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Design of fuzzy robust control strategies for a distributed solar collector field

Antonio Bayas[a], Igor Škrjanc[b], Doris Sáez[a,*]

[a] Department of Electrical Engineering, University of Chile, Av. Tupper 2007, Santiago de Chile, Chile
[b] Faculty of Electrical Engineering, University of Ljubljana, Slovenia, Tržaška 25, 1000 Ljubljana, Slovenia

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ABSTRACT

This paper presents novel control strategies using Takagi–Sugeno fuzzy models combined with a parametric uncertainty robust control approach to address both the nonlinearities of a process and the disturbances that act on it. In contrast to other robust control approaches, such as the H_∞ norm optimization-based approach, the proposed techniques allow the uncertainty information provided by fuzzy confidence intervals to be used to derive controllers that take into account performance specifications, such as overshoot or disturbance rejection, and to ensure the robust stability of the system due to a study based on applying the generalized Kharitonov's theorem and Lyapunov's analysis from the solution of a linear matrix inequality (LMI). To test these novel strategies, a solar collector field, which is a nonlinear plant with several disturbances affecting its operation, is used. For this plant, fuzzy confidence intervals are derived that allow representing the uncertainties associated with these disturbances, different performance objectives are tested, and a methodology for deriving these controllers is developed. The effectiveness of this approach is demonstrated on the solar collector field under different solar radiation conditions, and promising results are obtained.

1. Introduction

Several industrial processes often have a complex dynamic that is difficult to describe from its phenomenological equations; therefore, modeling using reduced and simplified structures of linear or non-linear equations is an appealing option. However, there are always non-modeled dynamics and disturbances that are not considered, which increases the error between the model and the process. Studying the uncertainty associated with these processes is of great importance since it can affect the performance of the control strategies that could be used for such processes.

Robust control theory addresses this issue, offering guarantees of both stability and performance when disturbances have a known structure or behavior. Thus, it is possible to find models for representing the uncertainties that are added as input/output or measurement noises, which is addressed by the classical robust control H_∞ [1], or parametric uncertainty, where the models of the plant are interval transfer functions with bounded coefficients due to the uncertainties.

There are several examples of the application of H_∞ robust control in the literature. In [2], Imanari et al. propose applying this robust control to a hot-strip mill, which is a multivariable process described by its state variables and that is affected by several disturbances. Some suitable parameters for the controller are obtained to improve the frequency performance in terms of both the sensitivity and the complementary sensitivity function to achieve better disturbance rejection. This strategy is favorably compared against a conventional proportional integral strategy. Other uses of H_∞ robust control theory are in energy applications, such as in [3], where it is applied in load mitigation for floating wind turbines. In this work, a structural control is developed using H_∞ theory, reducing the main fatigue load and the generator power error. Meanwhile, a piezoelectric actuator H_∞ control is designed in [4], where the fuzzy system is used for representing a particular component of the actuator (hysteresis); thus, a particular design is described for this application. Furthermore, medical applications have been tested, such as in [5], where this theory is used to control the injection of insulin in diabetic patients. In this case, H_∞ control theory is used to design a switching controller for insulin injection for blood glucose, thus making it possible to work with noisy measurements.

Conversely, the models with parametric uncertainty use Kharitonov's theory to demonstrate that a family of functions given

* Corresponding author.

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by an interval transfer function is stable if certain criteria are met [6]. This conclusion has had a significant impact since it states that irrespective of the order of the interval polynomial, only four polynomials have to be analyzed to demonstrate its stability. However, this result cannot be directly applied to discrete-time systems. Given this issue, considerable effort has been devoted to improve this result in the design of robust controllers in both discrete-time and continuous-time processes. Indeed, in [7], methods to prove the stability of interval discrete-time transfer functions are analyzed and compared. In [8], a similar result of continuous-time Kharitonov's theorem is demonstrated for discrete-time interval polynomials of degree $n \leq 3$. For continuous-time analysis, an important result is the generalized Kharitonov's theorem proposed in [9], which can be used for designing robust controllers with interval transfer functions.

Thus, using the aforementioned methods requires appropriate representations of the process. A very popular and useful approach in this sense is the fuzzy model, which is a universal approximator to describe the dynamics of non-linear processes [10]. Fuzzy-model-based control strategies have been proven to be very efficient in different applications; therefore, integrating such strategies with robust control strategies is interesting, as shown in the specialized literature. Medical applications for this approach are tested in [11], where fuzzy robust strategies are used to help patients with paraplegia. In this work, a slide model controller based on fuzzy logic is designed. However, the fuzzy model is ad hoc for this controller structure and represents a part of the unknown dynamics. Additionally, in [12], a robust fuzzy adaptive approach is implemented in a manipulator arm for people with disabilities, tackling the problem of the absence of a precise mathematical model for the process. This work includes a simple fuzzy system for this modeling, and the controller is based on stable adaptive law. Meanwhile, in [13], a development that finds the sufficient conditions for the existence of an H_∞ controller is presented based on the solution of a linear matrix inequality (LMI) by using a Takagi–Sugeno fuzzy model in the state variables. In this case, the uncertainty is modeled as uncertain time-varying matrices that are added to the matrices of the nominal model. Moreover, in [14], matrices with added noise are used, and the tuning of a fuzzy controller with guaranteed stability is studied. Additionally, the presented scheme allows selecting the parameters to achieve the desired performance, which is an important topic since the study of stability alone could lead to very conservative parameters of the controller. Indeed, another approach to manage the conservatism in control parameter tuning is shown in [15], where a PID controller is designed for robust performance using conditions similar to the H_∞ problem. Optimization algorithms are also an interesting option to address both the conservatism of parameters and robustness in control problems, as shown in [16], where particle swarm optimization (PSO) is used to tune a robust PID controller for a system with uncertain but bounded parameters.

In most of the aforementioned works, a certain structure and values for the uncertainty associated with the models are assumed without describing a formal method for their modeling.

This last topic is addressed in terms of fuzzy theory in [17], where a description of the uncertainty based on fuzzy sets is proposed. In that work, a fuzzy possibility distribution is used, taking the uncertainties that affect the system into account. Then, using fuzzy numbers, it is possible to model the process and its disturbances as an interval transfer function and find some suitable controllers based on the pole-placement method. This approach is called fuzzy parametric uncertainty and is used in [18], where it enables a disturbed process to be modeled as an interval transfer function and then uses the generalized Kharitonov's theorem to tune PI and PD controllers, ensuring stability and performance through an evaluation of gain and phase margins. Conversely, in [19], a method to transform the uncertainty existing in a system, expressed in the form of expert knowledge or linguistic variables, is presented in numerical intervals that bound that uncertainty. Then, using Kharitonov's theorem, a fuzzy PID controller is derived that ensures robust stability of the process. Similarly, fuzzy intervals have been demonstrated to be an effective technique for modeling the uncertainty of a system. Indeed, in [20], this strategy is used to model a waste-water treatment plant, which is a system with a very non-linear behavior and significant disturbances. Additionally, in [21], the fuzzy interval models are used for fault detection purposes in uncertain systems.

