

# Modelling age replacement policy under multiple time scales and stochastic usage profiles



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## ABSTRACT

This paper offers an original methodology to set multi-dimensional maintenance policies for machines whose aging processes require using multiple time scales. It can be considered a generalization of the traditional approach, that usually employs a unique time scale and sets a single age limit to carry out preventive maintenance actions. The methodology also considers situations in which a set of machines are operated using multiple usage profiles. We define usage profile as the relationship between the use of a machine in terms of one main time scale and another scale. In our case study the age of a mining haul-truck component can be best modeled as a combination of operating hours and load cycles since the last overhaul. We compare the results obtained with respect to using a single time scale policy. The comparison shows the importance of the bias in decision making that may arise due to incomplete modelling of the components' aging process.

## 1. Introduction

Failures in productive systems usually imply high costs for companies. These costs result from maintenance materials, labor and associated downtime (Pascual et al., 2008). As a way to reduce the number of failures and their economic consequences, maintenance actions can be set at planned epochs (Al-Najjar and Alsayouf, 2003). The most common criteria to set maintenance policies are reliability and expected cost per time unit (Barlow and Proschan, 1962; Frickenstein and Whitaker, 2003; Kordonsky and Gertsbakh, 1994). However, it is recognized that maintenance decision making often requires a combined use of several other criteria, such as availability, quality and/or safety. Triantaphyllou et al. (1997). The relative importance of those criteria is often difficult to assess by companies. For example, surface mining haul-trucks play an important role for transporting fragmented rocks from several mining sectors to processing plants or dumps. The age of each component (and the corresponding age limit for overhaul) can be estimated in different time (usage) scales. Operating hours are used for alternators and wheel motors, load cycles are used for hoppers, cumulated fuel consumption is used for diesel engines. However, for some components a single scale may not be sufficient for a complete description of the aging processes (Kordonsky and Gertsbakh, 1995; Ciampoli and Ellingwood, 2002; Fin, 1991; Vardar and Ekerim, 2007). The situation calls for setting age limits in several

time scales. If any limit is attained, a preventive maintenance action is triggered. Fig. 1 illustrates the situation.

Machines are often employed by different users under different operating conditions (Kordonsky and Gertsbakh, 1995).

Haul trucks are used on different haul routes and under different load profiles (Chapman, 2012; Darling, 2011). In these situations, the notion of system age becomes difficult to handle. To deal with the system reliability assessment it is necessary to refer to usage profiles (Kordonsky and Gertsbakh, 1995). Particularly, we define usage profile as the relation between the use of a machine in terms of one main time scale and another scale. For instance, Fig. 1 shows haul-truck 1 with usage profile 1 crossing first the usage limit in fuel consumption at the point A. This implies that this haul-truck is mostly used in terms of usage scale 3. On the other hand, haul-truck 2 with usage profile 2 is crossing first the usage limit in operating hours at the point B and, therefore, this second haul-truck is mostly used in usage scale 1.

Optimal maintenance policies using multiple time scales are often used by equipment manufacturers. These policies allow them to offer proper guidelines to customers on when and how to maintain equipment (Caterpillar, 2012). As an example, Table 1 shows a snapshot of the inspection and maintenance manual for a mining wheel loader. The equipment has to be checked according to operating hours or monthly, whichever occurs first. On the other hand, the manufacturer based on an appropriate maintenance strategy may also minimize expected costs

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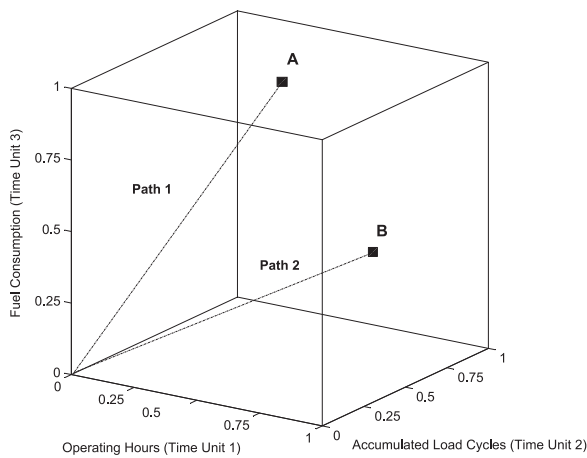


Fig. 1. Age evolution of a set of mining haul-trucks in three usage scales.

**Table 1**  
Snapshot of an inspection and maintenance manual for a mining wheel loader, adapted from Caterpillar (2012).

Interval	Subsystem/Component	Action
Every 250 operating hours or monthly	Battery	Clean
	Engine Crankcase Breather Belt	Inspect/Adjust/Replace
	Brake Accumulator	Check
	Differential and Final Drive	Test
	Oil Level	
	Braking System	Obtain
	Cooling System Coolant Sample (Level 1)	
	Engine Oil Sample	
	Drive Shaft Spline (Center)	Lubricate
	Quick Coupler	

of warranty service and maximize post-sale service profits (Manna et al., 2007).

