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Non-universal critical exponents in earthquake complex networks

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HIGHLIGHTS

- A time-based complex network approach is used to study seismic data sets.
- Two data sets are used, one containing a major earthquake, the other not.
- The networks are scale-free, and critical exponents γ (for connectivity), δ (betweenness centrality) and η (relating connectivity and betweenness centrality) are found.
- Data sets satisfy $\delta > (\gamma + 1)/2$, but δ is not universal.

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ABSTRACT

The problem of universality of critical exponents in complex networks is studied based on networks built from seismic data sets. Using two data sets corresponding to Chilean seismicity (northern zone, including the 2014 $M_w = 8.2$ earthquake in Iquique; and central zone without major earthquakes), directed networks for each set are constructed. Connectivity and betweenness centrality distributions are calculated and found to be scale-free, with respective exponents γ and δ. The expected relation between both characteristic exponents, $\delta > (\gamma + 1)/2$, is verified for both data sets. However, unlike the expectation for certain scale-free analytical complex networks, the value of δ is found to be non-universal.
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1. Introduction

Complex networks have been shown to be a useful tool to study various physical systems [1–4], including seismicity. In particular, scale-free complex networks are interesting, as various quantities can be shown to be distributed following power-law functions, with critical exponents which can characterize the network. These critical exponents are not expected to be independent, but should satisfy certain criticality relations or restrictions. For instance, the exponent γ, for the cumulative probability distribution of connectivity, can be related to δ, the exponent for the cumulative probability distribution of betweenness centrality [5]. It is important to note that γ and δ provide two relevant measures of the structure of scale-free networks. In this context, an interesting question is whether such exponents are universal, at least within a given class of systems. A claim in that sense was made in Ref. [6], where it was found that, for networks where γ is between 2

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and 3, δ is universal. However, Ref. [5] showed that this does not hold in general, finding instead that the exponent η which relates the connectivity to the betweenness centrality, is a more useful parameter to characterize networks.

So far, these analysis have been made for theoretical complex networks only, except for the World Wide Web [5], which is, however, a network where nodes and edges can appear easily, they do according to human needs, and where, given the size of the network, scale-free properties can be found across several decades [6–9]. Thus, it would be interesting to study this issue for less ideal networks, like earthquakes, an interesting and complex geophysical system [10–12].

In this paper we will do this for a complex network built from seismic data, where the hypocenter location of the event determines the node it belongs and edges are given by the temporal sequence between seismic events [13]. This complex network technique has been found to be a useful tool to study seismicity [13–17]. Furthermore, universal features have been found in these networks, in the sense that, in general, they are scale-free and/or small-world [14,15,17,18], regardless of the seismic zone. Other complex networks approaches to seismicity, such as visibility graphs [19,20], have shown similar scale-free features. In particular in [21] a universality between the Gutenberg–Richter b-value and the visibility graph based parameter, namely the slope of the least square line fitting the k-M plot (relationship between the magnitude M of each synthetic event and its connectivity degree k) was found.

Thus, in this paper we propose to complement previous studies on the complex earthquake network, by addressing the issue of the relationship between critical exponents, and their possible universalities.

2. Data and network

Two data sets are considered in this paper. The first set contains data from the central zone of Chile, and was measured during fifteen years, from February 10, 2000 to August 31, 2015, considering 33 160 seismic events. It was measured between 29.0° and 35.5° South Latitude and between 69.0° and 74.0° West Longitude.

The second set contains data from the northern zone of Chile, near Iquique, and was measured between January 1, 2012 and August 31, 2015, and includes the $M_w = 8.2$ earthquake of April 1, 2014. This seismic data set was measured between 18.0° and 22.0° South Latitude and between 68.0° and 72.0° West Longitude, and contains 8 601 seismic events.

