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The ordered weighted average in the theory of expertons

Salvador Linares-Mustarós[1] | **Joan C. Ferrer-Comalat**[1] | **Dolors Corominas-Coll**[1] | **José M. Merigó**[2,3]

[1]Department of Business Administration, University of Girona, Girona, Spain

[2]Department of Management Control and Information Systems, School of Economics and Business, University of Chile, Santiago, Chile

[3]Faculty of Engineering and Information Technology, School of Systems, Management and Leadership, University of Technology Sydney, Ultimo, NSW, Australia

Correspondence
Department of Business Administration, University of Girona, C. Universitat de Girona 10, Girona 17071, Spain.
Email: salvador.linares@udg.edu

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Abstract

This work presents a data-fusion mathematical object that incorporates the optimism level of a decision-making agent. The new fusion object is constructed by extending the ordered weighted averaging (OWA) operator in the process of creating an experton. The main advantage of this approach is that it can represent the attitudinal character of the decision maker in the construction of the experton. Therefore, this approach represents a new method for addressing multiperson problems by using optimistic and pessimistic perspectives. The work presents different practical examples based on the absolute hierarchical relationships of the "minimum of the bottom end of the intervals," "minimum of the top end of the intervals," and "minimum size of the interval." The work also considers a wide range of particular cases of the OWA-experton, including the minimum experton, the maximum experton, the average experton, and the olympic experton. In addition, the study presents software for the calculation of OWA-expertons. Finally, the paper ends with an application in business decision-making regarding the calculation of expected benefits.

1 | INTRODUCTION

Expert consultations are a matter of scientific interest with different available lines of inquiry. For example, we find research focused on who may be considered an expert,[1–4] the ideal number of experts to consult,[5–7] or the processing of expert information.[8–13] The current study falls into the latter line of research. It develops a new data-fusion mathematical object that, in a global way,

0 : The assertion is completely false
0.1: The assertion is mostly false
0.2: The assertion is almost false
0.3: The assertion is fairly false
0.4: The assertion is more false than true
0.5: The assertion is neither true nor false
0.6: The assertion is more true than false
0.7: The assertion is fairly true
0.8: The assertion is almost true
0.9: The assertion is mostly true
1 : The assertion is completely true

FIGURE 1 Typical Kaufmann and Gil-Aluja 11-point scale

presents a general view on the confidence levels of decision-makers' assertions. The framework of this research is offered by fuzzy logic,[14] a logic with which experts can reflect the degree of confidence in an assertion. This value, known as a valuation, offers a new dimension to initial data by allowing new mathematical modulations for classic decision problems.[15–18] The search for techniques to address expert valuations led to the appearance of a new line of research that birthed new mathematical objects called expertos.[19–21] Expertons allowed information expressed by a group of experts to be compiled in its entirety.

The starting point for constructing an experton consists of providing a semantic 11-point scale to evaluate a statement or assertion, as shown in Figure 1. Observe that the scale uses the idea of associating a linguistic value linked to the statement to one of the 11 possible values between 0 (the minimum value) and 1 (the maximum value). This type of scale was used for the first time by Kaufmann and Gil-Aluja.[22] Since then, hundreds of conferences, books, and scientific articles have utilized it.

The difficulty for experts to determine a precise numerical valuation led[23] to experts being asked for a closed interval rather than a single value. For example, if an expert did not have a precise valuation but had confidence within the range of 0.5 to 0.6, they would give the interval [0.5, 0.6]. Although this alternative increases uncertainty, it allows for a higher degree of confidence about the data provided by experts. The regular use of maximum confidence closed intervals as a valuation has led to the concept of an experton being historically associated with each expert providing a maximum confidence closed interval as a valuation.

The practical applications of the concept of the experton have been exploited by many fields. Gachechiladze and Panchvidze[24] used it in the problem of diagnosing malfunction failures in the supervisory control of a power system. Couturier and Fioleau[25] applied the experton concept to financial diagnoses. Cassú et al[26] used the concept to estimate the volatility of a set of stocks. Levrat et al[27] used it in the evaluation of car seat comfort. Burusco and Fuentes-González[28] used it to study cause and effect. Cassú et al[29] applied the concept to the study of growth prospects in economic sectors. Nait-Said and Loukia[30] presented a new approach in fuzzy mental workload modeling using the experton concept. Merigó et al[31] applied it to decision-making problems. Sirbiladze and Gachechiladze[32] used it to construct the image of the consonant structure of a syllable. Zalila et al[33] applied expertons to the sensory analysis of cell phone flaps. Delcea and Scarlat[34] used it in the search for causes of corporate bankruptcy. Lafuente and Bassa[35] applied expertons to determine customer needs. Merigó and Wei[36] applied it to an uncertain multiperson decision-making problem. Sirbiladze et al[37] applied it to a multicriteria decision-making problem. Merigó et al[38] used the concept in a multidecision problem. Sirbiladze et al[39] introduced it for an investment problem. Jaile et al[40] applied the experton concept to construct a neural network that allows for the prediction of future values of an economic variable. Yepes et al[41] apply it to the analysis of corporate social responsibility. Finally, Alfaro-Garcia et al[42] apply it to measuring innovation management.

The present work has the goal of obtaining a new experton, an information-fusion mathematical object, that also includes the degree of optimism of a decision maker and, therefore, complements the starting information in a decision-making problem.

The ordered weighted averaging (OWA) operator will be extended in the process of creating an experton in an effort to achieve our purpose. The new object will be called the "OWA-experton." Given its construction, the OWA-experton is a data-fusion object that provides new ways of validating the starting information of a decision-making problem.

The work has the following structure. First, in Section 2, the previous concepts of "expertons" and "OWAs" are presented. Next, Section 3 presents the method for calculating a new type of experton that contains the degree of optimism of a decision maker. Because the construction of these new expertons requires the confidence intervals proposed by the experts to be sorted and knowing that there is a multitude of different ways to sort these intervals, the new experton will depend on the proposed sorting method. Various practical examples shed light on this point. Then, in Section 4, the formalization of the new object is presented. Finally, conclusions and references are presented.

