Applied Soft Computing Journal 73 (2018) 254–262

Contents lists available at ScienceDirect

Applied Soft Computing Journal

journal homepage: www.elsevier.com/locate/asoc

A new method for identification of fuzzy models with controllability constraints

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HIGHLIGHTS

- Fuzzy identification method that heuristically imposes controllability is proposed.
- Controllability criterion used when identifying T&S consequence parameters.
- Criterion based on Sylvester matrix and fuzzy model linearization.
- Simulations show improved performance of MPC with models obtained with new method.

ARTICLE INFO

Article history:
Received 1 December 2017
Received in revised form 4 July 2018
Accepted 20 August 2018
Available online 28 August 2018

Keywords:
Controllability
Fuzzy models
System identification

ABSTRACT

Takagi–Sugeno fuzzy models are cataloged as universal approximators and have been proven to be a powerful tool for the prediction of systems. However, in certain cases they may fail to inherit the main properties of a system which may cause problems for control design. In particular, a non-suitable model can generate a loss of closed-loop performance or stability, especially if that model is not controllable. Therefore, ensuring the controllability of a model to enable the computation of appropriate control laws to bring the system to the desired operating conditions. Therefore, a new method for identification of fuzzy models with controllability constraints is proposed in this paper. The method is based on the inclusion of a penalty component in the objective function used for consequence parameter estimation, which allows one to impose controllability constraints on the linearized models at each point of the training data. The benefits of the proposed scheme are shown by a simulation-based study of a benchmark system and a continuous stirred tank: the stability and the closed-loop performances of predictive controllers using the models obtained with the proposed method are better than those using models found by classical and local fuzzy identification schemes.

1. Introduction

Nowadays, advances in control theory are closely related with the development of mathematical models that represent the dynamic behavior of physical systems. Several approaches have been reported to model dynamic systems in the specialized literature, being Takagi–Sugeno (TS) fuzzy models one of the most prominent ones to model non-linear dynamic systems [1]. Indeed, TS fuzzy models often are referred as universal approximators of non-linear functions [2,3], and thus of non-linear dynamics.

One of the main concerns about TS fuzzy models is structure selection and parameter identification. For example, Zhao et al. [4] described a methodology that integrated both structure selection and parameter identification with the selection of the inputs of the model. In this methodology, authors used the Gustafson–Kessel (GK) [5] and Compatible Cluster Merging (CCM) algorithms for cluster identification, cluster merging (to reduce the complexity of the resulting model and selection of the input variables) and identification of the parameters of the premises; whereas they used the least squares method to identify the parameters of the consequences. In [6], the authors pre-processed the information used in [4] to reduce the information loss during the identification procedure and to clearly define the membership functions to be used in the resulting model. Such pre-processing consisted in making a rotation of the input space which allowed defining auxiliary variables that account for the information that is not captured by the model itself. In [7], the authors proposed an approach based on linear piece-wise modeling. The state space is divided into several subspaces in accordance with the previous knowledge of the system, and a linear model is assigned to each subspace. Finally, the parameters of each local linear model are identified.

In addition, several modifications to the aforementioned approaches have been reported in the literature. For instance, clustering methodologies have been discussed in [8–10]. Specifically, in [8] the Gath–Geva algorithm was used, while the authors in [9]

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https://doi.org/10.1016/j.asoc.2018.08.023
1568-4946/

and in [10] respectively proposed the use of neural networks and gravitational search-based hyperplane. While the GK algorithm requires a priori definition of the number of rules (or a well-defined search domain), the works of [11–14] are more flexible and allow explicit elicitation of the number of rules during the training procedure. Local learning schemes, i.e. where the parameters of each rule are estimated separately, have been explored in [12–15]. It has been shown that local learning is usually more stable and homogeneous than global learning [16]. Algorithms that include uncertainty and noise have been proposed in [17] and [18]. Generalized TS fuzzy systems consider arbitrarily rotated rules and as such can show better performance than conventional TS fuzzy systems [19,20]. Online learning, including rule parameters and structure updating, in the so-called evolving TS models has been proposed and studied in [21–24]. Some new TS fuzzy models that use the Kronecker product have been proposed in [25,26]. Other relevant aspects and features for the identification of fuzzy models can be seen in [16,27,28].

Although TS fuzzy models are known as universal approximators, and despite the efforts done by the researchers to propose methodologies that allow capturing all features of the dynamic system under analysis, little attention has been paid to include in these methodologies typical properties of dynamic systems such as controllability, observability and stability. Indeed, current methodologies for the identification of TS fuzzy models focus on minimizing the prediction error of the model, and leave aside the study of the previously mentioned properties. It is important to remark that not including properties like controllability and observability could restrict the range of applications of the model. Specifically, the development of control strategies might not be possible if the model used to represent the system to be controlled is not controllable. In fact, controllability is a property that guarantees that a system could be driven towards a desired state from any initial state by means of a feasible control action [29]. Therefore, it is relevant to analyze the conditions that must be met by the TS fuzzy model to guarantee controllability.

In the literature, several approaches have been proposed to analyze the controllability of dynamic systems. For instance, the authors in [29] and [30] analyzed the controllability of linear dynamic systems, establishing the following condition for the absence of controllability: there exists a zero–pole cancelation in the model. Such condition can be verified using the Sylvester matrix as shown in [31]. Regarding non-linear systems, it is concluded in [32] that if a continuously differentiable continuous time nonlinear system, with continuous third order derivatives, is linearized around the origin and *(i)* the resulting model is controllable, and *(ii)* the free-response of the model is globally asymptotically stable, then the nonlinear system is controllable. The conditions however are too restrictive.

To the best knowledge of the authors of this manuscript, there are no works dealing with the controllability of discrete-time dynamical TS systems, whereas some tackle the controllability of TS fuzzy models of continuous time state space fuzzy models. The authors in [33] analyzed continuous time state space TS fuzzy models by regarding them as time-varying linear systems. The time-dependency of the parameters was determined by the membership functions and the inputs (and their changes) considered in the model, and controllability can be checked by means of the invertibility of a time-varying controllability matrix. The same class of models are analyzed in [34] by means of Lie algebra and a model relaxation. This relaxation considers the TS fuzzy models as linear systems with matrices contained in convex hulls. Lie algebras are also used for the analysis of continuous time TS models in [35]. In this case, the accessibility conditions were analyzed instead of the controllability conditions. Accessibility is a relaxation of controllability in which only a set of final/desired states are considered [36].