As shown above, a considerable amount of research has been conducted to develop a suitable model that can both manage uncertainties and model the nonlinear behavior of a process. Our objective in this work is to develop a novel robust fuzzy controller that can manage uncertainties and that has good performance in the case of nonlinear processes. The design procedure is based on the Takagi–Sugeno model with the uncertain parameters in the consequence transfer functions. This work also presents the methodology for tuning the robust fuzzy controller to achieve robust stability. The objective is to find a suitable controller for the plant by using Kharitonov's theorems, taking the performance and the robust stability into account. The fuzzy part of this approach enables modeling the nonlinear dynamics of the process, thus identifying different operation points of the plant, whereas the robust part ensures stability under disturbances. Furthermore, unlike other similar works, uncertainty is modeled using the parameters' covariances, which are obtained from the fuzzy intervals proposed in [22]. Indeed, by taking the covariances of the parameters of each rule in the Takagi–Sugeno model into account, the interval transfer functions are obtained, which define the upper and lower confidence intervals. These intervals quantify the uncertainties of the identified process in a certain way and enable the uncertainties to be used in the design process.

The proposed control design is tested on a distributed solar collector (DSC) simulator, which is located in Almeria, Spain [23]. The model of this plant is described using the continuous-time Takagi–Sugeno interval fuzzy model.

The structure of the interval fuzzy model is mandatory because of the strong nonlinear behavior of the plant and the disturbances, which cannot be neglected.

The remainder of this paper is organized as follows. Section 2 presents the theory for parametric uncertainty fuzzy models and the general design of the proposed fuzzy robust controller. The application of this scheme to a DSC field is described in Section 3, where a fuzzy identification of the plant is performed to derive the fuzzy robust controllers. Then, the results obtained through simulations are shown. Finally, the conclusions of this work are presented in Section 4.

2. Design of fuzzy robust control strategy

2.1. Fuzzy interval models with parametric uncertainty

A system with parametric uncertainty, also known as an interval plant, can be expressed as a model with transfer function coefficients that vary in a determined range [24]. Thus, the uncertainty associated with both disturbances and modeling errors is included in the model formulation.

Analytically, an interval transfer function can be written as follows:

$$H(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0} \tag{1}$$

where $b_p \in [\underline{b}_p, \bar{b}_p]$ and $a_p \in [\underline{a}_p, \bar{a}_p]$ for $p \in \{0, 1, 2, \ldots, m, \ldots, n\}$ denote the uncertain parameters of the system with $n > m$.

The goal of parametric robust control is to find an appropriate controller for the process defined by an interval transfer function, as shown in (1). The main challenge corresponds to finding the conditions for which an interval plant is stable because this interval modeling generates an infinite family of transfer functions to be analyzed.

In this topic, Kharitonov's theorem [6] and its generalization proposed in [9] are considered. Whereas the former is related to the stability analysis of a continuous-time interval transfer function, the latter is used in control design for systems with parametric uncertainty also described in a continuous-time model.

In this paper, these concepts are used to derive a robust fuzzy controller based on a Takagi–Sugeno fuzzy model with interval transfer functions in its consequences.

Takagi–Sugeno fuzzy models are based on a set of rules that relate the input variables or premises of a system in the form of cause-effect statements [25]. The Takagi–Sugeno model has the following structure:

$$R_j: \quad \textbf{If } x_1(t) \text{ is } A_1^j \text{ and } x_2(t) \text{ is } A_2^j \text{ and} \ldots \text{and } x_n(t) \text{ is } A_n^j \textbf{ then}$$

$$\hat{y}_j(t) = f_j(t) \tag{2}$$

where x_i are the inputs or premises of the model, A_i^j are the fuzzy sets associated with each premise, f_j is an arbitrary smooth function that is typically a linear model, and $\hat{y}_j$ denotes the output of the jth local model.

For the identification of Takagi–Sugeno models, fuzzy clustering is used to derive the premises and least squares is used for the parameters of the consequences when the selected functions f_j are linear models [26].

The uncertainty in model identification can be explained by two main reasons: disturbances affecting the entire process and modeling errors given by the finite set of data used for the identification. To take this uncertainty into account, a fuzzy interval model [22] is used.

Fuzzy confidence intervals use the covariance between observed data y_j and the output of each local system $\hat{y}_j$, which can be computed as follows:

$$cov(y_j - \hat{y}_j) = \hat{\sigma}_j^2 I + \hat{\sigma}_j^2 \varphi_j^T (\varphi_i \varphi_j^T)^{-1} \varphi_j \tag{3}$$

where $\hat{\sigma}_j^2 = \mathbb{E}\{e_i e_i^T\}$ is the variance of the noise of the jth local model and φ_j is the fuzzy local regression matrix for rule j. Additionally, the linear model for each rule j is as follows:

$$\hat{y}_j = \hat{\theta}^T \varphi_j \tag{4}$$

where $\hat{\theta}$ are the nominal parameters of the consequences with parameter covariance:

$$\hat{\sigma}_{\theta_j}^2 = \hat{\sigma}_j^2 (\varphi_j \varphi_j^T)^{-1} \tag{5}$$

This implies that

$$\theta_j \sim \mathcal{N}(\hat{\theta}_j, \hat{\sigma}_{\theta_j}^2) \tag{6}$$

Since a linear model is used as the consequence in each rule of the Takagi–Sugeno model, this analysis allows each local system to be written as an interval transfer function. Indeed, from expression (6), the lower and upper levels of each consequence parameter can respectively be written as follows:

$$\underline{\theta}_j = \hat{\theta}_j - \beta \hat{\sigma}_{\theta_j}$$

$$\bar{\theta}_j = \hat{\theta}_j + \beta \hat{\sigma}_{\theta_j} \tag{7}$$

where β is a tuning parameter that considers the percentage of data covered by the interval. This allows the consequences of the model (2) to be written as interval transfer functions. To address the closed-loop stability using these models, the generalized Kharitonov's theorem is used, which is briefly described in the following section.

2.2. The generalized Kharitonov's theorem

The general problem is to ensure the stability of a closed-loop system given a controller $C(s)$ with fixed parameters and a plant $H(s)$ whose parameters vary independently, i.e. [24],

$$C(s) = \frac{m(s)}{n(s)}$$

$$H(s) = \frac{b(s)}{a(s)} \tag{8}$$

where $a(s)$ and $b(s)$ are defined as interval polynomials with bounded parameters as in (1).

Thus, $b(s)$ and $a(s)$ generate a family of polynomials given by all the infinite combinations of their coefficients and accordingly, a family for $H(s)$ denoted $\mathbf{H}(s)$.

Using the notation presented in [24], the sets of the four Kharitonov's polynomials of $a(s)$ and $b(s)$ are $K_a(s)$ and $K_b(s)$, respectively.