The maintenance performance is perceived according to the perspective of the decision maker (Pintelon and Van Puyvelde, 1997). Fig. 2 illustrates the previous notion by means of the maintenance

value tree in a company. For example, a contract manager of mining haul-trucks is usually interested in performance indicators such as production and financial budget performance (long term objectives). On the other hand, maintainers are more interested in increasing uptime and decreasing labor and material costs as well as decreasing operational risks (operational objectives). From this, it can be noticed the common lack of alignment in terms of maintenance objectives between operational levels and strategic decision making levels within an organization.

To our knowledge there are no multiple-time scale models that set maintenance policies in a multiple criteria decision context. We are also unaware of any methodology to support maintenance decision making when the different actors involved do not have necessarily the same decision drivers in this context. The current paper intends to overcome such shortcoming. We propose a novel methodology to set maintenance policies suited to machines with multiple aging processes and multiple usage profiles. The rest of this article is organized as follows: first we investigate into the state of the art on multiple time-scale reliability modelling and maintenance planning. We then offer a for-purpose methodology to solve the problem under analysis. This is illustrated by means of a case study involving a fleet of haul-trucks of the mining industry. Finally, some conclusions are provided.

## 2. Literature review

We find in the literature two main approaches to handle reliability problems using multiple time scales: multivariate distribution models (Downton, 1970; Newby and Barker, 2006; Pievatolo and Ruggeri) and composite scale models (Barlow and Proschan, 1962; Frickenstein and Whitaker, 2003; Kordonsky and Gertsbakh, 1994). There are several reasons to prefer the second approach over the first. Firstly, a component failure usually depends on or can be influenced by multiple covariates or measures of time (Ciampoli and Ellingwood, 2002; Fin, 1991; Vardar and Ekerim, 2007). Secondly, such models usually offer better failure prediction capability. This is because the information capacity carried by the composite variable is larger than that carried by any individual variable or measure. Thus, the failure prediction based on the composite scale model is, in general, more accurate than that based on the single variable model. Furthermore, composite scale models are mathematically more tractable with respect to multivariate distribution models since the first ones are univariate models (Jiang and Jardine, 2006). However, Frickenstein and Whitaker (2003) also argue that composite scale models do not completely address the problems of maintenance in the original multiple scales. Particularly, it

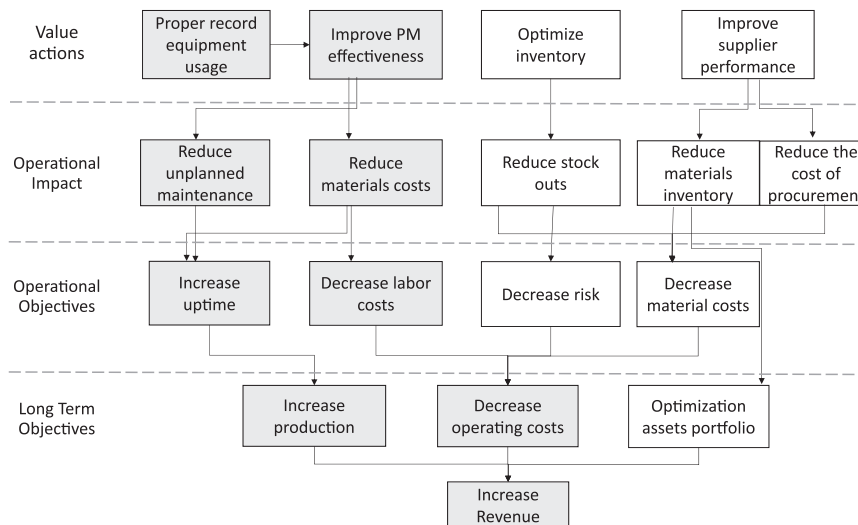


Fig. 2. Maintenance value tree adapted from Ahlmann (1984).

might be required to translate policies developed in composite scales to policies in the original scales. Taking into consideration the above, this work focuses on composite scale models. [Kordonsky and Gertsbakh \(1995\)](#) recognise that systems reliability depends strongly on the conditions of their use that include environmental factors, loading conditions and maintenance conditions. In particular, they deal with the problem of choosing the best time scale for calculating system age when machines operate under different conditions. Consider  $\Omega$  a set of all possible operating conditions which is in fact a multidimensional random parameter. When a particular machine starts operating, it chooses an operating condition  $\theta_j$  with probability  $p_j$ ,  $\theta_j \in \Omega$ . Consider also  $t_1$  and  $t_2$  two distinct observable and measurable lifetime scales for each operating machine. Both scales are related to a linear damage accumulation process  $t$ . Suppose a linear usage profile  $\theta$  with both time scales  $t_1$  and  $t_2$  (this assumption is widely used in the literature and makes sense in case of cyclic usage in fatigue life experiments ([Miner, 1945](#))). Then, it is possible to define “the equivalent age” as:

$$t = t_1 + \alpha t_2 \tag{1}$$

where

$$t_2 = \theta t_1 \tag{2}$$

and the weighting constant  $\alpha$  ( $0 \leq \alpha < \infty$ ) is set in some optimal way. The time scale  $U$  is called the ideal monitoring scale if for all  $\theta_j \in \Omega$  and all  $u > 0$ ,  $u_0 > 0$ , the probability  $\Pr\{U > u + u_0 | U > u_0; \theta_j\}$  is independent on  $\theta_j$ . This means

$$\Pr\{U > u + u_0 | U > u_0; \theta_j\} = \Pr\{U > u + u_0 | U > u_0\} \tag{3}$$