Notwithstanding the efforts done by the fuzzy systems community to provide an adequate procedure for parameter identification of TS fuzzy models and to derive conditions that allow analyzing properties such as controllability, little effort has been done to propose a methodology for parameter identification that includes a measurement of the controllability of the resulting model, so as to impose such a feature. Furthermore, there are no works dealing with the conditions to establish controllability for discrete time fuzzy models. Thus, the current paper proposes a methodology for parameters identification in which both the prediction error and the controllability of the model are considered. The methodology consists in a modification of the typical least squares identification, such that a barrier function is introduced into the objective function used for the identification of the parameters of the consequences, so that the cost function tends towards to infinity if the identified model is not controllable. More precisely, the TS fuzzy model is linearized at every point of the training data for the identification, and the Sylvester matrix is then used to determine if every local linearized model is controllable or not. Then, if the model is not controllable, a large penalty is applied. This procedure guarantees that each local linearized model is controllable. This is a heuristic approximation to impose the controllability of the identified fuzzy model. The proposed methodology is tested using two case studies: a benchmark system and the CSTR described in [37].

The remainder of this paper is organized as follows: Section 2 introduces the relevant preliminaries, i.e. theory regarding the controllability analysis of dynamic systems and Takagi & Sugeno fuzzy models; Section 3 presents an analysis of the controllability conditions for discrete-time dynamical TS fuzzy models; Section 4 presents the proposed methodology for parameter identification of TS fuzzy models; Section 5 describes the case studies used to validate the methodology; and Section 6 presents the concluding remarks and future work.

2. Preliminaries

This section discusses the controllability conditions for general non-linear systems, the concept is then specialized to linear systems, and finally introduces fuzzy Takagi & Sugeno models.

2.1. Controllability of non-linear systems

Consider a discrete-time dynamic system with a state equation given by

$$x(k+1) = f\left(x(k), u(k)\right), \tag{1}$$

where $f(\cdot) \in \mathbb{R}^M \times \mathbb{R}^U \to \mathbb{R}^M$, $x(k) \in \mathbb{R}^M$ denotes the state and $u(k) \in \mathbb{R}^U$ is the input. For system (1) controllability is defined as the ability to reach any desired state $x(k_1) = x_1$ from any initial state $x(k_0) = x_0$, by using an admissible sequence of control actions $u(k_0), \ldots, u(k_1)$, for some $k_1 > k_0$ [29,38]. In several cases these conditions are hard to verify or are not strictly fulfilled by a system, thus relaxed versions of the concept of controllability are usually considered.

One of the most used relaxations is given by the so called accessibility analysis. Let $R(x_0, T)$ denote the set of states that can be reached from x_0 in a finite time T. Then the set

$$R(x_0) = \bigcup_{T \geq 0} R(x_0, T) \tag{2}$$

denotes the set of states that can be reached from x_0. If $R(x_0) = \mathbb{R}^M$ then the system (1) is said to be controllable at x_0. If $R(x_0)$ contains only a neighborhood of x_0 the system (1) is said to be locally controllable [39]. Furthermore, if $R(x_0) = \mathbb{R}^M$ for all $x_0 \in \mathbb{R}^M$ then system (1) is controllable.

2.2. Controllability of linear systems

In the literature there are several approaches to determine if a linear system is controllable or not (see e.g. [29]). In this paper, only the method proposed in [31] is considered since it can be used in discrete-time dynamic systems represented as a transfer function from $u(q)$ to $y(q)$, being $y(q) \in \mathbb{R}$ the output of the system (with abuse of notation here $u(q) \in \mathbb{R}$). Moreover, since, in this paper, only Takagi & Sugeno fuzzy models with linear consequences have been considered, deriving the transfer function for each consequence is straightforward. Let $G(q)$ denote the transfer function of a discrete-time dynamic system. Then $G(q)$ is expressed as follows:

$$G(q) = \frac{y(q)}{u(q)} = \frac{\theta_{ny+1} q^{ny-1} + \cdots + \theta_{ny+nu} q^{ny-nu}}{\left(q^{ny} - \theta_1 q^{ny-1} - \cdots - \theta_{ny}\right)}, \tag{3}$$

with $n_y > n_u$ and $qy(k) = y(k+1)$. From (3) the controllability of a dynamic system is determined by the set of parameters $\{\theta_1, \ldots, \theta_{n_y+n_u}\}$. If the numerator and the denominator are not co-primes there is a zero–pole cancellation and therefore the system is non-controllable [30]. In fact, given this condition the system (3) behaves as a lower order system. Note that analyzing the existence of zero–pole cancellations is an easy way to determine if a system is controllable or not. However as the order of the system increases determining the zeros and poles is a more challenging task. An alternative to analyze the controllability of (3) consists in using the Sylvester matrix. From (3), the Sylvester matrix $S(y(q), u(q))$ between $y(q)$ and $u(q)$ is given by

$$S(y(q), u(q)) = \begin{bmatrix} 1 & 0 & \cdots & 0 & \theta_{ny+1} & 0 & \cdots & 0 \\ -\theta_1 & 1 & \ddots & \vdots & \vdots & \theta_{ny+1} & \ddots & \vdots \\ \vdots & -\theta_1 & \ddots & 0 & \theta_{ny+nu} & \vdots & \ddots & 0 \\ -\theta_{ny} & \vdots & \ddots & 1 & 0 & \theta_{ny+nu} & & \theta_{ny+1} \\ 0 & -\theta_{ny} & & -\theta_1 & \vdots & \ddots & \ddots & \vdots \\ \vdots & \ddots & -\theta_{ny} & \vdots & & & & \theta_{ny+nu} \\ 0 & \cdots & 0 & -\theta_{ny} & 0 & \cdots & & 0 \end{bmatrix}. \tag{4}$$

Then, if $\det(S(y(q), u(q))) = 0$ the numerator and the denominator in (3) are not co-prime. That is, if the Sylvester matrix is singular the system (3) is not controllable.