For both polynomials, $K_a(s)$ and $K_b(s)$, Kharitonov's line segments are introduced for an interval polynomial $p(s)$ as $S_p(s) = \{[K_p^1, K_p^2], [K_p^1, K_p^3], [K_p^2, K_p^4], [K_p^3, K_p^4]\}$, where the line segment between two polynomials (for instance, $\delta_1(s)$ and $\delta_2(s)$) is defined as the one parameter family of polynomials [27]:

$$[\delta_1(s), \delta_2(s)] = \{\delta_\lambda(s) : \delta_\lambda(s) = \lambda\delta_1(s) + (1-\lambda)\delta_2(s)\} \tag{9}$$

with $\lambda \in [0, 1]$.

Using the previous definitions, the extremal and Kharitonov's systems of $\mathbf{H}(s)$ are respectively defined as follows:

$$\mathbf{H}_E := \frac{K_a(s)}{S_b(s)} \bigcup \frac{S_a(s)}{K_b(s)} \tag{10}$$

$$\mathbf{H}_K(s) := \frac{K_a(s)}{K_b(s)} \tag{11}$$

With this, the generalized Kharitonov's theorem is introduced.

Theorem 1. Generalized Kharitonov's theorem [9]

(I) The controller $C(s)$ stabilizes the entire family $\mathbf{H}(s)$ if and only if $C(s)$ stabilizes every segment in $\mathbf{H}_E(s)$.

(II) (Vertex Condition): Moreover, if the numerator and denominator of $C(s)$ are of the form $s^t(as+b)U(s)Q(s)$, where $t \geq 0$ is an arbitrary integer, a and b are arbitrary real numbers, $U(s)$ is an anti-Hurwitz polynomial, and $Q(s)$ is an even or odd polynomial, then it is sufficient that $C(s)$ stabilizes the finite set of $\mathbf{H}_K(s)$ to ensure the stability of the system.

Using this theorem, a fuzzy robust control strategy is derived and described in the following section.

2.3. Fuzzy robust control design

In this section, the methodology for designing the proposed fuzzy robust controller is explained. The novelty of this approach is the use of the fuzzy confidence interval's information of disturbances in the design of the controller, allowing both fuzzy and parametric uncertainty approaches to be combined for a robustly stable scheme. In general terms, the design process for the controllers consists of three stages: the identification of a continuous-time model for the process, the determination of the ranges for the controller parameters that ensure robust stability, and finally, the definition of performance specifications.

In the next sections, the stages of the design are analyzed.

2.3.1. Continuous-time identification of a Takagi–Sugeno model with parametric uncertainty

Since Kharitonov's theorem is derived in general for continuous-time systems, the analysis starts by performing a continuous-time identification experiment on the plant to be controlled using a state variable filter (SVF) [28]. This method combines the least squares approach with filters on the input and output signals to identify the system.

In this case, the basic SVF that is used is given by the following:

$$L(s) = \left(\frac{\lambda}{s+\lambda}\right)^n \tag{12}$$

where λ is a parameter that takes the dynamics of the system into account and that is tuned using a value that is larger than the bandwidth of the process, and n is the estimated order of the plant.

Subsequently, using the fuzzy confidence interval method described in Section 2, the model can be written in a general form for an SISO plant as follows:

$$R_j: \quad \textbf{If } y_f^{(n)}(t) \text{ is } A_n^j \text{ and } y_f^{(n-1)}(t) \text{ is } A_{n-1}^j \text{ and...and}$$

$$\dot{y}_f(t) \text{ is } A_1^j \textbf{ then}$$

$$\mathbf{H}_j(s) = \frac{b_m^j s^m + b_{m-1}^j s^{m-1} + \cdots + b_1^j s + b_0^j}{a_n^j s^n + a_{n-1}^j s^{n-1} + \cdots + a_1^j s + a_0^j} \tag{13}$$

where u_f and y_f are the filtered versions of the input and output signals, respectively, and $\mathbf{H}_j(s)$ is the interval transfer function for the jth local model with $b_p^j \in [\underline{b}_p^j, \bar{b}_p^j]$ and $a_p^j \in [\underline{a}_p^j, \bar{a}_p^j]$.

Next, the design of a robust controller based on a Takagi–Sugeno fuzzy model with parametric uncertainty is described.

2.3.2. Determination of the robust stability ranges

For the Takagi–Sugeno model with parametric uncertainty shown in (13), a robust controller must be derived for each interval transfer function of the consequences. For this purpose, the generalized Kharitonov's theorem is used for determining the stability ranges for the parameters of the controllers. If the proposed controller $C(s)$ fulfills the vertex condition of the theorem, then Kharitonov's systems $\mathbf{H}_K(s)$ must be determined for the open-loop system; then, for each element of $\mathbf{H}_K(s)$, the resulting transfer functions at closed loop must be obtained. With this, a set of characteristic equations must be analyzed using the Routh–Hurwitz criterion, finding the conditions for the parameters of the controllers that stabilize each polynomial in the set. The results of this stage are the ranges in which it is possible to locate the parameters of the controller to ensure that the interval transfer function of each rule in the fuzzy model is robustly stable.

2.3.3. Performance conditions

The application of the performance specifications, such as overshoot, settling time and gain or phase margins, on the interval transfer functions is generally difficult since their parameters are not fixed. Therefore, it is necessary to characterize a transfer function family $\mathbf{H}_j(s)$ from fixed-coefficient transfer functions (see (13)). In this work, for each family, the study of its bounding systems is proposed by defining all the possible combinations of transfer functions that can be obtained using the lower and upper values of their uncertain parameters. With this, if an interval transfer function has U uncertain parameters, then 2^U possible combinations must be analyzed. Subsequently, a controller within the specifications is derived for each transfer function and then tested for the remaining transfer functions. Finally, the controller that presents the best performance for all the functions in terms of the previous specifications is selected for the respective interval transfer function.

The proposed methodology is summarized in Fig. 1. In this figure, the right side is related to the robust stability constraints, associated with the generalized Kharitonov's theorem, whereas the left side takes the performance conditions into account, which in this case are solved using the PSO algorithm.

Using these concepts, a fuzzy robust controller is applied for a DSC power plant in the following sections.

3. Application of fuzzy robust control strategy to the distributed solar collector field

3.1. Process description

The proposed scheme is tested in a simulator of a DSC plant, which is located in Almeria, Spain [23]. The DSC uses arrays of parabolic mirrors to concentrate sunlight into a receiver pipe to produce steam for a power generator. The controlled variable in the DSC field is the oil's temperature at the end of the pipeline. This process is very appropriate for testing the proposed controller since there are several disturbances that affect the plant operation, such as the environmental temperature, inlet oil temperature and dust on the mirrors, but the most important disturbance is the solar radiation.

Because of the challenges involved in the operation of the DSC, several control strategies have been studied and implemented for this plant, including fuzzy and robust approaches. Indeed, a fuzzy control strategy is proposed in [29], which is designed using linguistic variables. In [30], a fuzzy PI is designed and tested for the real plant using expert's knowledge. Both of these approaches achieve good results since they can address the non-linear dynamics and disturbances of the process. Advanced control techniques have also been tested in the solar plant, such as in [31], where a predictive control with fuzzy goals and constraints is used for a better interpretation of the control requirements.