Both seismic data sets were measured by the National Seismological Center (Centro Sismológico Nacional) of Chile [22], and include date, hypocenter position, and magnitude for each event. The hypocenter of a seismic event is characterized by three coordinates (Latitude, Longitude, Depth). Latitude and longitude data are converted to kilometers by assuming an Earth radius equal to 6370 km [23].

The largest magnitude in the seismic data set measured in the central zone of Chile is $M_w = 6.9$, thus this data set does not include large earthquakes (from the point of view of the damage they can produce in this particular seismic zone; we will discuss this issue later). On the other hand, the seismic data set measured in northern Chile, includes an event with $M_w = 8.3$.

Data points are shown in Fig. 1. For clarity, only the subset of data with magnitudes above $M_w = 4.0$ is shown.

Both catalogs are complete for $M_c \geq 3.0$, satisfying the Gutenberg–Richter over about three decades [23], as can be seen in Fig. 2, where they are shown to satisfy the Gutenberg–Richter law, with $b = 0.74 \pm 0.01$ for the data set measured for Iquique and $b = 1.03 \pm 0.01$, for the data set of the central zone of Chile.

To construct the network, the three dimensional volume where all events reside is divided into cubic cells with side Δ, and each cubic cell is a node if the hypocenter of a seismic event is inside the cell. Nodes in the network are connected according to their time sequence, *i.e.*, if the first event of the data set occurs in node, say, 10 and the second event occurs in node 450, a link from node 10 to node 450 is generated, and both nodes are thus connected. Two directed networks are constructed for both data sets in this way, one with $\Delta = 5$ km, and another with $\Delta = 10$ km [15,17]. The direction is given by the time.

3. Cumulative probability distribution

We analyze the relation between the cumulative probability distribution of connectivity (which measures the number of paths connecting a particular node to others, including eventual self-connections) and betweenness centrality g (which measures the number of shortest paths that goes through a given node) [2]. Fig. 3 shows a schematic representation of the connectivity (k) and the betweenness centrality (BC), for each node. It is known that the complex networks built with seismic data sets as described in Section 2 are scale-free [13,14,17], *i.e.*, the cumulative probability distribution of connectivity follows a power law ($P(< k) \sim k^{-\gamma}$, with γ as the characteristic exponent). It can also be shown that, for scale-free networks, the cumulative distribution of betweenness centrality also follows a power law ($P(< g) \sim g^{-\delta}$), [2,5] and that there is also a power law relating betweenness centrality and connectivity, [5,24]

$$g(k) \sim k^{\eta} . \quad (1)$$

As suggested in [5,24], these three characteristic exponents should be related by the expression