2.3. Takagi & Sugeno Fuzzy models

Takagi & Sugeno (TS) models are used in this work to represent the dynamic behavior of a dynamic system. These models define IF-THEN rules such as:

$$R_r: \textit{IF} z_1(k) \textit{ is } MF_1^r \textit{ and } \ldots \textit{ and } z_p(k) \textit{ is } MF_p^r \textit{ THEN}$$
$$y_r(z(k)) = f_r(z(k)) \tag{5}$$

where R_r denotes the rth rule of the fuzzy model, with $r \in \{1, \ldots, N_r\}$ and N_r the total number of rules; $y_r(z(k))$ is the consequence or local model; $z(k) = [z_1(k), \ldots, z_p(k)]$ is the premise vector at time k defined as $z(k) = \left[y(k-1), \ldots, y(k-n_y), u(k-1), \ldots, u(k-n_u)\right]^T$; $f_r(z(k))$ is a function of the model premises; and MF_i^r is the fuzzy set (membership function) of the ith premise corresponding to the rth rule. The membership grade $\mu_r(z_i(k))$ is of the ith premise $z_i(k)$ to the fuzzy set MF_i^r is defined as

$$\mu_r(z_i(k)) = \exp\left(-0.5 \cdot \left(a_{r,i} \cdot \left(z_i(k) - b_{r,i}\right)\right)^2\right) \tag{6}$$

where $a_{r,i}$ and $b_{r,i}$ are the membership function parameters. Linear consequences are considered in this work; the rth local model is given by:

$$f_r(z(k)) = \theta_r^T z(k) = \theta_{r,1} y(k-1) + \cdots + \theta_{r,n_y} y\left(k-n_y\right) + \theta_{r,n_y+1} u(k-1) + \cdots + \theta_{r,n_y+n_u} u(k-n_u) \tag{7}$$

where $\theta_r^T = \left[\theta_{r,1}, \ldots, \theta_{r,n_y+n_u}\right]^T$. Then, the fuzzy model output becomes:

$$y_{fuzzy}(k) = \sum_{r=1}^{N_r} h_r(z(k)) \cdot \theta_r^T z(k) \tag{8}$$

with $h_r(z(k)) = \frac{w_r(z(k))}{\sum_{l=1}^{N_r} w_l(z(k))}$ and $w_r(z(k)) = \prod_{i=1}^{n_y+n_u} \exp\left(-0.5 \cdot \left(a_{r,i} \cdot \left(z_i(k) - b_{r,i}\right)\right)^2\right)$.

To the best of the authors' knowledge, there are no identification procedures for TS discrete fuzzy models that enforce the controllability of the resulting dynamical model. Furthermore, the study of the conditions to ensure controllability of the fuzzy model has only been performed for continuous time models. Thus, the controllability of TS fuzzy models is studied in the next section, and a heuristic identification that considers the controllability of the resulting discrete time TS fuzzy model is proposed after.

3. Controllability of Takagi & Sugeno Fuzzy models

The controllability of discrete-time dynamical TS fuzzy models will be analyzed using the Sylvester matrix criterion. For this, a linear approximation and a transfer function from $u(k)$ to $y(k)$ are required.

Let z_0 be the linearizing point. Then the derivative of the fuzzy model with respect to each premise $z_i(k)$ is given by:

$$\frac{dy_{fuzzy}(k)}{dz_i(k)} = \sum_{r=1}^{N_r} \left\{ \frac{dh_r(z(k))}{dz_i(k)} \theta_r^T z(k) + h_r(z(k)) \cdot \theta_{r,i} \right\} \tag{9}$$

With

$$\frac{dh_r(z(k))}{dz_i(k)} = -\left(a_{r,i}\right)^2 \left(z_i(k) - b_{r,i}\right) \cdot h_r(z(k)) - h_r(z(k)) \sum_{l=1}^{N_r} -\left(a_{l,i}\right)^2 \left(z_i(k) - b_{l,i}\right) \cdot h_l(z(k))$$

Let $\hat{\theta}_i\left(\theta, a_{\cdot,i}, b_{\cdot,i}\right) = \left.\frac{dy_{fuzzy}(k)}{dz_i(k)}\right|_{z(k)=z_0}$, where θ is the vector of parameters of the consequences, and $a_{\cdot,i} = \left[a_{1,i}, \ldots, a_{N_r,i}\right]$ and $b_{\cdot,i} = \left[b_{1,i}, \ldots, b_{N_r,i}\right]$ are the parameters of the ith premise of each rule. Then (8) can be approximated by:

$$y_{approx}(k) = \sum_{i=1}^{n_y+n_u} \left\{ \hat{\theta}_i\left(\theta, a_{\cdot,i}, b_{\cdot,i}\right) \cdot \left(z_i(k) - z_{0,i}\right) \right\} + y(z_0) \tag{10}$$

To simplify the notation, let $\hat{\theta}_i = \hat{\theta}_i\left(\theta, a_{\cdot,i}, b_{\cdot,i}\right)$. Then, (10) can be rewritten as:

$$y_{approx}(k) = \sum_{i=1}^{n_y+n_u} \left\{ \hat{\theta}_i \cdot z_i(k) \right\} + \zeta(z_0), \tag{11}$$

where $\zeta(z_0) = y(z_0) - \sum_{i=1}^{n_y+n_u} \left\{ \hat{\theta}_i \cdot z_{0,i} \right\}$ is a constant value that depends on the value of z_0. Note that (11) is a linear model with a constant term (linear affine) [40]. Moreover, since $\zeta(z_0)$ is constant and independent of $u(k)$ such term can be omitted to compute the transfer function without affecting the result obtained by the analysis of the Sylvester matrix. Then, a transfer function (3) between $y_{approx}(k)$ and $u(k)$ can be computed from (11) by using the lag operator q^{-1} over the components of $z_i(k)$, where q^{-1} is such that $l(k-1)$ satisfies $l(k-1) = q^{-1}l(k)$ and $z_i(k)$ is such that $z(k) = [z_1(k), \ldots, z_p(k)] = \left[y(k-1), \ldots, y\left(k-n_y\right), u(k-1), \ldots, u(k-n_u)\right]^T$.

The Sylvester matrix criterion guarantees that the linearized model at z_0 is controllable if the Sylvester matrix associated with the linearized model is non-singular. Thus, a heuristic criterion for checking controllability of a discrete-time dynamical TS fuzzy model is proposed here: it is said that the system is fuzzy controllable in a set of values of $\{z_j\}$, if the linearized models at each z_j is controllable; otherwise, the model is said to be non-controllable and the parameters and/or the structure of the model have to be modified to guarantee its fuzzy controllability. This heuristic method is motivated by the fact that the controllability of the linearized model guarantees local controllability in continuous time models.