However, an ideal monitoring scale exists only when the operating conditions  $\theta_j$  in  $\Omega$  vary in a relatively small range. In this context, the authors propose the best monitoring scale as a time scale  $K$  in the family  $\{t_1 + \alpha t_2\}$  which in some sense would be the most closely related to the ideal scale  $U$  under the principle of minimal coefficient of variation. Specifically, [Kordonsky and Gertsbakh](#) consider  $K$  as family of time scales of type:

$$K = (1 - a)t_1 + \alpha t_2 \tag{4}$$

for  $a \in [0, 1]$  denoting  $\alpha = (1 - a)/a$ . We remark that this convex combination of both scales  $t_1$  and  $t_2$  leads to the addition of terms which are not expressed in an equivalent time unit but merely weighs them.

[Duchesne and Lawless \(2002\)](#) derive a rank-based estimator more efficient and robust than the traditional minimum coefficient of variation estimator ([Kordonsky and Gertsbakh, 1995](#)). Let  $t_1$  represent a fixed value of chronological time and let  $t_2(u)$  represent the value of usage or exposure at time  $t_1$ . Define  $\theta = \{t_2(u), 0 \leq u \leq t_1\}$  as usage profile. The ideal time scale is a function  $\Phi[\bullet, \bullet]$  of  $t_1$  and  $\theta(t_1)$  such that the conditional survivor function of the chronological time at failure,  $T_1$ , given the whole usage history,  $\theta = \lim_{t_1 \rightarrow \infty} \theta(t_1)$ , can be written as

$$\Pr\{T_1 > t_1 | \theta\} = G(\Phi[t_1, \theta(t_1)]) \tag{5}$$

where  $G(\bullet)$  is positive, 1-1 decreasing function. Both  $\Phi[\bullet, \bullet]$  and  $G(\bullet)$  can be fully specified given a vector of parameters, or one of  $\Phi[\bullet, \bullet]$  or  $G(\bullet)$  can be left arbitrary. A semiparametric inference is developed for the parameters of a time scale  $\Phi[\bullet, \bullet]$  when  $G(\bullet)$  is left unspecified.

[Jiang and Jardine \(2006\)](#) refer to the relative importance of scales in multiple time scales models. Consider a linear model described by:

$$t = \sum_{i=1}^n \alpha_i t_i \tag{6}$$

where

$$\sum_{i=1}^n \alpha_i = 1 \tag{7}$$

and  $0 \leq \alpha_i \leq 1$ . Using the constraint condition  $\mu_i = \mu_0$  to replace the constraint condition (7), where  $\mu_i$  is the mean of  $t$  and  $\mu_0$  is a constant,

the parameters  $\alpha_i$  can be estimated by minimizing the sample variance. Evaluating the value of  $t$  at  $\{t_i = \mu_i\}$  as follows

$$\mu_t = \mu_0 = \sum_{i=1}^n \alpha_i \mu_i \tag{8}$$

the scale  $t_i$  has a relative contribution value  $\nu_i = \alpha_i \mu_i / \mu_0$ . The larger the  $|\nu_i|$ , the more important the scale  $t_i$ . Consequently, it is possible to rank all the time scales of the linear model based on the values of  $|\nu_i|$ .

Regarding maintenance problems using a single time scale, [Barlow and Proschan \(1962\)](#) set an age replacement policy that balances the failure costs of a component during operation against the costs of planned replacements. Consider  $R(t) = 1 - F(t)$  the survivor function, and  $F(t)$  the failure function. Let  $C_c$  be the cost corresponding for each failed component that is replaced. This includes all costs resulting from the failure and its replacement. Assume that failures are instantly detected and replaced. Let  $C_p (< C_c)$  be the cost related to each nonfailed item that is replaced. Define the mean time between interventions  $MTBI$ , either preventive or corrective, such that if the replacement interval is set at  $t = t^p$

$$MTBI(t^p) = \int_0^{t^p} R(t) dt \tag{9}$$

Then, the expected maintenance cost per unit of time in the long run is

$$c(t^p) = \frac{C_p R(t^p) + C_c F(t^p)}{MTBI(t^p)} \quad t^p > 0 \tag{10}$$

where  $c(t^p)$  admits a unique and finite minimum  $t^{p*}$  if the failure rate is continuous and strictly increasing to infinity.

[Kordonsky and Gertsbakh \(1994\)](#), following the model by [Barlow and Proschan \(1962\)](#), address maintenance problems using multiple time scales. According to the equation (4), define the long-run average cost for a fixed  $a$  as

$$c_a(t^p) = \frac{C_p + (C_c - C_p)F_a(t^p)}{MTBI_a(t^p)} \quad t^p > 0 \tag{11}$$

The dimension of  $c_a(t^p)$  is maintenance cost per time unit in the scale  $t_a$ . As commented by [Frickestein and Whitaker \(2003\)](#), this kind of model does not completely address the problem of maintenance in the original multiple scales. The authors deal with this by converting  $c_a(t^p)$  into a cost function with dimension cost per main time unit (e.g., calendar time) of the form:

$$c_1(t^p) = \frac{c_a(t^p)MTBI_a(t^p)}{MTBI_1(t^p)} \quad t^p > 0$$

where  $MTBI_1(t^p)$  is the mean time between interventions in time unit 1. Then, the optimal policy minimizes over both  $a$  and  $t^p$ . Note also that an optimal policy as function of  $C_p/(C_c - C_p)$ , can lead to nonnested policies when varying the previous ratio ([Frickestein and Whitaker, 2003](#)).

