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Analysis of the Exponential Linear Pulse in Baseband Digital Communication Systems

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***Abstract*—In digital communication systems the use of filters that fulfill the first Nyquist criterion (Nyquist-I), guarantees that a sequence of pulses will not be affected by inter-symbol interference (ISI) if the receiver samples signals at optimum and uniformly spaced instants. In this manuscript the Nyquist-I pulse, called exponential linear pulse (ELP), is evaluated in the time and frequency domain using different evaluation tools and compared with other existing pulses. The eye diagram is simulated in presence of time sampling errors and the approximated average bit error rate (BER) is computed considering the ideal and truncated version of the pulses. Finally, the spectral energy distribution and spectral regrowth of the pulses are presented for comparison purposes. Numerical results show that the ELP outperforms other existing pulses in terms of the eye diagram opening and BER, evaluated for various symbol timing errors and roll-off factors. However, the good performance of the ELP in the time domain is at the expenses of introduction of out-of-band radiation compared to the traditional Raised Cosine pulse; therefore, a trade-off between BER and out-of-band radiation exists.**

***Keywords*—Bit error rate (BER), inter-symbol interference (ISI), Nyquist first criterion, Exponential Linear Pulse (ELP).**

I. Introduction

The current trends in wireless communications systems, lead us to design better spectral efficient digital communication systems, as data rate requirements are conservatively doubling each year [1]. Transmitting signals at high transmission rates introduces inter-symbol interference (ISI), which impacts negatively the communication performance. The design of ISI free signals in band-limited channels was a problem considered by Nyquist [2], [3]. Nyquist first criterion (Nyquist-I) guarantees that a sequence of pulses will be ISI-free by sampling signals at optimum and uniformly spaced instants. Additionally to the ISI-free prerequisite, pulse shaping filters have to show low sensitivity to timing errors. In practical receivers, it has been verified that the presence of errors in the sampling period may cause a deviation respect to sampling points; hence, symbol timing errors are produced and due to this effect the bit error rate (BER) increases. Therefore, it is expected that the tails of the filter must decay rapidly outside the pulse interval in order to eliminate the undesired effects of timing errors [3]–[5]. Further, the implementation of the filter in practical systems has to consider a finite or limited version of its impulse response which is accomplished through a truncation process, adding new challenges to the design of Nyquist-I filters because its frequency spectrum will be affected [6].

To overcome the prior concerns, several Nyquist-I pulses have been proposed. The most popular ISI-free Nyquist-I pulse for distortion-less transmissions is the traditional raised cosine (RC) pulse [2], [3]. The RC pulse has been proposed by the 3rd Generation Partnership Project (3GPP) as the pulse shaping filter to be implemented at the user equipment (UE) and at the base station (BS), for transmission and reception [7]. Besides the RC pulse, other Nyquist-I pulses with lower BER and wider eye openings have been proposed. The authors in [8] presented a family of ISI-free polynomial pulses that can have an asymptotic decay rate of t^{-k} for any integer value of k, whereas in [3], [9], [10] a new ISI-free linear combination of pulses with different decay rates has been proposed. Authors in [11] proposed a family of ISI-free pulses with senary piece-wise polynomial frequency characteristic. Other ISI-free pulses denoted as piece-wise flipped-exponential (PFE) have also been proposed in [12]. In [2], the authors presented several families of Nyquist-I pulses by using a parametric approach, adding more degrees of freedom in the design of ISI-free pulses. Furthermore, the proposed pulses in [2] incorporate reviewed pulses as special cases. To the authors best knowledge, the analysis of time-limited pulses in literature is very scarce, for example in [12], the practical implementation details of the truncated realization of the PFE is discussed and compared. Meanwhile, in [6] the design and implementation of the truncated version of the improved Nyquist filters are studied.