As previously mentioned, the solar collector field is subject to several disturbances that affect its operation. Consequently, robust control theory is an interesting alternative to address this issue, and some works applied to the solar plant can be found in the literature. In [32], an H_∞ controller is designed for the DSC integrated with an air conditioning system. In that work, an ARX model is used to represent the behavior of the entire process, and a controller is derived by solving an S/T sensibility problem, which attempts to increase the disturbance rejection and robustness of the scheme. Another approach is proposed by [33], where the robustness and performance of the controller are taken into account by using quantitative feedback theory (QFT). In that case, the selected model for the DSC is a continuous-time, second-order system with uncertain (but bounded) natural frequency and DC gain.

In most of the previously mentioned works, the DSC plant can be modeled, under general assumptions, by the following system of partial equations that describe its energy balance [34]:

$$\rho_m c_m A_m \frac{\partial T_m}{\partial t} = I\eta_0 D - H_l G(T_m - T_{env}) - LH_t(T_m - T_f)$$

$$\rho_f c_f A_f \frac{\partial T_f}{\partial t} + \rho_f c_f q \frac{\partial T_f}{\partial x} = LH_t(T_m - T_f) \tag{14}$$

where the subindex m refers to the metal of the pipe, f refers to the heat transfer fluid, and t and x are time and space, respectively. T is the temperature, T_{env} is the environment temperature, ρ is the density, c is the specific heat capacity, A is the cross-sectional area, D is the collector aperture, η_0 is its efficiency, G is the pipe diam-

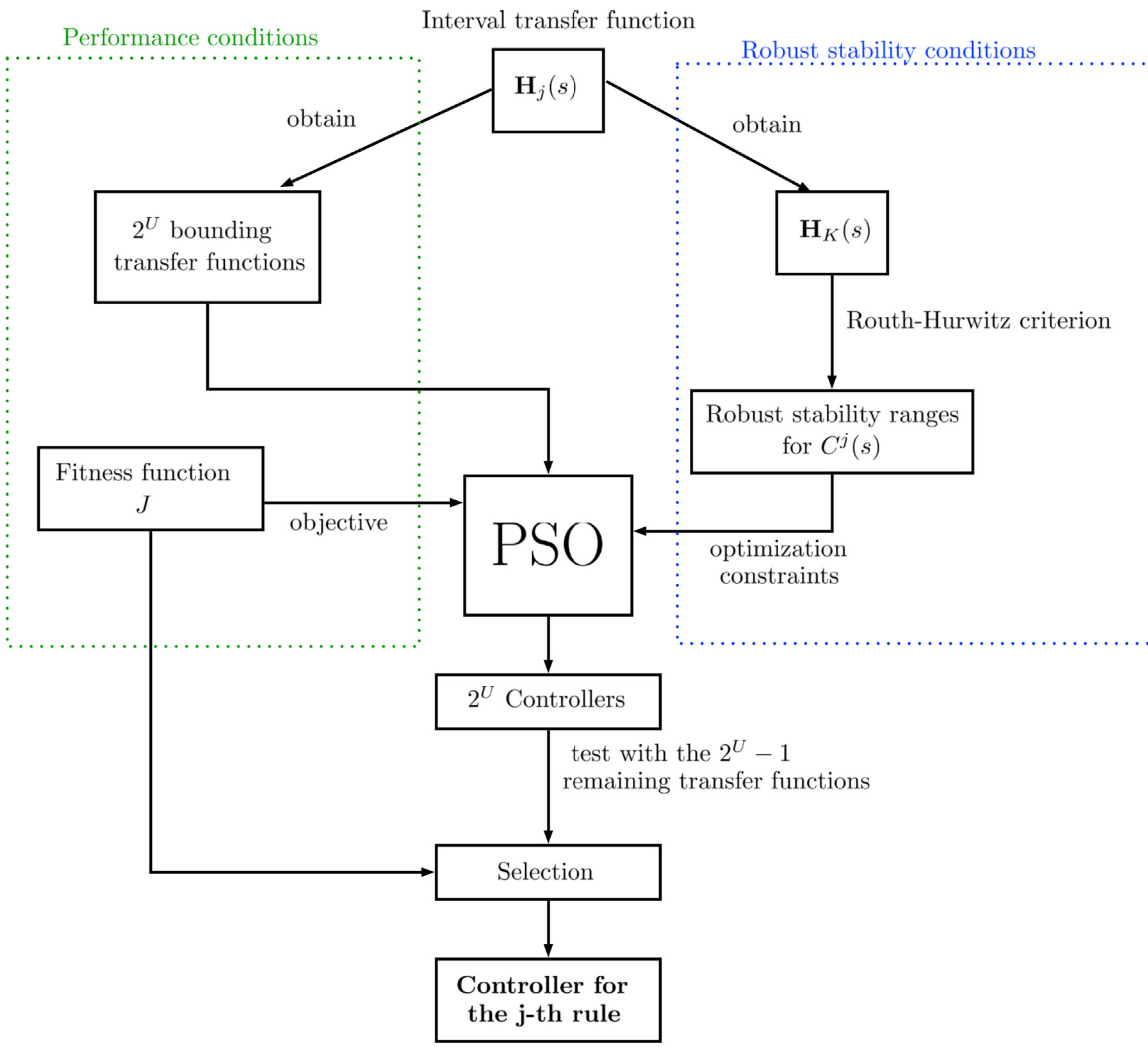

Fig. 1. Controller design diagram.

eter, I is the corrected direct solar irradiance, H_l is the convective thermal losses of the pipe exterior, H_t is the convective heat transfer coefficient of the pipe interior, and q is the flow rate of the heat transfer fluid.

The controlled variable in the DSC field is the temperature of the oil (T_f) at the end of the pipeline.

The main disturbances for this plant are the solar irradiance I, the inlet oil temperature T_{in} and the environmental temperature T_{env}. Since these variables are measurable and their effects produce changes in the output of the system (T_f at the end of the pipe) unrelated to the fluid flow q, it is desirable to add a control strategy that uses the measurement of the disturbances to reduce their effects on the controlled variable. This can be achieved using a feedforward control in series, letting the input of the feedforward term (T_{reff}: reference temperature) be the new manipulated variable for this system, as shown in Fig. 2. The sampling time for the process is 39 [s].

As mentioned in [23], several authors have modeled the DSC plant as a first-order system or overdamped second-order system, which is given by the open-loop response of the plant when a step is used as the input and considering the main disturbances in a quasi-steady state. Furthermore, the addition of a feedforward controller in series causes the entire plant to become an approximately linear system in some operation points.

3.2. Fuzzy interval modeling

With the purpose of performing an SVF-based Takagi–Sugeno identification as shown in Fig. 2, the bandwidth of the plant is required for tuning the parameter λ of the low-pass filter $L(s)$. Several experiments are conducted in the DSC simulator using inputs at different frequencies and operation points to obtain an empirical Bode diagram, which is shown in Fig. 3. Note that the largest bandwidth of the system occurs when the plant is operating at 150 [°C]. This bandwidth is approximately $b = 0.11$ [rad/seg]; therefore, the parameter λ is tuned using the SVF as $\lambda = 2b$.

