, affords a clearer understanding of the production system and how its configuration and operation can be optimized. The present article aims to demonstrate that Markov chains can contribute to the solution of complex problems, and in particular to the study of the productivity of a materials handling system as a function of its operating configuration. Thus, the proposed approach offers an alternative to the simulation of discrete events.

The theory behind Markov chains first appeared in a paper by Andréi

Markov in 1907, but attracted wider attention only after 1923 when the article was cited by Bernstein in his work extending the central limit theorem of probability theory [Basharin et al., 2004]. Since that time, Markov chains have been used to solve problems in a variety of fields. For example, the significant expansion of personal loans in the United States in the post-World War II era prompted Cyert and Thompson [1968] to develop a methodology for determining customer credit risk in the retail trade. Their approach was based on customers' historical behavior (ability to pay). The model they proposed divides the portfolio of a retail firm into multiple risk categories across which varying proportions of its customers are distributed. Another early application of Markov chains was a model proposed by Judge and Swanson [1962] for analyzing the evolution of the pig producer sector starting from a set of initial conditions. The study used data for 83 firms in Illinois over a period of 13 years. The strong simplifying assumptions of the model facilitated the paper's main purpose, which was to demonstrate the usefulness of the Markov chain concept for future studies in agricultural economics.

A paper by Burnham [1973] furthered the development of Markov chain models in agriculture. The author's model analyzed the effects of various policies in the southern Mississippi alluvial valley. In an application to manufacturing, given that perfectly reliable production systems are virtually non-existent, Abboud [2001] analyzed production and inventory in a system consisting of one machine producing a single good that is subject to breakdown. Output and demand rates were assumed constant while failure and repair rates were considered to be random. The author compared the Markov chain model to a simulation-based formulation, demonstrating that the two are strongly correlated.

The present article develops two stochastic models for solving the above-described problem of determining the best location for a third crusher. The first model uses discrete-time Markov chains while the second one uses a continuous-time formulation. Both versions are validated with a discrete-event simulation model.

2 The Problem

The production system at the Chuquicamata mine consists for present purposes of the following main items of equipment:

a) Power shovels. Used to load haul trucks inside the pit, these units are highly specialized pieces of equipment with a capacity of 100 metric tons and can load 5,000 metric tons per hour. Each shovel costs approximately US$20 million.

b) Haul trucks. These vehicles are designed for operation exclusively in mining applications and have a load capacity of 330 metric tons. They are built to order and sell for about US$4 million. The trucks are at their slowest when climbing fully loaded, and at their fastest when running empty on level ground.

c) Crushers. These are specialized units used to reduce the size of the ore-bearing rocks extracted from the pit. They are built to order and designed to fit the characteristics of a specific mine. Each crusher can process 6,200 metric tons per hour. Their basic selling price is US$15 million but to this must be added another $35 million for installation, mainly involving construction of the foundations. It is precisely this high installation cost that explains the significance of the present study.

d) Conveyor belts. The output of the crushers is transported by conveyor belts. Although more efficient than truck haulage, this method can only be used with ore that has already been crushed. At Chuquicamata, the in-pit and ex-pit crushers are each connected to the concentrator plant by a pair of parallel belts. The capacity of each belt is enough to carry the output of a crusher. A belt costs about US$2,500 per meter, but the final amount for in-pit belts is higher due to the tunnel construction costs.

e) Concentrator plant. This is a major installation and a strategic unit in the mining process, and is thus also a limiting factor in the mine's installed capacity. Concentrators consume huge quantities of energy and water, which in some cases constrains their construction and operation. A plant such as the one at Chuquicamata can today cost some US$ 8 million.

The problem we propose to solve covers the following mine processes:

P1: Loading of truck inside the pit by a shovel. There are about 15 loading units located at 4 different points around the pit.

P2: Transport of material with no economic value to the waste dump. This operation is carried out by about 80 trucks.

P3a: Transport of material with economic value to the in-pit crusher. After crushing it will be processed by the concentrator.

P3b: Transport of material to the ex-pit crusher, analogously to process P3a above. Processes P3a and P3b together use about 15 trucks.