This paper focuses on the numerical evaluation performance of the linear exponential pulse (ELP) in digital com-

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munication systems. Furthermore, it provides details regarding the effect of the impulse response truncation, which leads to the implementation in practical systems. The ELP was originally derived and optimized for peak-to-average power ratio (PAPR) reduction in single carrier orthogonal frequency division multiple access (SC-FDMA) [13]. Also, the ELP was used for mitigate the inter-carrier interference (ICI) in orthogonal frequency-division multiplexing (OFDM) systems [14]. The optimum ELP derived in [13] is evaluated and the results are compared with other recently proposed pulses in terms of the average BER, distribution of spectral energy and spectral regrowth, for various roll-off factors and symbol timing errors. Further, the eye diagram of the ELP will be evaluated and compared with the traditional RC pulse.

The remainder of the paper is organized as follows: in Section II, the ELP and its impulse response is presented. Section III describes the tools used to evaluate the pulses, and the results are presented for the ideal and time-limited version. Section IV shows the frequency characteristic and presents the spectral energy distribution for the ELP and the other evaluated pulses. Finally, conclusions are reported in Section V.

II. Exponential Linear Pulse

Nyquist first criterion for distortion-less transmissions within a band limited channel expressed the time-domain is defined as follows [2], [3], [9]

$$h\left(kT\right) = \begin{cases} 1, k = 0 \\ 0, k = \pm 1, \pm 2, \pm 3 \pm 4, \ldots, \end{cases} \tag{1}$$

where $h(t)$ is the impulse response, and T is the symbol period. The term ELP, is used to refer to a hybrid filter, composed of two main elements: a finite impulse response (FIR) filter and a Nyquist-I pulse [13]. The explicit time-domain expression of the ELP is given by

$$h(t)_{\text{ELP}} = e^{-\pi(\beta/2)(t/T)^2} \times \frac{\sin\left(\pi t/T\right)}{\left(\pi t/T\right)} \times \frac{\sin(\pi\alpha t/T)}{(\pi\alpha t/T)}. \tag{2}$$

The term α is the roll-off factor defined for $0 \leq \alpha \leq 1$, and *t/T* is the normalized time. The coefficient β is defined for $0 \leq \beta \leq 1$, and it is used to control amplitude of the central lobe and the side-lobes of the pulse. It can be seen that the hybrid ELP is the product of a exponential expression and the linear pulse (LP) for $n = 1$. The parameter n defines a family of pulses in the frequency domain, and for each value a new pulse is generated with an arbitrary excess of bandwidth α [2]. The $sinc(t/T)$ function in (2) is considered as a FIR filter. As we increase the value of β the amplitude of the side lobes of the pulse are decreased, but at expenses of having a narrower central lobe. Therefore throughout this manuscript, $\beta = \{0.5, 1\}$ will be used. The ELP with $\beta = 0.5$ is considered as a pulse with balance between the amplitude of the side lobes and the central lobe and the ELP with $\beta = 1$ as the pulse with the faster extinction and smaller side lobes. The family of pulses defined in (2), evaluated for $\lim_{t\to 0}(\cdot)$, and for any value of α, and β is always equal to one. Additionally, the ELP, evaluated for $k = \pm 1, \pm 2, \pm 3, \pm 4, \ldots$, and for any value of α and β is always equal to zero. Therefore, the family of pulses described in (2) fulfills Nyquist's ISI-free criterion, previously described in (1).

Figs. 1 and 2 present the impulse response of the $\text{ELP}_{\beta=1}$, $\text{ELP}_{\beta=0.5}$, the sinc parametric linear combination pulse (SPLCP), proposed in [5], and the traditional RC pulse with a roll-off factor equal to 0.35 and 0.50, respectively for comparison purposes. It can be seen that the impulse response of ELP pulse in both cases decays rapidly, having amplitude values close to zero from $t/T = 2$ and beyond. The behavior of the SPLCP pulse is similar to the $\text{ELP}_{\beta=1}$ for both values of α. The impulse response of the $\text{ELP}_{\beta=1}$ has smaller relative magnitudes in its side-lobes compared to the $\text{ELP}_{\beta=0.5}$ and the traditional RC pulse, but at expenses of having a narrower central lobe compared with both, $\text{ELP}_{\beta=0.5}$ and RC. Consequently, robustness against ISI, and a larger eye opening are expected for the $\text{ELP}_{\beta=1}$. Through the experiments realized it could be noticed that the trend is the same for other roll-off factors and values of β. Because the side-lobes of the ELP are rapidly diminished the undesired effects of jitter should decrease, and the pulse would be less sensitive to timing errors, resulting in a lower BER [2], [3], [8], [10].