The identification procedure to be presented in Section 4 will use this criterion for enforcing fuzzy controllability, where the set of linearization points is given by the training data points.

4. Fuzzy identification algorithm with controllability constraints

The proposed methodology for the identification of Takagi & Sugeno fuzzy models, considering the controllability conditions analyzed in Section 3, is described here.

4.1. General Fuzzy identification methodology

In general terms, the identification algorithm is based on the procedure shown in Fig. 1 [41]. Note that the contribution of this work lies in the fourth step (Parameter Identification with Controllability Constraints), precisely in the identification of the parameters of the consequences.

All steps are introduced in general terms in this section, whereas the procedure for identification of the consequence parameters considering the controllability condition will be presented in detail in Section 4.2.

The Identification Experiment Design consists in defining an identification signal so that the oscillation modes of the plant in the range of interest are excited [42,43]. Commonly, an amplitude modulated pseudo random binary signal (APRBS) [44] is considered, and is also used here. The output of the experiment is an input–output data-set that is divided into three subsets, namely, training, test, and validation data-subsets, that are further used in the selection of the model structure and in the computation of the model parameters. These sets are used in the selection of variables and the model structure, and posterior tests as is explained next.

The Variables Selection step consists in determining the regressors of the input and output to be considered in the model. Several approaches for this have been reported in the literature. For example, a methodology based on a sensitivity analysis is proposed in [41], heuristic methods are used in [45], and correlation/autocorrelation studies are performed to define the candidate variables in [42]. At the end of this step one obtains a set containing the most relevant variables to represent the behavior of the system.

In this work, the regressors are obtained by evaluating the corresponding fuzzy models for a range of combinations of (n_y, n_u), the maximum orders of the input and output regressors. More precisely, they are evaluated for $n_y = 1, \ldots, n_{y,\max}$ and $n_u = 1, \ldots, n_y - 1$. For each regressor combination (i.e. each combination of (n_y, n_u)), a fuzzy model is identified with the training subset, and the combination that provides the smallest prediction error for the validation subset is chosen.

The Structural Optimization step consists in defining the optimal number of rules N_r. For each different combination of (n_y, n_u),

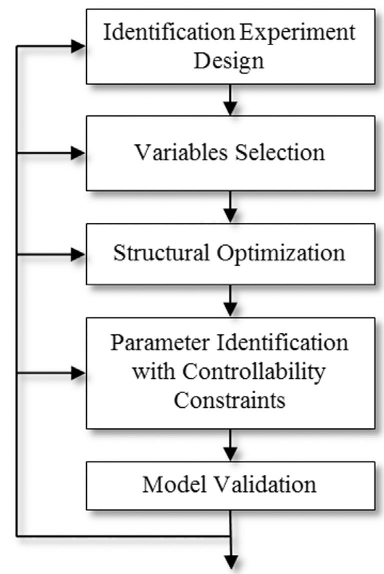

Fig. 1. Proposed methodology of Fuzzy Identification.

different structures are considered systematically, i.e. $N_r = 1, \ldots, N_{r,\max}$, and for each structure the Parameter Identification step proceeds, where the training subset is used to identify the parameters of the model. The number of rules that provides the smallest prediction error for the validation subset is chosen.

For each combination of (n_y, n_u) and structure of the fuzzy model, the Parameter Identification step attempts to find the parameters to minimize the squared error between the measured value and the model output [46,47] for the training data-set. It is noted that the clustering procedure is separated from the consequences parameters identification because a better representation of the operation conditions of the system can be obtained this way [48].

The well-known Gustafson–Kessel algorithm (GK) [5] for fuzzy clustering is selected for finding the membership functions (i.e. premises and their parameters) due to its ability to detect clusters with different shapes and orientation. However other methods for obtaining fuzzy clusters can be also used within the same proposed framework presented for controllability of TS models.

Once the membership functions are defined, one must find the parameters of the consequences. The least squared method is usually used, where the consequence parameters are computed as the solution of the following minimization problem:

$$\min_{\theta} J(\theta) = \sum_{k=1}^{m} \left(y(k) - y_{fuzzy}(k, \theta)\right)^2 \tag{12}$$

where θ is the vector of all consequences parameters, i.e., $\theta^T = \left[\theta_1^T, \ldots, \theta_{N_r}^T\right]$; $y(k)$ is the process output at time k; $y_{fuzzy}(k, \theta)$ is the predicted output obtained by the fuzzy model; and m is the number of measurements. One drawback of this method is that the resulting model is not necessarily controllable. This is the motivation of this work, which as a main contribution introduces a modification of problem (12) in order to enforce controllability of the resulting model. This new method will be presented in detail in Section 4.2.

As mentioned above, the obtained models for each combination of (n_y, n_u) and number of rules are assessed using the validation data-subset in order to find the optimal settings. The most used metrics are the root mean squared error (RMSE), and the comparison of the response of both the real system and the model.

Finally, the adequateness/usefulness of the identified models for control is studied using the test data-subsets.

4.2. Consequence parameters identification to impose controllability

The optimization procedure to find the consequences parameters is described here. The parameters of the premises are not modified here since changing these parameters might result in rules that do not necessarily fit with the operating conditions of the system.

To impose that the TS fuzzy model is controllable, the criterion presented in Section 3 is included into the optimization procedure, i.e. it is imposed that the Sylvester matrices associated with the linearized models at each of the training data points are not singular. Accordingly, the minimization problem (12) is reformulated as follows:

$$\min_{\theta} J(\theta) = \sum_{k=1}^{m} \left(y(k) - y_{fuzzy}(k,\theta)\right)^2 + \sum_{k=1}^{m} \varphi\left(S_{fuzzy}(k,\theta)\right) \tag{13}$$

where

$$\varphi\left(S_{fuzzy}(k,\theta)\right) = \begin{cases} \lambda \text{ if } S_{fuzzy}(k,\theta) \text{ is singular} \\ 0 \text{ if } S_{fuzzy}(k,\theta) \text{ is not singular} \end{cases} \tag{14}$$

where $\lambda \rightarrow \infty$ penalize the case when the model is not controllable; and $S_{fuzzy}(k,\theta)$ is the Sylvester matrix of the system linearized around the point in the training data-subset at time k given the parameters θ. The term $\sum_{k=1}^{m} \varphi\left(S_{fuzzy}(k,\theta)\right)$ guarantees that $S_{fuzzy}(k,\theta)$, $k = 1, \ldots, m$ are not singular by making λ large enough, thus the local systems obtained from the linearization around the points in the training data-subset at time k given the parameters θ are controllable. Thus, as discussed in Section 3, the resulting TS fuzzy model is said to be controllable. Recall that since this is a heuristic procedure, the optimization guarantees that the linearized models around each of the training points are controllable. Since the identification experiment should cover most of the space where the process will operate, this is a good approximation to actual controllability.