The DSC plant is a low-order system; thus, for the identification tests, the highest order derivative considered is $n = 2$. Then, the filter is given by the following:

$$L(s) = \left(\frac{0.22}{s + 0.22}\right)^2 \tag{15}$$

After the identification process, the best model is selected in terms of its mean squared error (MSE), which for the case of the solar plant has the form of a first-order system for each rule:

$R_j:$ If $\dot{y}_f(t)$ is A_1^j and $y_f(t)$ is A_2^j then:

$$\mathbf{H}_j(s) = \frac{\theta_2^j}{\theta_1^j s + 1} \tag{16}$$

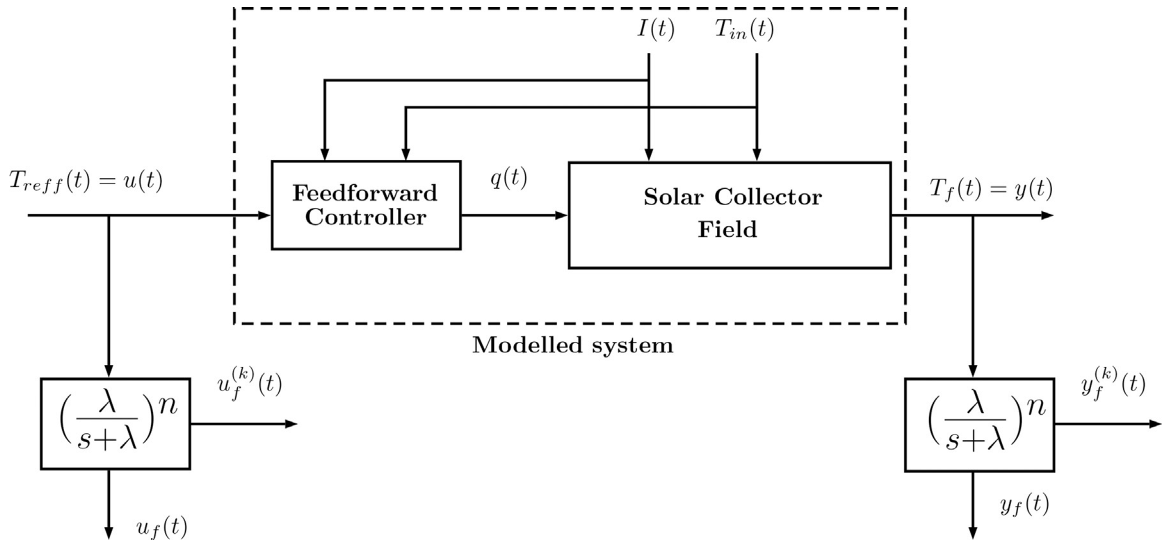

Fig. 2. Identification experiment.

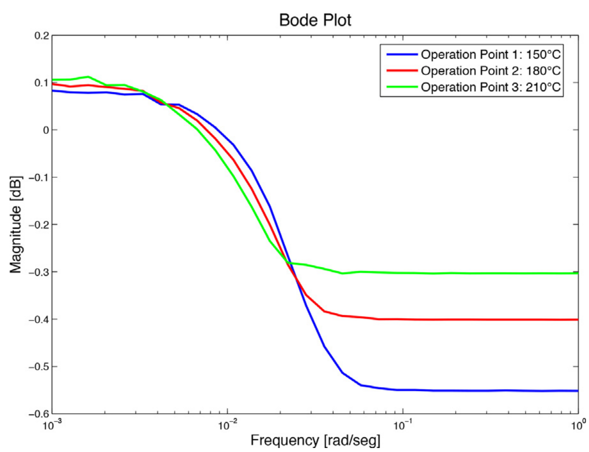

Fig. 3. Bode diagram for outlet temperature.

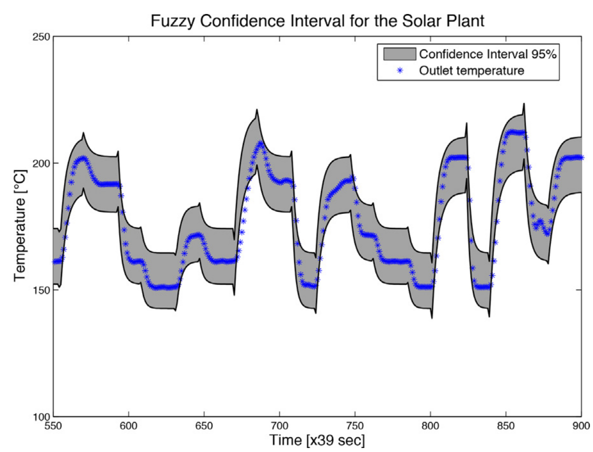

Fig. 4. Identified fuzzy interval model for the DSC plant.

with $j \in \{1, 2, 3, 4\}$, i.e., four rules. Then, using fuzzy intervals with 95% confidence, the model in (16) is written in Takagi–Sugeno form with the same premises of (16) and interval transfer functions as consequences with the following uncertain parameters:

$$\begin{aligned} &\theta_1^1 \in [126.7, 143.7], \ \theta_2^1 \in [1.009, 1.014] \\ &\theta_1^2 \in [77.83, 85.86], \ \theta_2^2 \in [1.022, 1.027] \\ &\theta_1^3 \in [156.3, 172.8], \ \theta_2^3 \in [0.9897, 0.995] \\ &\theta_1^4 \in [111.7, 124.5], \ \theta_2^4 \in [1.01, 1.014] \end{aligned} \tag{17}$$

The performance of the given fuzzy interval model with the parameters from (17) is presented in Fig. 4.

3.3. Fuzzy robust controller design

From the model with parametric uncertainty given in (16) and (17), a fuzzy PI controller that guarantees both robust stability and appropriate performance is designed using the generalized Kharitonov's theorem.

Note that from the definition given in the generalized Kharitonov's theorem, the PI controllers satisfy the vertex condition (Section 2.2). Thus, a finite set of polynomials must be tested to find the conditions for the parameters K_p^j and K_i^j of a PI controller that achieve the stability of the interval transfer functions of the Takagi–Sugeno model presented in (16) and (17). Using the Routh–Hurwitz criterion on the sets $\mathbf{H}_K(s)$ of the jth rule in this model, the corresponding closed-loop uncertain system is stable if

$$\begin{aligned} &K_i^j > 0 \\ &K_p^j > -\frac{1}{\theta_2^j} \end{aligned} \tag{18}$$

The conditions in (18) are used as constraints to derive PI controller parameters for each rule of the Takagi–Sugeno model.

To test different performance objectives for the control design, two fuzzy PI controllers are designed for the DSC plant. The first approach (Fuzzy Robust PI 1) is tuned using the pole-placement technique to obtain a small overshoot and a settling time of 600 s, whereas the second controller (Fuzzy Robust PI 2) is designed using the PSO algorithm to minimize the following fitness function:

$$J = M_p + \gamma P_m \tag{19}$$

where M_p is the overshoot, P_m is the phase margin, and γ = −0.01 is a scale factor to make both performance indices similar in magnitude. From a technical perspective, it is desirable for the DSC plant to have a small overshoot since it is related with the properties of the oil flows in its pipes. However, the inclusion of P_m permits a

Table 1
Fuzzy robust PI control parameters.