P4: Transport of crushed material from the in-pit and ex-pit crushers to the concentrator plant by the conveyor belts.

As noted in the introduction, the original configuration includes 1 in-pit and 1 ex-pit crusher, each served by a pair of belts. The concentrator plant capacity is equal to the combined capacities of the two crushers and is therefore one-half that of the four belts combined. Since each belt has a capacity equal to that of a crusher, the crushers' individual capacities are the limiting factor on the capacity of each of the 2 crusher-conveyor "systems". A flow chart depicting the operating configuration of the equipment and processes described above is shown in Figure 2.

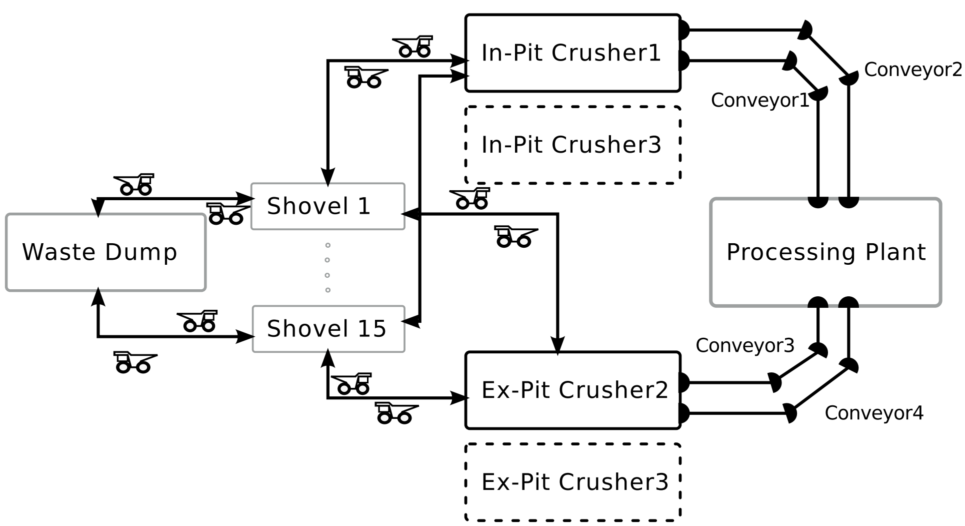

Figure 2: Operating configuration flow chart

The central elements in the third crusher decision problem are the randomness in the possibility of equipment failure and the cost considerations. Since the new crusher project did not contemplate additional conveyor belts, this meant that with two crushers installed at a single location, either inside or outside the pit, a belt failure would idle one of them. Originally, with just one crusher at each location, the failure of one belt did not affect production since the other belt could handle the unit's entire output.

If the crusher location alternatives are evaluated using the equipment's average availability rates, the optimal solution is to install the third crusher inside the pit. This is the case because the capacity of the two belts is sufficient to carry the output of the 2 crushers and belt transport is much cheaper than truck transport, the cost relationship being 1 to 5. Furthermore, if there were only 1 inside crusher, its failure would mean additional trucks would be needed to maintain production. With 2 in-pit units the number of trucks could be reduced.

On the other hand, a logistic factor that must be taken into account points to the opposite solution. The inside crusher location is 400 meters down into the 1,000-metre deep pit. For safety reasons, the trucks cannot descend when loaded so that material extracted by shovels operating above the 400-metre level could only be sent to the ex-pit crusher. This implies that a failure of a single outside crusher would cause a major interruption in global output. Thus, despite the greater operating expense, installing the third crusher outside the pit would ensure feed to the concentrator is more reliable.

In reality, however, there is significant variability in both crusher and belt failure rates, and this considerably complicates the third crusher location decision if average values are used. In what follows, therefore, we develop a methodology for addressing the problem of locating the third crusher that incorporates the randomness in the failure factors just described as well as the cost and logistic considerations.

3 The Methodology and the Model

Consider two production system configurations that include a third crusher. In the first configuration, denoted C1, there are two ex-pit crushers and one in-pit crusher while in the second configuration, called C2, there are two in-pit crushers and one ex-pit crusher.