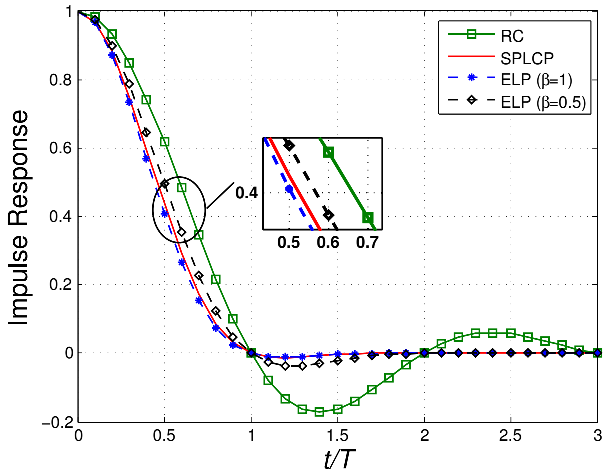

Fig. 1. Impulse response of the $\text{ELP}_{\beta=1}$, $\text{ELP}_{\beta=0.5}$, SPLCP and the RC pulse for an excess bandwidth of $\alpha = 0.35$.

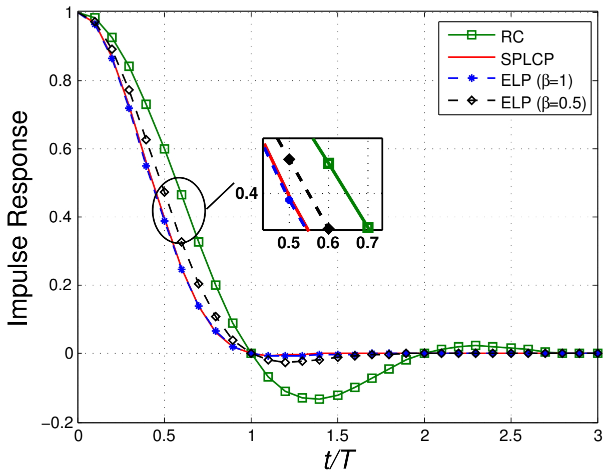

Fig. 2. Impulse response of the $\text{ELP}_{\beta=1}$, $\text{ELP}_{\beta=0.5}$, SPLCP and the RC pulse for an excess bandwidth of $\alpha = 0.5$.

III. PERFORMANCE EVALUATION

In this section the performance of the ELP is evaluated by using two main practical tools. One of the tools used to analyze the performance of the ELP is the eye diagram. The eye diagram is a useful tool to visually evaluate the susceptibility of the transmission systems due to ISI [15]. The eye diagrams were generated by superimposing 10^5 individual binary antipodal signaling sequences, and by inserting two consecutive symbol periods as was reported in the literature [10], [12]. Binary phase shift keying (BPSK) was the binary antipodal digital modulation used. In Figs 3 and 4 the eye diagrams of the ELP$_{\beta=1}$ and RC pulse with α equal to 0.35 and 0.5 are plotted respectively. The results show that ELP exhibits a much wider eye opening than the RC pulse for both values of α; therefore, a lower BER is expected because the ELP diminishes the undesired effects of jitter.