Some practical issues need to be addressed regarding the definition of the penalty term in (14). First, roundoff or observational errors play a crucial role here. Since a condition for singularity of a matrix A is $det(A) = 0$, it is clear to see an infinitesimal change of a parameter –either due to roundoff or to an observational/identification error – can make the matrix become non-singular and thus apparently controllable when it is not, or vice-versa. Second, there are cases where the determinant of a matrix may be close to zero when the matrix is not close to singular. For instance, consider the case of an $n \times n$ diagonal matrix with entries given by 10^{-1}. Clearly the value of the determinant will approach to zero as n becomes large, while the matrix is not singular. And third, it is desirable to use a criterion so that systems with Sylvester matrices that are close to singular – and thus are close to uncontrollable – will not appear as a solution of the problem. Such a system (assuming the absence of numerical and observational errors) may be technically controllable, but the time constant for one state may be extremely slow to be practical. Based on all these considerations, it is clear that an indicator of proximity of singularity is needed, and that is not given by the determinant. Thus, the condition number is considered, which for a matrix A, it is given by

$$\kappa(A) = \frac{\max_x \frac{\|Ax\|}{\|x\|}}{\min_x \frac{\|Ax\|}{\|x\|}} \tag{15}$$

If $\kappa(A)$ is large, then A is close to singular. In fact, it holds that $\kappa(A) = \|A\| \left\|A^{-1}\right\|$. As shown in [49], this condition for checking proximity to singularity is robust to all the issues mentioned above.

Thus, in order to penalize proximity to singularity, the penalty function (14) is redefined as

$$\varphi\left(S_{fuzzy}(k,\theta)\right) = \begin{cases} \lambda & \text{if } \kappa\left(S_{fuzzy}(k,\theta)\right) \geq M \\ 0 & \text{else} \end{cases} \tag{16}$$

for some large $M > 0$. If a local model has a condition number great than M it is regarded as close to non-controllable, and will be rejected due to the penalization.

The optimization problem (13) with the penalty function (16) is an unconstrained non-linear optimization problem due to the penalty terms. The LS problem (12) on the other hand is quadratic; in spite of its non-linear nature, $y_{fuzzy}(k,\theta)$ is a linear function of θ . In the examples below, the optimization problem (13) is solved with the Nelder–Mead simplex search method of [50,51] as implemented in the Optimization Toolbox of Matlab. This is an optimization method for unconstrained optimization without derivatives, and is based on the comparison of the function values at the *(n+1)* vertices of a general simplex (where *n* is the number of variables), followed by the replacement of the vertex with the highest value by another point. It is particularly suitable for non-continuous and non-smooth optimization problems.

A pseudocode of the proposed identification algorithm is presented next:

Algorithm 1: TS fuzzy identification with controllability constraints

0: Experiment design, data collection, separate training, validation and test sets.
1: set $n_{y,max}$, $N_{r,max}$, λ, M
2: $J^{opt} = \infty$
3: for $n_y = 1,\ldots,n_{y,max}$, $n_u = 1,\ldots,n_y - 1$
4: $\quad J^{opt}_{n_y,n_u} = \infty$
5: $\quad$ for $N_r = 1,\ldots,N_{r,max}$
6: $\qquad$ Perform clustering.
7: $\qquad$ Solve problem (13). Optimal solution is $\theta^*_{n_y,n_u,N_r}$
8: $\qquad$ if $J_{val}(\theta^*_{n_y,n_u,N_r}) < J^{opt}_{n_y,n_u}$, then $N_r^{opt} = N_r$, where $J_{val}(\theta^*_{n_y,n_u,N_r})$ corresponds to the evaluation of cost (13) using the validation set.
9: $\quad$ end
10: $\quad$ Optimal number of rules for (n_y, n_u) is N_r^{opt} and its cost is $J_{val}(\theta^*_{n_y,n_u,N_r})$
11: $\quad$ if $J_{val}(\theta^*_{N_r^{opt}}) < J^{opt}$, then $(n_y^{opt}, n_u^{opt}) = (n_y, n_u)$ and $N_r^{opt} = N_r$
12: end
13: The optimal solution is $\theta^*_{n_y,n_u,N_r}$ and it has (n_y^{opt}, n_u^{opt}) regressors and N_r^{opt} rules.

The complexity of the proposed algorithm is $n^2_{y,\max} N_{r,\max}$ $O(n_{y,\max}, N_{r,\max}, N, m)$, where $O(n_{y,\max}, N_{r,\max}, N, m)$ is the complexity of the optimization problem as a function of the maximum considered order of the dynamic system $n_{y,\max}$, the maximum considered number of rules of the model $N_{r,\max}$, the number of steps ahead in the optimization N and the data of the training set m. It is clear from Algorithm 1 that the main sources of computation requirements come from evaluating all possible combinations of (n_y, n_u, N_r), and of course the complexity of problem (13).

Algorithm 1, as it is proposed here is not suitable for application on real time as it requires the solution of optimization problem (13), which includes the whole training set. The use of the proposed algorithm for online application, by means of an iterative implementation, is indeed relevant and will be the focus of future research.

Since the contribution of this work lies in the fourth step, precisely in the identification of the parameters of the consequences enforcing controllability, the proposed method can still be used within fuzzy identification methodologies that choose different procedures for the other steps. For instance, different clustering or optimization methods could be used. Also, generalized TS fuzzy systems, that are able represent arbitrarily rotated rules, may be considered with the proposed methodologies.

In the next section, the proposed method is implemented in two case studies.

5. Case studies

Two case studies have been considered to analyze the proposed identification method: a benchmark controllable nonlinear system, and a continuous stirred tank reactor (CSTR) [52].