2 | BASIC CONCEPTS

2.1 | The experton

With Zadeh's introduction of fuzzy logic,[14] the foundations were laid for fuzzy thinking, which differs from binary thinking in that it can accept a partially true assertion. This flexibility allows us to arrive at solutions that are more similar to those of a human agent.

The introduction of partially true assertions into problems led to the birth of new techniques for aggregating and fusing data. The experton concept[19–21] is a clear example of this.

The idea of an experton that has been consolidated in scientific research emerges as a result of a procedure of aggregating the opinions of various experts with respect to the degree of truthfulness of an assertion. It is then evaluated through a confidence interval with extremes on an 11-point scale. The experton concept is intimately linked[43] with interval-valued fuzzy sets[44,47] (also called Φ-fuzzy sets) and Hirota's probabilistic fuzzy sets.[48]

To help understand the concept of an experton, we illustrate the construction of an experton from the 10 confidence intervals presented in Table 1.

TABLE 1 Confidence intervals of 10 experts

E_1: [0.2,0.3]	E_6: [0.8,1]
E_2: [0.5,0.6]	E_7: [0.4,0.8]
E_3: [0.1,0.7]	E_8: [0.4,0.5]
E_4: [0.3,0.4]	E_9: [0,0.1]
E_5: [0.6,0.6]	E_{10}: [0.2,0.4]

The construction process for an experton follows this method.

Step 1: We build a table that collects the absolute frequency or the number of times that a single valuation has been chosen from the lower end or from the upper end, as the table shows.

Step 2: We normalize the data in the absolute frequency table to get the table of relative frequencies.

Step 3: The relative frequencies obtained are interpreted as values in a probability density function, in which we obtain the strict function distributions.

Step 4: From the strict function distributions, we obtain the experton.

Tables 2–8 show each process in the creation of the experton in Table 1. At http://web2.udg.edu/grmfcee/experton.exe, one can execute a program for calculating an experton, which provides the final value of the experton. Its operation is detailed by Ferrer et al.[43]

With this construction, we can obtain the mathematical object of the experton. It is capable of offering the percentage of experts who agree that the truthfulness of the assertion is at least the given value. This is done level by level on an 11-point scale that ranges from 0 to 1. For example, if we examine the results for the value of the experton in level 0.4, it will tell us that the percentage of experts who are in favor of the assertion being true at least to a degree of 0.4 is between 50% and 80%.

Waltz and Llinas[49] claim that a data fusion mechanism is one that can visualize a single object of significant information that is more useful than the sum of its parts. Against this view, the previous observation justifies that the experton should be identified as a data fusion mechanism.

2.2 | OWA operators

Operators known as ordered weighted averages (OWAs)[50–52] are mathematical functions that are used to aggregate numerical data by providing a representative value to the series that considers the attitudinal character of the data. From its appearance, the OWA operator has been studied and applied to a wide range of problems, among which include situations with large uncertainty using fuzzy numbers.[53,54]

TABLE 2 Table of absolute frequencies

α	$F(\alpha)$	$F_*(\alpha)$
0	1	0
0.1	1	1
0.2	2	0
0.3	1	1
0.4	2	2
0.5	1	1
0.6	1	2
0.7	0	1
0.8	1	1
0.9	0	0
1	0	1

TABLE 3 Table of relative frequencies

α	$f(\alpha) = \frac{F(\alpha)}{NE}$ NE = number of experts	$f_*(\alpha) = \frac{F^*(\alpha)}{NE}$
0	0.1	0
0.1	0.1	0.1
0.2	0.2	0
0.3	0.1	0.1
0.4	0.2	0.2
0.5	0.1	0.1
0.6	0.1	0.2
0.7	0	0.1
0.8	0.1	0.1
0.9	0	0
1	0	0.1

Definition 1 Given a numerical series (a_i) and an associated length vector n,

W = (w_1, w_2,..., w_n), where w_j is a member of the interval [0,1] j and $\sum_{j=1}^{n} w_j = 1$. A descending OWA operator (DOWA) is defined as a function F of R^n over R such that

$$F(a_1, a_2, ..., a_n) = \sum_{j=1}^{n} w_j b_j, \quad (1)$$

where b_j is the jth largest value in the finite sequence a_i.

Definition 2 Given a numerical series (a_i) and an associated length vector n,

W = (w_1, w_2,...,w_n), where w_j is a member of the interval [0,1] ∀ j and $\sum_{j=1}^{n} w_j = 1$. An ascending OWA operator (AOWA) is defined as a function F of R^n over R, such that

TABLE 4 Table of strict distribution functions

α	$C(\alpha) = 0 + \sum_{\alpha'<\alpha} f_x(\alpha')$	$C_*(\alpha) = 0 + \sum_{\alpha'<\alpha} f_{x^*}(\alpha')$
0	0	0
0.1	0.1	0
0.2	0.2	0.1
0.3	0.4	0.1
0.4	0.5	0.2
0.5	0.7	0.4
0.6	0.8	0.5
0.7	0.9	0.7
0.8	0.9	0.8
0.9	1	0.9
1	1	0.9

$$F(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j, \tag{2}$$

where b_j is the jth lowest value in the finite sequence a_i.

The only difference between the two operators is in the sorting process. For the AOWA, we see that the arguments b_j are ascendingly ordered such that $b_1 \leq b_2 \leq \cdots \leq b_n$. However, for the DOWA, the order is descending such that $b_1 \geq b_2 \geq \cdots \geq b_n$.[55] Given the construction carried out in the definition of both operators, it is trivial to check that the weights are related according to the expression $w_{j'} = w_{n-j}$, where w_j' is the jth weight of the AOWA and w_{n-j} is the n–jth weight of the DOWA. In the current paper, given that the levels in expertons are usually displayed in ascending order from 0 to 1, we have chosen to work with the ascending OWA.

3 | THE OWA-EXPERTON CONCEPT

We take set of n experts E_1, E_2,..., E_n who offer a confidence interval for the truthfulness of an assertion that is identified as follows. E_i: $[a_i,b_i]$, $i = 1,\ldots, n$, and a set of weights $W = (w_1,w_2,\ldots,w_n)$, where w_j belongs to the interval [0,1] $\forall j$ and $\sum_{j=1}^{n} w_j = 1$. It is determined by a decision maker and reflects an attitudinal character. We construct the OWA-experton using the following procedure.