At the location with two crushers, whether inside or outside the pit, each crusher will be uniquely associated with one of the two belts. There are thus two crushing systems, each one composed of a belt and a crusher. A system will be said to be operative if both crusher and belt are operating normally. If either component fails, the system it belongs to will be said to be inoperative.

At the location with just one crusher, whether inside or outside the pit, the crusher will be associated with both belts. In this case there is a single crushing system consisting of two belts and a crusher. The system will be said to be operative if its crusher and at least one of its belts are operating normally. If either the crusher or both belts fail, the system will be said to be inoperative.

From the foregoing it is immediately evident that the mechanical reliability of the crushing systems depends on two factors: the mechanical reliability of the crushers and belts and the number of belts associated with each crusher.

An analysis of the mechanical performance of each crusher and belt at Chuquicamata over a period of a year revealed that the distributions of both the times between failures and the repair times follow a negative exponential function. Under these conditions, standard mechanical reliability theory (Dhillon [2002]) allows us to represent a set of equipment connected in series or in parallel can be represented as a single mechanical system. In our case, the crushing systems consist of one crusher and either one or two belts, depending on the configuration. Letting λ be the failure rate and μ the repair rate for each system, the actual rates calculated for the two configurations C1 and C2 are set out in Table 1, where I1 and I2 are the in-pit systems and E1 and E2 the ex-pit systems.

	C1			C2		
Parameter	I1	E1	E2	I1	I2	E1
λ [1/hr]	0.081	0.079	0.079	0.085	0.102	0.073
μ [1/hr]	0.850	0.250	0.250	0.690	0.230	0.630

Table 1: Parameter values for time between failures (λ) and repair time (μ)

3.1 Discrete-time Markov chains

Based on foregoing, we can model the two configurations using Markov chains. In each case there will be 8 states representing all possible combinations of the 3 crushing systems and their 2 possible states, operative and inoperative (see Table 2). Variable I1(t) is defined as 1 if crusher I1 is operative at time t and 0 otherwise; the other variables I2(t), E1(t) and E2(t) are defined analogously.

A schematic of the Markov chain for C1 and C2 with the possible transitions for a given period is shown in Figure 3. In the depicted case, the graph is complete because all of the transitions between states are possible, although as we will see later when discussing the infinitesimal period case, some of them are very unlikely.

We now want to determine the percentage of time in a steady state that a production system configuration is in each state. Let $\mathbf{P}$ be the transition

	C1			C2		
State	I1	E1	E2	I1	I2	E1
S1	1	1	1	1	1	1
S2	1	1	0	1	0	1
S3	1	0	1	0	1	1
S4	0	1	1	1	1	0
S5	0	0	1	0	1	0
S6	0	1	0	1	0	0
S7	1	0	0	0	0	1
S8	0	0	0	0	0	0

Table 2: State definitions by operational configuration

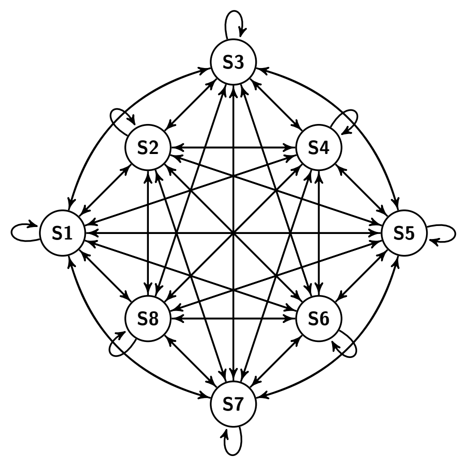

Figure 3: Discrete-time Markov chain graph

matrix whose elements p_{ij} are the probabilities of passing from state S_i to state S_j at time instant δt. If the Markov chain is irreducible and aperiodic, then $\pi = \pi \cdot \mathbf{P}$ where $\sum_j \pi_j = 1$.