Considering both case studies, the proposed identification procedure with controllability constraints (CCID) is compared with two other identification methods: the conventional least squares identification (LSID); and a local identification method (LLSID) [53]. In the latter method the parameters θ are found by minimizing:

$$\min_{\theta} J(\theta) = \sum_{r=1}^{N_r} \sum_{k=1}^{m} h_r\left(z(k)\right)\left(y(k) - y_r(k, \theta_r)\right)^2 \tag{17}$$

With some further manipulation, this can be decomposed in such a way that the parameters of each rule are determined separately by means of local least squares identification.

For each system, the performance of the models and the relevance of the imposition of controllability are evaluated through their use in control. While LSID provides the global solution of minimum prediction errors, LLSID provides better behaved models in the sense that they learn the trend embedded in the input–output observations better than LSID [53]. The comparison of CCID with these methods is then performed to verify the relevance of the imposition of controllability versus the model with the smallest error (LSID) and the one that best learns the trend of the input–output observations (LLSID).

Here, a Model Predictive Control (MPC) strategy is considered. Since MPC considers a minimization of the performance over the future N steps, for the LSID and CCID cases the optimization aims at minimizing the quadratic error of the N-steps-ahead predictions. However, only the minimization the quadratic error of the *1*-steps-ahead predictions can be considered for LLSID due to the structure of the local identification: if an N-step-ahead minimization is performed, the model will go through several rules, then the minimization cannot be performed separately for each local model.

An amplitude modulated pseudo random signal (APRBS) has been chosen as the input $u(t)$ in order to generate an input/output data set. This set is divided into the training (60%), validation (20%) and test (20%) sets. The training data set is used for finding the TS model parameters for all the considered settings of number of clusters and structure of the model (maximum delay of the past inputs and states). The validation data set is then used for choosing the best structure and number of clusters of the model. For the CCID procedure, the maximum allowed condition number is set to $M = 10^{10}$. Larger condition numbers are regarded as indicative of the system being close to non-controllable and a penalty is applied.

As mentioned above, an MPC strategy has also been considered for the assessment. The models obtained with the LSID, LLSID and CCID methods are used to obtain the predictions of an MPC controller that at each sample time solves the optimization problem given by

$$\begin{aligned} \min_{u_0,\ldots,u_{N-1}} \quad & \sum_{k=0}^{N-1} (y(k+1) - \text{ref}(k+1))^T Q (y(k+1) - \text{ref}(k+1)) \\ & + u(k)^T R u(k) \\ s.t. \quad & y(k) = f\left(y(k-1), \ldots, y(k-n_y),\right. \\ & \left. u(k-1), \ldots, u(k-n_u)\right), \quad k = 1, \ldots, N \end{aligned} \tag{18}$$

where ref is the reference for the output, $N = 10$ is the prediction horizon and $f\left(z(k)\right) = \sum_{r=1}^{N_r} h_r\left(z(k)\right)\theta_r^T z(k)$, with $z(k) = \left(y(k-1), \ldots, y(k-n_y), u(k-1), \ldots, u(k-n_u)\right)$. According to the receding horizon scheme, this optimization is solved at each sampling time k and only the first element of sequence of control actions, namely $u(k)$, is applied to the system.

For both systems, the fuzzy models are identified using $n_{y,\max} = 6$ and $N_{r,\max} = 5$.

5.1. Controllable nonlinear system: benchmark system

The following state space nonlinear system will be considered as a benchmark system to analyze the proposed identification method:

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) - x_1(t)\exp\left(x_1(t)\right) \\ \dot{x}_2(t) &= -x_1(t) - x_2(t)\exp\left(x_2(t)\right) + u(t) \\ y(t) &= h\left(x(t)\right) = x_1(t). \end{aligned} \tag{19}$$

Note that it can be rewritten as a time-varying linear system

$$\dot{x}(t) = A(t)x(t) + B(t)u(\mathrm{t}),$$

with

$$A(t) = \begin{bmatrix} -\exp(x_1(t)) & 1 \\ -1 & -\exp(x_2(t)) \end{bmatrix}, B(t) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Continuous-time linear time-varying systems are controllable if the controllability matrix $C = [\zeta_1, \ldots, \zeta_n]$, where $\zeta_1(t) = B(t)$, $\zeta_{i+1} = A(t)\zeta_i(t) - \dot{\zeta}_i(t)$, $i = 1, \ldots, n-1$ is full rank [54]. In this case, the controllability matrix is given by

$$C = \begin{bmatrix} 0 & 1 \\ 1 & -\exp\left(x_2(\mathrm{t})\right) \end{bmatrix},$$

which is full rank, thus proving that the system is controllable.

The CCID method is compared with LSID and LLSID using the controllable nonlinear benchmark system. The sampling time for the discrete-time dynamical fuzzy models and the MPC controllers is $T_s = 0.01[s]$.

Some general results regarding the controllability and the structure of the models obtained with the LSID, LLSID and CCID methods are shown in Table 1. Fuzzy models with 4 rules are obtained for LSID and CCID, but the optimal structures are different. LLSID has an optimal fuzzy model with 5 rules. The worst-case (among the different operation points given by the training data) condition number of the Sylvester matrix for CCID is just below the threshold. These matrices are then invertible and the model is controllable in all the operation points defined by the training points. On the other hand, the worst case for the condition number of the Sylvester matrix is above the defined threshold for the models obtained with LSID and LLSID. Then, the identified models are close to singular, implying they are close to uncontrollable and are thus rejected. All determinants are very close to zero, thus justifying the use of the condition number.

Table 2 presents a comparison between the measured and the output values provided by the fuzzy models for the three identification procedures. The root mean squared error (RMSE) considering 1 and 10 step-ahead-predictions was used as a performance index. As expected, the errors for 10-step-ahead predictions are larger than for 1-steps-ahead predictions for all methods. Also, the errors obtained with LSID are smaller than with CCID because the former method does not constrain the condition number and can find the unconstrained optimum. The errors found with LLSID are smaller than those obtained with CCID for 10-step-ahead predictions, but larger than those found by LSID. Indeed, LSID is

Table 1
Fuzzy Identification Results for the Benchmark System using the conventional (LSID), local (LLSID) or proposed (CCID) methods.