$$\delta > \frac{\gamma + 1}{2}, \quad (2)$$

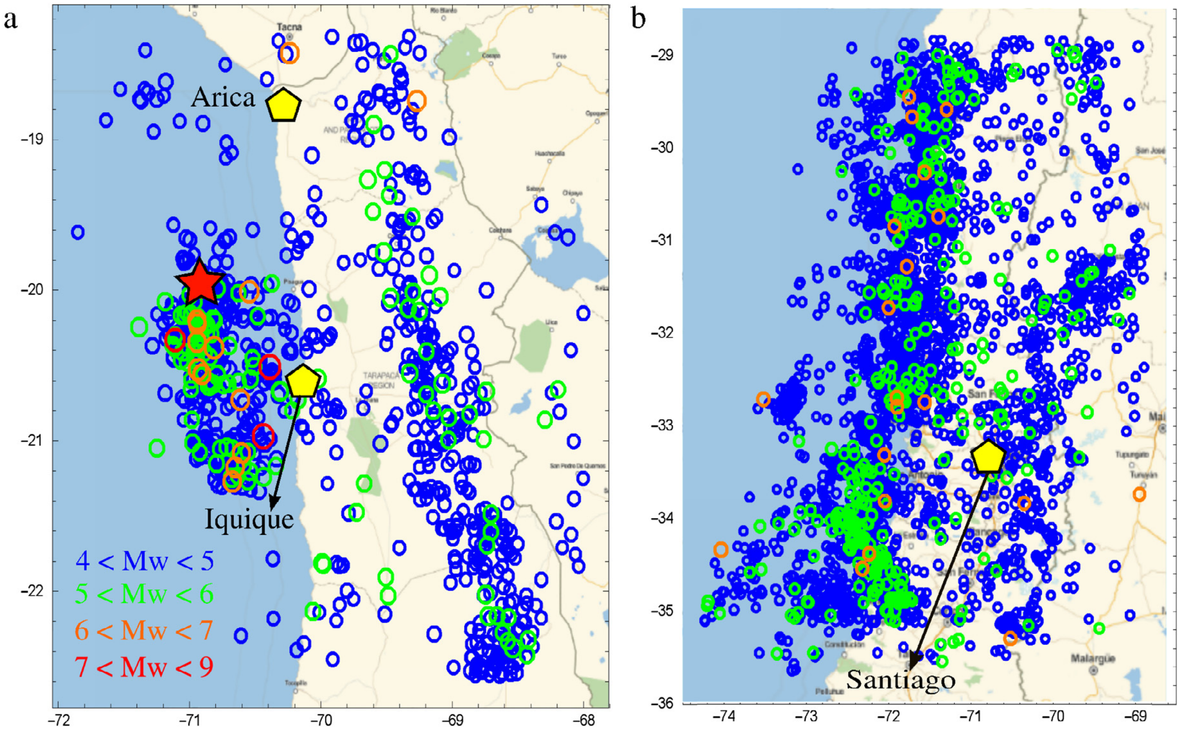

Fig. 1. Map of the northern zone (a) and the central zone (b) of Chile, with the seismic events with magnitudes greater than $M_w = 4.0$.

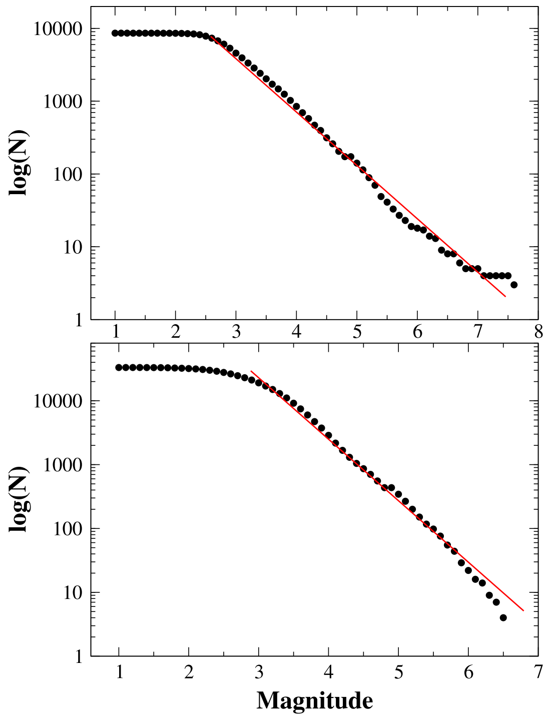

Fig. 2. Gutenberg–Richter law for the two catalogs studied. The completeness magnitude for the catalog measured in both northern Chile (top panel) and central Chile (bottom panel), is $M_w = 3.0$.

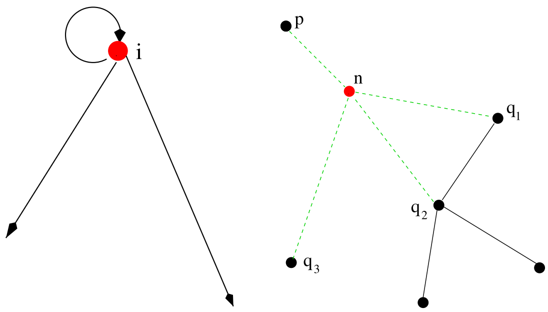

Fig. 3. Schematic representation of connectivity k_i for the ith node (left panel) and for the betweenness centrality, BC (right panel). For connectivity we count the links that go out, and the BC counts the number of shortest paths between two nodes p and q, that pass through node n.