Controller	Fuzzy robust PI 1		Fuzzy robust PI 2	
Rule/parameter	K_p	K_i	K_p	K_i
1	0.3697	0.0034	0.7789	0.0072
2	0.3193	0.0037	0.9537	0.0127
3	0.4280	0.0319	1	0.0077
4	0.4896	0.0039	0.9751	0.0102

better rejection of disturbances that affect the phase stability of the system.

The fuzzy robust PI controllers obtained using the proposed design procedures for the DSC plant are summarized in Table 1.

Subsequently, the stability of the controlled system is analyzed using Theorem 2 (Appendix A), as shown in the following section.

3.4. Global stability analysis

The constraints derived in (18) from the generalized Kharitonov's theorem allow concluding that every closed-loop local model in the Takagi–Sugeno scheme is stable. However, to draw conclusions about the stability of the global model with the fuzzy robust controller, a study of the closed-loop system must be performed.

In Appendix A, from Theorem 2, it can be observed that to prove the global stability of the controlled system, matrices $P>0$ and V_{jA} should be found (matrix V_{jA} can be chosen to be different for each rule).

Because the closed-loop system with a PI controller is a second-order plant, this analysis can be performed. However, if the controller system's order is higher, improved techniques should be used to solve the LMI since the size of the involved matrices increases.

In this case study, for both fuzzy robust PI controllers in Table 1, it is sufficient to consider the following matrix:

$$P=\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$$

with $V_{1A}=V_{2A}=V_{3A}=V_{4A}=diag(10^3,\ 10^3,\ 10^3,\ 10^3)$ to ensure that the LMI presented in (A.4) is satisfied. Then, the robustness of the closed-loop system with the proposed controllers is guaranteed.

3.5. Control strategies for comparison analysis

For the comparison analysis, two classical methods are considered: robust PI controller based on a crisp model with linear interval modeling and a fuzzy PI based on the Takagi–Sugeno model. Both controllers are described next.

The robust PI controller is designed from a crisp linear interval modeling of the DSC plant to compare its behavior with the proposed fuzzy robust controllers. This model is identified using the minimum RMSE criterion, and the uncertainty in its parameters is identified similarly to that shown in (7), in other words, using the standard deviation of the coefficients with $\beta=2$ to cover 95% of the data. With this, the model has the following structure:

$$H(s)=\frac{b_1}{a_2 s^2+a_1 s+1} \tag{20}$$

with $a_1 \in [576.21,\ \ 1339.13]$, $a_2 \in [125.08,\ \ 133.98]$ and $b_1 \in [0.9997, 1.0037]$.

The RMSE of the model in (20) is 7.3739 [°C], greater than the same index for the fuzzy model (16), which is 5.1641 [°C] when fixed parameters in the consequences are considered. This result occurs because the fuzzy approach provides a better approximation to every operation point of the plant, whereas the linear approach attempts to cover the entire behavior of the system in one single linear model. This last point can also be observed in the intervals, considering that the fuzzy interval is narrower than the one derived for the crisp model. Indeed, the fuzzy interval has a width of 33.8% (taking the upper and lower values of the measured data into account), whereas the interval from the crisp model has a width of 40.5%.

For the model in (20), a robust PI controller is derived following the same procedure used for the fuzzy robust PI controller, namely, identifying the bounding transfer functions and tuning a controller using the pole-placement technique for each of them. The best scheme is selected in terms of overshoot, resulting in the robust PI controller with the following parameters:

$$K_p=1.2162,\ \ K_i=0.0053798 \tag{21}$$

Finally, a fuzzy PI controller is also derived for the model in (16) (with fixed parameters in the consequences) using the pole-placement technique to compare the proposed schemes with a similar controller, in this case, without the constraints presented in Section 3.3. The parameters of this controller are shown in Table 2.

Table 2
Fuzzy PI control parameters.

Controller Rule/parameter	Fuzzy K_p	PI K_i
1	0.8237	0.0047
2	1.1231	0.0065
3	1.2373	0.0043
4	0.7456	0.0056

Table 3
Performance indices.

Controller	Overshoot (%)	Settling time (s)
Fuzzy robust PI 1	0	660
Fuzzy robust PI 2	0	410
Fuzzy PI	0	550
Robust PI	17	1090

3.6. Simulation results

The robust fuzzy controllers presented in Table 1 and the classical schemes defined in Section 3.5 are tested under different solar radiation scenarios: under a sunny day without the presence of clouds and under a cloudy day, which are common disturbances on the DSC field. Both radiation profiles are shown in Fig. 5.

The results for using the proposed and classical controllers for the sunny day are presented in Fig. 6, where a reference that covers different operating points is used.

Fuzzy robust controllers achieve better performance in terms of overshoot and settling time for each reference level compared to the robust PI controller, as shown in Table 3.

These results are explained because the proposed fuzzy controllers can manage the nonlinearities of the DSC plant and effectively identify its different operating points, deriving a proper controller for each of them, whereas the robust PI controller based on the linear crisp model attempts to provide a general model for all the operating points, thereby introducing more uncertainty into the identification. Additionally, the proposed technique achieves lower values of pumped oil flow to the solar collector field. In fact, a mean value of 4.26×10^{-4} [m^3/s] is obtained with both fuzzy robust approaches compared to 4.40×10^{-4} [m^3/s] and 4.37×10^{-4} [m^3/s] with the robust and fuzzy PI controllers, respectively; thus, less control energy is used. Recall that although the temperature to the feedforward controller was used as the manipulated variable of the identified system (Fig. 2), the pumped oil flow to the collector field

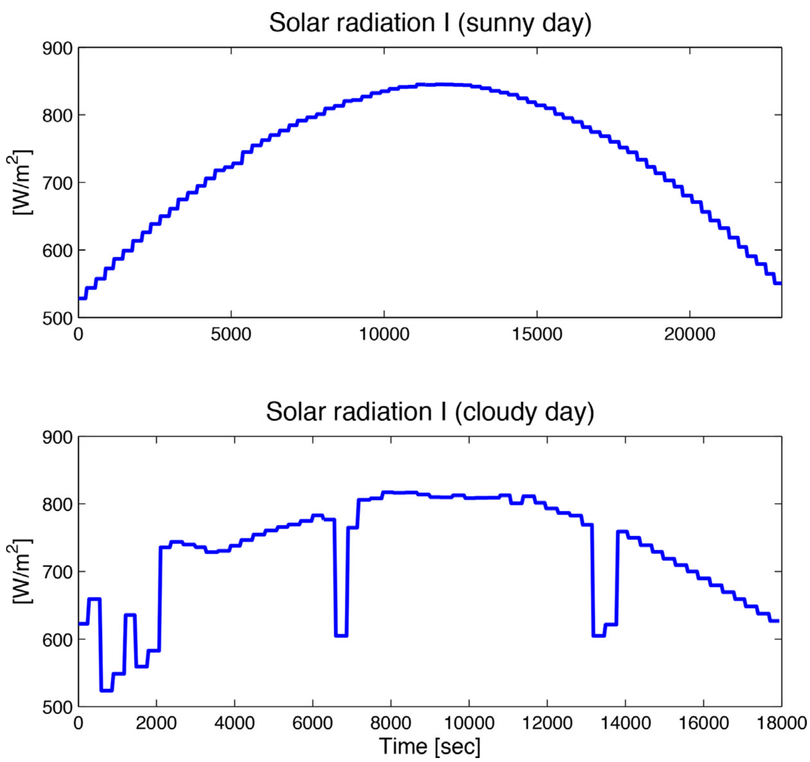

Fig. 5. Radiation profiles used in the tests.

is the manipulated variable to measure the control energy used in this plant.