Step 1: Sort the intervals given by the experts according to an absolute order relationship to be decided by the decision maker.

Step 2: Obtain a new interval series that fulfils the condition that the relative frequencies are the weights of the ascending OWA vector given by the decision maker.

Step 3: Given the new interval series, construct the experton associated with said series.

Given that the construction of a new object depends on the sorting method chosen by the decision maker, it is possible to use attitudinal criteria with regard to whether the interval average is a representative value in the sorting. With the aim of showing various examples, we will start from a relationship formulated by Yager.[56] It is defined as follows.

Definition 3 (**Yager's preordered relationship**) Given an interval $I_j = [a_j,b_j]$, the representative of the interval is defined as

$\mathrm{Rep}_\lambda(I_j) = (a_j + b_j)/2 + \lambda(b_j - a_j)/2 = m_j + \lambda \cdot r_j/2$,
where mj is the average of the interval, r_j is the statistical range or intervals size, and λ is a real numerical variable that ranges between −1 and 1. Observe that if $\lambda = -1$, we do not believe that the average is a representative value of the segment and that the representative value is maximally far below the average) we obtain $\mathrm{Rep}_{-1}(I_j) = a_j$. If $\lambda = 1$, we do not believe that the average is a representative value of the segment and that the representative value is maximally far above the average and we obtain $\mathrm{Rep}_1(I_j) = b_j$. If $\lambda = 0$, we believe that the average is a representative value of the segment, and we obtain $\mathrm{Rep}_0(I_j) = m_j$.

This relationship is a presorting because it clearly verifies reflexive and transitive properties. To achieve a total order relationship, it is necessary to add some other condition to satisfy the properties of antisymmetry and completeness. In the following sections, we show the different

TABLE 5 Experton

α	$a(\alpha) = 1 - C(\alpha)$	$b(\alpha) = 1 - C_*(\alpha)$
0	1	1
0.1	0.9	1
0.2	0.8	0.9
0.3	0.6	0.9
0.4	0.5	0.8
0.5	0.3	0.6
0.6	0.2	0.5
0.7	0.1	0.3
0.8	0.1	0.2
0.9	0	0.1
1	0	0.1

examples of OWA-expertons based on the three complete sorting methods. In all of them, we will start from the intervals provided by experts in Table 1.

3.1 | The OWA-experton with order relationship "minimum of the bottom end of the intervals"

The OWA-experton with the order relationship "minimum of the bottom end of the intervals" is an experton created by reordering the confidence intervals reported by experts according to the order relationship based on sorting the minimum values of the bottom end of the intervals that are equivalent to $\lambda = -1$ for Yager's preordered relationship.

The relationship is defined in the following way.

$$\mathrm{E}_i : [a_i, b_i] \leq \mathrm{E}_j : [a_j, b_j] \Leftrightarrow (a_i < a_j) \text{ or } (a_i = a_j \text{ and } b_i \leq b_j). \tag{3}$$

A decision maker should use this sorting method when they wish to construct an OWA-experton where the focus is on the lower ends of the interval.

Example 1 Suppose a decision maker decides to create this type of OWA-experton from Table 1 with the following weights: W = (0.4, 0.3, 0.2, 0.1, 0, 0, 0, 0, 0, and 0). Given the values of the weighting vector, we can infer that the decision maker has adopted an attitudinal criterion to highlight the intervals with the most pessimistic lower ends. The construction of the OWA-experton would be determined as follows.

Step 1: Reorder the intervals given by the experts from Table 1 according to the total order relationship presented in the paper by Skjong and Wentworth.[1] With the total order relationship criteria, we get the following reordering.

$\mathrm{E}_9 \leq \mathrm{E}_3 \leq \mathrm{E}_1 \leq \mathrm{E}_{10} \leq \mathrm{E}_4 \leq \mathrm{E}_8 \leq \mathrm{E}_7 \leq \mathrm{E}_2 \leq \mathrm{E}_5 \leq \mathrm{E}_6$.

Step 2: From the W weighting vector, we obtain a new series of intervals that satisfy the condition that their relative frequencies are exactly the given weights. In this example, we would obtain the following series of intervals. Γ.

TABLE 6 OWA-experton

α	$a(\alpha)$	$b(\alpha)$
0	1	1
0.1	0.6	1
0.2	0.3	0.6
0.3	0	0.6
0.4	0	0.4
0.5	0	0.3
0.6	0	0.3
0.7	0	0.3
0.8	0	0
0.9	0	0
1	0	0

Abbreviation: OWA, ordered weighted averaging.

Step 3: Through the four general steps described in section 2, we would obtain the OWA-experton as follows (Table 6).

With the goal of simplifying the OWA-experton calculations, the following address http://web2.udg.edu/grmfcee/OWAExperton.exe allows one to access a calculation program created by the authors. Its interface is shown in Figure 2. It presents an intuitive operation that is similar to the previously mentioned program "Experton.exe."

3.2 | The OWA-experton with order relationship "minimum of the top end of the intervals"

The OWA-experton with the order relationship "minimum of the top end of the intervals" is an experton created from reordering of the confidence intervals given by experts according to a total order relationship based on sorting the minimum value of the top end of the intervals that are equivalent to $\lambda = 1$ for Yager's preordered relationship. This relationship is defined as follows.

$$\mathrm{E}_i : [a_i, b_i] \leq \mathrm{E}_j : [a_j, b_j] \Leftrightarrow (b_i < b_j) \text{ or } (b_i = b_j \text{ and } a_i \leq a_j). \tag{4}$$

A decision maker should use this sorting method when they want to construct an experton where the focus is on the upper ends of the interval

Example 2 Suppose a decision maker creates this kind of OWA-experton from λ with the following weights: W = (0, 0, 0, 0, 0, 0.03, 0.07, 0.2, 0.3, and 0.4) to focus on the more optimistic values. The construction of the OWA-experton would be determined as follows.