Table 1
Values of γ for three subsets with different thresholds in magnitude and two values of the cell size, $\Delta = 5$ km and $\Delta = 10$ km, for the central zone of Chile. $M_{w,\min}$ is the magnitude threshold for the subset, $k_{\min}$ is the minimum value of the connectivity for the MLE fit, and $k_{\max}$ is its maximum value.

Δ (km)	$M_{w,\min}$	γ	$k_{\min}$	$k_{\max}$	N_{fit}	N_{total}
5	3.0	2.21 ± 0.02	3	25	24	106
	3.5	2.19 ± 0.03	1	13	13	26
	4.0	3.6 ± 0.2	2	5	4	7
10	3.0	2.21 ± 0.03	3	40	37	229
	3.5	2.4 ± 0.1	3	25	22	50
	4.0	2.4 ± 0.2	3	10	7	14

which holds if $\eta < 2$, where η is the critical exponent that relates the betweenness centrality and the connectivity, as defined in Eq. (1). The limit case $\eta = 2$ occurs for tree networks [5]. In Refs. [6,24], it is claimed that δ has the universal value 2.2 for large scale-free complex networks, when the value of γ is between 2 and 3. This is shown for theoretical networks and the World Wide Web, and we intend to test this universality in scale-free seismic complex networks.

Since one strong difference between our network and those previously studied in Refs. [6,24], is the number of nodes, we will consider size effects by taking three subsets of the seismic data, corresponding to three cut-offs in magnitude: $M_w > 3.0$, $M_w > 3.5$ and $M_w > 4.0$. Then, directed networks for each subset are built as described above, and the power law exponents of event distribution functions of connectivity and betweenness centrality are calculated.

3.1. Connectivity

For the data set from the central zone of Chile, the cumulative probability distribution of connectivity is scale-free for each of the three magnitude thresholds, as shown in Fig. 4(a). We should notice that, due to the small number of events, the scale-free feature of the network is less clear for the largest magnitude threshold (blue curve in Fig. 4(a)). However, as discussed below, our results will be valid for all chosen subsets, where scale-freeness is more evident.

The values of γ were evaluated using the Maximum Likelihood Estimation (MLE) for discrete data sets, considering values of the connectivity between $k_{\min}$ and $k_{\max}$, a range which was set using a Kolmogorov–Smirnov test [25]. N is the total number of points that were considered for the γ estimation. The resulting values are presented in Table 1, for each value of the cell size Δ and magnitude threshold $M_{w,\min}$. The error is estimated by the standard deviation of the values of γ that are obtained by 100 resamplings of $N/2$ data points in the accepted range $k_{\min} < k < k_{\max}$ (a bootstrap method). The number of data between $k_{\min}$ and $k_{\max}$ is N_{fit}, the total number of data for the cumulative distribution is N_{total}.

Similar results are found for the data set from northern Chile. The connectivity distribution is shown in Fig. 4(b), and a scale-free behavior is found for the three-magnitude thresholds as well. The characteristic exponent γ is calculated as above, and results are shown in Table 2.

3.2. Betweenness centrality (BC)

The betweenness centrality (BC) measures the importance of a node in the network considering the number of paths through a given node, as measured along the links of the network. The cumulative probability distribution of the BC for the central zone and the northern zone of Chile is shown in Fig. 5(a) and (b). A scale-free behavior is found, as expected from the scale-free behavior of the connectivity distribution observed in Fig. 4.

The characteristic exponent δ is calculated as before, using the MLE method, and results are shown in Tables 3 (central zone of Chile) and 4 (northern zone of Chile). The error is computed as before using the bootstrap method.