[Kordonsky and Gertsbakh \(1994\)](#) do not consider the different usage profiles possible for a set of machines aging in multiple time scales. In this regard, [Frickestein and Whitaker \(2003\)](#) propose maintenance policies in two time scales along several linear profiles. When machines age along  $m$  linear usage profiles, let  $\mathcal{M} = \{(t_j, \theta_j t_j): 0 < t_j < \hat{t}_j\}$  (for  $j=1, \dots, m$ ) be a composite policy given a maintenance time vector  $\hat{t}^p = (\hat{t}_1, \hat{t}_2, \dots, \hat{t}_m)$ . Maintenance actions are performed on machines of usage profile  $j$  upon failure or when their usages reach  $\hat{t}_j$ , whichever occurs first. The policy  $M$  is such that if it prescribes maintenance action for a machine of a particular age, it also prescribes maintenance action of any older machine; if it does not prescribe maintenance action of the machine, it does not prescribe maintenance action of any younger machine either. The above mentioned implies that  $\theta_j \hat{t}_j \geq \theta_i \hat{t}_i$  and  $\hat{t}_j \leq \hat{t}_i$  for  $i < j$  in  $\{1, \dots, m\}$ . The cost of policy  $M$  with corresponding maintenance time vector  $\hat{t}^p$  by taking the weighted average of costs of the profile-specific policies  $\hat{t}_j$  is as follows

$$c(\hat{t}^p) = \sum_{j=1}^m c(\hat{t}_j^p) p_j \quad \hat{t}_j^p > 0; \quad j = 1, \dots, m. \quad (12)$$

Frickenstein and Whitaker (2003) set as many ages limits as usage profiles are considered. Consequently, the proposed model does not lead to set centralized maintenance decision making for fleets being used in multiple usage profiles.

Finally, Zhang (1994) develops a novel variant of the previous models by a bivariate replacement policy  $(t^p, N)$  under which the system is maintained at age  $t^p$  or at the time of  $N$ th failure, whichever occurs first. The problem is to choose an optimal maintenance policy  $(t^p, N)^*$  such that the long-run average cost per time unit is minimized. Because the total lifetime of the repairable system is limited, the minimum of the long-run average cost per time unit exists. The author also proves that under some mild conditions an optimal policy  $(t^p, N)$  is better than the optimal policy  $N^*$  or the optimal policy  $t^{p*}$ .

### 3. Proposed methodology

Let  $t$  be equivalent age of a machine according to Equation (1). For reasons of simplicity, consider the aging process as a linear combination of two scales  $t_1$  and  $t_2$ . Also from Equation (1) let  $\alpha$  be a fixed parameter that transforms time unit 2 ( $tu_2$ ) into equivalent time unit ( $etu$ ). Assume that the machine has linear usage profile  $\theta_j$  with probability  $p_j$ . Define a rectangular region  $\mathcal{R} = \{(t_1, t_2): 0 \leq t_1 \leq t_1^p, 0 \leq t_2 \leq t_2^p\}$  in which the values  $\{t_1^p, t_2^p\}$  are the usage limits of the machine that define an optimal maintenance policy. Maintenance actions are performed if the usage profile of the machine crosses the region  $\mathcal{R}$  or upon failure, whichever occurs first.

Let  $\hat{t}_j$  be the limit equivalent age when the machine with usage profile  $\theta_j$  crosses the boundary of region  $\mathcal{R}$  such that:

$$\hat{t}_j = \min \{t_j(\theta_j, t_1^p); t_j(\theta_j, t_2^p)\} \quad (13)$$

$$t_j(\theta_j, t_1^p) = t_1^p (1 + \alpha \theta_j) \quad (14)$$

$$t_j(\theta_j, t_2^p) = t_2^p \left( \frac{1}{\theta_j} + \alpha \right) \quad (15)$$

From Eqs. (13)–(15) one can see that  $\hat{t}_j$  triggers preventive maintenance actions if the usage profile  $\theta_j$  crosses the usages limits  $t_1^p$  or  $t_2^p$ , whichever occurs first. In this regard, Fig. 3 shows the linear usage profile  $\theta_1$  of a machine crossing the points A and B. Both points define the machine's invariant equivalent ages  $t_A(\theta_1, t_1^p)$  and  $t_B(\theta_1, t_2^p)$ , respectively. As  $t_A < t_B$ , then  $\hat{t}_1 = t_A(\theta_1, t_1^p)$ . This implies that the usage profile  $\theta_1$  first crosses the region  $\mathcal{R}$  through the usage limit  $t_1^p$ . Similarly, Fig. 4 shows the linear usage profile  $\theta_2$  of machine ( $\theta_2 > \theta_1$ ) to illustrate the case  $\hat{t}_2 = t_C(\theta_2, t_2^p)$ , when the usage profile  $\theta_2$  first crosses the region  $\mathcal{R}$  through the usage limit  $t_2^p$ .