Note that the fuzzy PI controller also achieves good results in this test. Since this scheme is derived without the restrictions for the robust stability and the test does not have major disturbances, the performance of such a controller could be equal to or better than that of the robust approach in this case.

Finally, the performance of the controllers on a cloudy day is shown in Fig. 7, where a constant reference of 180 [$^\circ$C] is used.

As shown in Fig. 7, the proposed controllers have a better disturbance rejection than the robust PI controller and fuzzy PI controller, with approximately 4 or 6[$^\circ$C] of difference in the peaks due to the passing clouds. Additionally, good results were obtained for the oil flow. Indeed, the mean values of the oil flow for the fuzzy robust controllers are 3.77×10^{-4} [m^3/s] and 3.76×10^{-4}[m^3/s] for PI 1 and 2, respectively, and 4.174×10^{-4}[m^3/s] and 4.167×10^{-4}[m^3/s] for the robust PI controller derived from the crisp model and the fuzzy PI controller, respectively. Thus, control energy savings is obtained when the fuzzy robust PI controller is used.

Meanwhile, note that in this case, the fuzzy robust controllers achieve better results in terms of disturbance rejection than the fuzzy PI version. This is because of the conditions of robust stability and performance used in the derivation of the proposed schemes, which permits a better performance when disturbances occur.

Finally, to draw conclusions about the effectiveness of the proposed controllers, they are tested under 100 different radiation profiles for a constant reference of 180 [$^\circ$C]. The results of this test are summarized in Table 4.

As shown in Table 4, the fuzzy robust PI controller exhibits better performance in terms of reference tracking (from the average outlet temperature $\bar{T}_f$). Additionally, the average oil flow demanded from the pump ($\bar{q}$) demonstrates that this scheme allows savings in the control energy.

Table 4
Validation results: 100 different radiation profiles.

Controller	Temperature [$^\circ$C]		Flow [m^3/s] $\times 10^{-4}$	
	$\bar{T}_f$	$std(T_f)$	$\bar{q}$	$std(q)$
Fuzzy robust PI 1	179.9936	1.1111	3.8874	0.1163
Fuzzy robust PI 2	179.9432	1.5737	3.7378	0.5746
Fuzzy PI	179.9207	2.0284	3.7413	0.5852
Robust PI	179.9001	2.1869	4.0283	0.6054

4. Conclusions

In this paper, a novel robust control strategy based on Takagi–Sugeno fuzzy interval models was proposed and tested on a DSC field. This new scheme is designed to address the nonlinearities of the process and to ensure that the closed-loop system is robustly stable by taking the disturbances affecting its operation into account. The stability analysis is performed using the generalized Kharitonov's theorem together with a stability study based on Lyapunov's analysis, which is the basis of the methodology proposed to tune these controllers. The latter enables the use of fuzzy confidence intervals for control purposes since they permit a characterization of the disturbances that affect a plant.

As a case study, fuzzy robust PI controllers for a DSC plant simulator were tested. The proposed approach enables robustly controlling the DSC plant, taking into account performance objectives such as settling time, overshoot or disturbance rejection, as well as ensuring the stability of the closed-loop system within the bounds of the fuzzy intervals. The effectiveness of the control design was demonstrated through tests performed under different conditions for the solar radiation and for several days, achieving better results than a conventional linear controller derived from a crisp model in terms of reference tracking, disturbance rejection and control energy, as shown in the results. Additionally, the proposed

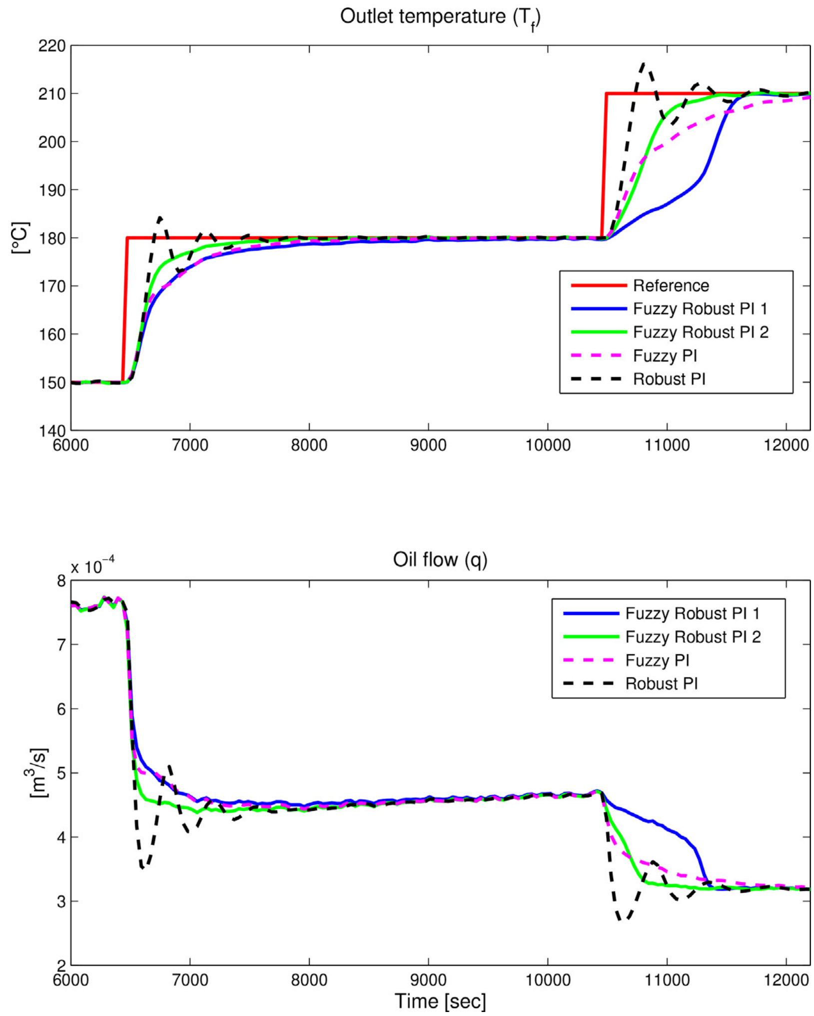

Fig. 6. Results for the controllers under a sunny day.

controllers exhibit better performance compared with a conventional fuzzy PI when disturbances, such as passing clouds, occur.