Method/Set	$\max\left(\mathrm{cond}\left(S_{fuzzy}(k)\right)\right)$	$\min\left(\det\left(S_{fuzzy}(k)\right)\right)$	(n_y, n_u)	Number ofRules
LSID	2.4154e+15	5.8591e−38	(5,5)	4
LLSID	5.4276e+15	9.0710e−46	(6,5)	5
CCID	9.7036e+09	1.2965e−34	(6,2)	4

The second column informs the maximum value of the condition number of the Sylvester matrix computed for every data point in the training sets. The third column informs the minimum value of the Sylvester matrix computed for the same points. The fourth and fifth columns present the optimal output and input orders (n_y, n_u) and number of rules, respectively.

Table 2
RMSE results for benchmark system.

Method/Steps ahead	Training	Validation	Test
LSID (1 step ahead)	1.3139e−05	1.7663e−05	1.3332e−05
LLSID (1 step ahead)	1.3145e−05	1.5754e−05	1.2286e−05
CCID (1 step ahead)	8.0931e−05	1.0094e−04	9.5322e−05
LSID (10 steps ahead)	0.0022	0.0030	0.0023
LLSID (10 step ahead)	0.0026	0.0032	0.0024
CCID (10 steps ahead)	0.0038	0.0049	0.0047

Table 3
Closed-Loop Performance Results for the Benchmark System controlled with MPC using the model from conventional (LSID), local (LLSID) or proposed (CCID) methods.

Method/Set	avg(J_{cl})	max(J_{cl})	min(J_{cl})	std(J_{cl})
LSID	5.5395e+08	3.6325e+09	1.1257e+03	7.2789e+08
LLSID	6.1014e+08	3.8865e+09	1.1122e+03	7.9900e+08
CCID	4.6995e+08	3.8102e+09	168.1958	7.2806e+08

The table reports the avg, max, min values and the standard deviation of the evaluation of J_{cl} over the different initial conditions and references for which MPC with the LSID model is stable.

the global optimum. However, since LLSID learns better the trend of the input–output dynamics, it generalizes better, and obtains almost the same errors as LSID for the training data, but obtains smaller errors for the validation and test data sets.

It is important to remark that the fuzzy models are of greater order than the original system (five or six of the fuzzy models versus two of the original system). Although this often generates an over-fitting in the response of the model, this happens naturally in the identification procedure to capture all non-linear behaviors introduced by the exponential functions present in the original system (see Eq. (19)).

The application of MPC with the identified models is used to test the relevance of the CCID method. The MPC parameters are $Q = 10^8$, $R = 1$ and $N = 10$. MPC is applied with the models obtained with LSID (MPC-LSID), LLSID (MPC-LLSID) and CCID (MPC-CCID) for 36 different initial conditions (all the crossed combinations of $x_1 = 0, 0.2, 0.4, 0.6, 0.8, 1$ and $x_2 = -1, -0.6, -0.2, 0.2, 0.6, 1$ and 7 different references ($ref = 0, 0.2, 0.4, 0.6, 0.8, 1, 1.2$). The closed-loop performance, which is recorded for each setting, is given by $J_{\mathrm{cl}} = \sum_{k=0}^{K-1}(y(k+1) - \mathrm{ref})^T Q(y(k+1) - \mathrm{ref}) + u(k)^T Ru(k)$, where $K = 100$ is the total simulation runtime.

The average and minimum performances of MPC-CCID are better than that of MPC-LSID. Particularly, the average performance of MPC-CCID is 15.16% smaller than that of MPC-LSID and 22.98% smaller than that of MPC-LLSID.

Thus MPC-CCID consistently, in average and the best case, yielded better performances than both LSID and LLSID. These results validate the relevance of the controllability of the models, since better closed-loop performances can be obtained when controllability is forced, in spite of the facts that the training identification errors are smaller for LSID and LLSID and that models are smoother and correctly follow the output–input trend in LLSID.

5.2. Continuous stirred tank reactor

A continuous stirred tank reactor (CSTR) is also used to evaluate the proposed identification method. The model of the CSTR is described by the following equations [52]:

$$\dot{x_1}(t) = -x_1(t) + K_1(t) \cdot (1 - x_1(t)) - K_2(t)\, x_2(t)$$
$$\dot{x_2}(t) = u(t) - x_2(t) + 5\left[K_1(t) \cdot (1 - x_1(t)) - K_2(t)\, x_2(t)\right]$$
$$K_1(t) = K_{10} \exp\left(-\frac{5000}{x_2(t)}\right); K_2(t) = K_{20} \exp\left(-\frac{7500}{x_2(t)}\right) \tag{20}$$

where $x(t) = [x_1(t), x_2(t)]^T$ is the state of the system, with $x_1(t)$ the conversion and $x_2(t)\,[^\circ K]$ the temperature inside the reactor. $K_{10} = 3 \cdot 10^5$, $K_{20} = 6 \cdot 10^7$ are the Arrhenius constants of the reactions present in the reactor. The manipulated variable $u(t)\,[^\circ K]$ corresponds to the temperature of the inlet flow of cooling fluid and is limited to the interval $[300, 490]\,[^\circ K]$ .The controllable variable, $y(t)$, is the conversion rate, i.e., $y(t) = x_1(t)$.

The sampling time for the discrete-time fuzzy dynamical models and the MPC controllers is $T_s = 0.05[s]$. Some general results regarding the controllability and the structure of the models obtained with the LSID, LLSID and CCID methods are shown in Table 4. The worst-case (among the different operation points given by the training data) condition number of the Sylvester matrix for CCID is well below the threshold. These matrices are then invertible and the model is controllable in all the operation points defined by the training points. For LSID and LLSID, on the other hand, the worst case for the condition number of the Sylvester matrix is above the defined threshold. Then, the identified models are close to singular, implying they are close to uncontrollable and are thus rejected.

Table 5 presents a comparison between the measured and the output values provided by the fuzzy model for the three identification procedures. The RMSE considering 1 and 10 steps ahead predictions was used as a performance index. As expected, just like for the SNLC, the errors for 10-step-ahead predictions are larger than for 1-step-ahead predictions for all methods. Also, the errors obtained with LSID are smaller than with CCID because the former method does not constrain the condition number and can find the unconstrained optimum. The errors found with LLSID are smaller than those obtained with CCID, but larger than those found by LSID for 10-step-ahead predictions. Indeed, LSID is the global optimum for the N-step-ahead predictions. However, the errors are smaller for LSSID for 1-step-ahead predictions because it generalizes better than LSID.