Step 1: Reorder the intervals given by the experts from Table 1 according to the total order relationship presented in [2]. With the total order relationship criteria, we get the following reordering. $\mathrm{E}_9 \leq \mathrm{E}_1 \leq \mathrm{E}_{10} \leq \mathrm{E}_4 \leq \mathrm{E}_8 \leq \mathrm{E}_2 \leq \mathrm{E}_5 \leq \mathrm{E}_3 \leq \mathrm{E}_7 \leq \mathrm{E}_6$.

Step 2: From the *W* vector weights we obtain a new series of intervals that satisfy the condition that their relative frequencies are exactly the given weights. In this example, we would

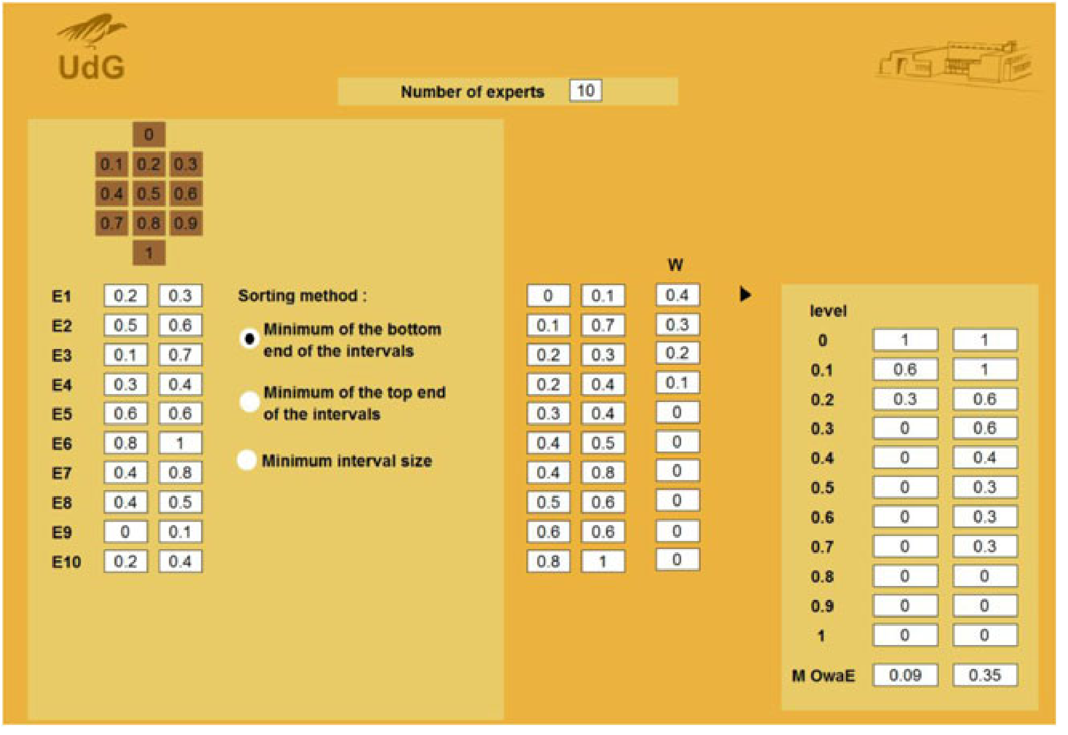

FIGURE 2 Calculation of an OWA-experton with the program "OWAExperton.exe." OWA, ordered weighted averaging [Color figure can be viewed at wileyonlinelibrary.com]

obtain the following series of intervals. E_2, ...$^{3)}$, E_2, E_5, ...$^{7)}$, E_5, E_3, ...$^{20)}$,E_3,E_7,...$^{30)}$,E_7, E_6,...$^{40)}$, E_6.

Step 3: Through the four general steps described in section 2, we would obtain the OWA-experton as follows (Table 7).

TABLE 7 OWA-experton

α	$a(\alpha)$	$b(\alpha)$
0	1	1
0.1	1	1
0.2	0.8	1
0.3	0.8	1
0.4	0.8	1
0.5	0.5	1
0.6	0.47	1
0.7	0.4	0.9
0.8	0.4	0.7
0.9	0	0.4
1	0	0.4

Abbreviation: OWA, ordered weighted averaging.

3.3 | The OWA-experton with relationship order "minimum interval size"

The OWA-experton with the relationship order "minimum interval size" is an experton created by reordering the confidence intervals reported by experts according to a total relationship order based on the minimum value for the size of the intervals (minimum entropy), and, in the case of equal entropy, ordered based on the average value of the estimate that are equivalent to $\lambda = 0$ for Yager's preordered relationship. This relationship is defined as follows.

$$\mathrm{E}_i\ :[a_i, b_i] \le \mathrm{E}_j\ :[a_j, b_j] \Leftrightarrow (b_i - a_i < b_j - a_j) \text{ or } \left(b_i - a_i = b_j - a_j \text{ and } \frac{a_i + b_i}{2} \le \frac{a_j + b_j}{2}\right). \quad (5)$$

A decision maker should use this sorting method when they wish to construct an experton that focuses on the precision of the predictions.

Example 3 Suppose a decision maker decides to create this kind of OWA-experton from Table 1 with weights that are almost proportional to the different sizes of the intervals. The construction of the OWA-experton would be determined as follows.

Step 1: Reorder the intervals given by the experts from Table 1 according to the total order relationship presented in the paper by Summers et al.[3] First, we calculate the length of the intervals as follows.

E_1: [0.2,0.3]	E_2 : [0.5,0.6]	E_3 : [0.1,0.7]	E_4 : [0.3,0.4]	E_5 : [0.6,0.6]
$b_i - a_i = 0.1$	$b_i - \mathrm{a}_i = 0.1$	$b_i - a_i = 0.6$	$b_i - a_i = 0.1$	$b_i - a_i = 0$
E_6 : [0.8,1]	E_7 : [0.4,0.8]	E_8 : [0.4,0.5]	E_9 : [0,0.1]	E_{10}: [0.2,0.4]
$b_i - a_i = 0.2$	$b_i - a_i = 0.4$	$b_i - a_i = 0.1$	$b_i - a_i = 0.1$	$b_i - a_i = 0.2$

With the total order relationship criteria, we get the following reordering.

$$\mathrm{E}_5 \le \mathrm{E}_9 \le \mathrm{E}_1 \le \mathrm{E}_4 \le \mathrm{E}_8 \le \mathrm{E}_2 \le \mathrm{E}_{10} \le \mathrm{E}_6 \le \mathrm{E}_7 \le \mathrm{E}_3.$$

Step 2: First, we find the weights that will be distributed proportionally between the sizes. As we have five different sizes, we have to construct the following weighting series.