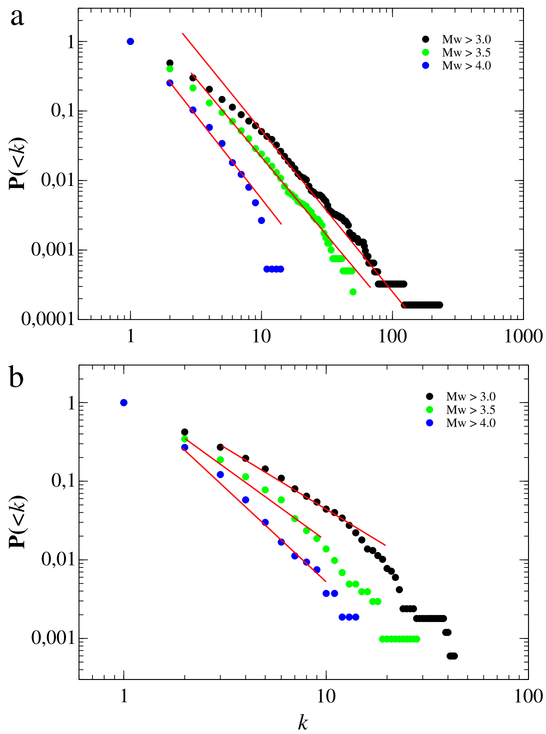

Fig. 4. Cumulative probability distribution for the directed network of the seismic data set, for three different thresholds: black $M_{w,\min} = 3.0$, red $M_{w,\min} = 3.5$ and blue $M_{w,\min} = 4.0$. Cell size is $\Delta = 10$ km. (a) Central zone of Chile; (b) northern Chile. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

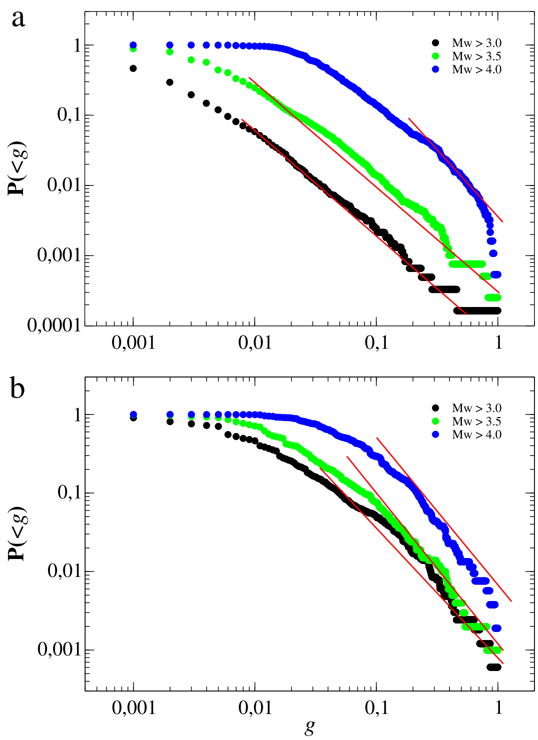

Fig. 5. Same as Fig. 4, but for the distribution of betweenness centrality.

Table 2
Same as Table 1, but for northern Chile.

Δ (km)	M_w	γ	k_{min}	k_{max}	N_{fit}	N_{total}
5	3.0	2.8 ± 0.2	3	10	8	10
	3.5	2.9 ± 0.2	2	5	4	10
	4.0	3.7 ± 0.3	2	6	5	8
10	3.0	1.9 ± 0.1	3	20	18	43
	3.5	2.1 ± 0.1	2	10	8	28
	4.0	2.6 ± 0.2	2	10	8	14

Table 3
Table with values of the characteristic exponent δ for the magnitude thresholds and two values of Δ, 5 km and 10 km, for the central zone of Chile. $M_{w,\min}$ is the magnitude threshold for the subset, $g_{\min}$ is the minimum value of the connectivity for the MLE fit, and $g_{\max}$ is its maximum value.