Let  $c$  be the expected maintenance cost per equivalent age unit,

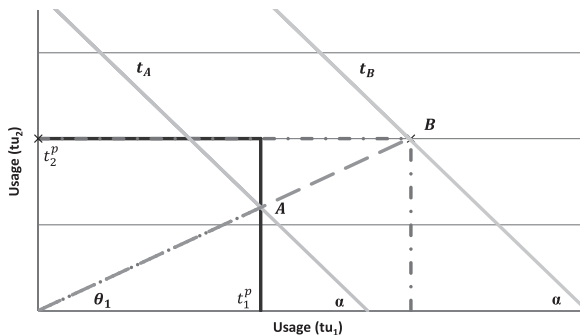


Fig. 3. Limit equivalent age when  $t_1^p$  sets the usage limit of machine.

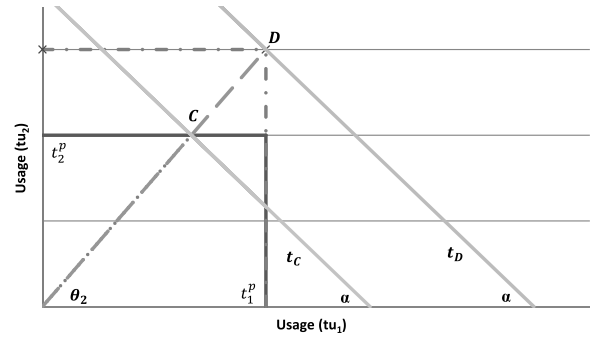


Fig. 4. Limit equivalent age when  $t_2^p$  sets the usage limit of machine.

$$c(t_1^p, t_2^p) = \sum_{j=1}^J \left( \frac{R_j(\hat{t}_j) C_p(\hat{t}_j) + F_j(\hat{t}_j) C_c(\hat{t}_j)}{MTBI(\hat{t}_j)} \right) p_j \quad (16)$$

Particularly, two types of optimal policies can be defined. Policy I minimizes the expected maintenance cost per time unit 1:

$$c_1(t_1^{p(I)}, t_2^{p(I)}) = \sum_{j=1}^J \left( \frac{R_j(\hat{t}_j) C_p(\hat{t}_j) + F_j(\hat{t}_j) C_c(\hat{t}_j)}{MTBI_1(\hat{t}_j)} \right) p_j \quad (17)$$

and policy II minimizes the expected maintenance cost per time unit 2:

$$c_2(t_1^{p(II)}, t_2^{p(II)}) = \sum_{j=1}^J \left( \frac{R_j(\hat{t}_j) C_p(\hat{t}_j) + F_j(\hat{t}_j) C_c(\hat{t}_j)}{MTBI_2(\hat{t}_j)} \right) p_j \quad (18)$$

where  $MTBI_1$  and  $MTBI_2$  are the mean times between interventions in time unit 1 and 2, respectively, of the following forms:

$$MTBI_1(\hat{t}_j) = \frac{MTBI(\hat{t}_j)}{(1 + \alpha \theta_j)} \quad (19)$$

$$MTBI_2(\hat{t}_j) = \frac{MTBI(\hat{t}_j)}{\left( \frac{1}{\theta_j} + \alpha \right)} \quad (20)$$

Note that as the number of machines increases, one can expect a larger number of usage profiles. In such cases using stochastic usage profiles can be recommended to set better policies. For this purpose, let the machine's usage profile  $\theta$  have a continuous density function  $f(\theta)$ . Then, rewrite Equations (16)–(18), respectively as follows:

$$c(t_1^p, t_2^p) = \int \left( \frac{R(\hat{t}(\theta)) C_p + F(\hat{t}(\theta)) C_c}{MTBI(\hat{t}(\theta))} \right) f(\theta) d\theta \quad (21)$$

$$c_1(t_1^{p(I)}, t_2^{p(I)}) = \int \left( \frac{R(\hat{t}(\theta)) C_p + F(\hat{t}(\theta)) C_c}{MTBI_1(\hat{t}(\theta))} \right) f(\theta) d\theta \quad (22)$$

$$c_2(t_1^{p(II)}, t_2^{p(II)}) = \int \left( \frac{R(\hat{t}(\theta)) C_p + F(\hat{t}(\theta)) C_c}{MTBI_2(\hat{t}(\theta))} \right) f(\theta) d\theta \quad (23)$$

Note also that other optimization criteria can be easily considered in our proposed methodology by replacing  $C_p$  and  $C_c$  for those relevant weighting factors for a particular optimization case. For example, consider the minimization of expected system unavailability. In this case, the relevant weighting factors are the times to repair in preventive and corrective actions, respectively,  $T_p$  and  $T_c$ . Then, the policy I that minimizes the expected system unavailability per time unit 1 is:

$$D_1(t_1^{p(I)}, t_2^{p(I)}) = \int \left( \frac{R(\hat{t}(\theta)) T_p + F(\hat{t}(\theta)) T_c}{MTBI_1(\hat{t}(\theta))} \right) f(\theta) d\theta \quad (24)$$

and the policy II that minimizes the expected system unavailability per time unit 2 is:

**Table 2**  
Case study parameters.

j	$\theta$	$n_j$
1	0.5	11
2	0.9	24
3	1.3	37
4	1.7	51
5	2.1	32

$$D_2(t_1^{p(i)}, t_2^{p(i)}) = \int \left( \frac{R(\hat{t}(\theta))T_p + F(\hat{t}(\theta))T_c}{MTBI_2(\hat{t}(\theta))} \right) f(\theta) d\theta \quad (25)$$

with  $MTBI_1$  and  $MTBI_2$  given by Eqs. (19) and (20), respectively.