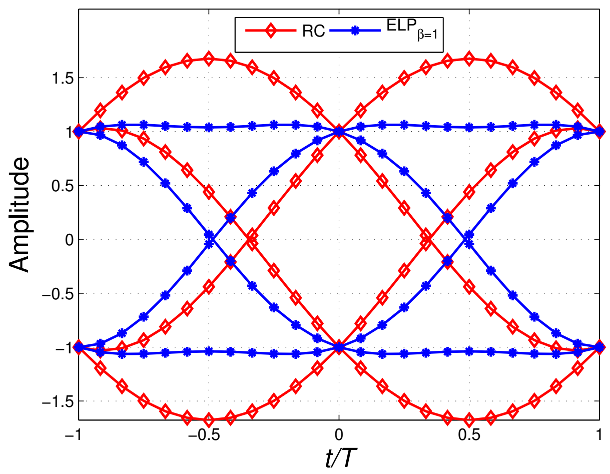

Fig. 3. Average envelopes of the eye diagram of the ELP$_{\beta=1}$ and RC pulses for an excess bandwidth of $\alpha = 0.35$.

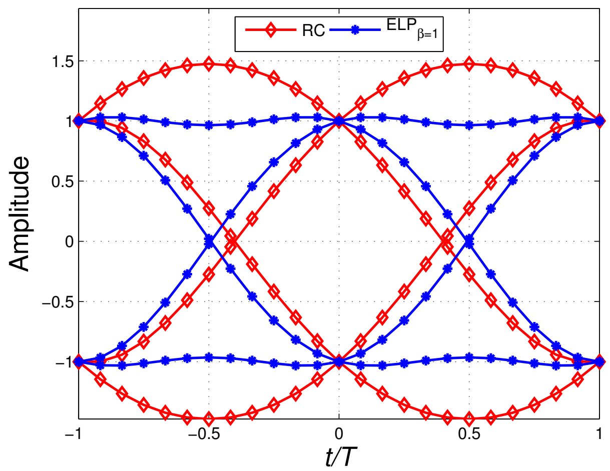

Fig. 4. Average envelopes of the eye diagram of the ELP$_{\beta=1}$ and RC pulses for an excess bandwidth of $\alpha = 0.5$.

The next step of the evaluation process involves the calculation of the BER in the presence of time sampling errors, different roll-off factors for the ideal and truncated response of the pulses. The BER is certainly the most important metric of performance in digital communication systems because it considers the effects of additive Gaussian white noise (AWGN), distortion, synchronization, among other physical phenomenon. To determine the BER of the ELP in the presence of time-sampling errors, the truncated Fourier series proposed in [16] is used as follows

$$P_e(\eta) = \frac{1}{2} - \frac{2}{\pi} \times \sum_{\substack{m=1 \\ m=\text{odd}}}^{M} \left\{ \frac{\exp\left(-m^2 w^2/2\right)\sin(mwg_0)}{m} \right\} \prod_{\substack{k=N_1 \\ k\neq 0}}^{N_2} \cos(mwg_k). \tag{3}$$

The latter truncated Fourier series is the de facto evaluation metric used in the literature to determine BER in the presence of symbol timing errors [2], [5], [10], [12]. The expression given in (3) assumes AWGN in the channel, and BPSK binary antipodal signaling. In (3), $w = 2\pi/T_f$ is the period used in the series, M represents the number of coefficients used to converge the truncated Fourier series, whereas N_1 and N_2 indicate the number of interfering symbols before and after the transmitted symbol, respectively. For the expression $g_k = p(kT + \eta)$, $p(t)$ is the ISI-free pulse evaluated at the receiver at time kT plus symbol timing error η. The parameters used to make the truncated Fourier series converge comply with the parameters used in [4], [5], [12], [17], and are depicted in Table I. The BER for the complete impulse response in the presence of time-sampling errors was determined for the ELP, the RC pulse, the recently proposed PFE [12], as well as the recently evaluated SPLCP [5] and the improved parametric linear combination pulse (IPLCP) [4]. To the authors best knowledge, the PFE, SPLCP, and IPLCP are the pulses with the best BER performance found in the literature. A signal-to-noise ratio (SNR) of 15 dB has been assumed, while 2^{10} interfering symbols were generated.