Δ (km)	$M_{w,\min}$	δ	g_{min}	g_{max}	N_{fit}	N_{total}
5	3.0	1.64 ± 0.06	0.0025	0.251	226	999
	3.5	1.60 ± 0.07	0.005	0.158	153	999
	4.0	1.95 ± 0.02	0.079	0.631	552	999
10	3.0	1.5 ± 0.1	0.0031	0.316	313	999
	3.5	1.4 ± 0.1	0.0063	0.316	310	999
	4.0	1.3 ± 0.1	0.0251	0.794	769	999

Table 4
Same as Table 3, but for northern Chile.

Δ (km)	$M_{w,\min}$	δ	g_{min}	g_{max}	N_{fit}	N_{total}
5	3.0	1.61 ± 0.06	0.025	0.631	60	99
	3.5	2.09 ± 0.02	0.039	0.631	59	99
	4.0	2.34 ± 0.06	0.1	0.631	53	99
10	3.0	1.87 ± 0.02	0.01	0.15	148	999
	3.5	1.65 ± 0.06	0.031	0.199	168	999
	4.0	1.83 ± 0.03	0.079	0.251	172	999

Table 5
Values of η, δ and $(\gamma + 1)/2$ for the central zone of Chile, for the different magnitude thresholds and cell sizes considered.

Δ (km)	$M_{w,\min}$	γ	η	$(\gamma + 1)/2$	δ
5	3.0	3.0 ± 0.6	1.5 ± 0.01	2.0 ± 0.3	2.35 ± 0.02
	3.5	3.0 ± 0.1	1.5 ± 0.02	2.0 ± 0.1	2.30 ± 0.02
	4.0	3.9 ± 0.2	1.5 ± 0.05	2.5 ± 0.1	3.0 ± 0.1
10	3.0	1.9 ± 0.1	1.7 ± 0.02	1.5 ± 0.1	2.08 ± 0.03
	3.5	2.1 ± 0.1	1.6 ± 0.02	1.6 ± 0.1	1.98 ± 0.03
	4.0	2.8 ± 0.1	1.5 ± 0.03	1.9 ± 0.1	2.07 ± 0.04

4. Relation between the connectivity (k) and the BC (g)

We now intend to study if Eq. (2) holds for the seismic complex networks we are considering. Also, since we have shown that $2 \leq \gamma \leq 3$, we are interested in the possible universality of δ claimed in Ref. [6] for scale-free networks with γ within such range of values.

In order to do so, we have plotted g versus k in Fig. 6 for the two seismic data sets used, corresponding to central and northern Chile.

The characteristic exponent η was evaluated by linear regression, and the right hand side of Eq. (2) was calculated using Fig. 4. and compared with the value of δ obtained in Tables 3 and 4.

The results for the central zone of Chile are shown in Table 5 and the results for northern Chile are shown in Table 6.

It can be observed that the values of the characteristic exponent δ satisfy Eq. (2) for all magnitude thresholds for the seismic data set measured in the central zone of Chile.

In the case of the seismic data measured in northern Chile, as for the central zone of Chile, Eq. (2) is satisfied. However, the value of δ is not the same for these two seismic data sets.

On the other hand, it is interesting to notice, regarding the critical exponent η, that not only it is always less than 2 as expected, but it does not seem to depend strongly on cell size and magnitude thresholds, unlike γ and δ. This is particularly clear in Table 5, but also, to a lesser extent, in Table 6. It is suggestive that Table 5 corresponds to data without major earthquakes, whereas Table 6 considers the 2014 earthquake in Iquique. After all, our labeling of the magnitude 8.2 event as the only major earthquake is dependent on the seismic features of the zone. On average, seisms below magnitude 6 do