E_5 : [0.6,0.6]	E_9 : [0,0.1]	E_1 : [0.2,0.3]	E_4 : [0.3,0.4]	E_8 : [0.4,0.5]
$b_i - a_i = 0$	$b_i - a_i = 0.1$	$b_i - a_i = 0.1$	$b_i - a_i = 0.1$	$b_i - a_i = 0.1$
weight: $5a$	weight: $4a$	weight: $4a$	weight: $4a$	weight: $4a$
E_2 : [0.5,0.6]	E_{10} : [0.2,0.4]	E_6 : [0.8,1]	E_7 : [0.4,0.8]	E_3 : [0.1,0.7]
$b_i - a_i = 0.1$	$b_i - a_i = 0.2$	$b_i - a_i = 0.2$	$b_i - a_i = 0.4$	$b_i - a_i = 0.6$
weight: $4a$	weight: $3a$	weight: $3a$	weight: $2a$	weight: a

Since $5a + 4a + 4a + 4a + 4a + 4a + 3a + 3a + 2a + a = 1$ must be satisfied, we get a $\approx$ 0.0294. Considering that they must add up to 1 and it is possible we will have to modify the weights of some fractions, we get the following weightings.

E_5: [0.6,0.6]	E_9: [0,0.1]	E_1: [0.2,0.3]	E_4: [0.3,0.4]	E_8: [0.4,0.5]
$b_i - a_i = 0$	$b_i - a_i = 0.1$	$b_i - a_i = 0.1$	$b_i - a_i = 0.1$	$b_i - a_i = 0.1$
weight: 0.1474	weight: 0.1176	weight: 0.1176	weight: 0.1176	weight: 0.1176
E_2: [0.5,0.6]	E_{10}: [0.2,0.4]	E_6: [0.8,1]	E_7: [0.4,0.8]	E_3: [0.1,0.7]
$b_i - a_i = 0.1$	$b_i - a_i = 0.2$	$b_i - a_i = 0.2$	$b_i - a_i = 0.4$	$b_i - a_i = 0.6$
weight: 0.1176	weight: 0.0882	weight: 0.0882	weight: 0.0588	weight:0.0294

From the W vector's weights, we obtain a new series of intervals that satisfy the condition that their relative frequencies are the exact weights. In this example, we obtain the following series of intervals.

E_5, …$^{1474)}$, E_5, E_9, …$^{1176)}$, E_9, E_1, …$^{1176)}$, E_1, E_4, …$^{1176)}$, E_4,E_8, …$^{1176)}$, E_8, E_2, …$^{1176)}$, E_2, E_{10}, …$^{882)}$, E_{10}, E_6, …$^{882)}$,E_6, E_7, …$^{588)}$, E_7, E_3, …$^{294)}$, E_3, where the confidence interval of expert E_5 appears 1474 times, E_9's appears 1176 times, and so on.

Step 3: Through the four general steps described in section 2, we would obtain the OWA-experton that follows (Table 8).

4 | FORMULATING THE OWA-EXPERTON

To get a correct mathematical formulation of the OWA-experton concept, following the formalization of the experton concept proposed by Ferrer et al,[43] it is necessary to have previously defined the problem we are modeling. In our case, we start from a finite set of "m" experts that we represent with $E = \{E_1, E_2,..., E_m\}$ and who we will ask to give a subjective numerical estimation on the confidence level that an imprecise characteristic ω offers. To be coherent with usual practice, we will accept that every estimation may be given through a closed interval within the interval [0, 1]. The extremes of which take values within the set $I = \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\} \subset [0, 1]$, which correspond to the 11 possible values of the 11-point scale. With the following definition of interval-valued fuzzy set, we will be able to model the starting set.

TABLE 8 OWA-experton

α	$a(\alpha)$	$b(\alpha)$
0	1	1
0.1	0.8824	1
0.2	0.853	0.8824
0.3	0.6472	0.8824
0.4	0.5296	0.7648
0.5	0.3532	0.559
0.6	0.2356	0.4414
0.7	0.0882	0.1764
0.8	0.0882	0.147
0.9	0	0.0882
1	0	0.0882

Abbreviation: OWA, ordered weighted averaging.

Definition 4 (Interval-valued fuzzy sets) An interval-valued fuzzy set $\tilde{A}$ of referential Ω is a set of ordered pairs $\tilde{A} = \{(x, \ \mu_{\tilde{A}}(x))/x \in \Omega\}$, where $\mu_{\tilde{A}}$ is a function.

$$\mu_{\tilde{A}} : \ \Omega \quad \rightarrow \quad P([0, 1])$$

$$x \rightarrow \mu_{\tilde{A}}(x) = [a_x, \ \ b_x],$$

such that in each x$\in \Omega$, there is a corresponding interval $[_{ax,}\ b_x] \subseteq [0, 1]$. Similar to fuzzy sets, the function $\mu_{\tilde{A}}$ is called a membership function of the interval-valued fuzzy set $\tilde{A}$.

Following the definition, we can establish that our starting set is the following interval-valued fuzzy set of referential E.

$$\tilde{A} = \{(\mathrm{E}_1, [a_1, b_1]), (\mathrm{E}_2, [a_2, b_2]), \ldots, (\mathrm{E}_m, [a_m, b_m])\},$$

where the membership value of every expert to the interval-valued fuzzy set is determined by the confidence interval that is derived from the valuation of the given expert.

Given that it is compulsory to have a total order relationship to order the closed intervals belonging to the interval [0, 1], we can consider a new interval-valued fuzzy set of the same referential E as follows.

$\tilde{A}' = \{(\mathrm{E}_{k_1}, [a_{k_1}, b_{k_1}]), (\mathrm{E}_{k_2}, [a_{k_2}, b_{k_2}]), \ldots, (\mathrm{E}_{k_m}, [a_{\mathrm{k}_m}, b_{\mathrm{k}_m}])\}$, with the feature that this set has the same elements as set $\tilde{A}$, but in this case the elements are perfectly ordered with a total order relationship using the elements' membership function.