TABLE I. SYSTEM SIMULATION PARAMETERS

Parameter	Value
M	100
T_f	60
Interfering Symbols	2^{10}
Channel	AWGN
Digital Modulation	BPSK
Signal-to-noise ratio	15 dB
Symbol timing errors, t/T	$\pm 0.05, \pm 0.10, \pm 0.20$
Roll-off factor, α	$0.25, 0.35, 0.5$

Ideal Impulse Response

Table II presents the obtained results for the different pulses and scenarios. In general, the ELP for $\beta = 0.5$ and $\beta = 1$ performed well for different roll-off factors and timing offsets compared to the other pulses. Further, the ELP$_{\beta=1}$ has the smallest error rates for $\alpha = 0.25$ and $\alpha = 0.35$, among all of the evaluated timing offsets compared to the RC and the pulses proposed in [4], [5], [12]. This behavior is consistent with the wider eye opening of the ELP and because its side-lobes vanish rapidly compared to the other pulses. For the case of α equal to 0.50, SPCLP has the best performance for all timing offsets, even though the the ELP$_{\beta=1}$ was very close. To clarify this point, it can be seen from Fig 2 that the SPLCP, beside having a wider central lobe, its tails have smaller amplitude than the ELP$_{\beta=1}$. So the contributions of the adjacent pulses in the compute of BER would be smaller. In general, it can be seen that increasing the value of the time-sampling error, for a fixed excess bandwidth α, the BER increases for all pulses. Further, for a fixed sampling time error, a larger BER is obtained with a smaller roll-off factor due to the increase of the tails of the impluse response.

TABLE II. BIT ERROR PROBABILITY FOR 2^{10} INTERFERING SYMBOLS AND SNR$= 15dB$

α	Pulse	$t/T = 0.05$	$t/T = 0.10$	$t/T = 0.20$
0.25	RC	8.2189e-08	2.8184e-06	9.7472e-04
	PFE	4.5110-e08	7.9603e-07	1.9140e-04
	SPLCP	1.3870e-08	4.4260e-08	2.4530e-06
	IPLCP	1.5232e-08	5.8295e-08	3.7486e-06
	ELP ($\beta = 0.5$)	1.7221e-08	8.0914e-08	6.0494 e-06
	ELP ($\beta = 1$)	**1.3444e-08**	**3.9914e-08**	**2.1059e-06**
0.35	RC	3.9253e-08	5.4021e-07	1.0129e-04
	PFE	3.0130e-08	3.3720e-08	5.6450e-05
	SPLCP	1.3380e-08	3.9432e-07	2.0438e-06
	IPLCP	1.4476e-08	5.0372e-08	3.0138e-06
	ELP ($\beta = 0.5$)	1.6175e-08	6.8666e-08	4.7534e-6
	ELP ($\beta = 1$)	**1.3323e-08**	**3.8656e-08**	**1.9911e-06**
0.50	RC	2.4134e-08	1.8580e-07	2.0878e-05
	PFE	1.8921e-08	1.1615e-07	1.3072e-05
	SPLCP	**1.2867e-08**	**3.4079e-08**	**1.5437e-06**
	IPLCP	1.3437e-08	3.9955e-08	2.0958e-06
	ELP ($\beta = 0.5$)	1.4595e-08	5.1589e-08	3.1092e-06
	ELP ($\beta = 1$)	1.3174e-08	3.6974e-08	1.8220e-06

TABLE III. BIT ERROR PROBABILITY FOR THE TRUNCATED PULSE VERSION IN [-5.5 t/T; 5.5 t/T] FOR 2^{10} INTERFERING SYMBOLS AND SNR$= 15dB$