Now let $\mathbf{P_1}$ be the transition matrix for configuration C1 and $\mathbf{P_2}$ the transition matrix for configuration C2. To calculate the transition probabilities p_{ij} we use the failure and repair rates of each crushing system. For example, the value of p_{12} for configuration C1 is calculated as follows:

$$p_{12} = P\{I1(t{+}\delta t) = 1, E1(t{+}\delta t) = 1, E2(t{+}\delta t) = 0 | I1(t) = 1, E1(t) = 1, E2(t) = 1\}$$

Since the crushing systems' respective failure probabilities are independent of each other,

$$p_{12} = P\{I1(t+\delta t) = 1|I1(t) = 1\} \cdot P\{E1(t+\delta t) = 1|E1(t) = 1\} \cdot P\{E2(t+\delta t) = 0|E2(t) = 1\} \quad (1)$$

where

$$P\{I1(t + \delta t) = 1|I1(t) = 1\} = e^{-\lambda_{I1}\delta t} \quad (2)$$

$$P\{E1(t + \delta t) = 1|E1(t) = 1\} = e^{-\lambda_{E1}\delta t} \quad (3)$$

$$P\{E2(t + \delta t) = 0|E2(t) = 1\} = 1 - e^{-\lambda_{E2}\delta t} \quad (4)$$

Substituting (2), (3) and (4) into (1), we have

$$p_{12} = e^{-\lambda_{I1}\delta t} \cdot e^{-\lambda_{E1}\delta t} \cdot (1 - e^{-\lambda_{E2}\delta t}) \quad (5)$$

If we chose a small value for δt of 1/13 of an hour, $p_{12} = 0.60\%$ and the rest of the p_{ij} can be calculated in a similar manner for the two configurations. Transition matrix $\mathbf{P_1}$ is given in Table 3 and transition matrix $\mathbf{P_2}$ in Table 4. Note that the 0% entries in the matrices represent probabilities that are small but non-zero.

$P_1 =$

98.18%	0.60%	0.60%	0.61%	0.00%	0.00%	0.00%	0.00%
1.86%	96.92%	0.01%	0.01%	0.00%	0.60%	0.59%	0.00%
1.86%	0.01%	96.92%	0.01%	0.60%	0.00%	0.59%	0.00%
6.29%	0.04%	0.04%	92.50%	0.56%	0.56%	0.00%	0.00%
0.12%	0.00%	6.21%	1.75%	91.32%	0.01%	0.04%	0.56%
0.12%	6.21%	0.00%	1.75%	0.01%	91.32%	0.04%	0.56%
0.04%	1.83%	1.83%	0.00%	0.01%	0.01%	95.68%	0.60%
0.00%	0.12%	0.12%	0.03%	1.73%	1.73%	6.13%	90.15%

Table 3: Transition matrix for configuration C1

$P_2 =$

98.02%	0.77%	0.64%	0.55%	0.00%	0.00%	0.01%	0.00%
1.77%	97.03%	0.01%	0.01%	0.00%	0.54%	0.64%	0.00%
5.14%	0.04%	93.53%	0.03%	0.52%	0.00%	0.74%	0.00%
4.69%	0.04%	0.03%	93.88%	0.62%	0.74%	0.00%	0.00%
0.25%	0.00%	4.47%	4.92%	89.58%	0.04%	0.04%	0.71%
0.08%	4.64%	0.00%	1.69%	0.01%	92.93%	0.03%	0.61%
0.09%	5.09%	1.68%	0.00%	0.01%	0.03%	92.58%	0.52%
0.00%	0.24%	0.08%	0.09%	1.61%	4.87%	4.43%	88.67%

Table 4: Transition matrix for configuration C2

Clearly both Markov chains have the ergodic property (aperiodic and connected). We can then solve the equation $\pi = \pi \cdot \mathbf{P}$ for each configuration.

The time percentages π^{C1} for configuration 1 and π^{C2} for configuration 2 are given in Table 5.

States	π^{C1}	π^{C2}
S_1	52.1%	55.3%
S_2	16.8%	24.3%
S_3	16.8%	6.9%
S_4	5.1%	6.5%
S_5	1.6%	0.8%
S_6	1.6%	2.8%
S_7	5.4%	3.0%
S_8	0.5%	0.4%

Table 5: Steady-state probabilities for discrete-time Markov chain, by configuration

3.2 Continuous-time Markov chains

If we assume δt is an infinitesimal time interval, the probability that two or three crushing systems change state in any one interval is negligible. The problem can then be modeled as a continuous-time Markov chain.