Finally, with the associated vector of size n, $W = (w_1, \ w_2, \ldots, w_n)$, we can consider a new interval-valued fuzzy set.

$$\begin{aligned} \tilde{A}'' = \{&(\mathrm{E}_{k_1}, [a_{k_1}, b_{k_1}]), \overset{10^k \cdot w_1 \text{ times)}}{\ldots}, (\mathrm{E}_{k_1}, [a_{k_1}, b_{k_1}]), \\ &(\mathrm{E}_{k_2}, [a_{k_2}, b_{k_2}]), \overset{10^k \cdot w_2 \text{ times)}}{\ldots}, (\mathrm{E}_{k_2}, [a_{k_2}, b_{k_2}]), \\ &\ldots(\mathrm{E}_{k_\mathrm{m}}, [a_{k_\mathrm{m}}, b_{k_\mathrm{m}}]), \overset{10^k \cdot w_\mathrm{m} \text{ times)}}{\ldots}, (\mathrm{E}_{k_\mathrm{m}}, [a_{k_\mathrm{m}}, b_{k_\mathrm{m}}])\}. \end{aligned}$$

This is a new referential with auxiliary E′ such that the interval series satisfies the condition that its relative frequencies coincide with the weights of the ascending OWA vector given by the decision maker. Note that if $10^k \cdot w_1 + 10^k \cdot w_1 + 10^k \cdot w_1 + \cdots + 10^k \cdot w_1 \neq 10^k$, we will have to round some weights.

This interval-valued fuzzy set will be the starting set for the construction of the OWA-experton. Then, we present a basic definition for the final construction as follows. Although Hirota proposes a more general definition for probabilistic fuzzy sets,[48] we take a more concrete case that simplifies the definition of the OWA-experton concept.

Definition 5 (**Probabilistic fuzzy sets**) Given that Ω is a referential, a probabilistic fuzzy set $\hat{A}$ of Ω is a set of ordered pairs $\hat{A} = \{(x, \mu_{\hat{A}}(x))/x \in \Omega\}$, where for each x $\in \Omega$, $\mu_{\hat{A}}(x)$ is a random variable defined in the interval [0, 1].

Given that in probabilistic fuzzy sets, the image through $\mu_{\hat{A}}(x)$ of an element of σ-algebra is also an element of σ-algebra, we will always be able to find the image of a given interval in [0, 1] with the function $\mu_{\hat{A}}(x)$ for all x.

If we suppose the referential set Ω is $\Omega = \{\omega\}$, where ω is an imprecise characteristic, we can use the previous definition to formally construct the following probabilistic fuzzy sets. $\hat{A} = \{(\omega, \ \mu_{\hat{A}}(\omega))\}$ and $\hat{B} = \{(\omega, \mu_{\hat{B}}(\omega))\}$, where $\mu_{\hat{A}}(\omega)$ and $\mu_{\hat{B}}(\omega)$ are random variables from the set $I = \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\}$, that have the following respective probability density functions.

$$\text{Prob } [\mu_{\hat{A}}(\omega) = \alpha] = p_\alpha = \frac{1}{10^k} \cdot \text{card } \{a_i/a_i = \alpha, \ i = 1, 2, \ldots, k\}. \tag{6}$$

$$\text{Prob } [\mu_{\hat{B}}(\omega) = \alpha] = p_\alpha^* = \frac{1}{10^k} \cdot \text{card}\{b_i/b_i = \alpha, \ i = 1, 2, \ldots, k\}. \tag{7}$$

We then consider the respective strict distribution functions starting from these probability density functions.

$$C(\alpha) = \text{Prob } [\mu_{\hat{A}}(\omega) < \alpha] = \sum_{\alpha' < \alpha} p_{\alpha'}. \tag{8}$$

$$C_*(\alpha) = \text{Prob } [\mu_{\hat{B}}(\omega) < \alpha] = \sum_{\alpha' < \alpha} p_{\alpha'}^*. \tag{9}$$

We will take the following complementary functions $a(\alpha) = 1 - C(\alpha)$ *i* $b(\alpha) = 1 - C_*(\alpha)$ to definitively formalize the concept of OWA-experton.

Definition 6 (**OWA-experton associated with an interval-valued fuzzy set**) Given an imprecise variable ω and an interval-valued fuzzy set given by $\tilde{A} = \{(\mathrm{E}_1, \ [a_1, b_1]), (\mathrm{E}_2, [a_2, \ b_2]), \ldots, (\mathrm{E}_m, \ [a_m, b_m])\}$, we apply the previous process to obtain the new interval valued fuzzy set $\tilde{A}'' = \{(\mathrm{E}_{k_1}, \ [a_{k_1}, \ b_{k_1}])^{10^k \cdot w_1 \text{ times})}, \ldots, (\mathrm{E}_{k_\mathrm{m}}, [a_{k_\mathrm{m}}, \ b_{k_\mathrm{m}}])^{10^k \cdot w_\mathrm{m} \text{ times})}\}$, which is made up of ordered valuations and a weighting vector W. We call the associated OWA-experton $\tilde{A}$ and represent it as $\dot{\tilde{A}} = \{ \ (\alpha, [a(\alpha), b(\alpha)]), \alpha \in I\}$ in the interval-valued fuzzy set on the referential with the membership function $\mu_{\dot{A}}(\alpha) = [a(\alpha), b(\alpha)] = [1 - C(\alpha), \ 1 - C_*(\alpha)]$ with α∈I, where $C(\alpha)$ and $C_*(\alpha)$ are functions (3) and (4), respectively.

From this definition and considering that $C(\alpha)$ and $C_*(\alpha)$ are strict probability density functions (therefore, growing functions where $C(0) = 0$ and $C_*(0) = 0$), it is evident that the following conditions are satisfied for an OWA-experton.

1. $\forall\alpha\in$I, where $a(\alpha) \leq b(\alpha)$.
2. $\forall\alpha,\alpha'\in$I, where $\alpha < \alpha'$, which verifies $a(\alpha) \geq a(\alpha')$ and $b(\alpha) \geq b(\alpha')$.
3. $a(0) = b(0) = 1$.