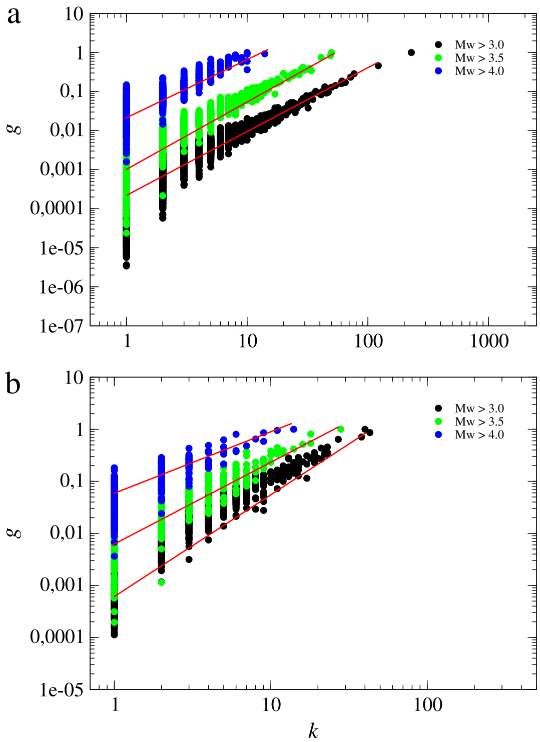

Fig. 6. Relation between connectivity (k) and BC for the seismic network constructed for (a) central zone of Chile, and (b) northern Chile, for $\Delta = 10$ km.

Table 6
Same as Table 5, but for northern Chile.

Δ (km)	M_w	γ	η	$(\gamma+1)/2$	δ measured
5	3.0	2.8 ± 0.2	1.55 ± 0.02	1.9 ± 0.1	2.12 ± 0.04
	3.5	2.9 ± 0.2	1.43 ± 0.04	2.0 ± 0.1	2.39 ± 0.05
	4.0	2.4 ± 0.1	1.3 ± 0.1	1.7 ± 0.1	3.2 ± 0.3
10	3.0	1.9 ± 0.1	1.50 ± 0.03	1.5 ± 0.1	1.88 ± 0.05
	3.5	1.9 ± 0.1	1.42 ± 0.04	1.5 ± 0.1	1.8 ± 0.1
	4.0	2.1 ± 0.3	1.40 ± 0.06	1.6 ± 0.2	1.9 ± 0.3

not cause major damage in the zone, mostly due to it being a subduction zone, so that large seisms are typically very deep. Thus, in this sense, "large" means a seism that has a major effect on the zone near the rupture, and it would make sense that the complex network of seisms (partially formed by the subsequent seismic events) should have its topology (in this case given by η) altered. In any case, more studies are needed in order to establish whether this is significative or not. Beyond this issue, though, our results suggest that η seems to have more universal features for seismic events than the other critical exponents, which is consistent with previous suggestions [5,26].

5. Discussion and conclusions

A complex network analysis of two seismic data sets measured in Chile was done, one corresponding to the central zone of Chile (without any major earthquake) and other for the northern zone of Chile (including the $M_w = 8.2$ earthquake of April 1st 2014 in Iquique). The earthquake complex networks contain spatiotemporal features: the spatial distribution of hypocenters yields its nodes, and the time sequence yields its links. Both networks were used to analyze the relation between three critical exponents in complex networks.

The connectivity distribution of the constructed complex networks are shown to be scale-free, and thus it is expected that the betweenness centrality distribution is also scale-free, and that a power law relates connectivity and betweenness centrality. The corresponding characteristic exponents are expected to satisfy Eq. (2).

Considering various cell sizes to build the complex network, and various magnitude thresholds for the data, we find that the characteristic exponent δ for betweenness centrality is not universal, as previously claimed [6], but more consistent with later results for ideal networks [5,26].

On the other hand, Eq. (2) is actually verified for the data. Results also suggest that the exponent η relating betweenness centrality and connectivity has a more universal behavior in networks built from seismic events, with respect to cell size and magnitude threshold. There is a slight difference between η values for both data sets, but more detailed studies are needed in order to identify whether this is an effect of the major earthquake included in one of the data sets or not.

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