**4. Case study**

Consider an original equipment manufacturer of mining haul-trucks that provides maintenance service in five open-pit sites of a single mining company. There are  $n_j$  mining haul-trucks with usage profile  $j$  in site  $j$  for  $j = 1, \dots, 5$ . Table 2 shows the parameters for this case study. Let

$$p_j = \frac{n_j}{\sum_{j=1}^5 n_j}$$

be the probability that a haul-truck is being used with usage profile  $\theta_j$  expressed in terms of average number of load cycles per hour. A major component is usually overhauled regarding the use of haul-truck in thousands of operating hours since the last overhaul. In this traditional approach, the optimization is done by minimizing the expected maintenance cost per operating hour. This is an operational level decision made with the purpose of reducing the operating budget of the company. The aging process of a major component is associated to thousands of accumulated operating hours ( $t_1$ ) and also depends on thousands of load cycles ( $t_2$ ). In this context, the above approach does not consider the existence of a second usage scale of the haul-truck, and therefore, the existence of different usage profiles. Then, the mining company agrees on a novel two-dimensional maintenance policy for the component under analysis. This considers both deterministic and stochastic usage profiles. The company also recognizes that minimization of maintenance costs per Ton (or load cycle) is an interesting goal.

The company pays the equipment manufacturer  $C_p = 1$  and  $C_c = 4$  for preventive and corrective actions, respectively. Table 3 shows the failure data where  $i$  represents the  $i$ th failure of the set of components. Fig. 5 illustrates the failure data according to the usage profiles.

**Table 3**  
Failure data adapted from Paredes (2012).

i	$t_1$	$t_2$
1	13.5	17.0
2	20.0	10.2
3	16.3	8.4
4	15.9	21.0
5	16.8	15.3
6	18.1	30.0
7	15.1	26.0
8	19.1	25.5
9	17.6	22.6
10	12.1	24.9
11	13.3	23.1
12	13.9	13.2
13	16.0	34.3
14	16.8	27.4
15	13.3	6.9

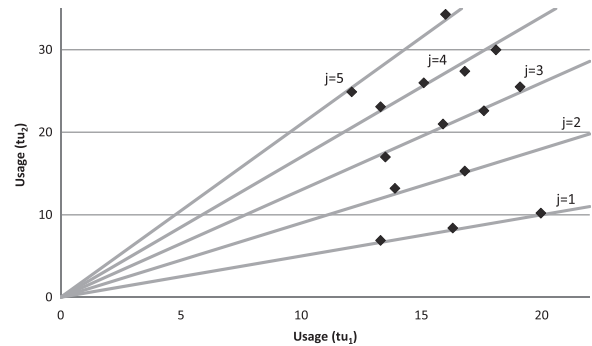


Fig. 5. Failure data according to the usage profiles of components.

**4.1. Component reliability model**

To determine the optimal parameter  $\alpha$  that relates both scales  $t_1$  and  $t_2$ , several statistical techniques can be used on the reliability data (Kordonsky and Gertsbakh, 1995). We choose the method of maximum coefficient of determination. In this method, we obtain all parameters by maximizing the coefficient of determination. For that, a sensitivity analysis with respect to  $\alpha$  is performed (Fig. 6) using the failure data shown in Table 3. We obtain a Weibull reliability model in multiple time scales,  $R(t)$  with parameters  $\beta_1 = 4.8$ ,  $\eta_1 = 35$  etu and  $\alpha = 0.80$  etu/tu<sub>2</sub>; with  $t$  as set in Eq. (1). Fig. 7 illustrates the difference between the Weibull fit in the case that we use single time scale (in operating hours). The results are:  $\beta_0 = 7.9$ ,  $\eta_0 = 16.9$  tu<sub>1</sub>. The use of combined time scales has allowed an improvement of the coefficient of determination from 95.2% for  $\alpha=0$  to 99.3% for  $\alpha=0.80$ .

**4.2. Optimal policies using deterministic usage profiles**

Fig. 8 shows the result of applying the proposed methodology under Policy I of minimum cost per hour by solving Eq. (17) and under Policy II of minimum cost per load cycles based on Eq. (18). Particularly, under both policies I and II the lives of components with usage profiles  $j=1,2$  are limited by their usage in thousands of operating hours ( $tu_1$ ). Components with usage profiles  $j = 3, 4, 5$  are limited by their usage in thousands of load cycles ( $tu_2$ ). Note that the usage profile  $\theta$  of a component crosses the maintenance region  $\mathcal{R}$  through the usage limit in operating hours or load cycles depending on its slope and the optimal usages limits set in both scales.