In analogous fashion to the discrete-time model, we define the 8 possible states of each production system configuration as a function of the 3 crushing systems' 2 operating states, operative (1) and inoperative (0). Simplifying the notation, the failure rate and the repair rate of crushing system i are denoted λ_i and μ_i, respectively. The continuous-time Markov chain can then be graphed as in Figure 4. Each node in the graph must be in equilibrium, that is, the inflow to any node must be equal to the outflow from it. The flow conservation equations for each node in the steady state are given in the following system:

$$\begin{aligned}
\pi_{(1,1,1)} \cdot (\lambda_1 + \lambda_2 + \lambda_3) &= \pi_{(0,1,1)} \cdot \mu_1 + \pi_{(1,0,1)} \cdot \mu_2 + \pi_{(1,1,0)} \cdot \mu_3 \\
\pi_{(1,1,0)} \cdot (\lambda_1 + \lambda_2 + \mu_3) &= \pi_{(0,1,0)} \cdot \mu_1 + \pi_{(1,0,0)} \cdot \mu_2 + \pi_{(1,1,1)} \cdot \lambda_3 \\
\pi_{(1,0,1)} \cdot (\lambda_1 + \mu_2 + \lambda_3) &= \pi_{(0,0,1)} \cdot \mu_1 + \pi_{(1,1,1)} \cdot \lambda_2 + \pi_{(1,0,0)} \cdot \mu_3 \\
\pi_{(0,1,1)} \cdot (\mu_1 + \lambda_2 + \lambda_3) &= \pi_{(1,1,1)} \cdot \lambda_1 + \pi_{(0,0,1)} \cdot \mu_2 + \pi_{(0,1,0)} \cdot \mu_3 \\
\pi_{(0,0,1)} \cdot (\mu_1 + \mu_2 + \lambda_3) &= \pi_{(1,0,1)} \cdot \lambda_1 + \pi_{(0,1,1)} \cdot \lambda_2 + \pi_{(0,0,0)} \cdot \mu_3 \\
\pi_{(0,1,0)} \cdot (\mu_1 + \lambda_2 + \mu_3) &= \pi_{(1,1,0)} \cdot \lambda_1 + \pi_{(0,0,0)} \cdot \mu_2 + \pi_{(0,1,1)} \cdot \lambda_3 \\
\pi_{(1,0,0)} \cdot (\lambda_1 + \mu_2 + \mu_3) &= \pi_{(0,0,0)} \cdot \mu_1 + \pi_{(1,1,0)} \cdot \lambda_2 + \pi_{(1,0,1)} \cdot \lambda_3 \\
\pi_{(0,0,0)} \cdot (\mu_1 + \mu_2 + \mu_3) &= \pi_{(1,0,0)} \cdot \lambda_1 + \pi_{(0,1,0)} \cdot \lambda_2 + \pi_{(0,0,1)} \cdot \lambda_3.
\end{aligned} \tag{6}$$

If we then add the condition that the π_j values are a probability distribution, so that $\sum_j \pi_j = 1$, we obtain a system of 8 linearly independent