A fuzzy robust PI control design was used in this paper, but other strategies can be used, such as proportional (P) or proportional derivative (PD) controllers. PID controllers require additional analysis to ensure robust stability since they do not satisfy the vertex condition shown in the generalized Kharitonov's theorem. However, a procedure similar to that presented in this work can be used to design a fuzzy robust PID.

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Appendix A. Robust stability conditions for fuzzy closed-loop systems

To draw conclusions about the robustness of this approach, it is necessary to prove the global stability of the proposed scheme.

For the stability study, the closed-loop system in each rule of the controlled fuzzy model is written in state space using the controllable canonical form.

In the following analysis, the nominal system's parameters for the jth rule (state, matrices and vectors) will be denoted with the subindex $j0$. Then, considering the nominal closed-loop transfer functions in each rule of the fuzzy model, the state space representation is given by the following:

$$\dot{x}_{j0}(t) = A_{j0}x_{j0}(t) + B_{j0}u(t)$$

$$y_{j0}(t) = C_{j0}x_{j0}(t) \tag{A.1}$$

The required notation and theorems for studying the interval system stability are introduced next [35].

The upper and lower matrices, derived from the uncertain parameters, can respectively be written as follows:

$$\bar{A}_j = \begin{bmatrix} \bar{a}_{j11} & \cdots & \bar{a}_{j1n} \\ \vdots & \ddots & \vdots \\ \bar{a}_{jn1} & \cdots & \bar{a}_{jnn} \end{bmatrix}$$

$$\underline{A}_j = \begin{bmatrix} \underline{a}_{j11} & \cdots & \underline{a}_{j1n} \\ \vdots & \ddots & \vdots \\ \underline{a}_{jn1} & \cdots & \underline{a}_{jnn} \end{bmatrix}$$

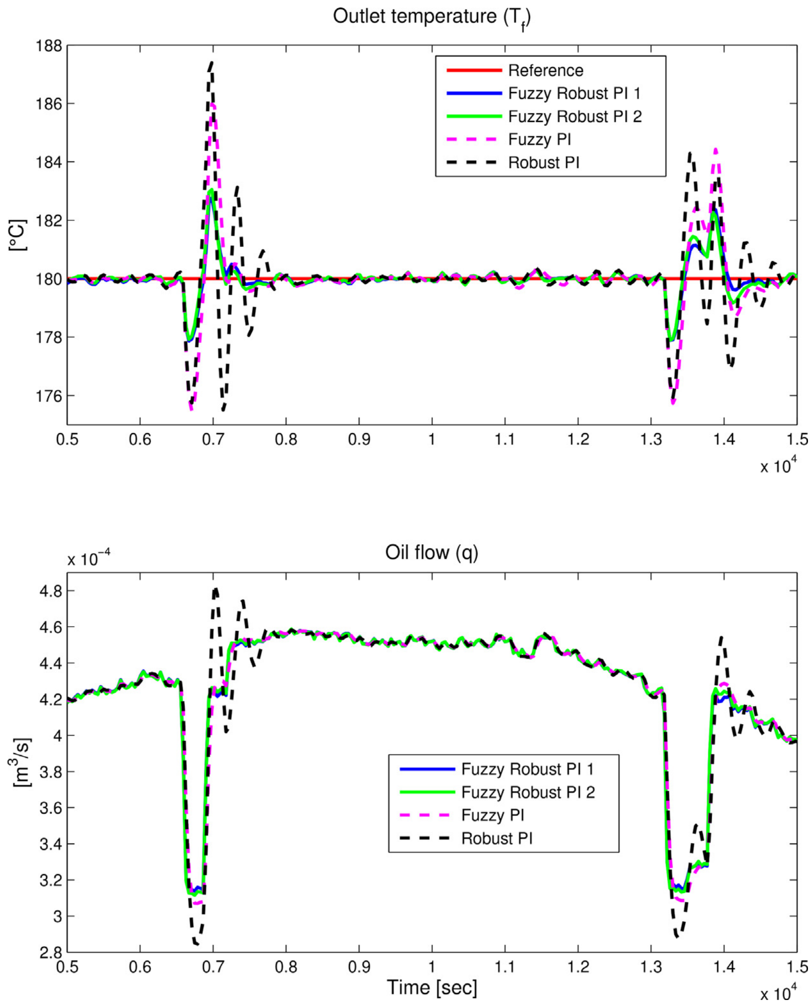

Fig. 7. Results for the controllers under a cloudy day.

An interval matrix is then defined as $\hat{A}_j \in [\underline{A}_j, \bar{A}_j]$ with:

$$\hat{A}_j = A_{j0} + \sum_{v=1}^{n}\sum_{w=1}^{n} e_v \hat{f}_{jvw} e_w^T, \ \ |\hat{f}_{jvw}| \le \delta_{jvw}^A \tag{A.2}$$

where:

$$\delta_{jvw}^A = \frac{\bar{a}_{jvw} - \underline{a}_{jvw}}{2}$$

$$A_{j0} = \frac{1}{2}(\bar{A}_j + \underline{A}_j)$$

$$e_v = [0, 0, \ldots, 1, \ldots, 0]^T$$

in e_v the number one is located at the v-th element.

From these definitions and using Lyapunov's analysis, the fuzzy closed-loop system with parametric uncertainty is asymptotically stable if a positive definite matrix $P > 0$ exists such that

$$\hat{A}_j^T P + P\hat{A}_j < 0 \tag{A.3}$$

for all $j \in \{1, 2, \ldots, N_r\}$

The following theorem proposes the robust stability conditions in LMI form.

Theorem 2. Robust stability of fuzzy systems with interval uncertainties [35]

Let us assume that there exist matrices $P > 0$ and $V_{jA} = diag(\lambda_{j11}, \ldots, \lambda_{j1n}, \ldots, \lambda_{jn1}, \ldots, \lambda_{jnn}) > 0$, which satisfy the following LMI for all $j \in \{1, 2, \ldots, N_r\}$:

$$\begin{bmatrix} PA_{j0}^T + A_{j0}P + E\Delta_j^A F V_{jA} F \Delta_j^A E^T & PE \\ * & -V_{jA} \end{bmatrix} < 0 \tag{A.4}$$

where

$$\Delta_j^A = diag(\delta_{j11}^A, \ldots, \delta_{jn1}^A, \delta_{j12}^A, \ldots, \delta_{j1n}^A, \ldots, \delta_{jnn}^A)$$

$E = (I_n, I_n, \ldots, I_n)$ n times, with I_n nxn identity

$*$ is $(PE)^T$

and F is a permutation matrix such that

$$FV_{jA}F = diag(\lambda_{j11}, \ldots, \lambda_{in1}, \lambda_{j12}, \ldots, \lambda_{j1n}, \ldots, \lambda_{jnn})$$

then, the model under study is asymptotically stable.

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