The MPC controller parameters are $Q = 10^{10}$, $R = 1$ and $N = 10$. MPC-LSID, MPC-LLSID and MPC-CCID are applied for 6 different initial conditions ($x_1 = 0, 0.2, 0.4, 0.6, 0.8, 1$, $x_2 = 351$) and all the combinations with the references $ref = 0.4, 0.45, 0.5, 0.55, 0.6, 0.65$. The closed-loop performance J_{cl} is recorded for each

Table 4
Fuzzy Identification Results for the CSTR using the conventional (LSID), local (LLSID) or proposed (CCID) methods.

Method/Set	max (cond ($S_{fuzzy}(k)$))	min (det ($S_{fuzzy}(k)$))	(n_y, n_u)	Number of Rules
LSID	2.2085e+15	4.0007e−42	(4,4)	4
LLSID	5.4276e+15	9.0710e−46	(6,5)	5
CCID	1.3848e+07	1.7608e−13	(2,1)	5

The second column informs the maximum value of the condition number of the Sylvester matrix computed for every data point in the training sets. The third column informs the minimum value of the Sylvester matrix computed for the same points. The fourth and fifth columns present the optimal output and input orders (n_y, n_u) and number of rules, respectively.

Table 5
RMSE results for benchmark system.

Method/Steps ahead	Training	Validation	Test
LSID (1 step ahead)	8.4325e−05	8.5083e−05	8.1722e−05
LLSID (1 step ahead)	7.9448e−05	8.0607e−05	7.6701e−05
CCID (1 step ahead)	2.4790e−04	2.2314e−04	2.2554e−04
LSID (10 steps ahead)	0.0044	0.0038	0.0037
LLSID (10 step ahead)	0.0052	0.0052	0.0047
CCID (10 steps ahead)	0.0075	0.0063	0.0067

Table 6
Closed-Loop Performance Results for the CSTR controlled with MPC using the model from conventional (LSID), local (LLSID) or proposed (CCID) methods.

Method/Set	avg(J_{cl})	max(J_{cl})	min(J_{cl})	std(J_{cl})
LSID	1.6921e+10	5.9734e+10	5.8818e+08	1.4032e+10
LLSID	1.7661e+10	6.3513e+10	6.1223e+08	1.4657e+10
CCID	1.5406e+10	5.4524e+10	2.6907e+08	1.2800e+10

The avg, max, min and values and the standard deviation of the evaluation of J_{cl} over the different initial conditions and references are reported.

initial condition and reference, with $K = 120$ (associated with a simulation time of $6[s]$).

The results for J_{cl} for both methods and all initial conditions and references are reported in Table 6. The average, maximum and minimum performances of MPC-CCID are better than that of MPC-LSID and MPC-LLSID. Particularly, the average performance of MPC-CCID is 8.95% smaller than that of MPC-LSID and 12.77% smaller than that of MPC-LLSID. This happens in spite of the fact that the identification error for training and validation subsets are smaller for LSID and LLSID and that models are smoother and correctly follow the output–input trend in LLSID. This validates the relevance of controllability of the identified model just like in the case of SNLC.

5.3. Discussion

LSID and LLSID, in both examples, obtain a maximum conditioning number (among the training data set points) of around 10^{15} (see Tables 1 and 4), which is large compared to the maximum allowed value of 10^{10}. Though this value is arbitrary, it is useful for illustrating the relevance of controllability of the identified models. CCID obtains models for which the maximum conditioning number is less than 10^{10} (see Tables 1 and 4), and in order to satisfy this constraint, it sacrifices predictive accuracy with respect to LSID and LLSID (see Tables 2 and 5). This sacrifice is performed with the goal of obtaining controllability, which is a desired property in control, which allows that all states can be reached given a feasible sequence of inputs. For this reason, the different methods are tested in a setting that uses an MPC controller with the obtained models. MPC is one of the most popular advanced control methods based in models. It minimizes the performances of the system over a prediction horizon, then applies the first control action of the sequence, and then repeats the procedure the next instant.

LSID is the conventional global least squares minimization based identification, which finds the global optimum for the prediction errors. However by doing so may yield rules that by themselves do not appropriately approximate the dynamics of the system in the vicinity of the center of the cluster associated to that rule. On the other hand, LLSID is a local least squares minimization method, which sacrifices prediction accuracy for ensuring that the local models of each rule approximate the local dynamics of the system. Due to this, it may compete in the errors for 1-step-ahead predictions when the optimization is performed for 10-step-ahead predictions. Additionally, from this property one could expect that imposing controllability for the fuzzy model may not be needed.

However, simulation results show (see Tables 3 and 6) that the models obtained with CCID consistently reach better closed loop performances (as evaluated by the same performance metric used in MPC) than those obtained with the other models. This shows that both priorities (reaching the global optimum for the prediction error and ensuring local models that resemble the real dynamics) are overshadowed by the relevance of imposing controllability when using the models for control, and confirms the relevance of the inclusion of controllability in fuzzy identification of dynamical systems for control. If the model is controllable, then the predicted states in the optimization within the controller can reach the desired operation points, which will then allow better quality control inputs to be applied to the systems.

It is relevant to note that, as mentioned before, this work highlights the relevance of imposing controllability when the models are used for control. However it does not claim that it is better than other methods well established in the literature. Instead, it points out a relevant aspect for the identification of fuzzy systems, which can be included in different identification frameworks.

6. Conclusions

A new fuzzy identification methodology that imposes controllability on the fuzzy models has been proposed in this work. The key step of the methodology is to include, in addition to the classical cost measuring the error between the model estimation and real data, a penalty term that heuristically penalizes models that are not controllable. Two case studies were used for validating the effectiveness of proposed method, and the simulation results showed that the closed-loop responses, in terms of stability and of quantifiable performance, are better when imposing controllability. In particular, these results highlight the role of controllability in predictive controllers design: controllability helps obtaining better closed-loop performance. Future research will focus on developing improvements for computational complexity, such as defining heuristics for finding the optimal regressors order and number of clusters, and the extension of the developed methodology for online application by means of an iterative implementation.

Acknowledgments

This work has been partially supported by Solar Energy Research Center (SERC) CONICYT/FONDAP/15110019, Complex Engineering Systems Institute (CONICYT – PIA – FB0816), and FONDECYT-CONICYT Postdoctorado N° 3170040.

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