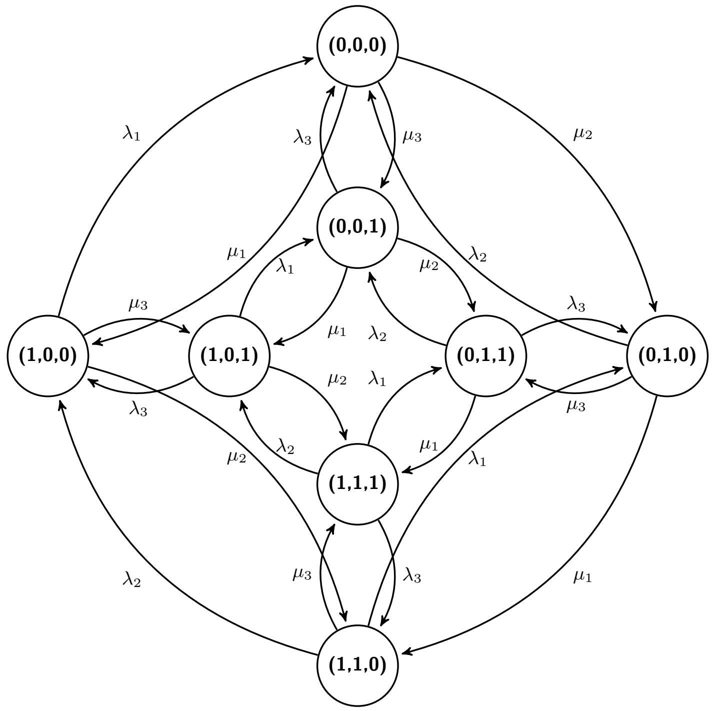

Figure 4: Continuous-time Markov chain graph

equations in 8 unknowns. The π value for each state represents the fraction of time the system is in the corresponding state over the long term. Solving the system yields the following π values:

$$\begin{aligned}
\pi_{(1,1,1)} &= \frac{\mu_1 \cdot \mu_2 \cdot \mu_3}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)(\lambda_3+\mu_3)}\\
\pi_{(1,1,0)} &= \frac{\mu_1 \cdot \mu_2 \cdot \lambda_3}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)(\lambda_3+\mu_3)}\\
\pi_{(1,0,1)} &= \frac{\mu_1 \cdot \lambda_2 \cdot \mu_3}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)(\lambda_3+\mu_3)}\\
\pi_{(0,1,1)} &= \frac{\lambda_1 \cdot \mu_2 \cdot \mu_3}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)(\lambda_3+\mu_3)}\\
\pi_{(0,0,1)} &= \frac{\lambda_1 \cdot \lambda_2 \cdot \mu_3}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)(\lambda_3+\mu_3)}\\
\pi_{(0,1,0)} &= \frac{\lambda_1 \cdot \mu_2 \cdot \lambda_3}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)(\lambda_3+\mu_3)}\\
\pi_{(1,0,0)} &= \frac{\mu_1 \cdot \lambda_2 \cdot \lambda_3}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)(\lambda_3+\mu_3)}\\
\pi_{(0,0,0)} &= \frac{\lambda_1 \cdot \lambda_2 \cdot \lambda_3}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)(\lambda_3+\mu_3)}.
\end{aligned} \tag{7}$$

From these results we derive the π^{C1} values for configuration C1 and the π^{C2} values for configuration C2, set out in Table 6. Since the transition period defined for the discrete-time Markov chain is relatively short, the solutions for the steady-state probabilities in the two cases are similar.

States	π^{C1}	π^{C2}
S_1	52.4%	55.6%
S_2	16.8%	24.3%
S_3	16.8%	6.8%
S_4	4.9%	6.4%
S_5	1.6%	0.8%
S_6	1.6%	2.8%
S_7	5.4%	3.0%
S_8	0.5%	0.3%

Table 6: Steady-state probabilities for continuous-time Markov Chain, by configuration

More generally, for a system of n components where each component is subject to failure at a rate λ_i and can be repaired at a rate μ_i, the probability of being in state $(k_1, k_2, \ldots, k_n)$ with $k_i \in \{0, 1\}$ is given by the following formula:

$$\pi_{(k_1,k_2,\ldots,k_n)} = \prod_{i=1}^{n} \frac{\lambda_i^{1-k_i} \cdot \mu_i^{k_i}}{(\lambda_i^{1-k_i}\mu_i^{k_i} + \lambda_i^{k_i}\mu_i^{1-k_i})}.$$

4 Model Validation

To validate our results we modeled configurations C1 and C2 using the discrete-event simulation software Promodel. The simulation period was one year, consisting of 8,610 hours discretized into intervals of 1/13 of an hour. For each configuration, 100 realizations were simulated.

The results of the simulations were contrasted with those obtained by the Markov model in Figure 5 for C1 and in Figure 6 for C2. The bars in the figures represent the ranges of the realizations while the markers indicate their expected values.

The goodness of fit between the value predicted by our Markov model and the average simulated realization value was calculated using the well-known coefficient of determination indicator R^2 (Draper and Smith [1981]). For C1, the result was $R^2 = 0.990$ while for C2 it was $R^2 = 0.981$.