α	Pulse	$t/T = 0.05$	$t/T = 0.10$	$t/T = 0.20$
0.25	RC	8.2158e-08	2.8157e-06	9.7340e-04
	PFE	4.2680e-08	6.8789e-07	1.5600e-04
	SPLCP	1.3867e-08	4.4257e-08	2.4534e-06
	IPLCP	1.5232e-08	5.8295e-08	3.7486e-06
	ELP ($\beta = 0.5$)	1.7222e-08	8.0914e-08	6.0494e-06
	ELP ($\beta = 1$)	**1.3444e-08**	**3.9914e-08**	**2.1059e-06**
0.35	RC	5.9982e-08	1.3886e-06	3.9043e-04
	PFE	2.8305e-08	2.9460e-07	4.6706e-05
	SPLCP	1.3380e-08	3.9428e-08	2.0426e-06
	IPLCP	1.4476e-08	5.0372e-08	3.0138e-06
	ELP ($\beta = 0.5$)	1.6175e-08	6.8666e-08	4.7534e-06
	ELP ($\beta = 1$)	**1.3323e-08**	**3.8656e-08**	**1.9911e-06**
0.50	RC	3.9721e-08	5.4880e-07	1.0213e-04
	PFE	1.8071e-08	1.0345e-07	1.1500e-05
	SPLCP	**1.2864e-08**	**3.4066e-08**	**1.5397e-06**
	IPLCP	1.3436e-08	3.9954e-08	2.0958e-06
	ELP ($\beta = 0.5$)	1.4595e-08	5.1589e-08	3.1092e-06
	ELP ($\beta = 1$)	1.3174e-08	3.6974e-08	1.8220e-06

Time-limited impulse response

The Nyquist pulses are digitally implemented in practical systems using a truncated version of the impulse response that result in a same or lower BER performance, because amplitude values beyond the truncation are not considered [18]. Due to the truncation process, the pulse does not fulfill the first Nyquist criterion anymore and as consequence even if the transmitter and receiver are perfectly synchronized errors due to ISI will arise. Table III shows the obtained results for different Nyquist pulses truncated at [-5.5 t/T, 5.5 t/T] and for different roll-off factors following the procedure detailed in [12]. The performance of all pulses improves compared to the pulses with the ideal impulse response, but except for those with side-lobes that rapidly become extinguished and the truncation operation does not affect them. It is desirable that the tails of the pulse decay rapidly in order to allow quick truncation, and decrease the number of coefficients that represent the digital filter.

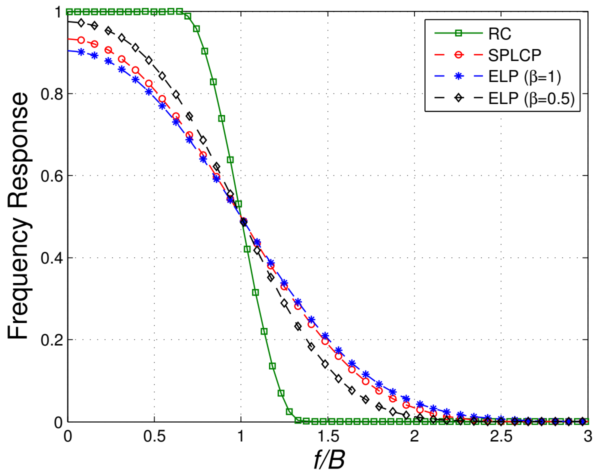

Fig. 5. Frequency response of $ELP_{\beta=1}$, $ELP_{\beta=0.5}$ and RC pulses for $\alpha = 0.35$ considering the complete impulse response .

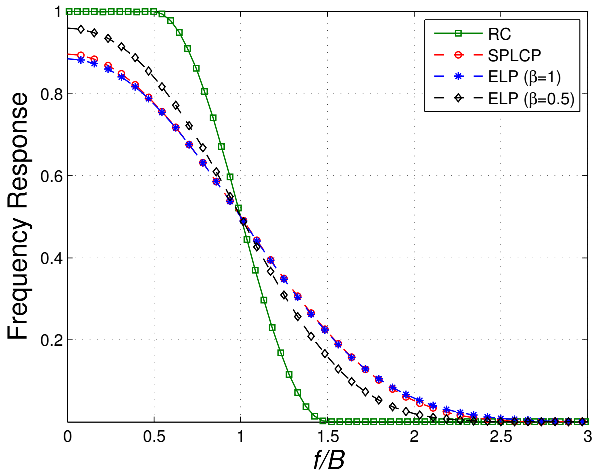

Fig. 6. Frequency response of $ELP_{\beta=1}$, $ELP_{\beta=0.5}$ and RC pulses for $\alpha = 0.5$ considering the complete impulse response .