Finally, we reference two particularly relevant cases.

1) In the case that there is only one expert to consult and only a single confidence interval as an expression of the expert's valuation, we will have an OWA-experton where $a(\alpha)$ and $b(\alpha)$ will only have values of 0 or 1.
2) If the expert valuations are expressed through a fuzzy set instead of an interval-valued fuzzy set (that is, each expert gives the result of their valuation with a single value instead of a confidence interval), we will get $a_i = b_i$ ($\forall i = 1, 2, \ldots, m$). Therefore, $C(\alpha) = C^*(\alpha)$ and $a(\alpha) = b(\alpha)$, and the OWA-experton will be expressed in this case as a fuzzy set of I. Thus, the OWA-experton becomes the OWA probabilistic set.

5 | PARTICULAR CASES OF OWA-EXPERTONS

In this section, we will study different concrete cases of OWA operators applied to an experton. The main OWA operator families that we will consider in this work are the families with minimum, maximum, average, olympic-OWA, and window-OWA weighting vectors.[57,58] In addition, note that if the intervals used in the construction of the OWA-experton become crisp numbers, we get the OWA probabilistic set, which is a generalization of Hirota's probabilistic sets[48] by using OWA operators.

Case minimum: When the weighting vector $W = (1, 0, 0, \ldots, 0)$ is used, the OWA-experton coincides with the particular case where we calculate an experton using the bottom interval given by the experts.

Case maximum: When the weighting vector $W = (0, 0, 0, \ldots, 1)$ is used, the OWA-experton coincides with the particular case where we calculate an experton using the top interval given by the experts.

Case average: When the weighting vector $W = (\frac{1}{n}, \frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})$ is used, the OWA-experton coincides with the particular case of the classic experton. Thus, we observe that the classic calculation of an experton is a particular case of the general calculation of an OWA-experton.

Case olympic-OWA: When the weighting vector $W = (0, \frac{1}{n-2}, \frac{1}{n-2}, \ldots, \frac{1}{n-2}, 0)$ is used, the OWA-experton coincides with the particular case of a classic experton calculation after the maximum and minimum intervals have been removed.

Case window-OWA: When the weighting vector $W = (0, \text{k})\ldots, 0, \frac{1}{n-k-s}, \frac{1}{n-k-s}, \ldots, \frac{1}{n-k-s}, 0, \text{s})\ldots, 0)$ where $k > 1$, $s > 1$ and $n - k - s > 1$ is used, the OWA-experton coincides with the calculation of a classic experton after the first k smallest interval and the first s largest intervals have been removed.

6 | APPLICATION IN BUSINESS DECISION MAKING

In what follows, we develop an example of an uncertain multiperson decision-making problem in strategic management by using the OWA-experton. Although the example does not use real data, it represents a common real-world situation.

First, let us assume that a United States–based company is analyzing its general strategy for the next year and is considering expanding into a new market. After careful review with the board of directors, it considers that, to avoid cash flow risks, expansion into the new market will be viable if the profits forecast in the traditional market are at least three million dollars. Given that the country's economic situation is changing with respect to previous years, the company's financial background is not considered reliable for making such a future prediction. Therefore, it is decided they will use the OWA-experton method to evaluate the viability of the expansion proposal. The board decides that it will be the members of the management committee who will use their intuition to evaluate the possibility of achieving the said level of profits in the coming year.

Second, let us assume we have seven people on the board who offer their opinions regarding the validity of the assertion, "the company's profits in the traditional market for the coming year will be in excess of three million dollars." To help evaluate the assertion we ask the members of the board to use a two-phase 11-point scale.[59] To do this, we first ask the board members to choose one of the following options.

N: I am convinced that it is impossible to achieve the forecasted profits
B: I am convinced it is unlikely that the forecasted profits will be achieved
M: I am convinced it is moderately likely that the forecasted profits will be achieved
H: I am convinced it is considerably likely that the forecasted profits will be achieved
T: I am absolutely convinced that the forecasted profits will be achieved

Then, in a second phase, if option B was chosen, one of the following three options must be chosen.

Bb: It is practically impossible to achieve the forecasted profits
Bm: It is very unlikely that the forecasted profits will be achieved
Bh: It is unlikely that the forecasted profits will be achieved

TABLE 9 Eleven-point scale

	B			**M**			**H**			
N	**Bb**	**Bm**	**Bh**	**Mb**	**Mm**	**Mh**	**Hb**	**Hm**	**Hh**	**T**
0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1

TABLE 10 Table of absolute frequencies

α	$F(\alpha)$	$F(\alpha)$
0	0	0
0.1	0	0
0.2	0	0
0.3	25	0
0.4	25	0
0.5	0	0
0.6	50	50
0.7	0	50
0.8	0	0
0.9	0	0
1	0	0

In the case that M was chosen, one of the following three options must be chosen.

Mb: I am slightly more convinced that the profits will not be achieved than they will
Mm: I can't say if the profits will be achieved or not
Mh: I am slightly more convinced that the profits will be achieved than they will not

In the case that H was chosen, one of the following three options must be chosen.

Hb: It is considerably likely that the forecasted profits will be achieved
Hm: It is highly likely that the forecasted profits will be achieved
Hh: It is almost certain that the forecasted profits will be achieved

The final assignment is obtained from the following relationship (Table 9).

TABLE 11 Table of relative frequencies

α	$f(\alpha) = \frac{F(\alpha)}{\mathrm{NE}}$ NE = number of experts	$f_*(\alpha) = \frac{F^*(\alpha)}{\mathrm{NE}}$
0	0	0
0.1	0	0
0.2	0	0
0.3	0.25	0
0.4	0.25	0
0.5	0	0
0.6	0.5	0.5
0.7	0	0.5
0.8	0	0
0.9	0	0
1	0	0

TABLE 12 Table of strict distribution functions

α	$C(\alpha) = 0 + \sum_{\alpha' < \alpha} f_x(\alpha')$	$C_*(\alpha) = 0 + \sum_{\alpha' < \alpha} f_{x^*}(\alpha')$
0	0	0
0.1	0	0
0.2	0	0
0.3	0	0
0.4	0.25	0
0.5	0.5	0
0.6	0.5	0
0.7	1	0.5
0.8	1	1
0.9	1	0
1	1	0

TABLE 13 OWA-experton

α	$a(\alpha) = 1 - C(\alpha)$	$b(\alpha) = 1 - C_*(\alpha)$
0	1	1
0.1	1	1
0.2	1	1
0.3	1	1
0.4	0.75	1
0.5	0.5	1
0.6	0.5	1
0.7	0	0.5
0.8	0	0
0.9	0	0
1	0	0

Abbreviation: OWA, ordered weighted averaging.