5 Results

In our specification of the problem, the truck transport distance depends on three factors: first, the location of the shovels; second, the location of

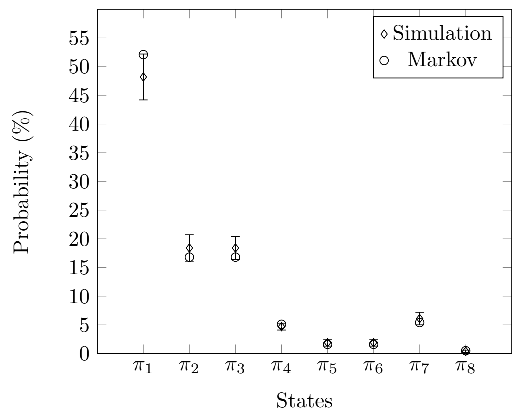

Figure 5: Markov versus discrete-event simulation, configuration 1

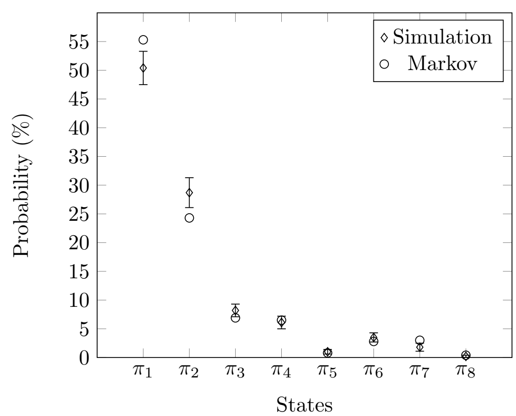

Figure 6: Markov versus discrete-event simulation, configuration 2

the crushers; and third, the crushers' mechanical availability. Thus, we associated a transport distance d_S with each state S. Since the shovels must be relocated periodically as ore extraction advances, d_S will vary over time.

To evaluate the best location of the planned third crusher, we defined a time horizon of 4 years. For each year, the distances associated with each state were calculated. The values so computed for each configuration are summarized in Table 7.

The average distance d is a function of the time percentage π and the distances d_S. This relationship determines the productivity of the system as a function of the operating configuration, the failure rates and the crushing

States	π^{C1}	Distance(m) d_S^{year1}	d_S^{year2}	d_S^{year3}	d_S^{year4}	π^{C2}	Distance(m) d_S^{year1}	d_S^{year2}	d_S^{year3}	d_S^{year4}
S_1	52.1%	4,310	3,207	3,666	3,366	55.3%	4,295	3,173	3,663	3,364
S_2	16.8%	4,310	3,207	3,666	3,366	24.3%	4,295	3,173	3,663	3,364
S_3	16.8%	4,310	3,207	3,666	3,366	6.9%	4,295	3,173	3,663	3,364
S_4	5.1%	6,328	5,203	6,094	4,395	6.5%	4,513	4,579	3,987	4,910
S_5	1.6%	6,328	5,203	6,094	4,395	0.8%	4,513	4,579	3,987	4,910
S_6	1.6%	6,328	5,203	6,094	4,395	2.8%	4,513	4,579	3,987	4,910
S_7	5.4%	4,910	4,677	3,990	4,912	3.0%	6,924	5,306	6,094	4,395
S_8	0.5%	6,928	6,673	6,418	5,941	0.4%	7,143	6,713	6,419	5,941

Table 7: Distance associated with states, by configuration and year

system repair rates. Therefore, using the continuous-time Markov chain solution π and the distance vector d_S, we calculated the average distance with the following formula:

$$d = \frac{\mu_1 \cdot \mu_2 \cdot \mu_3 \cdot d_{(1,1,1)} + \mu_1 \cdot \mu_2 \cdot \lambda_3 \cdot d_{(1,1,0)} + \mu_1 \cdot \lambda_2 \cdot \mu_3 \cdot d_{(1,0,1)} + \lambda_1 \cdot \mu_2 \cdot \mu_3 \cdot d_{(0,1,1)}}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)(\lambda_3 + \mu_3)} + \frac{\lambda_1 \cdot \lambda_2 \cdot \mu_3 \cdot d_{(0,0,1)} + \lambda_1 \cdot \mu_2 \cdot \lambda_3 \cdot d_{(0,1,0)} + \mu_1 \cdot \lambda_2 \cdot \lambda_3 \cdot d_{(1,0,0)} + \lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdot d_{(0,0,0)}}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)(\lambda_3 + \mu_3)}.$$