IV. FREQUENCY CHARACTERISTIC

To design a modulation filter, the designer needs to consider carefully both representations of the signal, in the time and frequency domain. Thus, in the designing process, time or frequency representation has to be compromised at the expenses of the other, and it is a particular application that defines the final compromise [18].

Ideal Frequency Response

Figs. 5 and 6 show the frequency response of the ELP$_{\beta=0.5}$, ELP$_{\beta=1}$, SPLCP, and the traditional RC pulse for roll-off factors 0.35 and 0.5 respectively considering the ideal impulse response. It can bee seen from the frequency characteristic that both ELP pulses will introduce additional out-of-band radiation compared to the RC pulse, which has a stop band value of

TABLE IV. DISTRIBUTION OF THE SPECTRAL ENERGY IN PERCENTAGE FOR THE ELP$_{\beta=1}$, ELP$_{\beta=0.5}$, SPLCP, AND RC FOR $\alpha = 0.25, 0.35, 0.5$ CONSIDERING THE IDEAL IMPULSE RESPONSE.

α	Pulse	$0 \leq f/B < 1-\alpha$	$1-\alpha \leq f/B < 1+\alpha$	$1+\alpha \leq f/B$
0.25	RC	74.81	25.16	0.03
	SPLCP	64.37	24.93	10.71
	ELP (β=0.5)	67.66	24.96	7.39
	ELP (β=1)	62.15	24.90	12.95
0.35	RC	65.10	34.88	0.02
	SPLCP	56.34	34.95	8.71
	ELP (β=0.5)	59.60	34.93	5.47
	ELP (β=1)	54.83	34.91	10.27
0.5	RC	50.53	49.46	0.02
	SPLCP	43.27	49.50	7.22
	ELP (β=0.5)	46.54	49.50	3.97
	ELP (β=1)	42.87	49.36	7.77

$f/B = 1+\alpha$. This out-of-band radiation can be interpreted as a transfer of energy from the low spectral region ($f/B \leq 1-\alpha$) to the high spectral region ($f/B \geq 1+\alpha$) resulting in a more open receiver eye in the eye diagram and a faster extinction of the tails in the impulse response, giving better results in therm of BER [19]. Therefore, there is a trade-off between out-of-band radiation and lower BER due to ISI.

Table IV presents the energy contained in different intervals for the normalized frequency considering the pulses with the best BER performance and the RC for comparison purposes for different roll-off factors. The energy of the RC pulse is allocated completely in the low spectral zone, having the highest energy for the main lobe ($0 \leq f/B < 1-\alpha$) compared to the other pulses for all the roll-off factors. Considering the ELP$_{\beta=1}$, and the SPLCP, its energy distribution is almost the same, but the ELP concentrates a bit more energy (less than 2 %) in the high spectral zone, ($f/B > 1+\alpha$) than the SPLCP. Comparing the distribution of energy of the ELP$_{\beta=0.5}$ and the ELP$_{\beta=1}$ it can be seen that the out-of-band radiation can be controlled modifying β, and for lower values the radiation decreases.

The frequency response of the ELP pulse is not explicit known, because the filter was formulated in the time domain, so for the calculation of the spectrum starting from the impulse response, numerical methods like fast Fourier transform (FFT) need to be used. Thus the relation between the filter stop-band and parameters of the pulse is not explicit, so for every α and β parameter selection, the stop-band value needs to be found.