The difference of applying to the case study the traditional policy by resolving Eq. (10), instead of the proposed policies I and II by resolving Eqs. (17) and (18), respectively, is illustrated in Fig. 9 and Table 4. Note that under traditional policy, the optimal usage limit of the set of components which is supposed to minimize the expected maintenance cost per hour is at  $t_1^{p(0)} = 11.5$  tu<sub>1</sub>. However, under Policy I that also minimizes the usage profile-weighted maintenance cost-per-hour, it is set maintenance actions at equivalent age of 21.3 tu<sub>1</sub>. On the other hand, to compare the results of both maintenance strategies, the traditional

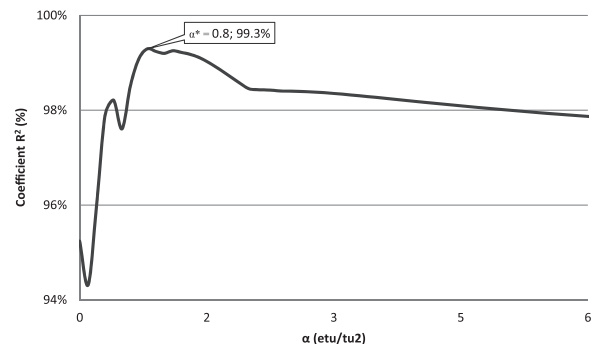


Fig. 6. Optimal parameter  $\alpha$  by the maximization of coefficient  $R^2$ .

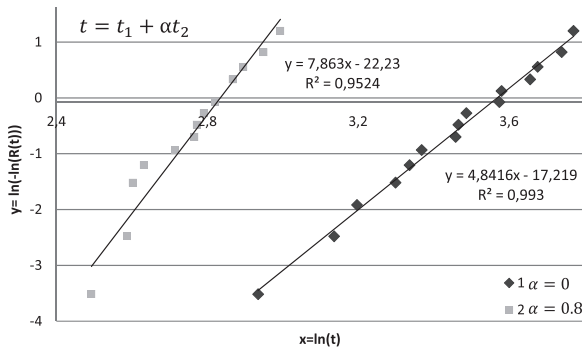


Fig. 7. Weibull diagram for  $\alpha=0$  and  $\alpha=0.8$ .

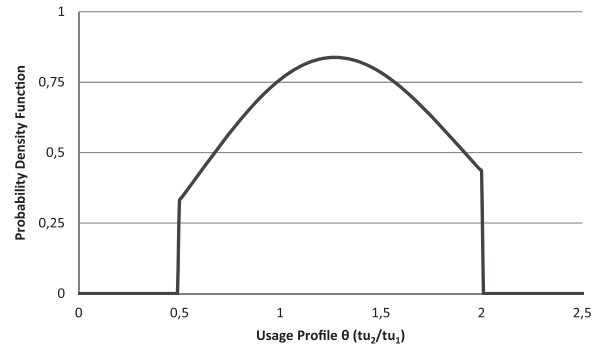


Fig. 10. Probability density function of usage profile  $\theta$ .

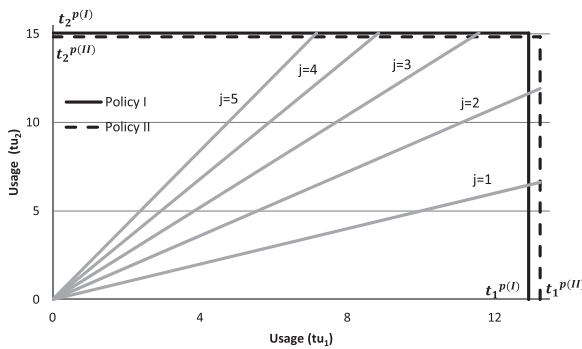


Fig. 8. Optimal policies of minimum cost using discrete usage profiles.

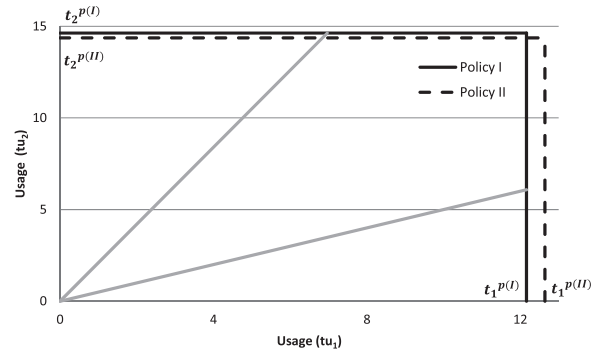


Fig. 11. Optimal policies of minimum cost using continuous usage profiles.

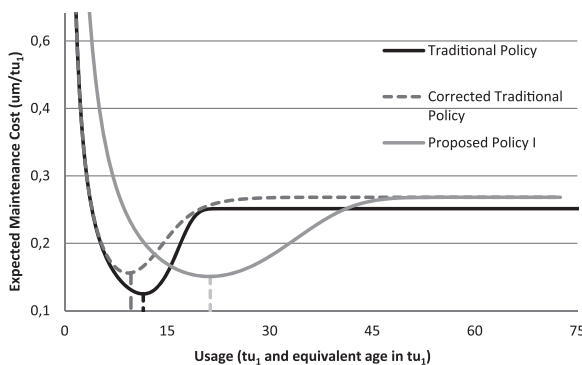


Fig. 9. Comparison between the traditional and proposed policies.

Table 4  
Resulting maintenance policies under a traditional approach and proposed policy I.