The truck cycle time was then identified from the values for the transport distance and the average velocity, the latter computed for each of the 4 years. The productivity of the transport system was defined as the average hourly tonnage per truck transported to the crusher. The truck requirement (number of trucks) was determined by two factors: first, the tonnage to be transported; and second, the transport system productivity as just defined. For a given tonnage, the greater the productivity the smaller the truck requirement, and vice versa. The truck requirement for each configuration and period along with the transport distance, cycle time and productivity are set forth in Table 8.

Year	Tonnage [kton]	Config.C1 distance [meters]	cycle time [min]	productivity [ton/hr]	trucks [unit]	Config.C2 distance [meters]	cycle time [min]	productivity [ton/hr]	trucks [unit]
Y1	44,816	4,524	36.24	497	19	4,408	34.03	529	17
Y2	44,941	3,471	29.76	605	15	3,393	28.71	627	15
Y3	44,817	3,900	30.57	589	15	3,779	29.69	606	15
Y4	44,569	3,549	28.80	625	15	3,561	28.81	625	15

Table 8: Truck requirement by operational configuration and year

As regards the economic evaluation, the operating cost items included wages, maintenance, fuel and truck tires, all of which are directly related to the truck requirement. Capital costs were also taken into account, although they are not affected by randomness and are therefore not directly incorporated into our model. These items included crusher installation and truck acquisition. Note particularly that installing a crusher inside the pit costs US$20 million more than doing so outside it. The operating costs were calculated for a 4-year time horizon and both cost categories were discounted at a rate of 8% following CODELCO's accounting practices. The resulting present values were MUS$ -123.953 for C1 and MUS$ -129.949 for C2 and are shown in Table 9, broken down by individual cost item. On the basis of these results, configuration C1 was recommended.

Economic Breakdown	Config.C1 [MUS$]	Config.C2 [MUS$]
Manpower	-12.193	-11.776
Fuel	-37.380	-35.011
Tires	-11.678	-11.440
Maintenance	-40.686	-39.314
Truck CAPEX	-7.407	0.000
Crusher Installation	-13.889	-32.407
Total NPV (8%)	-123.233	-129.949

Table 9: Economic evaluation by cost item and configuration

6 Conclusions

This paper presented a solution methodology for the decision problem faced by Chile's Chuquicamata open pit copper mine regarding the location of a third crusher. The two available alternatives were to situate it next to either one of the two existing units, one installed inside the pit and the other outside it. The alternatives posed a complex trade-off between the acquisition and operating costs of the crushers and associated equipment and their reliability or probability of failure.

The proposed methodology was built around a Markov chain model that incorporated failure as a randomness factor in determining the productivity of the crusher systems. Two different configurations, one each for the two location alternatives just described, were modelled using discrete-time simulation and the results were compared with those derived from the Markov chains approach. The goodness-of-fit measurements found that the coefficient of determination for the inside and outside installation configurations were 0.990 and 0.981, respectively, demonstrating that the Markov chain model could replace the simulation model for productivity calculations.

A continuous-time Markov model was then employed to determine how each relevant variable affected the behaviour of the crusher systems in the two configurations so that an economic evaluation determining which of the configurations was more cost effective could be made and one of the configurations recommended. This model also showed how the different variables interacted and was thus able to deliver insights into the dynamics of the crusher systems that a simulation could not provide, despite the latter approach's greater cost in terms of a series of factors relating to the many complexities of modeling, solving and calibrating discrete-time formulations.

Finally, the methodology was successfully applied to the Chuquicamata case in 2010 and the optimal of the two configurations was implemented at the mine. Some of the specific data presented in this study were changed for reasons of confidentiality.

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