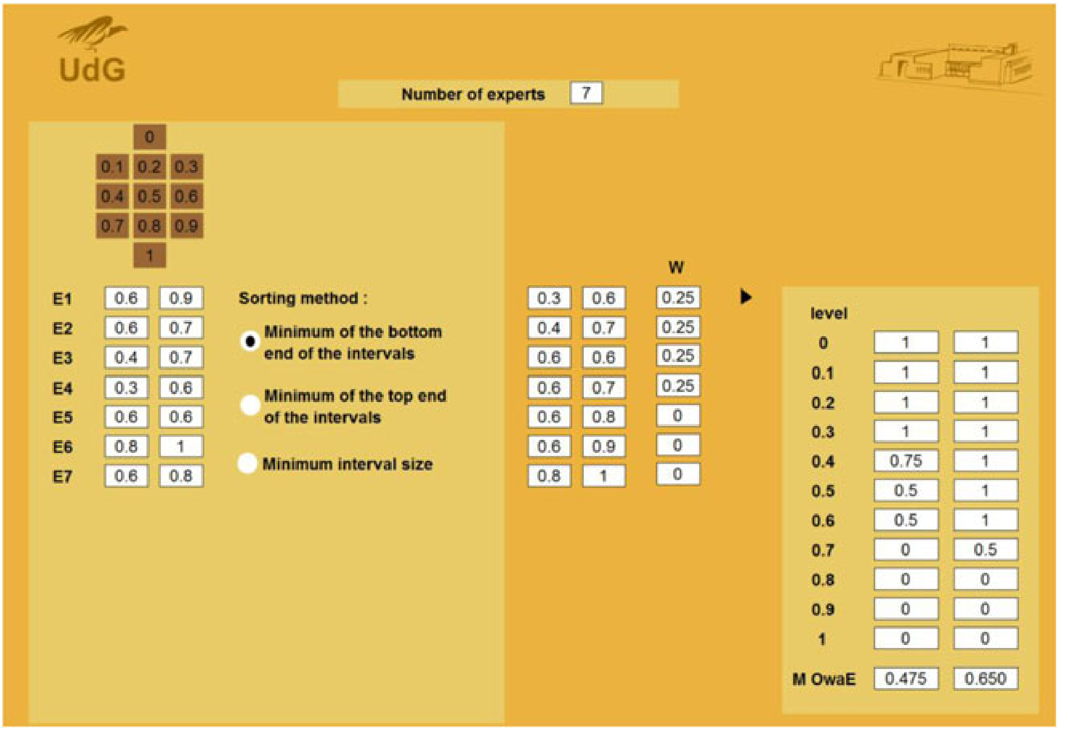

FIGURE 3 Example of calculating the OWA-experton using the "OWAExperton.exe" application, OWA, ordered weighted averaging [Color figure can be viewed at wileyonlinelibrary.com]

Assuming uncertain choices of values in the scale, suppose the board members provide the following confidence intervals: [0.6, 0.9], [0.6, 0.7], [0.4, 0.7], [0.3, 0.6], [0.6, 0.6], [0.8, 1], and [0.6, 0.8].

Finally, the board decides to construct an OWA-experton from the previous information, with a weighting vector $w = (0.25, 0.25, 0.25, 0.25, 0, 0, 0)$ and intervals sorted according to the total order relationship "minimum of the bottom end of the intervals." This is because the board errs on the side of caution by focusing on the values of the pessimistic majority. Therefore, the OWA-experton is calculated according to the following sorting method.

$$E_{k_1}:[0.3,0.6] \leq E_{k_2}:[0.4,0.7] \leq E_{k_3}:[0.6,0.6] \leq E_{k_4}:[0.6,0.7] \leq E_{k_5}:[0.6,0.8] \leq E_{k_6}:[0.6,0.9] \leq E_{k_7}:[0.8,1],$$

which, together with the weighting vector, determines the following starting interval-valued fuzzy set made up of 100 elements.

Tables 10 to 12 show the creation of the OWA-experton. Table 13 shows the OWA-experton, which could also have been determined using the software presented in the paper, as is shown in Figure 3.

We can observe from the OWA-experton object that at least 50% of the most pessimistic board of directors are more convinced than not that the projected profits will be achieved in the following year (level 0.6). These values can be presented to shareholders as justification for why the board opts to invest in the new market.

7 | CONCLUSIONS

This paper has presented the OWA-experton, which is a new information fusion mathematical object. The OWA-experton uses the AOWA in the process of constructing an experton. The main advantage of this approach is that it can represent the attitudinal character of the decision maker in the construction of the experton. The work has shown a possible mathematical formulation of the new object, various easy-to-apply numerical examples and immediate properties that the object satisfies. In addition, note that if the interval-valued fuzzy sets used in the analysis become crisp fuzzy sets, the OWA-experton becomes the OWA probabilistic set.

The work also presents some software that enables the calculation of the OWA-experton using the three sorting methods presented. The example application in Adobe Flash Professional CS6 can be downloaded in its executable form at the following address: http://web2.udg.edu/grmfcee/OWAExperton.zip.

Finally, this work has also presented an example of the full development of a use of the OWA-experton to solve a multiperson decision-making problem in strategic management under conditions of great uncertainty. With this example, we believe the OWA-experton demonstrates its application potential in decision theory, justifying this article's exhaustive presentation of the concept.

In future research, other aggregation operators will be used in the construction of the experton, including induced and generalized aggregation operators,[58] weighted averages[60] and probabilities.[61] Moreover, several other applications will be considered, including economics,[62] business,[63] and engineering.[64]

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