Case	$t_1^{p(I)}$	$t_2^{p(I)}$	$t_1^{p(II)}$	$t_2^{p(II)}$
Base case	11.5	–	–	–
Case study	12.9	15	13.2	14.8

policy has to be corrected since it considers neither the degradation of components due to load cycles, nor the different usage profiles. This means to consider  $\beta_1, \eta_1, t_1 = t_1^{p(0)}$  and  $t_2 = \theta_j t_1^{p(0)}$  for all  $j$  in Eq. (17) to compare with Policy I, or in Equation (18) to compare with Policy II. Then, the proposed policies I and II result in cost savings respect to the corrected traditional policy of 11% and 9%, respectively.

#### 4.3. Optimal policies using stochastic usage profiles

Consider that the most significant component of the haul-trucks has

Table 5  
Resulting maintenance policies for case studies 4.3 and 4.4.

Decision criterion	$t_1^{p(I)}$	$t_2^{p(I)}$	$t_1^{p(II)}$	$t_2^{p(II)}$
Maintenance Cost	12.2	14.6	12.6	14.4
System Unavailability	13.2	16.1	13.8	15.6

usage profile  $\theta$  with density function  $f(\theta)$  for  $\theta \in [0.5, 2.1]$ . Consider also that  $f(\theta)$  follows a Weibull distribution with parameters  $\beta_a = 2.6$  and  $\eta_a = 1.5$ , see Fig. 10. Fig. 11 and Table 5 show the optimal usages limits of the component after applying the proposed methodology. This is Policy I of minimum cost per hour obtained from Eq. (22) and Policy II of minimum cost per load cycles based on Eq. (23). Then, policies I and II result in cost savings respect to the corrected traditional policy of 6% and 5%, respectively. Consider the corrected traditional policy by replacing  $\beta_1, \eta_1, t_1 = t_1^{p(0)}$  and  $t_2 = \theta t_1^{p(0)}$  for all valid  $\theta$  in Eq. (22) to compare with Policy I, or in Eq. (23) to compare with Policy II.

#### 4.4. Optimal policies using system unavailability as criterion

Finally, we consider optimal maintenance policies in the case that the equipment manufacturer and the client agree on a maintenance service which minimizes the expected system unavailability. For this purpose, consider the times to repair  $T_p = 12$  and  $T_c = 36$ . Once again we consider that the major component has usage profile  $\theta$  with the density function  $f(\theta)$  that follows a Weibull distribution with parameters  $\beta_a = 2.6$  and  $\eta_a = 1.5$ , see Fig. 10. Fig. 12 and Table 5 show the optimal usages limits of the component after applying the proposed methodology. This is Policy I of minimum expected unavailability per hour by solving Eq. (24), and Policy II of minimum expected system unavailability per load cycles based on Eq. (25). Both policies I and II result in unavailability savings of 4% with respect to the corrected traditional policy. Once again consider the corrected traditional policy by replacing  $\beta_1, \eta_1, t_1 = t_1^{p(0)}$  and  $t_2 = \theta t_1^{p(0)}$  for all valid  $\theta$  in Eq. (24) to compare with Policy I, or in Eq. (25) to compare with Policy II.

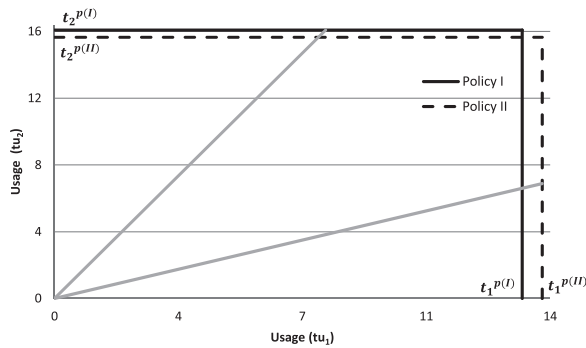


Fig. 12. Optimal policies of minimum downtime using continuous usage profiles.

From Table 5, it can be seen that the proposed methodology provides a decision support system for setting maintenance policies. Operational and strategic decision-making levels may study and decide different maintenance policies.

## 5. Closure

This paper proposes a novel decision-making aid framework to set up optimal maintenance policies for machines whose aging processes can be modeled using multiple usage scales as well as setting centralized decision making for fleets with multiple usage profiles. Furthermore, the proposed methodology encompasses a decision support system that provides integrated maintenance decision making using a set of criteria reflecting different organizational levels objectives for a given company. This set of maintenance criteria can be easily customized in order to attend specific needs of companies operating in different contexts, thus increasing the likelihood of obtaining better long term results (e.g., increase in production, decrease in operating costs) in asset-intensive sectors such as mining, power generation and distribution, and military defense. The methodology is applied to a case study involving an original equipment manufacturer of mining haul-trucks that provides maintenance service in several open-pit sites of a single mining company. The results show that the proposed methodology provides maintenance policies leading to significant cost savings with respect to the traditional approach that only considers one time scale. On the other hand, in this paper we have considered a fixed, known distribution  $f(\theta)$  whereas future research should consider it in a class, e.g. Beta distribution in an interval, and use data to estimate its parameters (and, possibly, experts' opinion if a Bayesian approach is chosen).

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