Spectral Regrowth

When the ideal impulse response of the pulses is truncated at certain value, its representation in the frequency domain changes (this phenomena is called spectral regrowth) introducing harmonics that can spread frequency components over the stop-band value. In Figs. 7 and 8 the spectral regrowth is presented for the ELP$_{\beta=1}$, ELP$_{\beta=0.5}$, SPLCP and RC for roll-off factor 0.35, 0.5 and truncation at [-3.5 *t/T*, 3.5 *t/T*] for demonstration purposes. From the figure we can see that the spectral-regrowth affects only the RC pulse, because its tails decay at a lower rate than the ELP, increasing the stop-band value beyond $f/B = 1+\alpha$. For higher values of α, the spectral regrowth decreases. An increase in the truncation length (i.e. the use of more taps for represent the digital filter) is necessary in order to have both, low spectral re-growth and reliable error probability performance. Moreover, the increase of the truncation length determines bigger latency and at the same time the increase of hardware complexity, so a trade-off exist and is a task for the designer to choose the optimal combination for the concrete application.

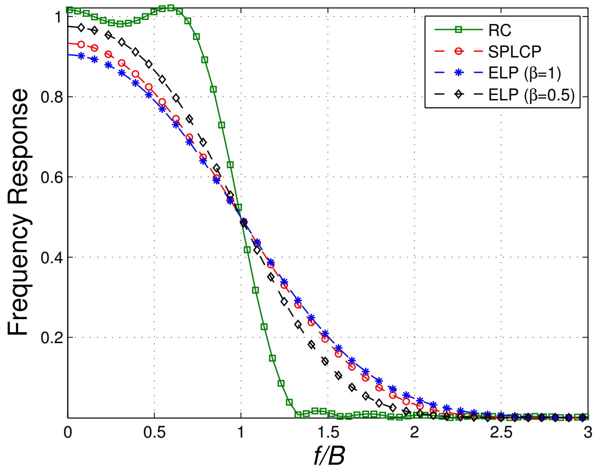

Fig. 7. Frequency response of $ELP_{\beta=1}$, $ELP_{\beta=0.5}$, SPLCP and RC pulses for $\alpha = 0.35$ considering the time-limited impulse response.

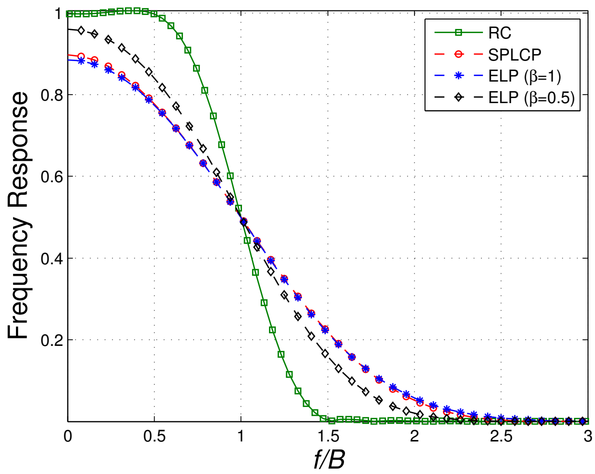

Fig. 8. Frequency response of $ELP_{\beta=1}$, $ELP_{\beta=0.5}$, SPLCP and RC pulses for $\alpha = 0.5$ considering the time-limited impulse response.

V. CONCLUSION

In this work the ELP pulse was numerically analyzed and compared with respect to the latest pulses given in the literature and the traditional RC pulse for reference in the time and frequency domains. The eye diagram of each impulse response in the transmitter side and the the BER in the receiver side were used as evaluation tools. For the ideal impulse response the ELP for $\beta = 1$ generates the smaller BER compared to the other evaluated pulses for $\alpha = 0.25, 0.35$. For $\alpha = 0.5$ only the Sinc Parametric Linear Combination Pulse (SPLCP) outperform the ELP$_{\beta=1}$. For the time-limited version of the pulses, the behavior improves in terms of BER for the pulses with tails that decays slowly. Considering the Frequency response, the ELP introduces additional out-of-band radiation compared to the RC pulse. The excess of bandwidth introduced by the family of pulses analyzed in this manuscript explains the good performance in the time domain, in terms of BER and wider eye opening. Finally, for the truncated frequency response, the ELP does not present additional spectral regrowth. The performance of the ELP could potentially be improved by using optimization techniques, specifically designed for BER reduction in presence of time sampling errors.

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